

TORSTEN HOEFLER

# Parallel Programming Parallel Sorting (A taste of parallel algorithms!)

## On the usage of sorting networks to control greenhouse climatic factors

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### Abstract

The goal of this article is the application of a non-adaptive classification algorithm to support the variable management process for internal climate control. The protected agriculture has given many advantages for the care and improvement in the production of almost any food. This work is focused on improving a control system for climate variables. The decision for activating an actuator for the correct care of the crop is very important. A sorting network technique with

## Sorting networks and their applications - ACM Digital Library

<https://dl.acm.org/citation.cfm?id=1468121>

by KE Batcher - 1968 - Cited by 2919 - Related articles

David C. Van Voorhis, An economical construction for sorting networks, Proceedings of the May 6-10, 1974, national computer conference and exposition, May ...

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## Optimal-depth sorting networks

- |          |                                      |   |
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We prove depth optimality of sorting networks from "The Art of Computer Programming". Sorting networks possess symmetry that can be used to generate a few representatives. These representatives can be efficiently encoded using regular expressions. We construct SAT formulas whose unsatisfiability is sufficient to show optimality. Resulting algorithm is orders of magnitude faster than prior work on small instances. We solve a 40-year-old open problem on depth optimality of sorting networks. In 1973, Donald E. Knuth detailed sorting networks of the smallest depth known for  $n \geq 16$  inputs, quoting optimality for  $n \geq 8$  (Volume 3 of "The Art of Computer Programming"). In 1989, Parberry proved optimality of networks with  $9 \leq n \leq 10$  inputs. We present a general technique for obtaining such results, proving optimality of the remaining open cases of  $11 \leq n \leq 16$  inputs. Exploiting symmetry, we construct a small set  $R_n$  of two-layer networks such that: if there is a depth- $k$  sorting network on  $n$  inputs, then there is one whose first layers are in  $R_n$ . For each network in  $R_n$ , we construct a propositional formula whose satisfiability is necessary for the existence of a depth- $k$  sorting network. Using an off-the-shelf SAT solver we prove optimality of the sorting networks listed by Knuth. For  $n \geq 10$  inputs, our algorithm is orders of magnitude faster than prior ones.

## Leadership Computing for Europe and the Path to Exascale Computing

May 29, 2018 by [Rich Brueckner](#) [Leave a Comment](#)



In this video from the [NVIDIA GPU Conference](#), Thomas Schulthess from [CSCS](#) presents: *Leadership Computing for Europe and the Path to Exascale Computing*.

"With over 5000 GPU-accelerated nodes, [Piz Daint](#) has been Europe's leading supercomputing systems since 2013, and is currently one of the most performant and energy efficient supercomputers on the planet. It has been designed to optimize throughput of multiple applications, covering all aspects of the workflow, including data analysis and visualisation. We will discuss ongoing efforts to further integrate these extreme-scale compute and data services with infrastructure services of the cloud. As Tier-0 systems of PRACE, Piz Daint is accessible to all scientists in Europe and worldwide. It provides a baseline for future development of exascale computing. We will present a strategy for developing exascale computing technologies in domains such as weather and climate or materials science."



Thomas Schulthess is the Director of CSCS in Switzerland.



# Today: Parallel Sorting

## (one of the most fun problems in CS)



Quick-sort with Hungarian (Küküllőmenti legényes) folk dance

1,318,280 views

 14K 
  186 
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Insert-sort with Romanian folk dance

595,979 views

 3.8K 
  57 
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## Literature

- D.E. Knuth. The Art of Computer Programming, Volume 3: Sorting and Searching, Third Edition. Addison-Wesley, 1997. ISBN 0-201-89685-0. Section 5.3.4: Networks for Sorting, pp. 219–247.
- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. Introduction to Algorithms, Second Edition. MIT Press and McGraw-Hill, 1990. ISBN 0-262-03293-7. Chapter 27: Sorting Networks, pp.704–724.

google

"chapter 27 sorting networks"

## How fast can we sort?

Heapsort & Mergesort have  $O(n \log n)$  worst-case run time

Quicksort has  $O(n \log n)$  average-case run time

These bounds are all tight, actually  $\Theta(n \log n)$

So maybe we can dream up another algorithm with a lower asymptotic complexity, such as  $O(n)$  or  $O(n \log \log n)$

This is unfortunately **IMPOSSIBLE!**

But why?

# Permutations

Assume we have  $n$  elements to sort

For simplicity, also assume none are equal (i.e., no duplicates)

How many permutations of the elements (possible orderings)?

Example,  $n=3$

$a[0] < a[1] < a[2]$	$a[0] < a[2] < a[1]$	$a[1] < a[0] < a[2]$
$a[1] < a[2] < a[0]$	$a[2] < a[0] < a[1]$	$a[2] < a[1] < a[0]$

In general,  $n$  choices for first,  $n-1$  for next,  $n-2$  for next, etc.  $\rightarrow n(n-1)(n-2)\dots(1) = n!$  possible orderings

## Representing every comparison sort

Algorithm must “find” the right answer among  $n!$  possible answers

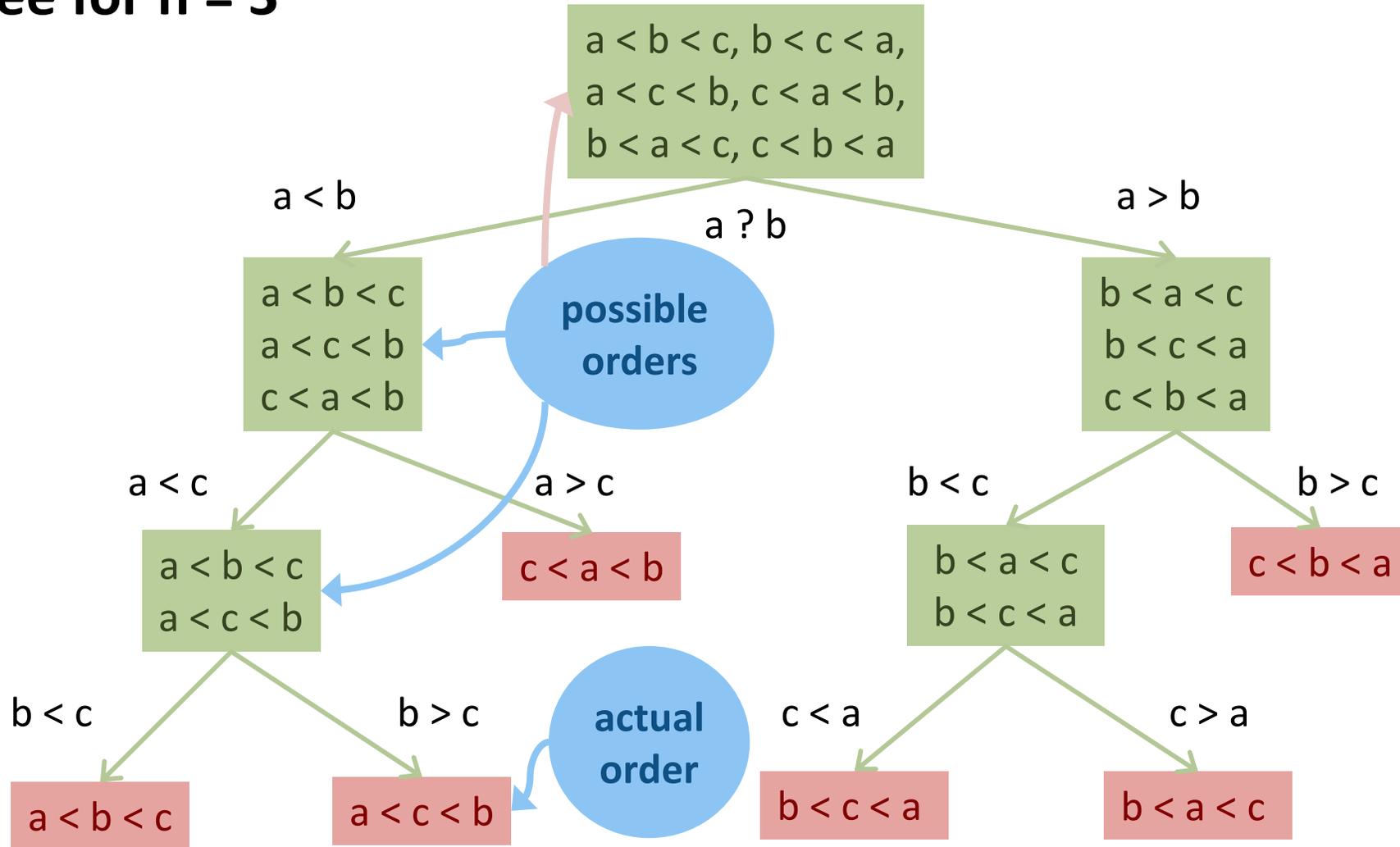
Starts “knowing nothing” and gains information with each comparison

Intuition is that each comparison can, at best, eliminate half of the remaining possibilities

Can represent this process as a decision tree

- Nodes contain “remaining possibilities”
- Edges are “answers from a comparison”
- This is not a data structure but what our proof uses to represent “the most any algorithm could know”

# Decision tree for n = 3



The leaves contain all possible orderings of a, b, c

## What the decision tree tells us

### *Binary* tree because

- Each comparison has binary outcome
- Assumes algorithm does not ask redundant questions

Because any data is possible, any algorithm needs to ask enough questions to decide among all  $n!$  answers

- Every answer is a leaf (no more questions to ask)
- So the tree must be big enough to have  $n!$  leaves
- Running any algorithm on any input will at best correspond to one root-to-leaf path in the decision tree

So no algorithm can have worst-case running time better than the height of the decision tree

## Where are we

**Proven:** No comparison sort can have worst-case better than the height of a binary tree with  $n!$  leaves

- Turns out average-case is same asymptotically
- So how tall is a binary tree with  $n!$  leaves?

**Now:** Show a binary tree with  $n!$  leaves has height  $\Omega(n \log n)$

- $n \log n$  is the lower bound, the height must be at least this
- It could be more (in other words, a comparison sorting algorithm could take longer but can not be faster)

**Conclude that:** (Comparison) Sorting is  $\Omega(n \log n)$

## Lower bound on height

The height of a binary tree with  $L$  leaves is at least  $\log_2 L$

So the height of our decision tree,  $h$ :

$$\begin{aligned}
 h &\geq \log_2 (n!) \\
 &= \log_2 (n \cdot (n-1) \cdot (n-2) \dots (2)(1)) \\
 &= \log_2 n + \log_2 (n-1) + \dots + \log_2 1 \\
 &\geq \log_2 n + \log_2 (n-1) + \dots + \log_2 (n/2) \\
 &\geq (n/2) \log_2 (n/2) \\
 &\geq (n/2)(\log_2 n - \log_2 2) \\
 &\geq (1/2)n \log_2 n - (1/2)n \\
 &\text{"=" } \Omega(n \log n)
 \end{aligned}$$

property of binary trees

definition of factorial

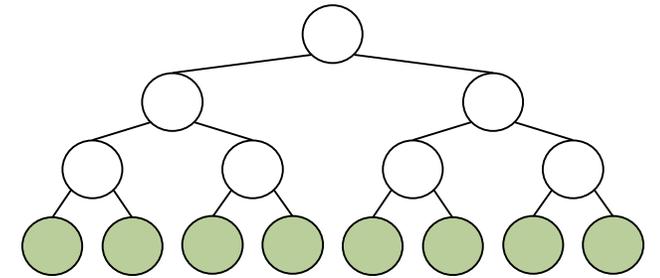
property of logarithms

keep first  $n/2$  terms

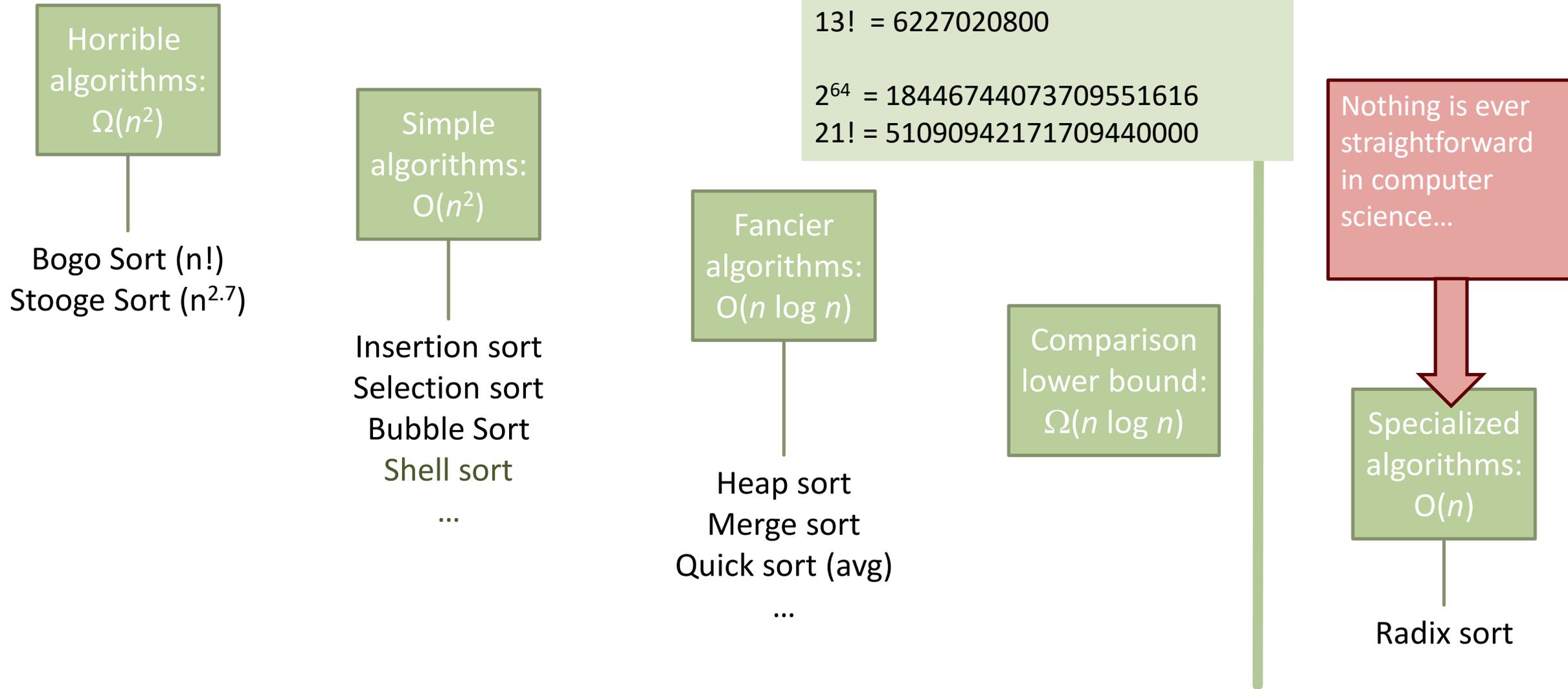
each of the  $n/2$  terms left is  $\geq \log_2 (n/2)$

property of logarithms

arithmetic



# Breaking the lower bound on sorting



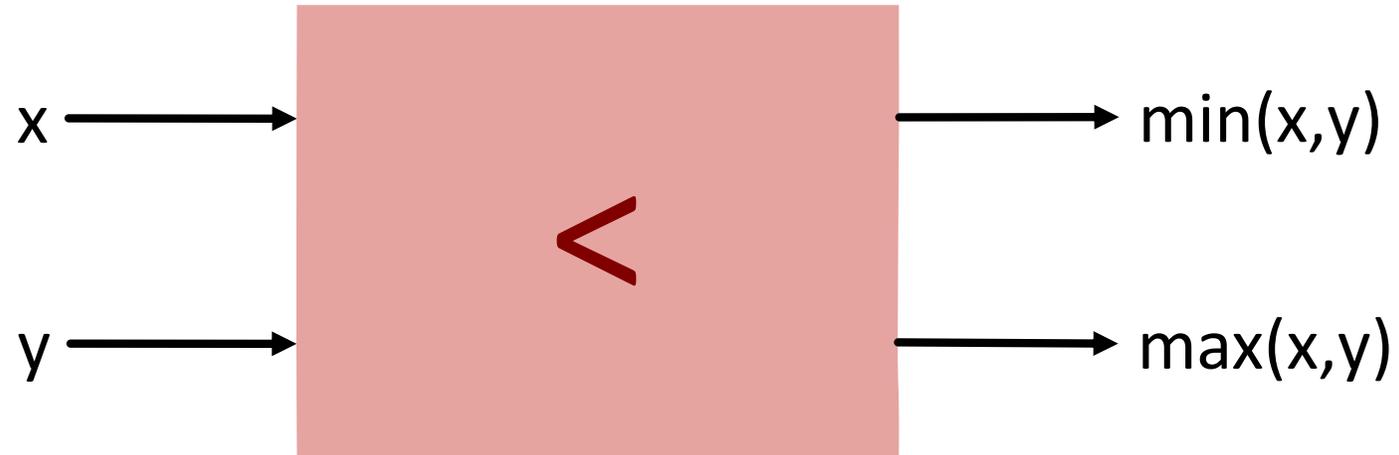
Assume 32/64-bit Integer:

$2^{32} = 4294967296$   
 $13! = 6227020800$

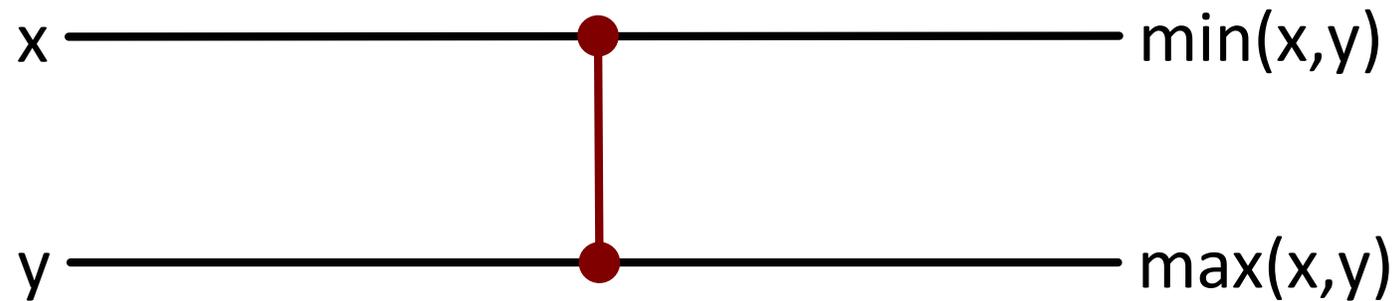
$2^{64} = 18446744073709551616$   
 $21! = 51090942171709440000$

# SORTING NETWORKS

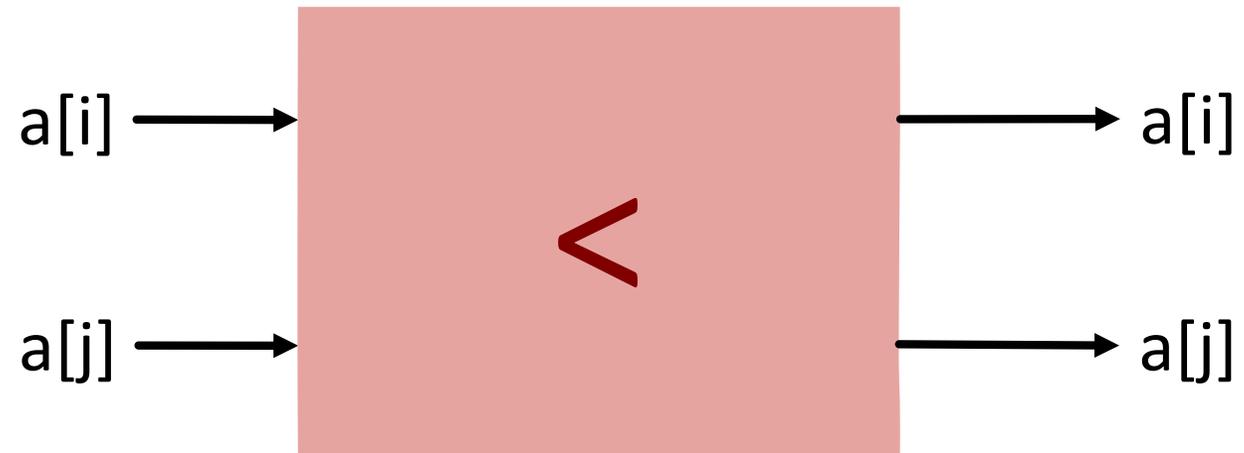
# Comparator – the basic building block for sorting networks



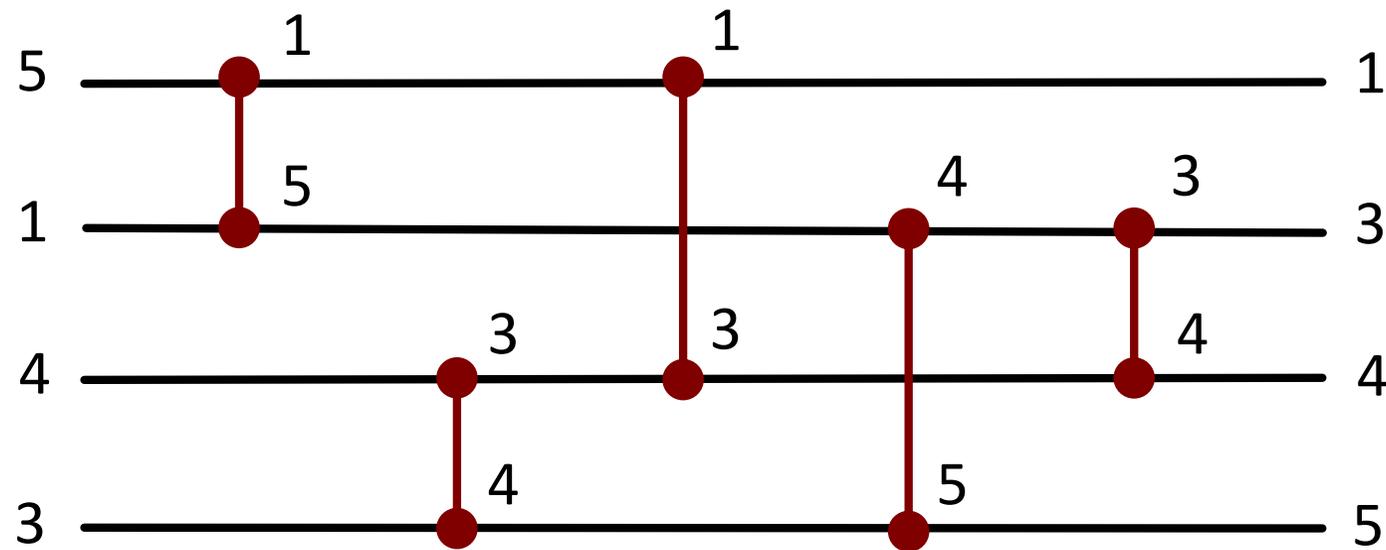
shorter notation:



```
void compare(int[] a, int i, int j, boolean dir) {  
    if (dir==(a[i]>a[j])){  
        int t=a[i];  
        a[i]=a[j];  
        a[j]=t;  
    }  
}
```

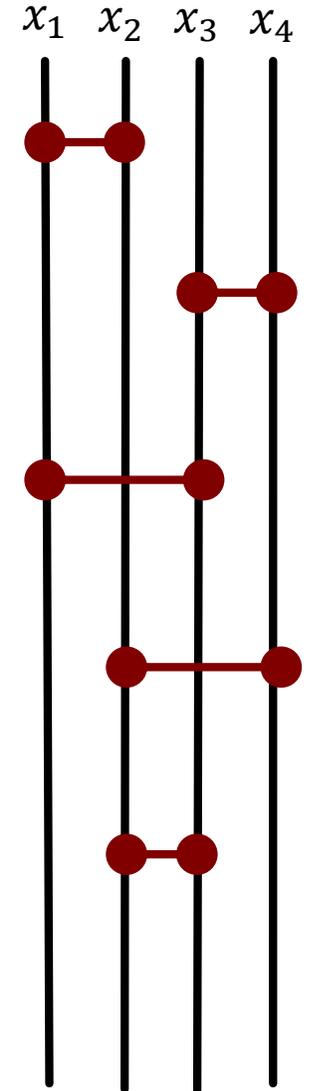
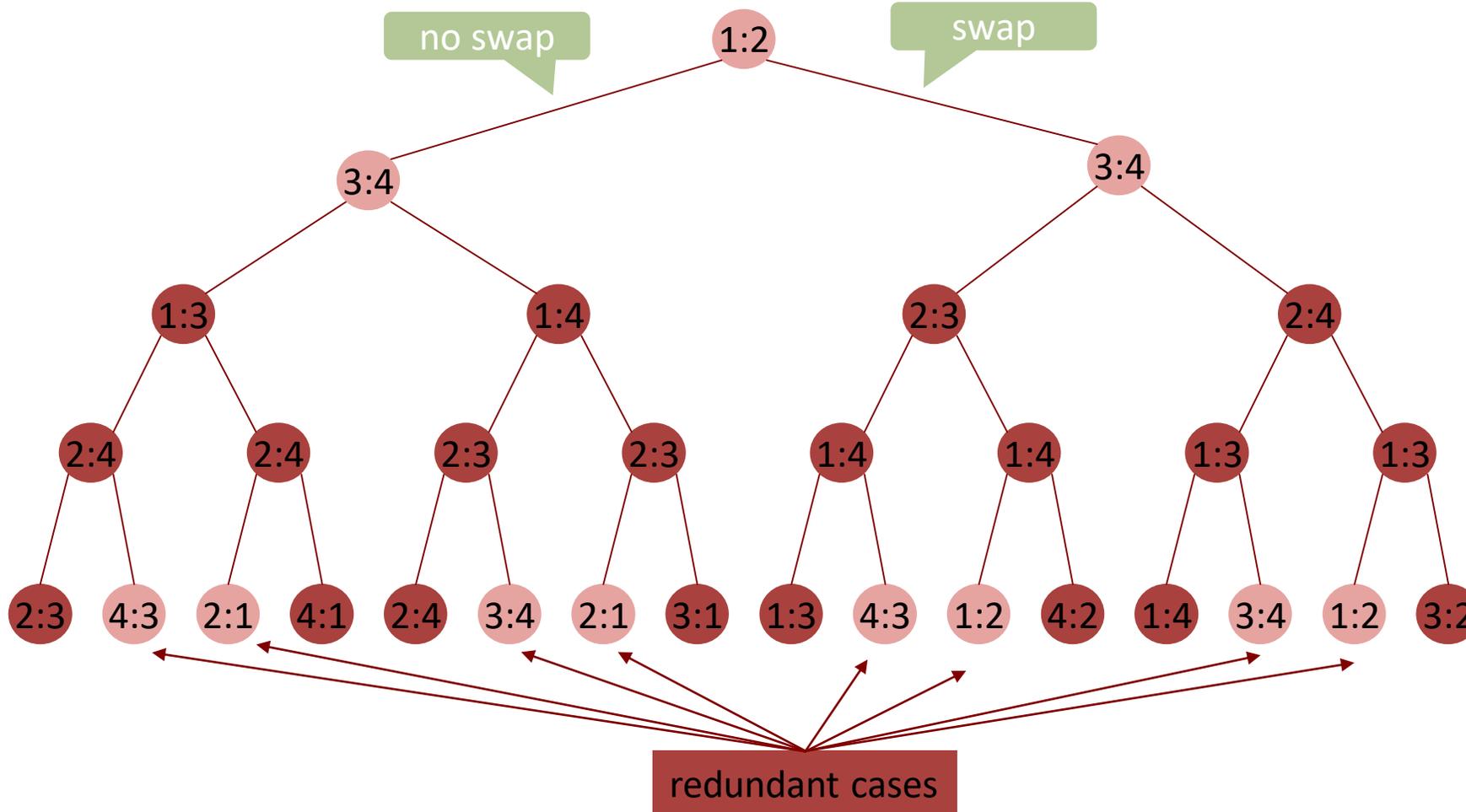


# Sorting networks



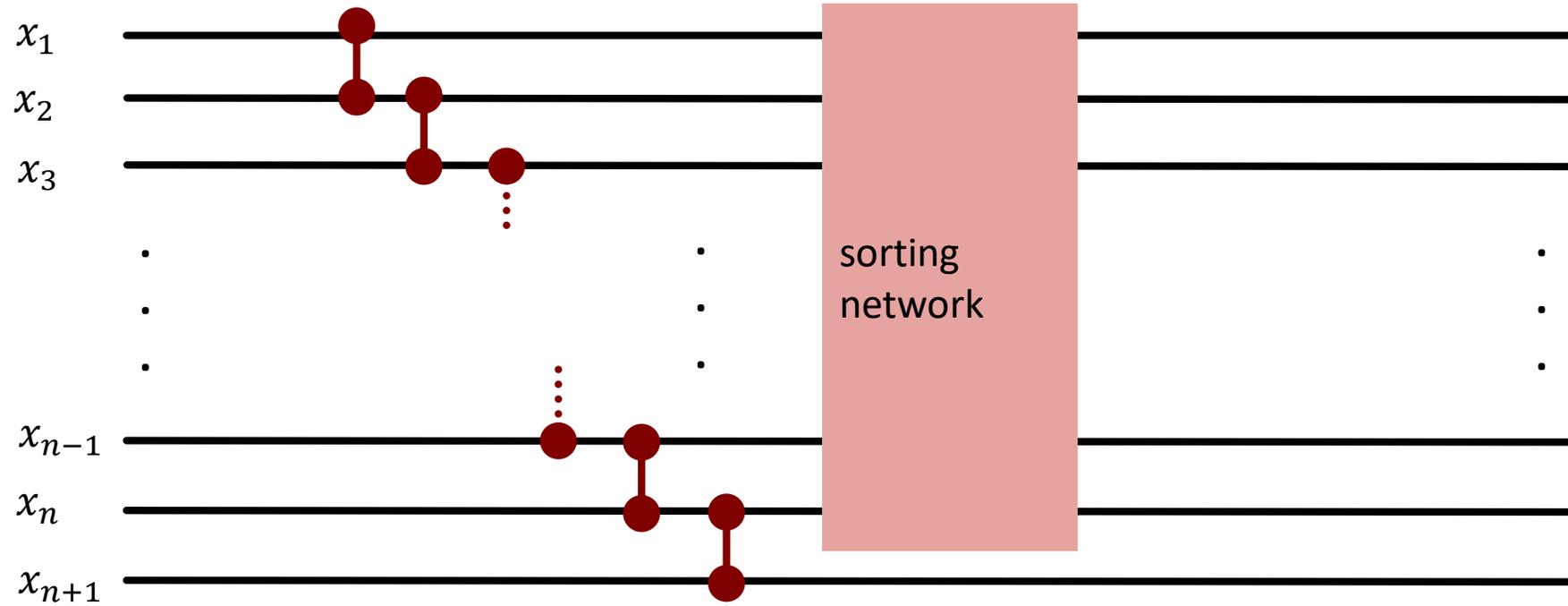
# Sorting networks are *data-oblivious* (and redundant)

Data-oblivious comparison tree

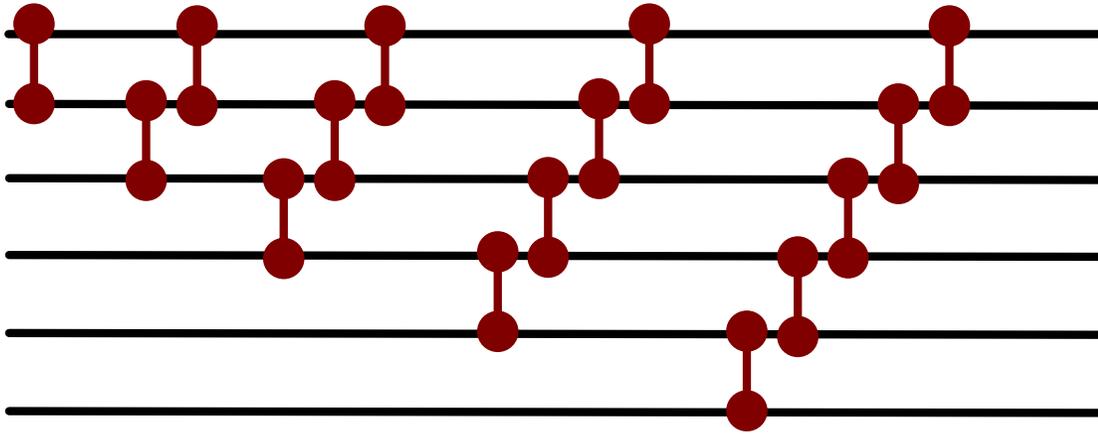




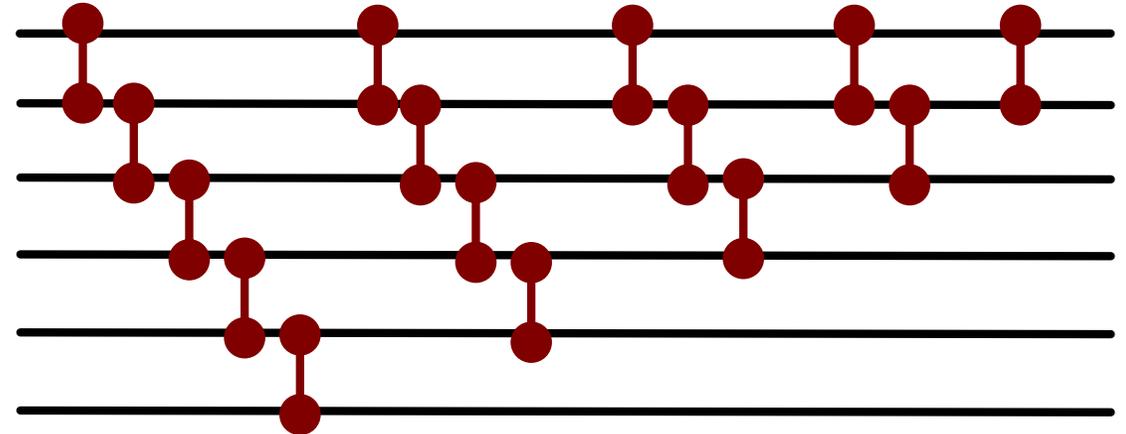
# Recursive construction: Selection



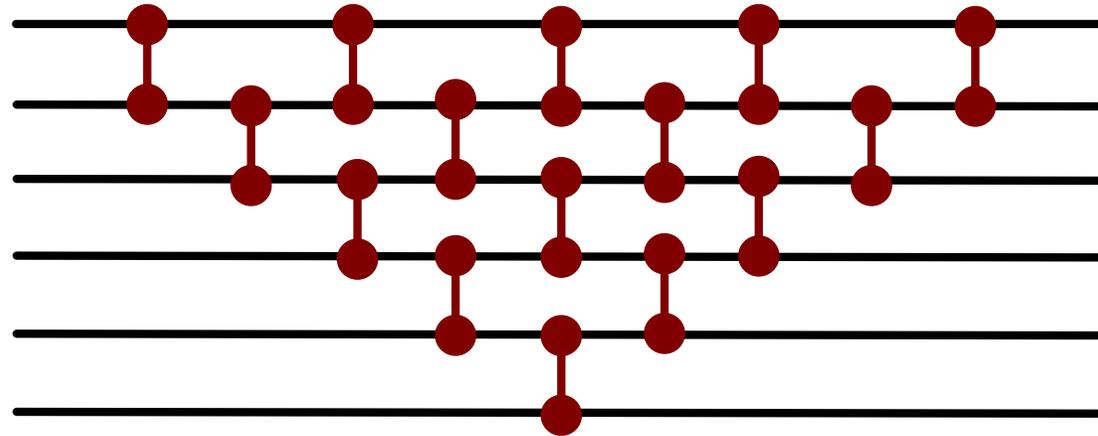
# Applied recursively..



insertion sort



bubble sort



with parallelism: insertion sort = bubble sort !

## Question

How many steps does a computer with infinite number of processors (comparators) require in order to sort using parallel bubble sort (depth)?

Answer:  $2n - 3$

Can this be improved ?

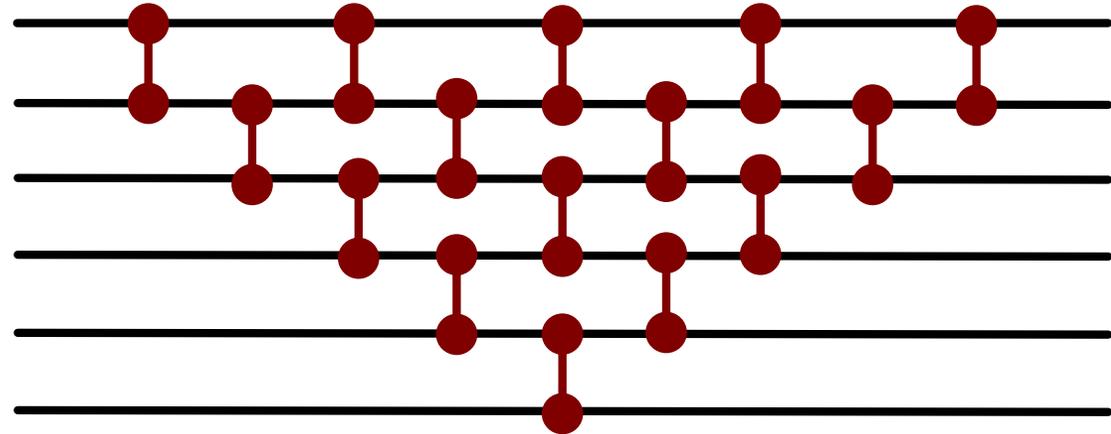
How many comparisons ?

Answer:  $(n-1) n/2$

How many comparators are required (at a time)?

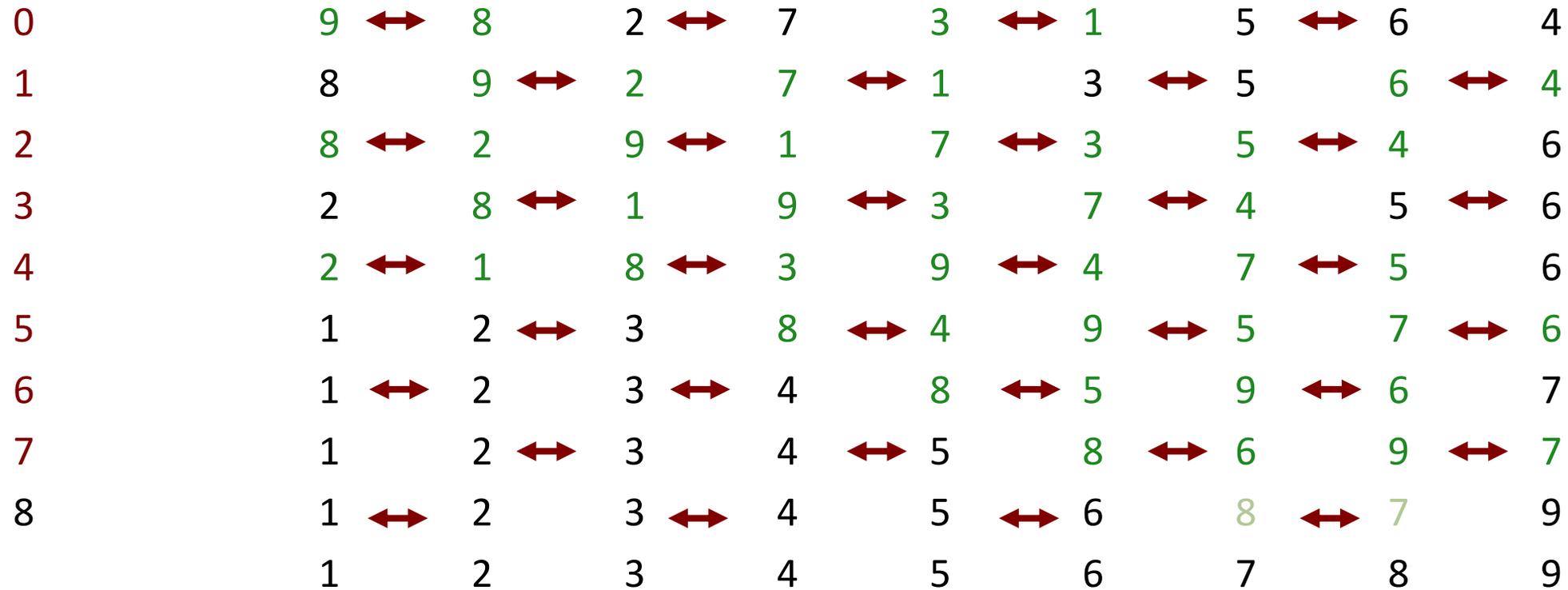
Answer:  $n/2$

Reusable comparators:  $n-1$

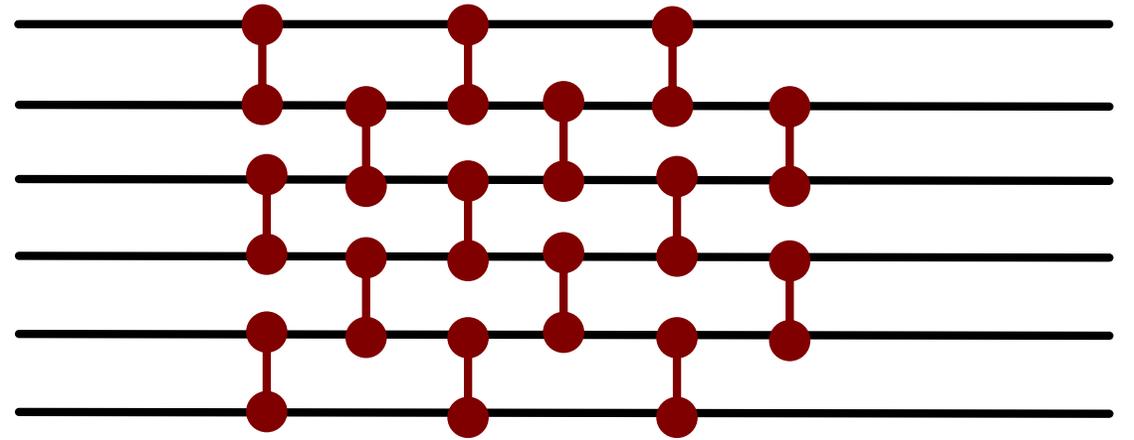


# Improving parallel Bubble Sort

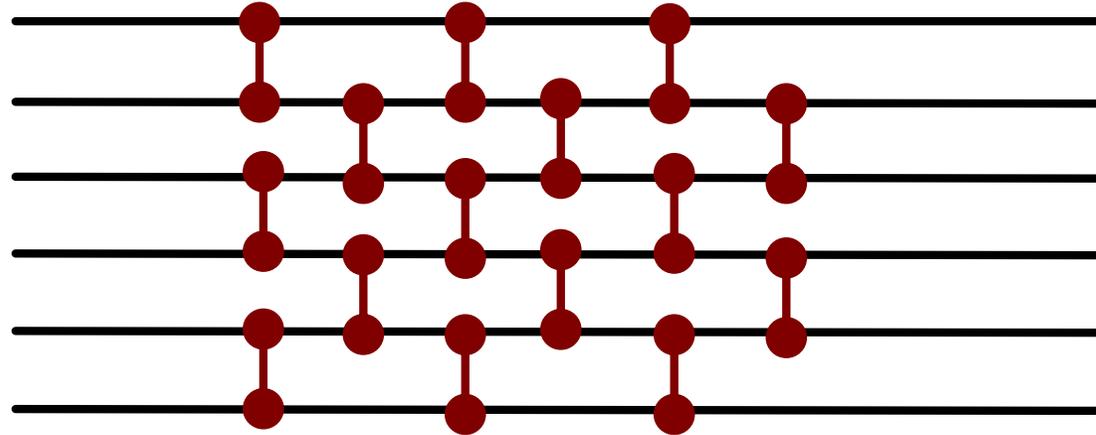
Odd-Even Transposition Sort:



```
void oddEvenTranspositionSort(int[] a, boolean dir) {  
    int n = a.length;  
    for (int i = 0; i < n; ++i) {  
        for (int j = i % 2; j + 1 < n; j += 2)  
            compare(a, j, j + 1, dir);  
    }  
}
```



## Improvement?



Same number of comparators (at a time)

Same number of comparisons

But less parallel steps (depth):  $n$

In a massively parallel setup, bubble sort is thus not too bad.

But it can go better...

# How to get to a sorting network?

- **It's complicated** 😊
  - In fact, some structures are clear but there is a lot still to be discovered!
- **For example:**
  - What is the minimum number of comparators?
  - What is the minimum depth?
  - Tradeoffs between these two?

## Optimal sorting networks [\[edit\]](#)

Source: wikipedia

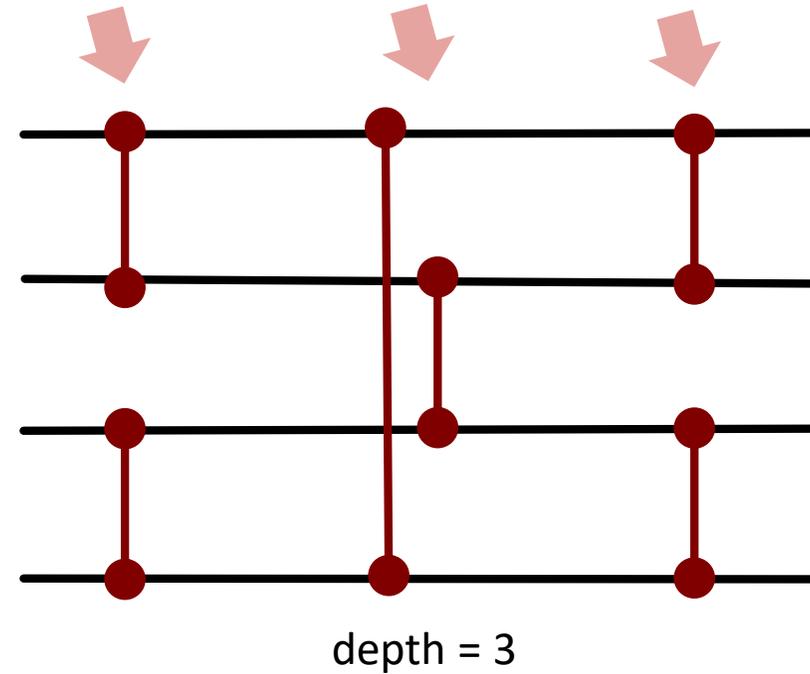
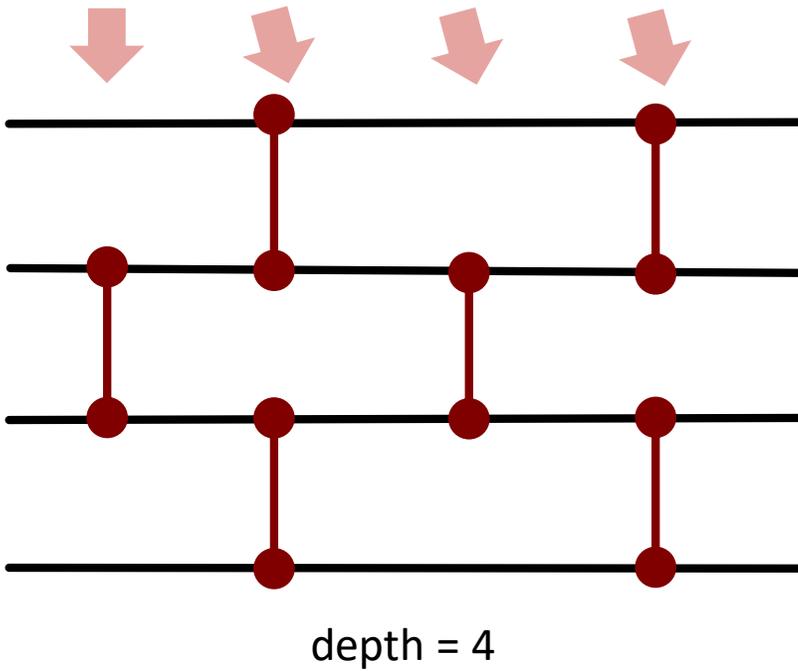
For small, fixed numbers of inputs  $n$ , *optimal* sorting networks can be constructed, with either minimal depth (for maximally parallel execution) or minimal size (number of comparators). These networks can be used to increase the performance of larger sorting networks resulting from the [recursive](#) constructions of, e.g., Batcher, by halting the recursion early and inserting optimal nets as base cases.<sup>[9]</sup> The following table summarizes the known optimality results:

$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Depth <sup>[10]</sup>	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10
Size, upper bound <sup>[11]</sup>	0	1	3	5	9	12	16	19	25	29	35	39	45	51	56	60	71
Size, lower bound (if different) <sup>[11]</sup>											33	37	41	45	49	53	58

The first sixteen depth-optimal networks are listed in Knuth's *Art of Computer Programming*,<sup>[1]</sup> and have been since the 1973 edition; however, while the optimality of the first eight was established by Floyd and Knuth in the 1960s, this property wasn't proven for the final six until 2014<sup>[12]</sup> (the cases nine and ten having been decided in 1991<sup>[9]</sup>).

For one to ten inputs, minimal (i.e. size-optimal) sorting networks are known, and for higher values, lower bounds on their sizes  $S(n)$  can be derived inductively using a lemma due to Van Voorhis:  $S(n + 1) \geq S(n) + \lceil \log_2(n) \rceil$ . All ten optimal networks have been known since 1969, with the first eight again being known as optimal since the work of Floyd and Knuth, but optimality of the cases  $n = 9$  and  $n = 10$  took until 2014 to be resolved.<sup>[11]</sup>

# Parallel sorting



Prove that the two networks above sort four numbers. Easy?

## Zero-one-principle

**Theorem:** If a network with  $n$  input lines sorts all  $2^n$  sequences of 0s and 1s into non-decreasing order, it will sort any arbitrary sequence of  $n$  numbers in non-decreasing order.

# Proof

Argue: If  $x$  is sorted by a network  $N$  then also any monotonic function of  $x$ .

e.g.,  $\text{floor}(x/2)$



Show: If  $x$  is **not** sorted by network  $N$ , **then** there is a monotonic function  $f$  that maps  $x$  to 0s and 1s and  **$f(x)$  is not sorted** by the network



$x$  not sorted by  $N \Rightarrow$  there is an  $f(x) \in \{0,1\}^n$  not sorted by  $N$

$\Leftrightarrow$

$f$  sorted by  $N$  for all  $f \in \{0,1\}^n \Rightarrow x$  sorted by  $N$  for all  $x$

## Proof

Assume a monotonic function  $f(x)$  with  $f(x) \leq f(y)$  whenever  $x \leq y$  and a network  $N$  that sorts. Let  $N$  transform  $(x_1, x_2, \dots, x_n)$  into  $(y_1, y_2, \dots, y_n)$ , then it also transforms  $(f(x_1), f(x_2), \dots, f(x_n))$  into  $(f(y_1), f(y_2), \dots, f(y_n))$ .

All comparators must act in the same way for the  $f(x_i)$  as they do for the  $x_i$

Assume  $y_i > y_{i+1}$  for some  $i$ , then consider the monotonic function

$$f(x) = \begin{cases} 0, & \text{if } x < y_i \\ 1, & \text{if } x \geq y_i \end{cases}$$

→  $N$  converts

$(f(x_1), f(x_2), \dots, f(x_n))$  into  $(f(y_1), f(y_2), \dots, f(y_i), f(y_{i+1}), \dots, f(y_n))$

1

0

## Bitonic sort

Bitonic (Merge) Sort is a parallel algorithm for sorting

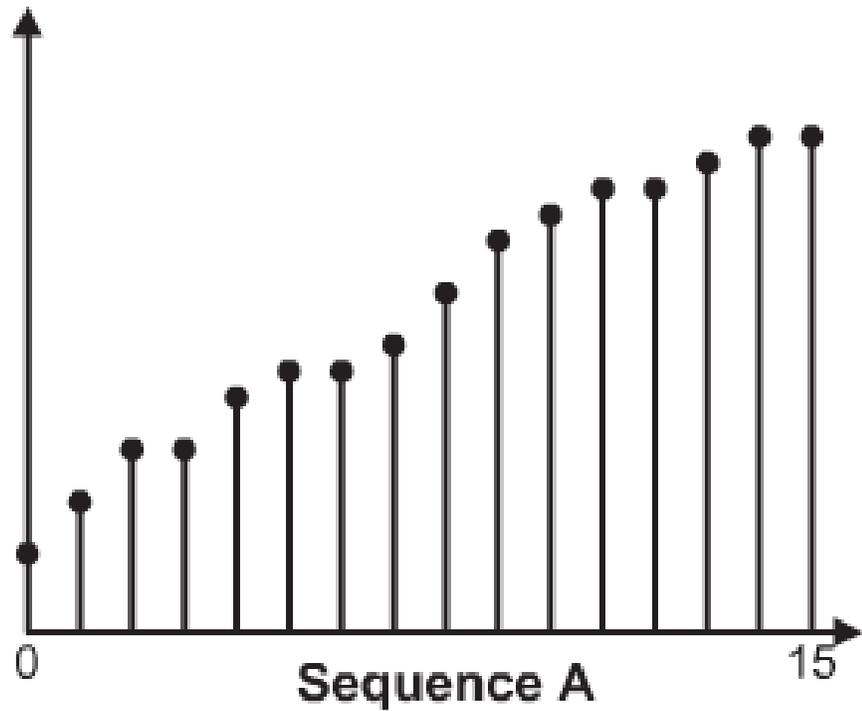
If enough processors are available, bitonic sort breaks the lower bound on sorting for comparison sort algorithm

Time complexity of  $O(n \log^2 n)$  (sequential execution)

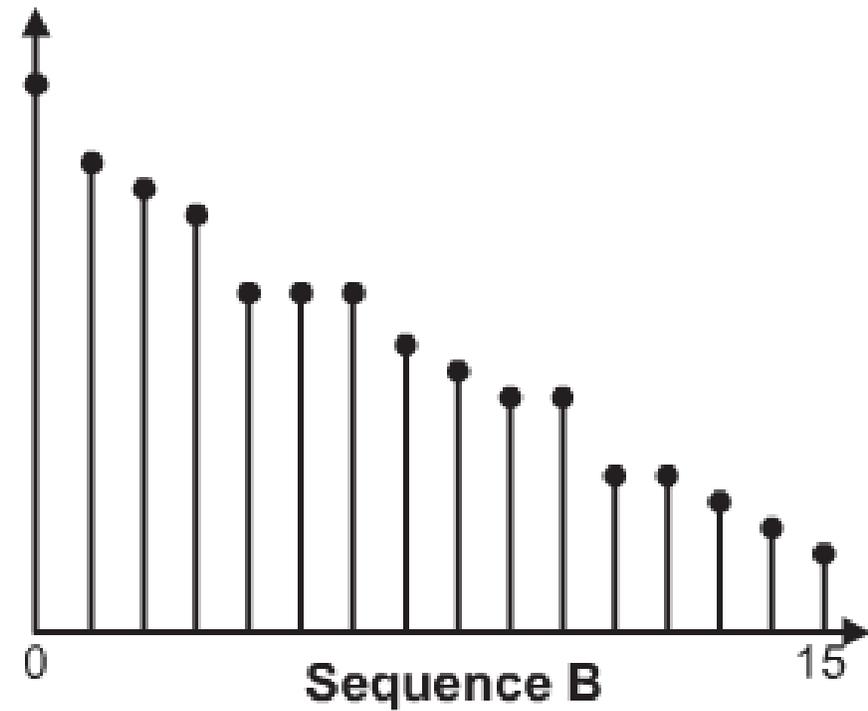
Time complexity of  $O(\log^2 n)$  (parallel time)

Worst = Average = Best case

# What is a Bitonic sequence?

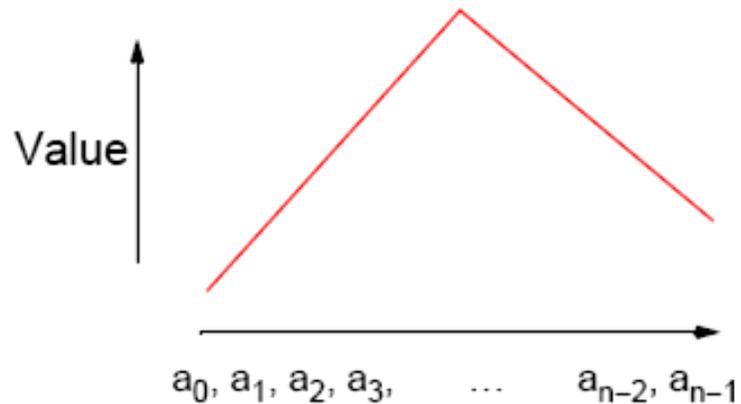


Monotonic ascending sequence

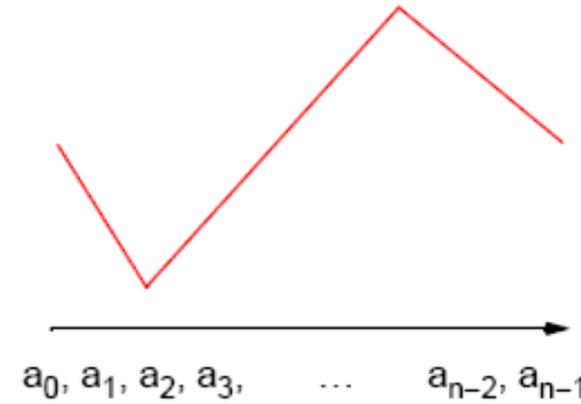


Monotonic descending sequence

## Bitonic Sequences Allow Wraparound



(a) Single maximum



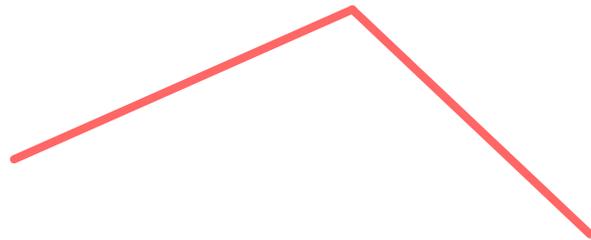
(b) Single maximum and single minimum

A *bitonic sequence* is defined as a list with no more than one **Local maximum** and no more than one **Local minimum**.

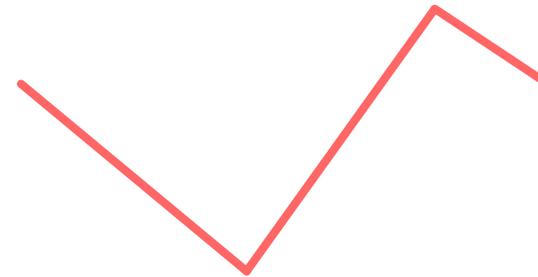
## Bitonic (again)

Sequence  $(x_1, x_2, \dots, x_n)$  is bitonic, if it can be circularly shifted such that it is first monotonically increasing and then monotonically decreasing.

$(1, 2, 3, 4, 5, 3, 1, 0)$



$(4, 3, 2, 1, 2, 4, 6, 5)$



# Bitonic 0-1 Sequences

$$0^i 1^j 0^k$$

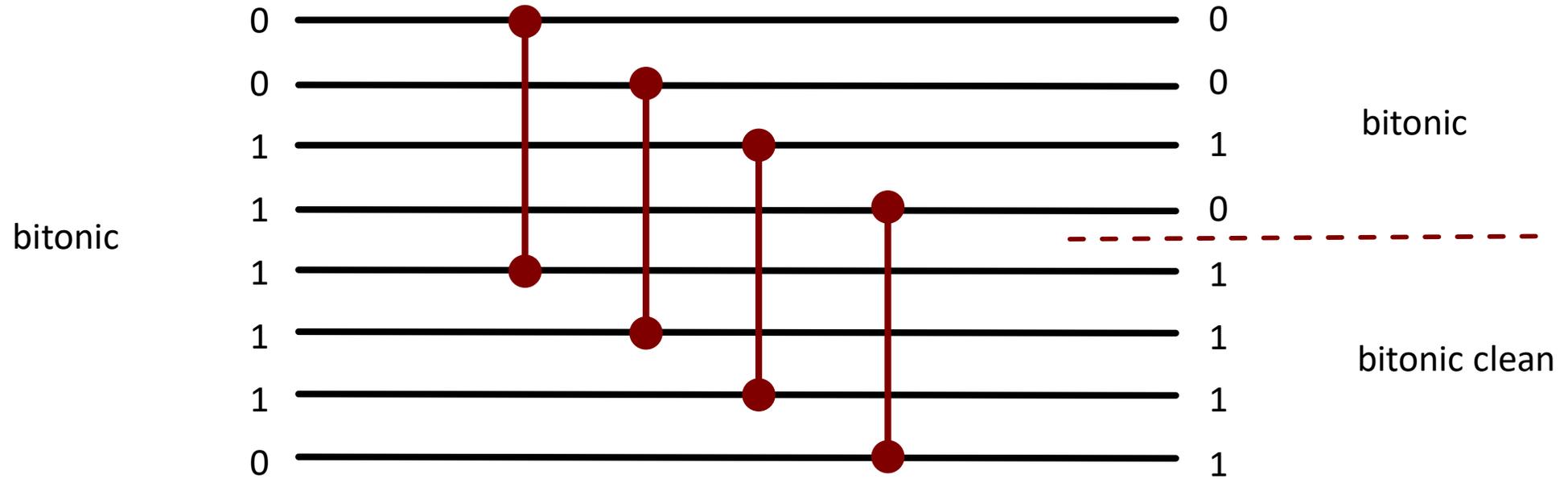
$$1^i 0^j 1^k$$

## Properties

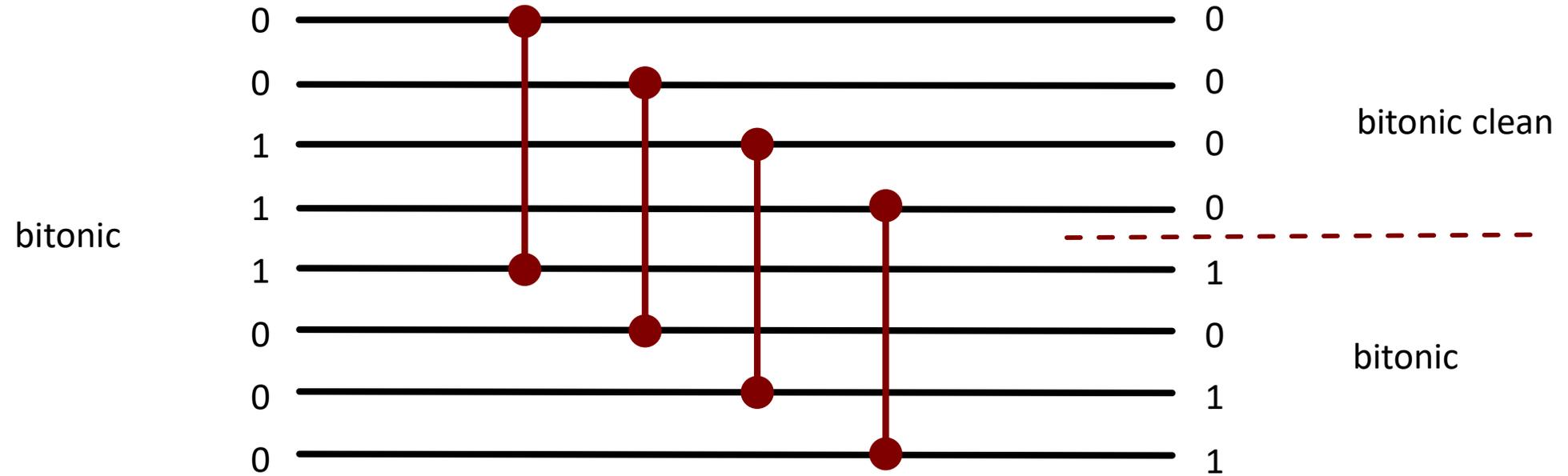
If  $(x_1, x_2, \dots, x_n)$  is monotonically increasing (decreasing) and then monotonically decreasing (increasing), then it is bitonic

If  $(x_1, x_2, \dots, x_n)$  is bitonic, then  $(x_1, x_2, \dots, x_n)^R := (x_n, x_{n-1}, \dots, x_1)$  is also bitonic

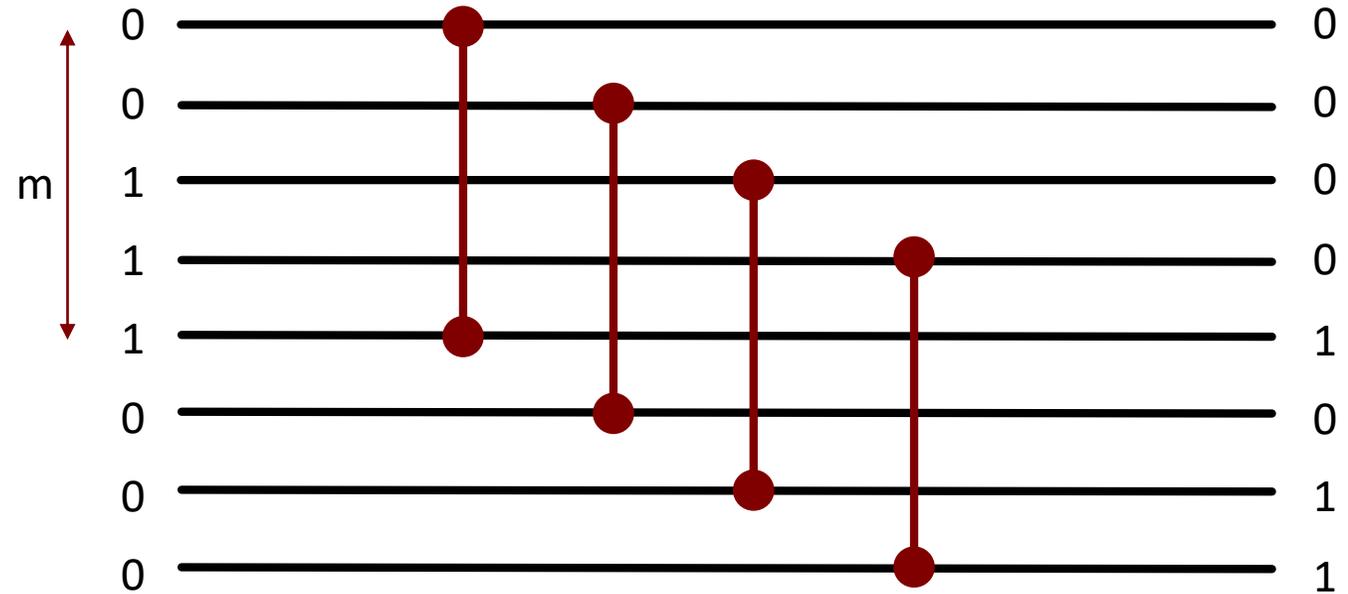
# The Half-Cleaner



# The Half-Cleaner

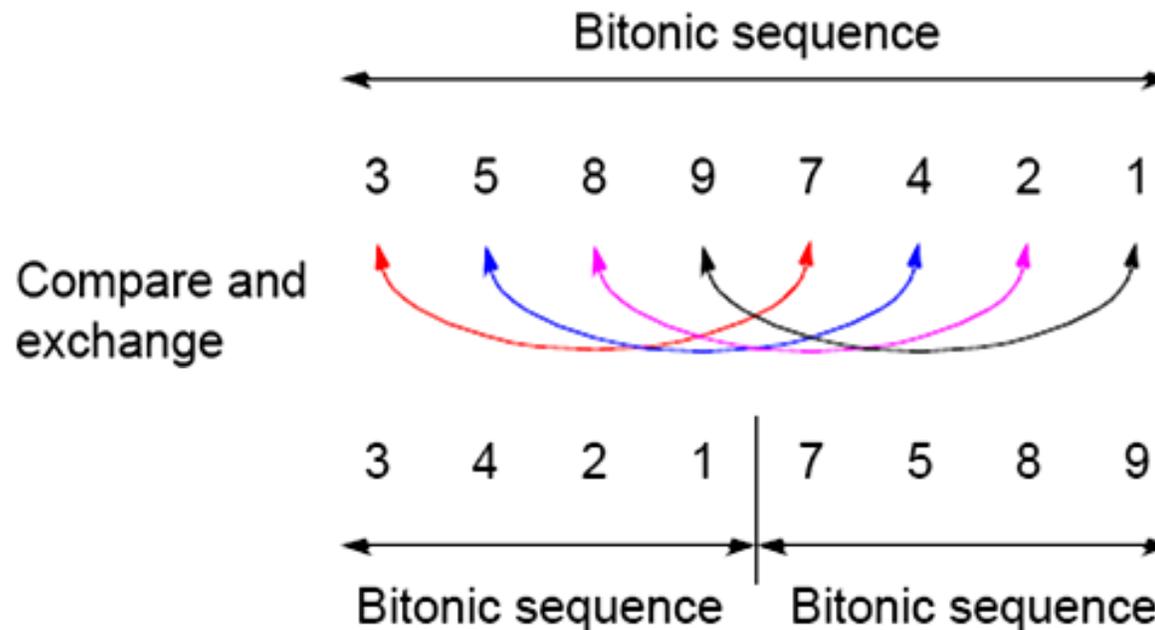


```
void halfClean(int[] a, int lo, int m, boolean dir)
{
    for (int i=lo; i<lo+m; i++)
        compare(a, i, i+m, dir);
}
```



## Binary Split: Application of the Half-Cleaner

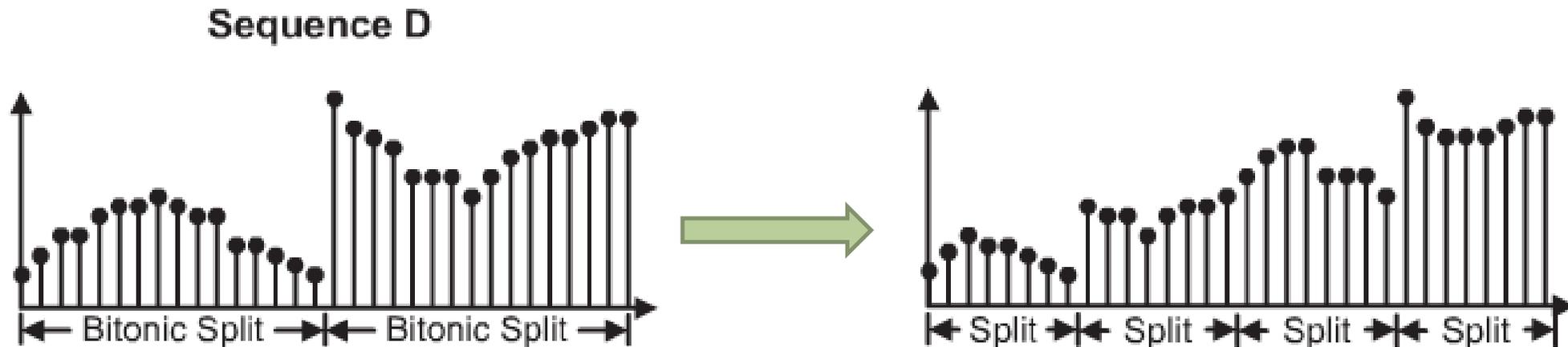
1. Divide the bitonic list into two equal halves.
2. Compare-Exchange each item on the first half with the corresponding item in the second half.



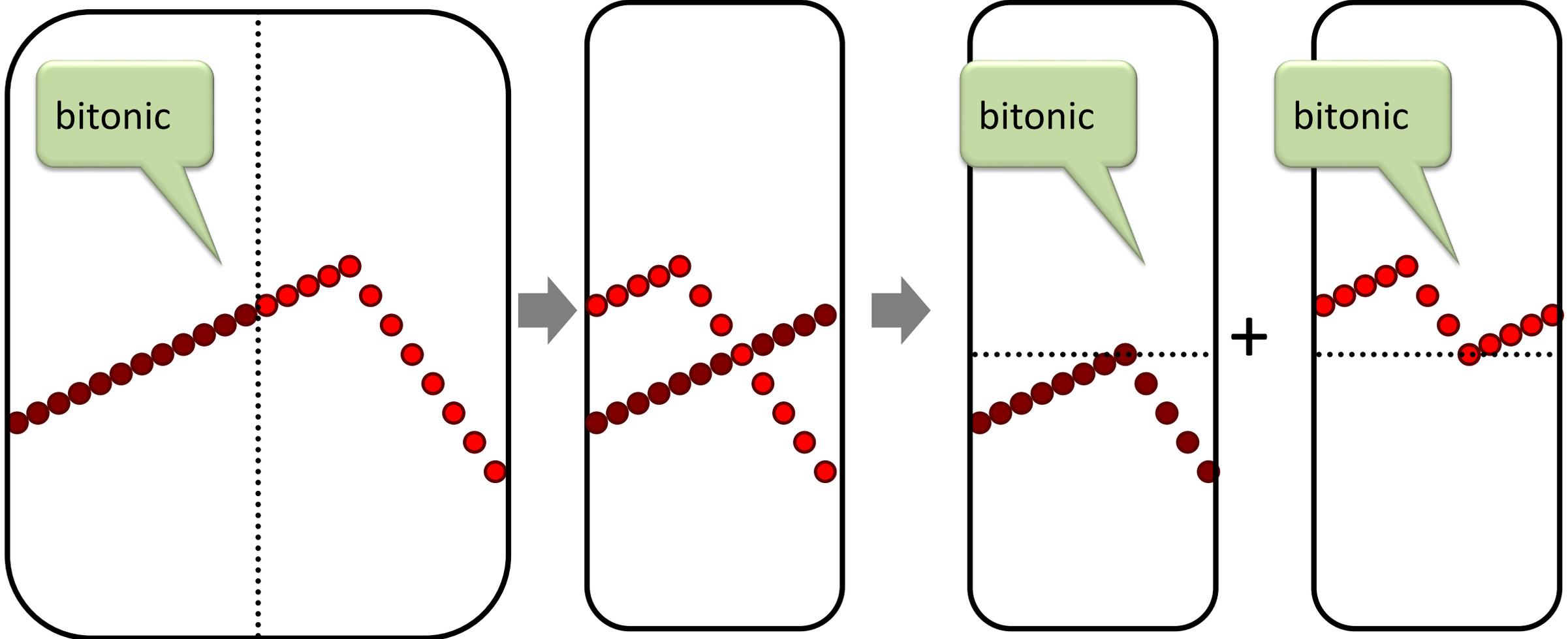
## Binary Splits - Result

Two *bitonic* sequences where the numbers in one sequence are all less than the numbers in the other sequence.

Because the original sequence was *bitonic*, every element in the lower half of new sequence is less than or equal to the elements in its upper half.



# Bitonic Split Example

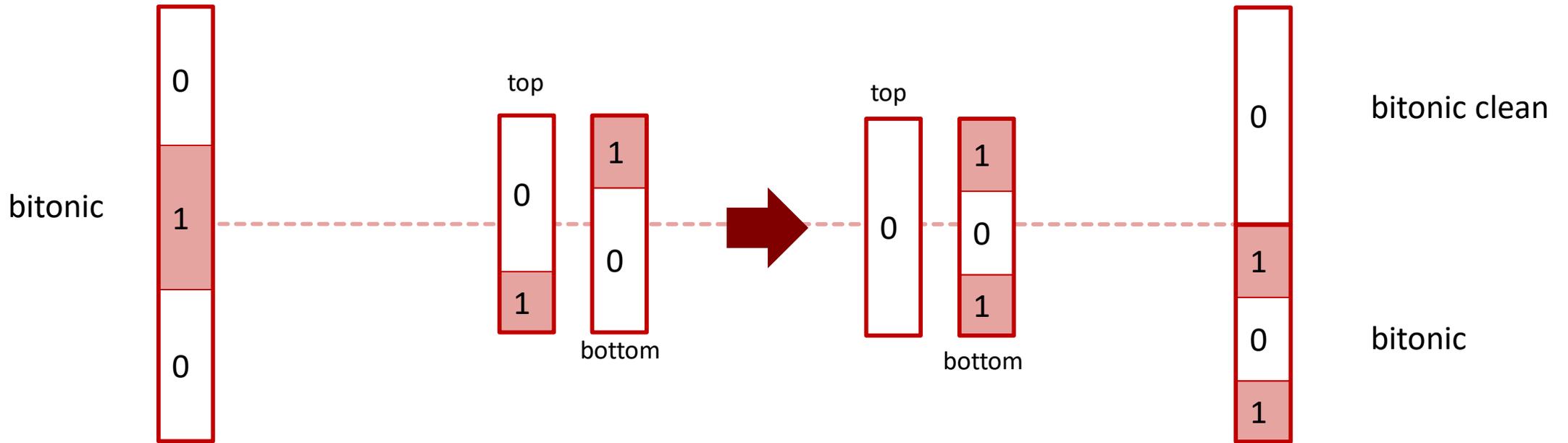


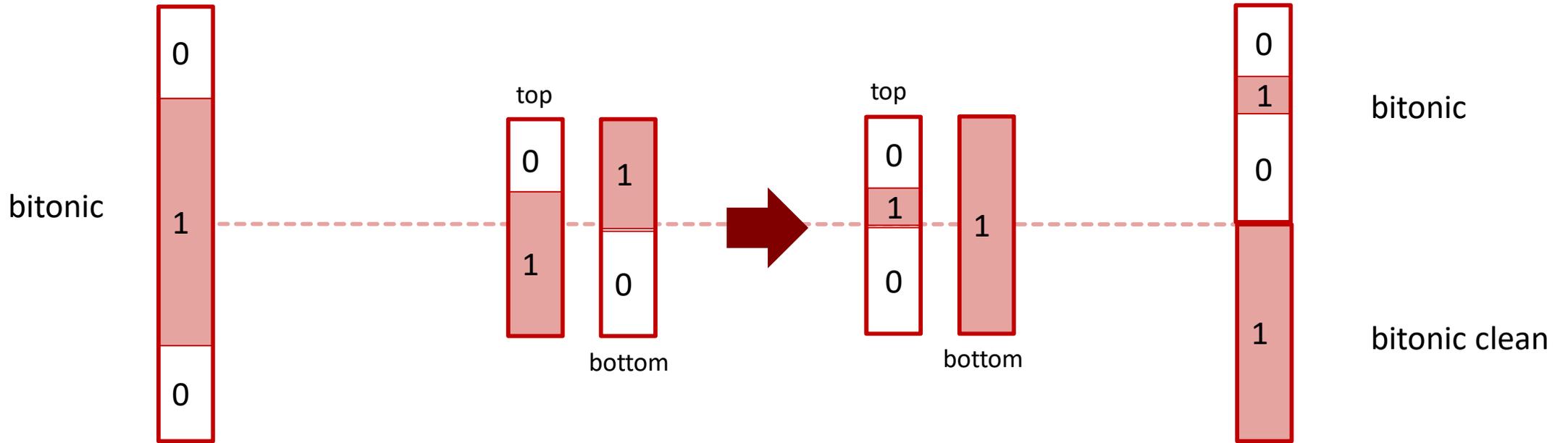
## Lemma

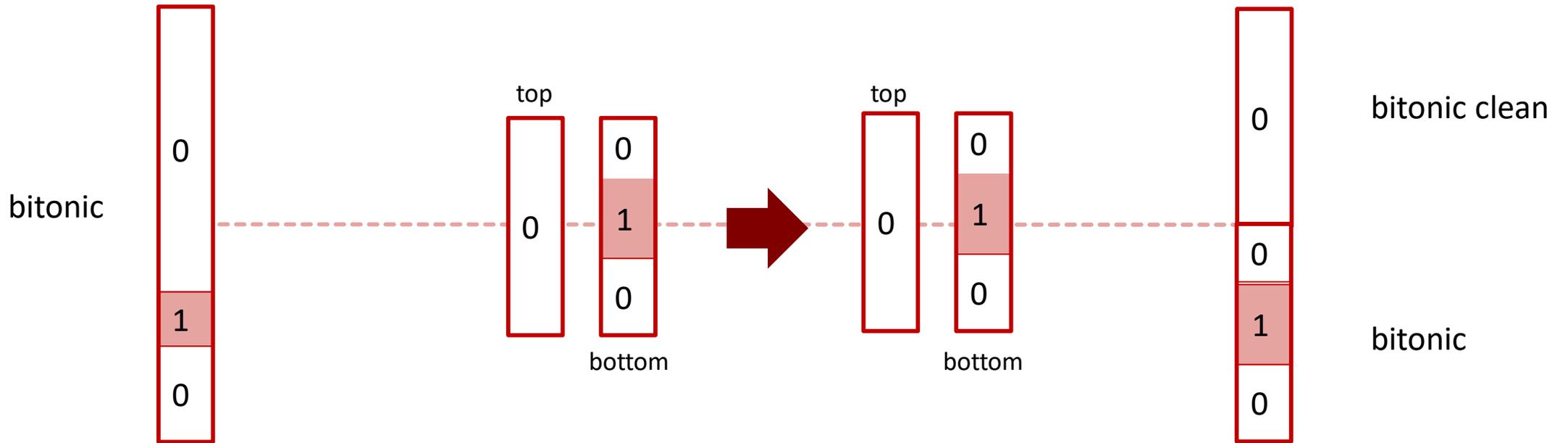
Input bitonic sequence of 0s and 1s, then for the output of the half-cleaner it holds that

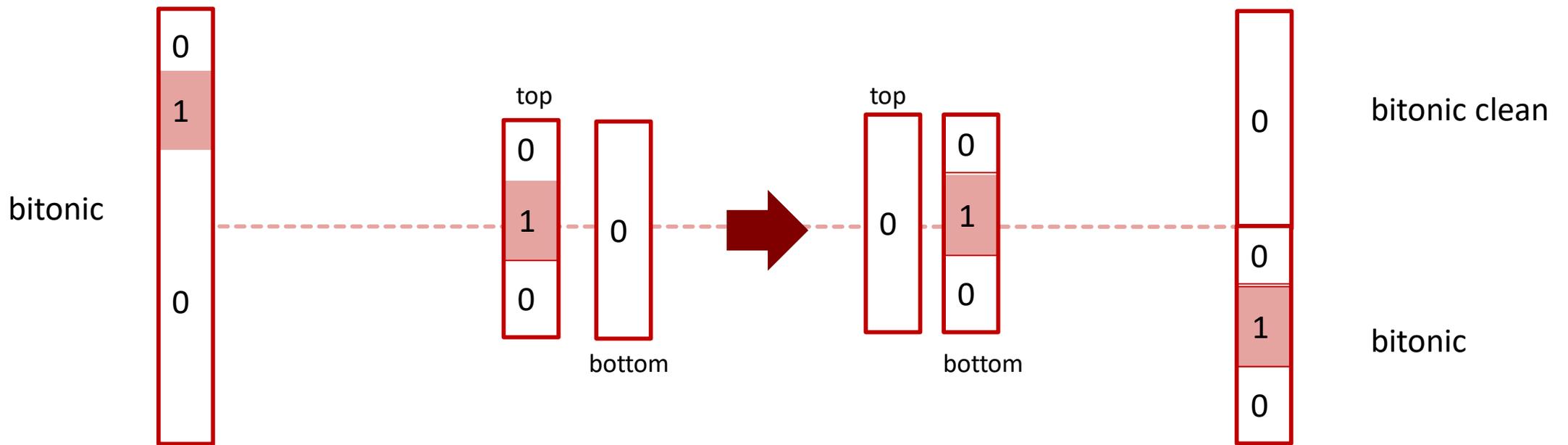
- Upper and lower half is bitonic
- One of the two halves is bitonic clean
- Every number in upper half  $\leq$  every number in the lower half

# Proof: All cases

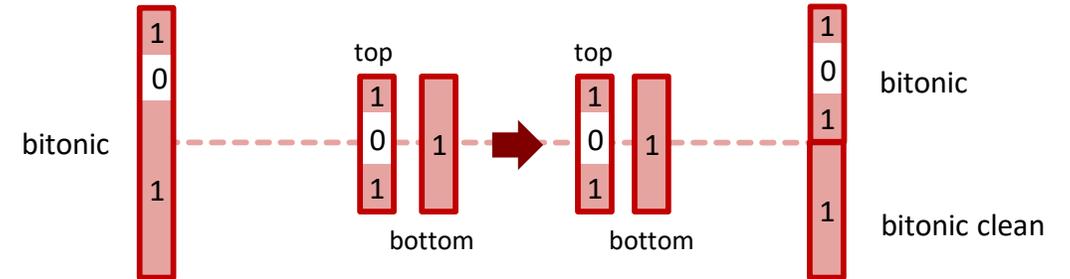
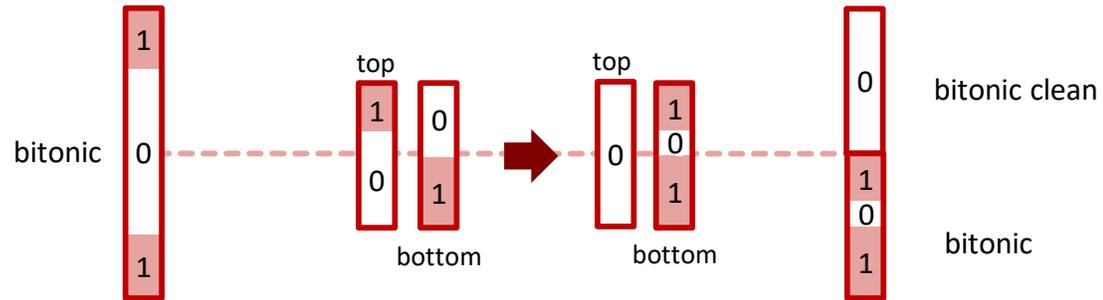
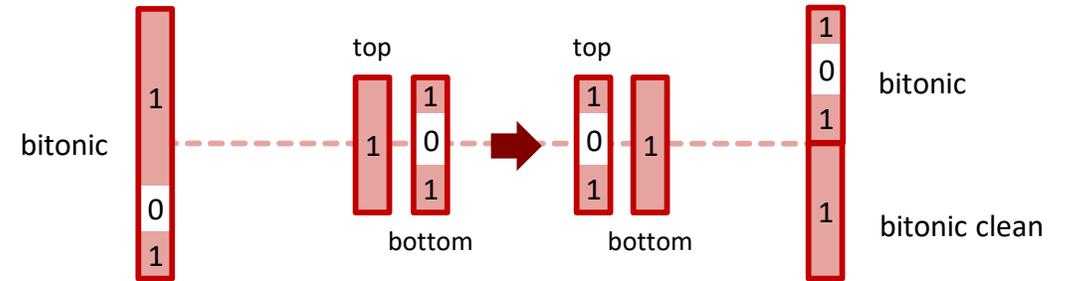
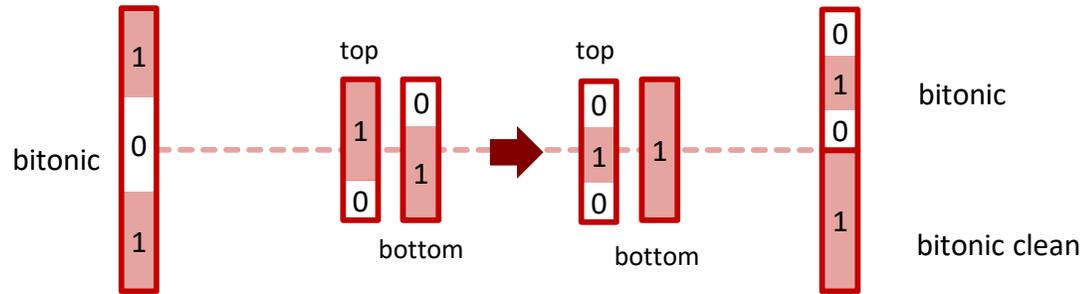




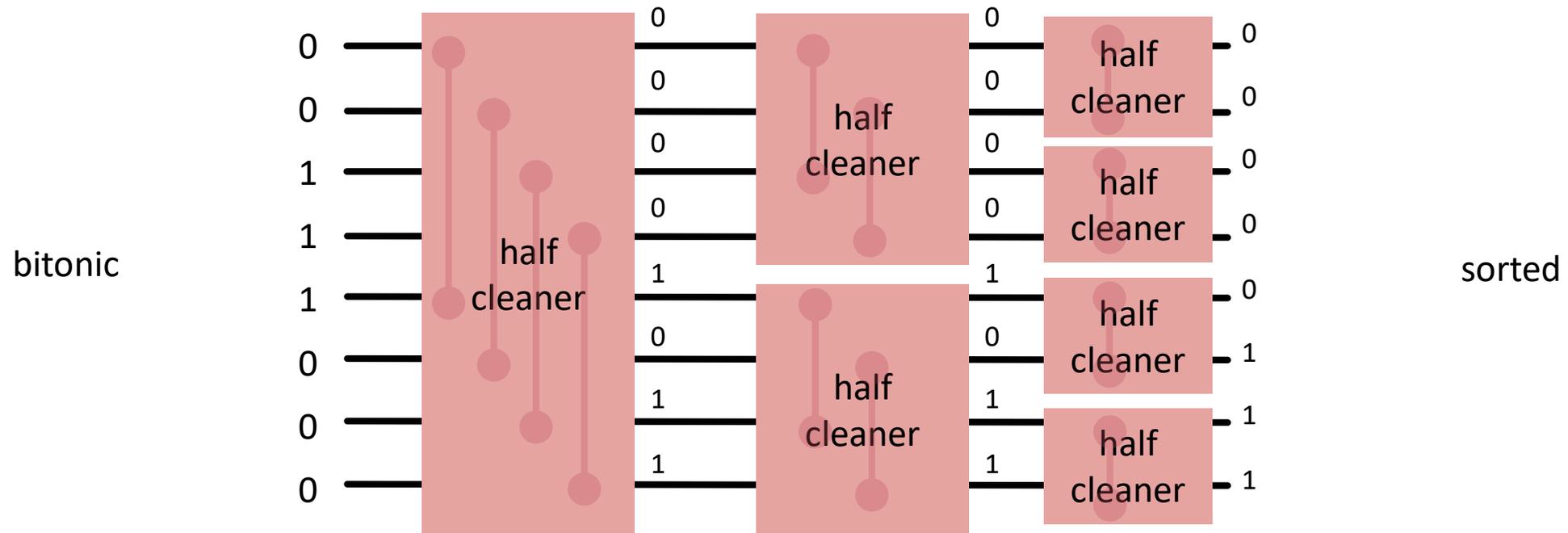




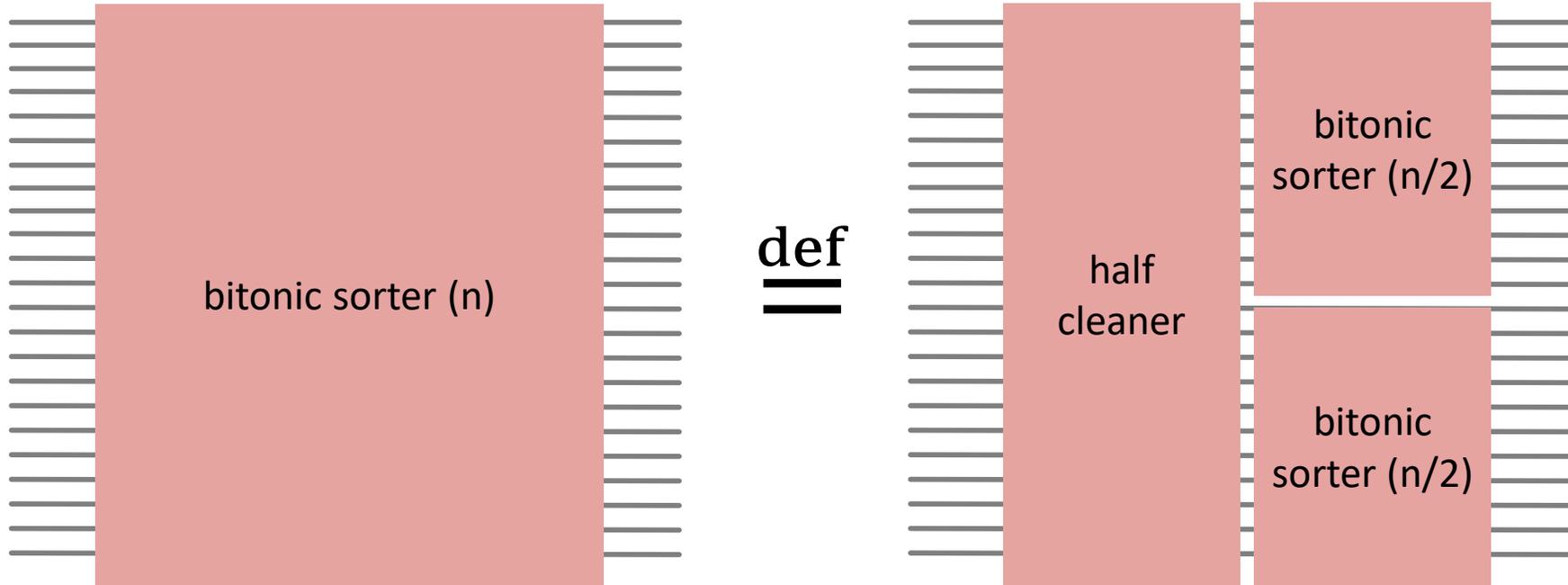
# The four remaining cases (010 → 101)



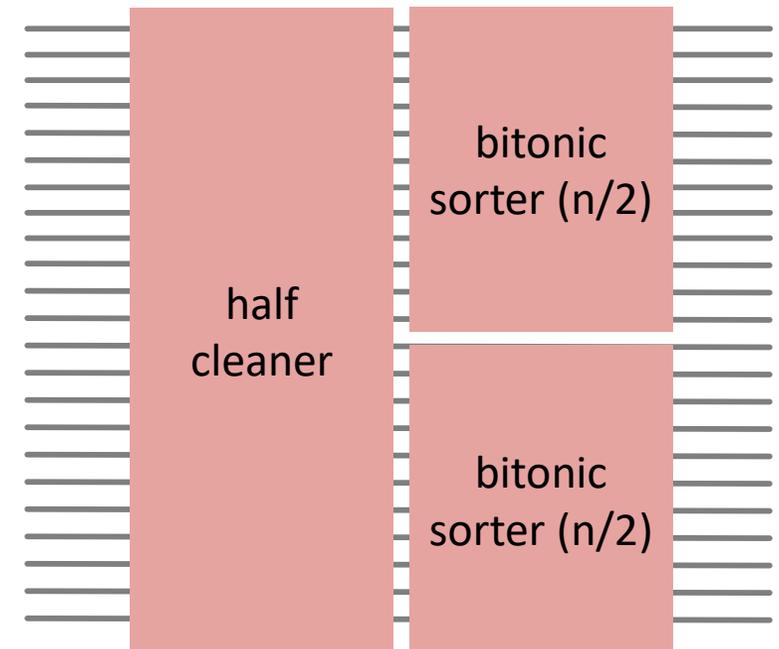
# Construction of a Bitonic Sorting Network



# Recursive Construction

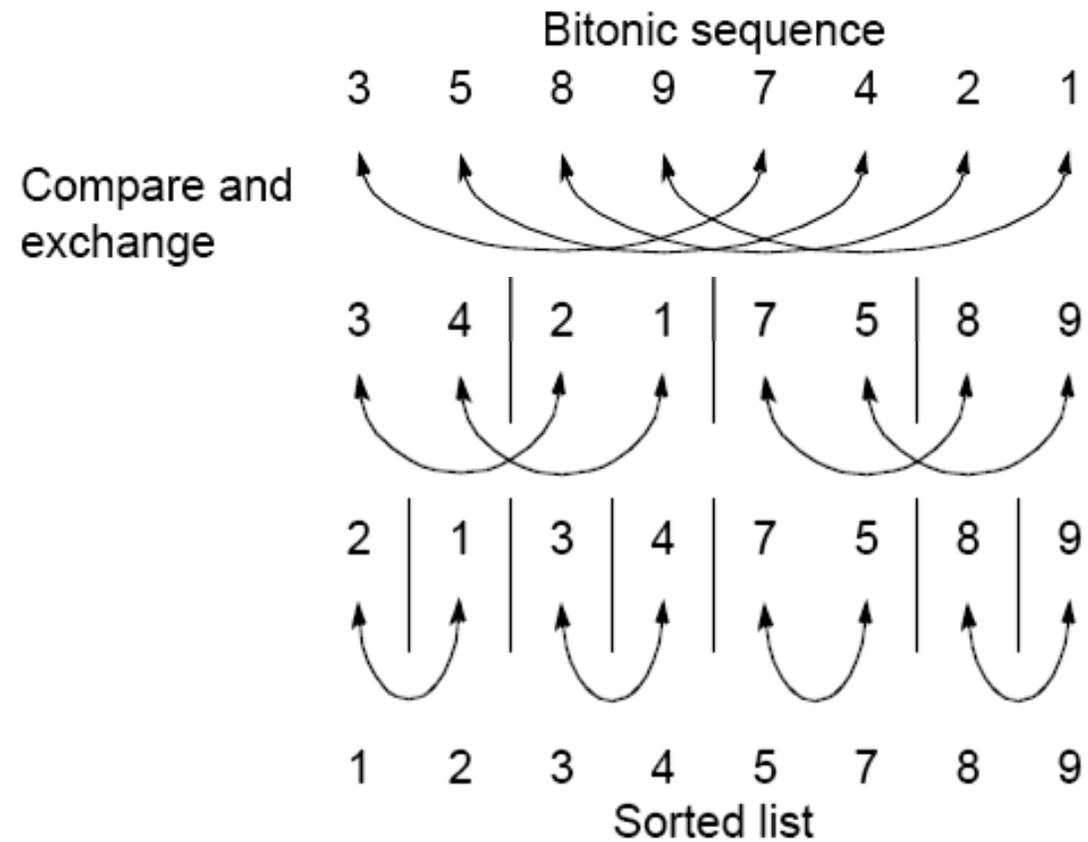


```
void bitonicMerge(int[] a, int lo, int n, boolean dir)
{
    if (n>1) {
        int m=n/2;
        halfClean(a, lo, m, dir);
        bitonicMerge(a, lo, m, dir);
        bitonicMerge(a, lo+m, m, dir);
    }
}
```

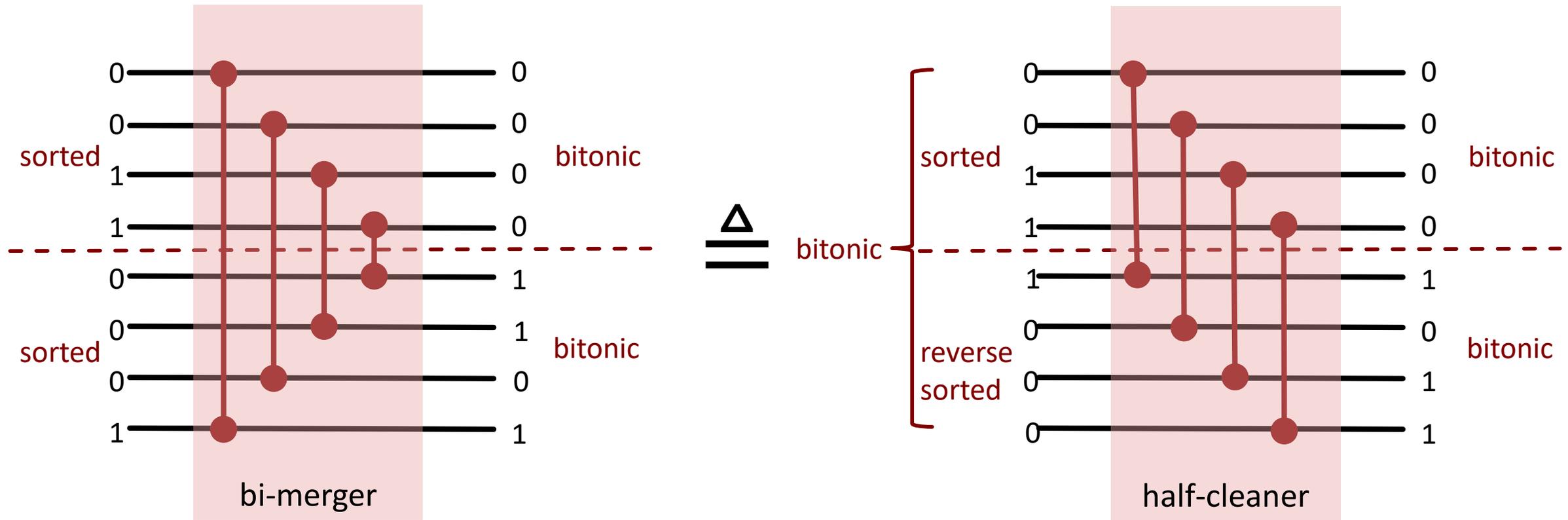


# Bitonic Merge

- Compare-and-exchange moves smaller numbers of each pair to left and larger numbers of pair to right.
- Given a *bitonic* sequence, recursively performing '*binary split*' will sort the list.

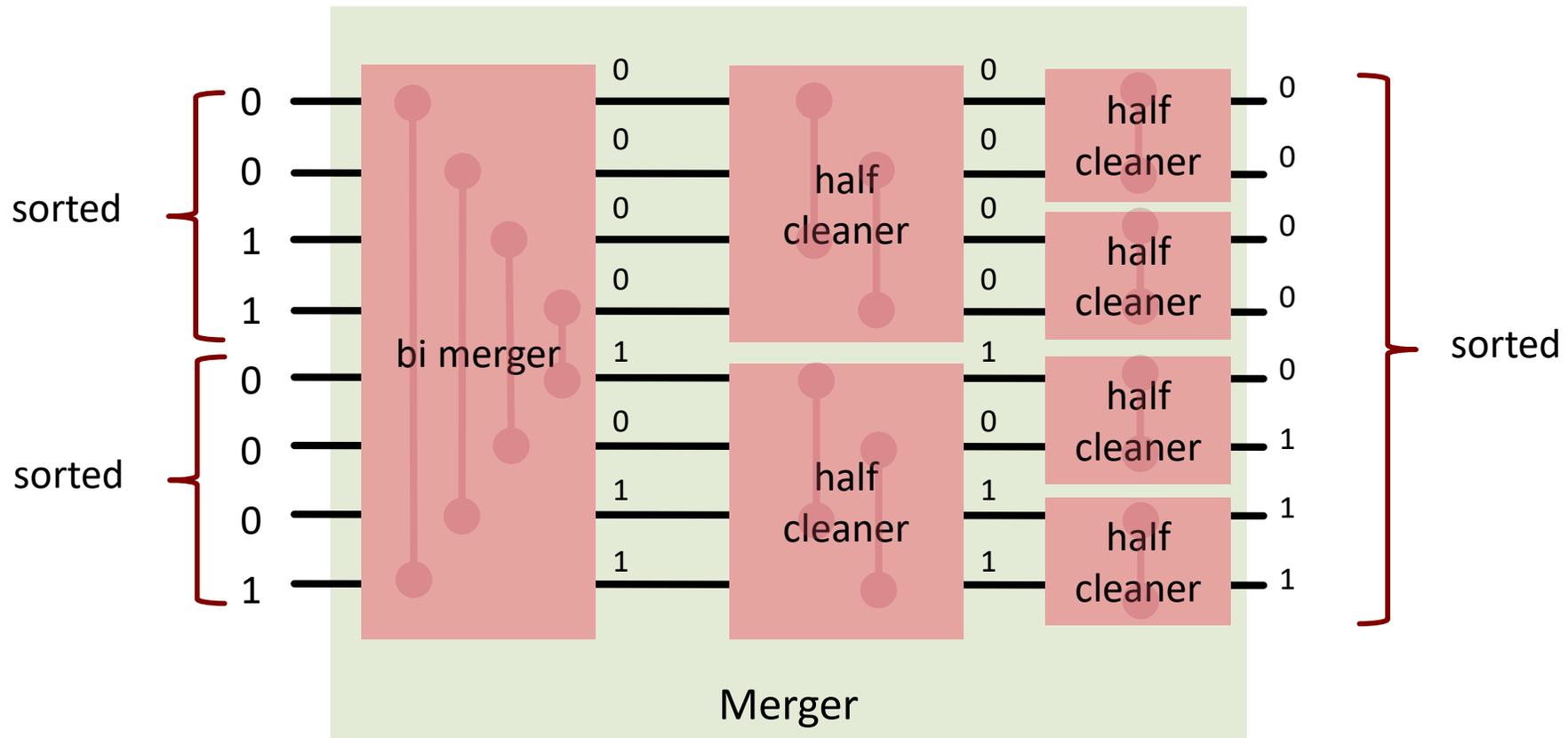


# Bi-Merger

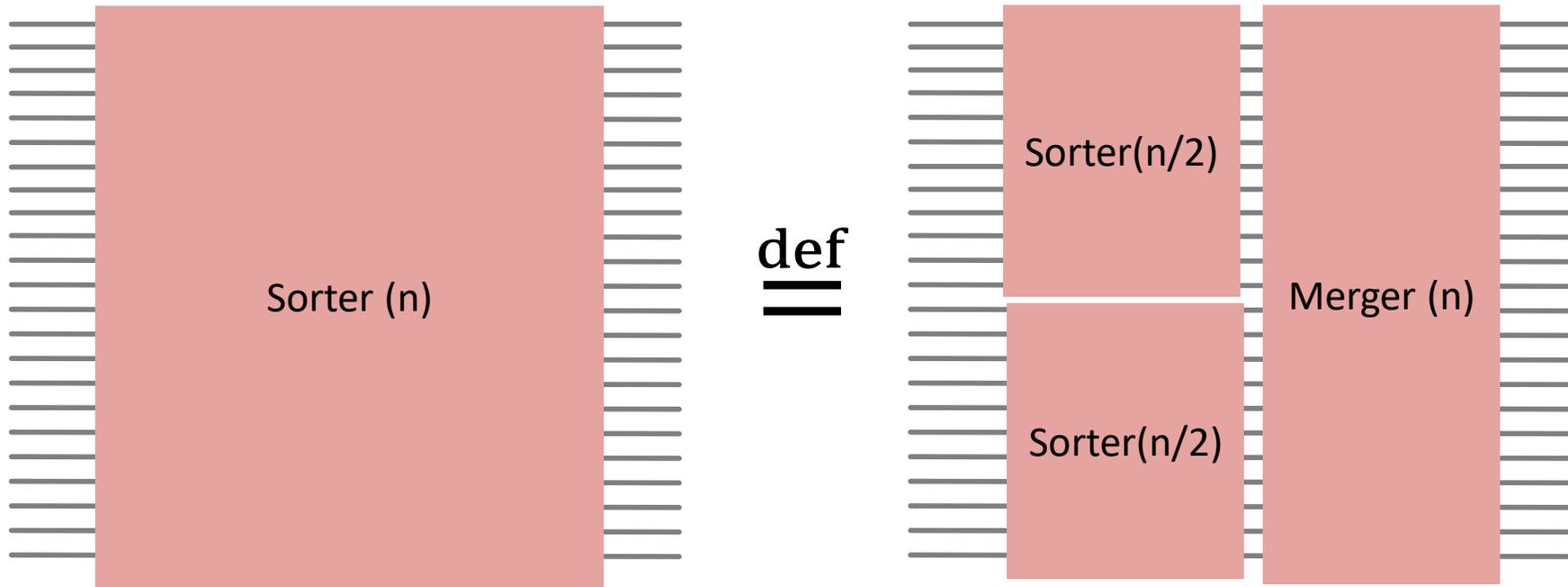


Bi-Merger on two sorted sequences acts like a half-cleaner on a bitonic sequence (when one of the sequences is reversed)

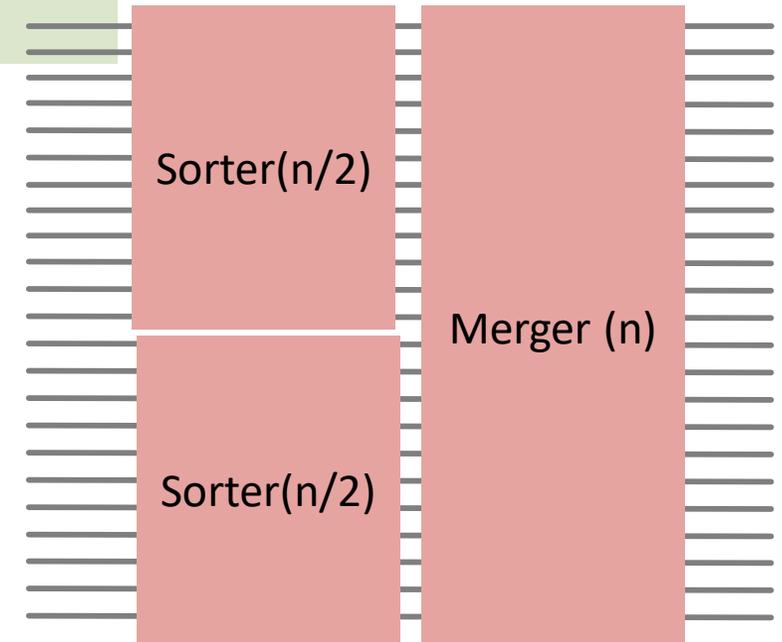
# Merger



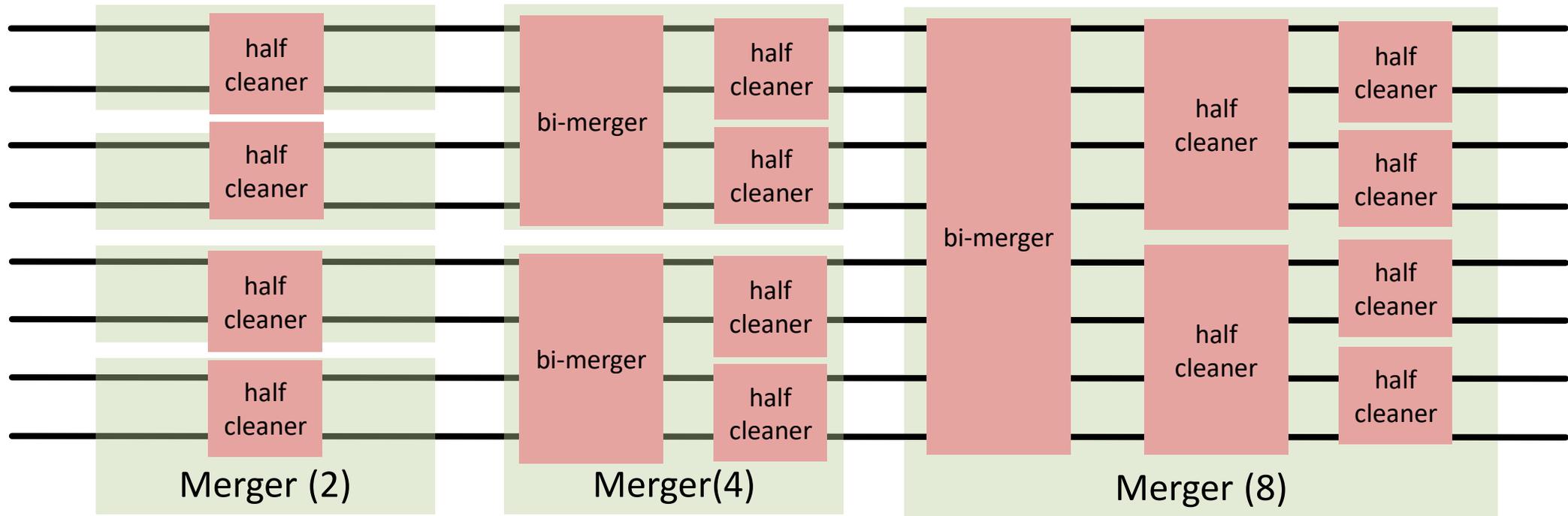
# Recursive Construction of a Sorter



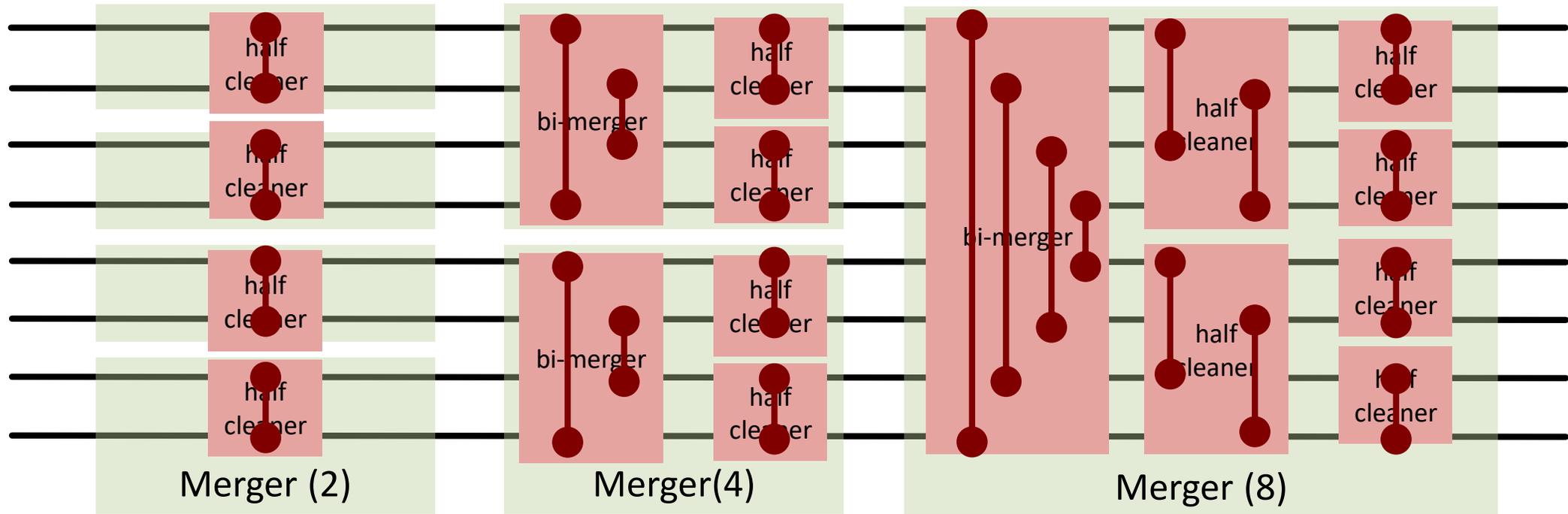
```
private void bitonicSort(int a[], int lo, int n, boolean dir) {  
    if (n>1){  
        int m=n/2;  
        bitonicSort(a, lo, m, ASCENDING);  
        bitonicSort(a, lo+m, n, DESCENDING);  
        bitonicMerge(a, lo, n, dir);  
    }  
}
```



# Example



# Example



# Bitonic Merge Sort

## How many steps?

#mergers

$$\sum_{i=1}^{\log n} \log 2^i = \sum_{i=1}^{\log n} i \log 2 = \frac{\log n \cdot (\log n + 1)}{2} = O(\log^2 n)$$

#steps /  
merger

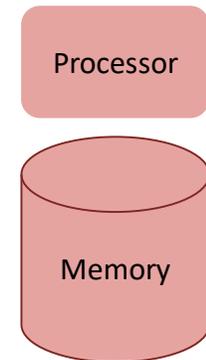
## Interlude: Machine Models

### RAM : Random Access Machine

- Unbounded local memory
- Each memory has unbounded capacity
- Simple operations: data, comparison, branches
- All operations take unit time

Time complexity: number of steps executed

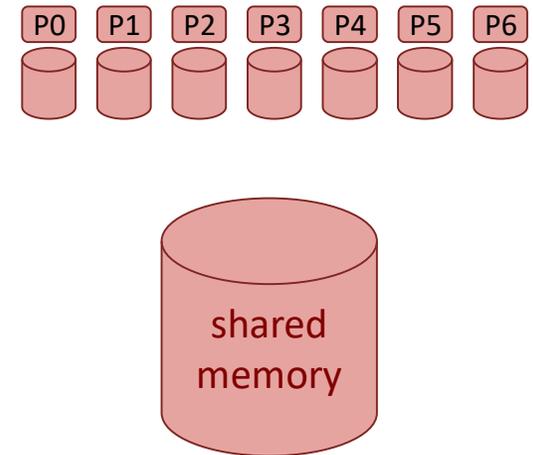
Space complexity: (maximum) number of memory cells used



## Machine Models

### PRAM : *Parallel Random Access Machine*

- Abstract machine for designing algorithms applicable for parallel computers
- Unbounded collection of RAM processors  $P_0, P_1, \dots$
- Each processor has unbounded registers
- Unbounded shared memory
- All processors can access all memory in unit time
- All communication via shared memory



## Shared Memory Access Model

**ER:** processors can simultaneously read from distinct memory locations

**EW:** processors can simultaneously write to distinct memory locations

**CR:** processors can simultaneously read from any memory location

**CW:** processors can simultaneously write to any memory location

Specification of the machine model as one of EREW, CREW, CRCW

## Example: Why the machine model can be important

### Find maximum of $n$ elements in an array $A$

Assume  $O(n^2)$  processors and the **CRCW model**

For all  $i \in \{0, 1, \dots, n - 1\}$  in parallel do

$P_{i0}: m_i \leftarrow true$

For all  $i, j \in \{0, 1, \dots, n - 1\}, i \neq j$  in parallel do

$P_{ij}: if A_i < A_j then m_i \leftarrow false$

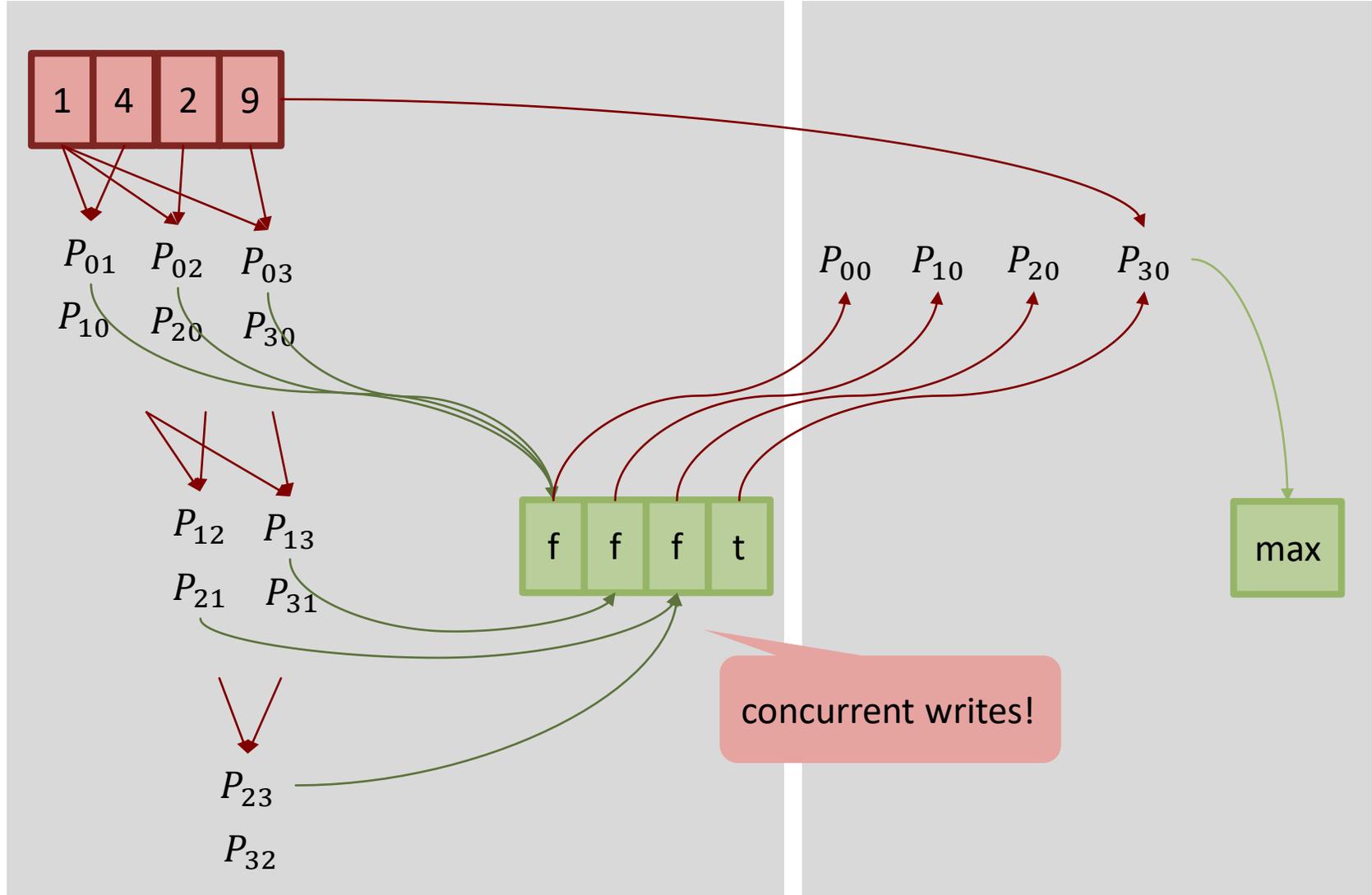
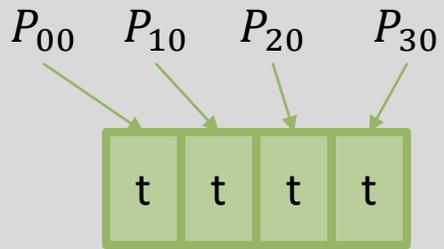
For all  $i \in \{0, 1, \dots, n - 1\}$  in parallel do

$P_{i0}: if m_i = true then max \leftarrow A_i$

$O(1)$  time  
complexity!

# Illustration

1. Init



## CREW

Q: How many steps does max-find require with CREW?

Using CREW only two values can be merged into a single value by one processor at a time step: number of values that need to be merged can be halved at each step  $\rightarrow$  Requires  $\Omega(\log n)$  steps

There is a lot of interesting theoretical results for PRAM machine models (e.g., CRCW simulatable with EREW) and for PRAM based algorithms (e.g., cost optimality / time optimality proofs etc). We will not go into more details here.

In the following we assume a CREW PRAM model -- and receive in retrospect a justification for the results stated above on parallel bubble sorting.

# How to compute fast?



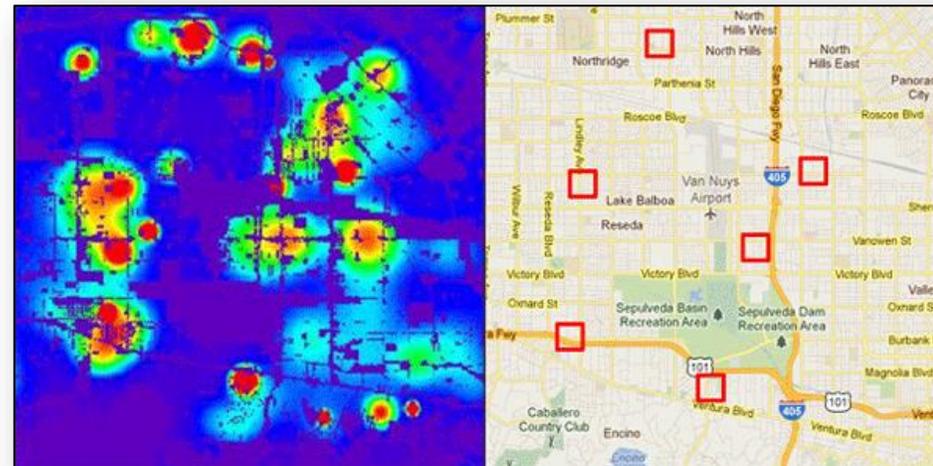
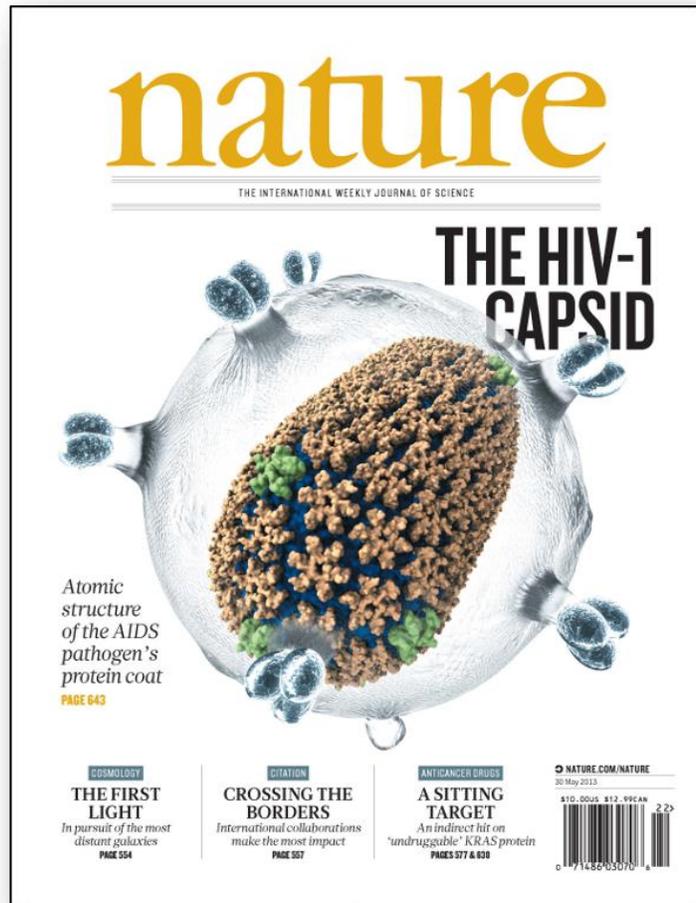
March 2015

## Last lecture -- basic exam tips

- **First of all, read all instructions**
- **Then, read the whole exam paper through**
- **Look at the number of points for each question**
  - This shows how long we think it will take to answer!
- **Find one you know you can answer, and answer it**
  - This will make you feel better early on.
- **Watch the clock!**
  - If you are taking too long on a question, consider dropping it and moving on to another one.
- **Always show your working**
- **You should be able to explain most of the slides**
  - Tip: form learning groups and present the slides to each other
  - If something is unclear:
    - Ask your friends*
    - Read the book (Herlihy and Shavit for the second part)*
    - Ask your TAs*

# Why computing fast?

- Computation is the third pillar of science



# But why do I care!!?? Maybe you like the weather forecast?



## Swiss Weather Forecasting Achieves 1.1km Resolution on 'Piz Kesch'

By John Russell

April 1, 2016

After six months of tweaking – producing a 20 percent reduction in time-to-solution for weather forecasting – MeteoSwiss, the Federal Office of Meteorology and Climatology, today reported its next generation COSMO-1 forecasting system is now operational. COSMO-1 requires 20 times the computing power of COSMO-2 and runs on the hybrid CPU-GPU supercomputer, Piz Kesch, operated by the Swiss National Supercomputing Centre (CSCS) and custom built in collaboration with Cray and NVIDIA.

COSMO-1 was put into service last September (see, [Today's Outlook: GPU-accelerated Weather Forecasting, HPCwire](#)) and improves resolution from 2.2 km to 1.1 km over COSMO-2, an important advance, particularly for Alpine topography forecasts where high spatial resolution is required to accurately predict local weather events such as thunderstorms and thermally induced mountain and valley wind systems.

## Swiss First to Tap GPUs to Improve National Weather Forecasts

September 15, 2015 by ROY KIM

Ten years ago, Hurricane Katrina devastated New Orleans. Three years ago, Hurricane Sandy battered New York City. Hundreds lost their lives. Damages were in the billions.

Wherever you live, predicting the weather is a high-stakes game.

Now, thanks to GPU-accelerated computing, the Swiss have made significant advancements in their ability to predict storms and other weather hazards with higher levels of accuracy.

The Swiss Federal Office of Meteorology and Climatology, [MeteoSwiss](#), is the first major national weather service to deploy a GPU-accelerated supercomputer to improve its daily weather forecasts.



Tobias Gysi,  
PhD Student @SPCL

## MeteoSchweiz und das CSCS gewinnen den Swiss ICT Award

Date of publication	15 November 2016
Topics	<a href="#">About us</a>
Type	Press release

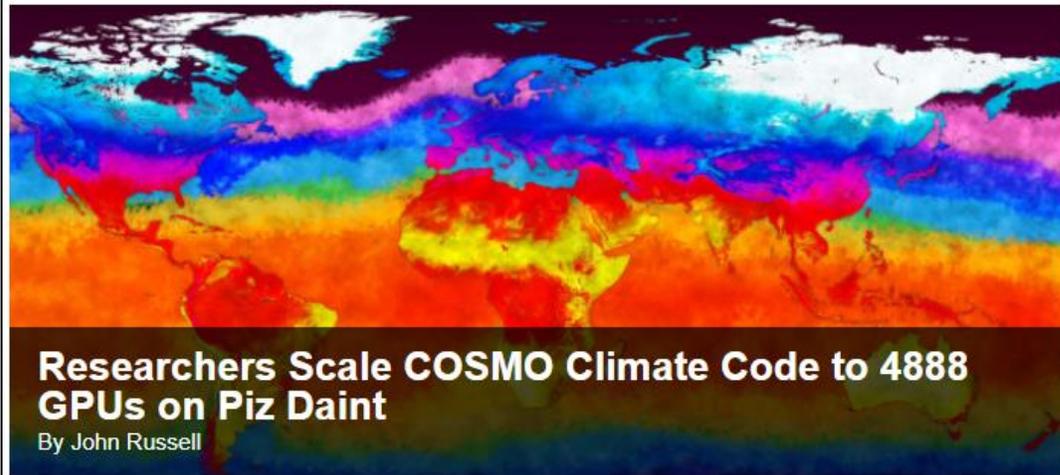
Die diesjährige Auszeichnung für ein besonderes IT-basiertes Produkt oder Service der Schweizer Informatikbranche geht an das Nationale Hochleistungszentrum der Schweiz (CSCS) und das Bundesamt für Meteorologie und Klimatologie MeteoSchweiz für ihr gemeinsames Projekt „Super-Wetterrechner“. Nach Ansicht der Jury ist die neue Art der Berechnung riesiger Wetter-Datenmengen auf einem Supercomputer richtungsweisend.

## MeteoSwiss and CSCS pave the way for more detailed weather forecasts

Date of publication	15 September 2015
Topics	<a href="#">Hazards</a> <a href="#">Weather</a>
Type	Press release

At the Swiss National Supercomputing Centre (CSCS) in Lugano the new "super weather computer" of the Swiss Federal Office of Meteorology and Climatology MeteoSwiss has started its operation. MeteoSwiss is the first meteorological service which has switched to a new GPU based computer architecture. Thus, the new supercomputer is able to calculate weather models with a resolution twice as high more efficiently and quicker than before.

# Or you wonder about the future of the earth?



## Researchers Scale COSMO Climate Code to 4888 GPUs on Piz Daint

By John Russell

October 17, 2017

Effective global climate simulation, sorely needed to anticipate and cope with global warming, has long been computationally challenging. Two of the major obstacles are the needed resolution and prolonged time to compute. This month a group of researchers from ETH Zurich, MeteoSwiss, and the Swiss National Supercomputing Center (CSCS) report scaling popular COSMO code to run on all 4888 GPUs of CSCS's Piz Daint supercomputer and achieving ultra-high resolution.

In their [paper](#), 'Near-global climate simulation at 1 km resolution: establishing a performance baseline on 4888 GPUs with COSMO 5.0', posted on the open access site, Geoscientific Model Development Discussion, authors present their rather extensive efforts necessary to port the code. Previously COSMO had only been scaled to 1000 GPUs on [Piz Daint](#).

## Near-global climate simulation at 1 km resolution: establishing a performance baseline on 4888 GPUs with COSMO 5.0

Oliver Fuhrer<sup>1</sup>, Tarun Chadha<sup>2</sup>, Torsten Hoefler<sup>3</sup>, Grzegorz Kwasniewski<sup>3</sup>, Xavier Lapillonne<sup>1</sup>, David Leutwyler<sup>4</sup>, Daniel Lüthi<sup>4</sup>, Carlos Osuna<sup>1</sup>, Christoph Schär<sup>4</sup>, Thomas C. Schulthess<sup>5,6</sup>, and Hannes Vogt<sup>6</sup>

<sup>1</sup>Federal Institute of Meteorology and Climatology, MeteoSwiss, Zurich, Switzerland

<sup>2</sup>ITS Research Informatics, ETH Zurich, Switzerland

<sup>3</sup>Scalable Parallel Computing Lab, ETH Zurich, Switzerland

<sup>4</sup>Institute for Atmospheric and Climate Science, ETH Zurich, Switzerland

<sup>5</sup>Institute for Theoretical Physics, ETH Zurich, Switzerland

<sup>6</sup>Swiss National Supercomputing Centre, CSCS, Lugano, Switzerland

**Correspondence:** Oliver Fuhrer (oliver.fuhrer@meteoswiss.ch)

Received: 16 September 2017 – Discussion started: 5 October 2017

Revised: 7 February 2018 – Accepted: 8 February 2018 – Published: 2 May 2018

**Abstract.** The best hope for reducing long-standing global climate model biases is by increasing resolution to the kilometer scale. Here we present results from an ultrahigh-resolution non-hydrostatic climate model for a near-global setup running on the full Piz Daint supercomputer on 4888 GPUs (graphics processing units). The dynamical core of the model has been completely rewritten using a domain-specific language (DSL) for performance portability across different hardware architectures. Physical parameterizations and diagnostics have been ported using compiler directives. To our knowledge this represents the first complete atmospheric model being run entirely on accelerators on this scale. At a grid spacing of 930 m (1.9 km), we achieve a simulation throughput of 0.043 (0.23) simulated years per day and an energy consumption of 596 MWh per simulated year. Furthermore, we propose a new memory usage efficiency (MUE) metric that considers how efficiently the memory bandwidth – the dominant bottleneck of climate codes – is being used.

in the availability of water resources and the occurrence of droughts (Pachauri and Meyer, 2014).

Current climate projections are mostly based on global climate models (GCMs). These models represent the coupled atmosphere–ocean–land system and integrate the governing equations, for instance, for a set of prescribed emissions scenarios. Despite significant progress during the last decades, uncertainties are still large. For example, current estimates of the equilibrium global mean surface warming for doubled greenhouse gas concentrations range between 1.5 and 4.5 °C (Pachauri and Meyer, 2014). On regional scales and in terms of the hydrological cycle, the uncertainties are even larger. Reducing the uncertainties of climate change projections, in order to make optimal mitigation and adaptation decisions, is thus urgent and has a tremendous economic value (Hope, 2015).

How can the uncertainties of climate projections be reduced? There is overwhelming evidence from the literature that the leading cause of uncertainty is the representation of clouds, largely due to their influence upon the reflection of

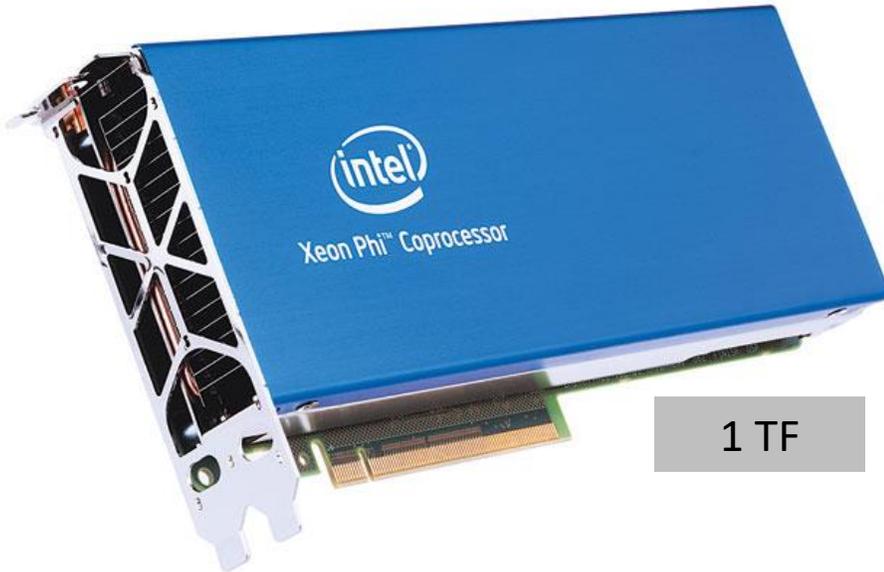
1 Teraflop in 1997



\$67 Million

# 1 Teraflop 17 years later (2014)

Want to play with any of these?



1 TF

*“Amazon.com by Intel even has the co-processor selling for just \$142 (plus \$12 shipping) though they seem to be now out of stock until early December.” (Nov. 11, 2014)*



[Update 2018]  
7.8 Tflop/s double precision  
15.7 Tflop/s single precision  
125 Tflop/s half precision

## 1 Teraflop 20 years later (2017)

TECHNOLOGY

# Intel's new chip puts a teraflop in your desktop. Here's what that means

It's as fast as a turn-of-the-century supercomputer.

By Rob Verger June 1, 2017



# 1 Teraflop 25 years later (2022)



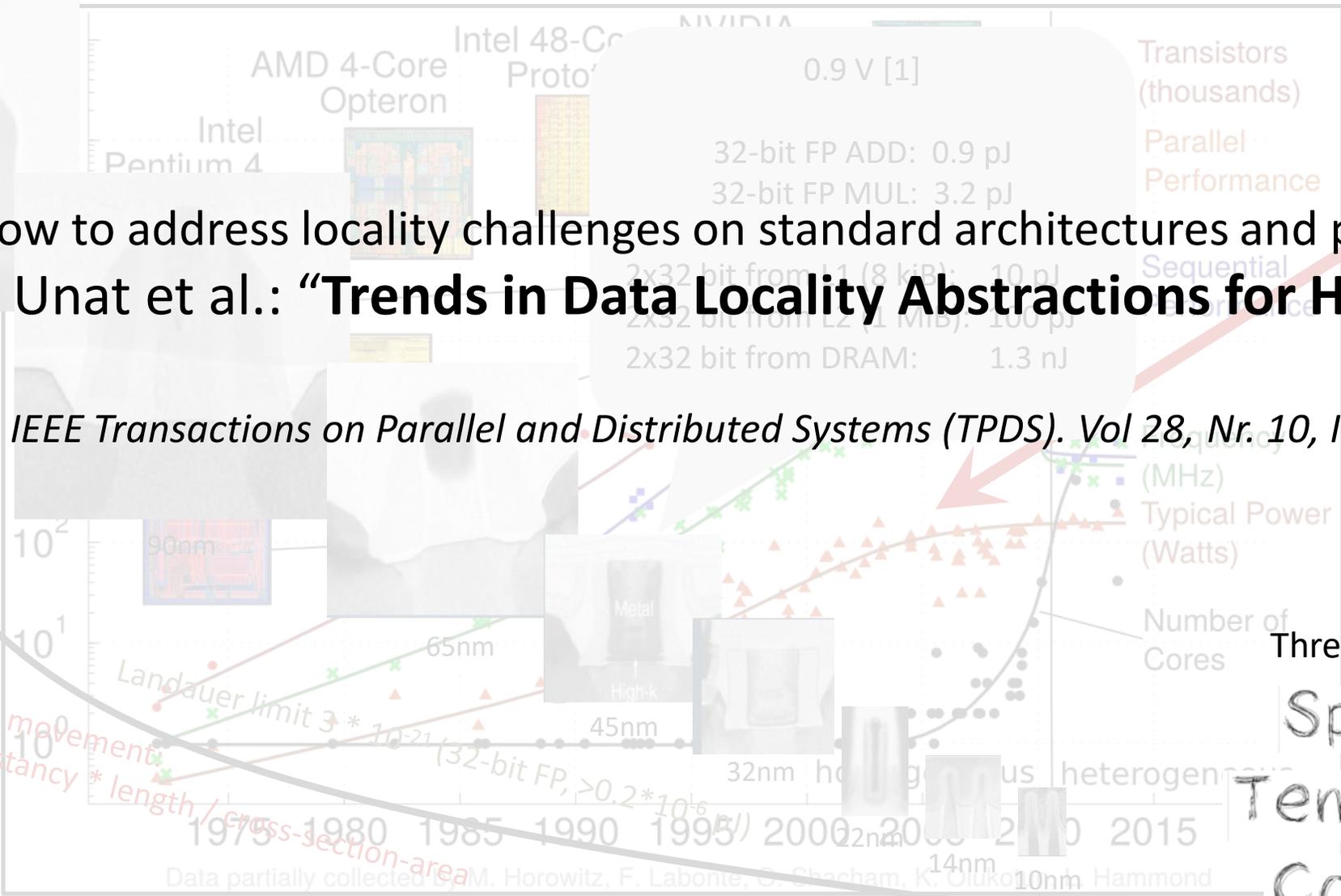
# 1 Petaflop 35 years later (2032???)



**Not so fast ...**  
(or: performance became interesting again)

# Changing hardware constraints and the physics of computing

How to address locality challenges on standard architectures and programming?  
 D. Unat et al.: **“Trends in Data Locality Abstractions for HPC Systems”**



*IEEE Transactions on Parallel and Distributed Systems (TPDS). Vol 28, Nr. 10, IEEE, Oct. 2017*

Moore's law really is dead this time  
 The chip industry is no longer going to treat Gordon Moore's law as the target to aim for.  
WILLIAM BRONK | 31 OCTOBER 2012 6:22 AM

Three Ls of modern computing:  
 Spatial Locality  
 Temporal Locality  
 Control Locality

Electrical data movement loss ~ resistancy \* length / cross-section-area

[1]: Marc Horowitz, Computing's Energy Problem (and what we can do about it), ISSC 2014, plenary  
 [2]: Moore: Landauer Limit Demonstrated, IEEE Spectrum 2012

# Load-store vs. Dataflow architectures

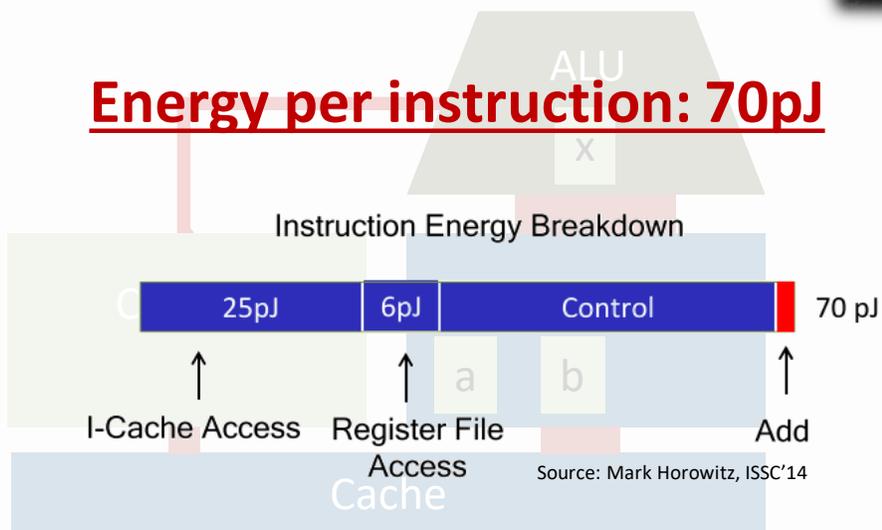
Turing Award 1977 (Backus): "Surely there must be a less primitive way of making big changes in the store than pushing vast numbers of words back and forth through the von Neumann bottleneck."

## Load-store ("von Neumann")



$$x = a + b$$

**Energy per instruction: 70pJ**

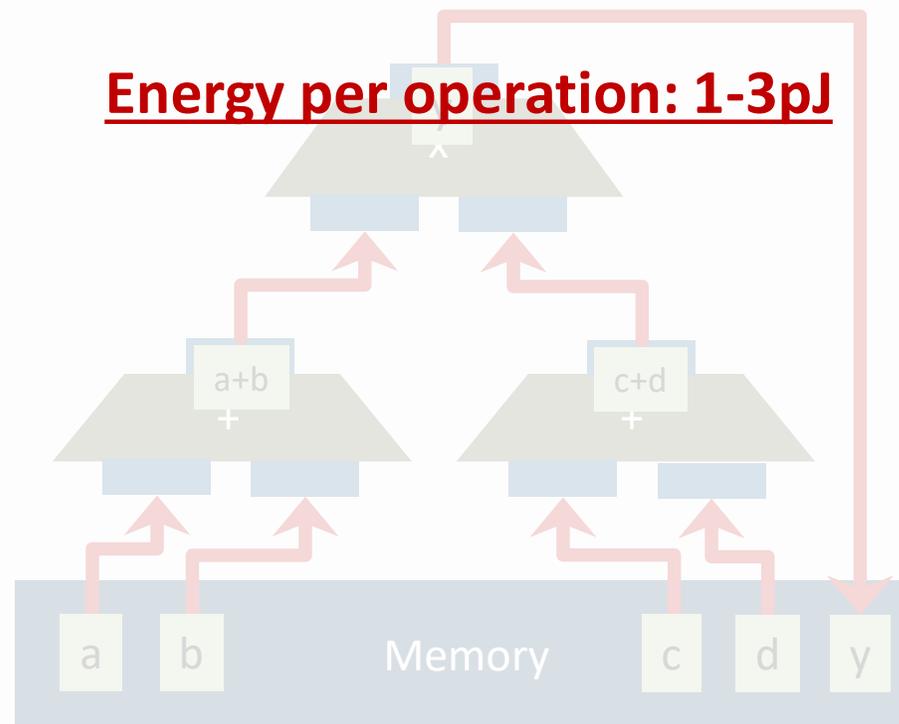


## Static Dataflow ("non von Neumann")



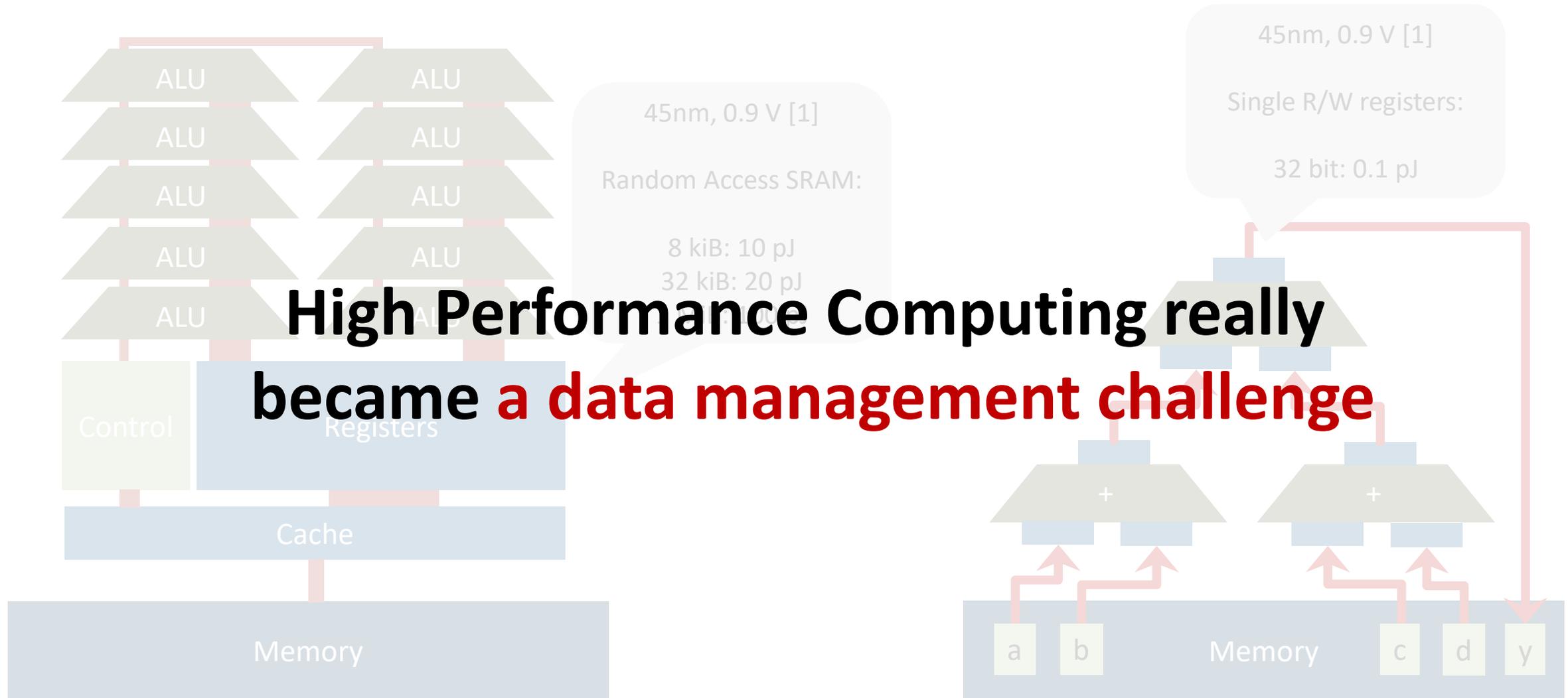
$$y = (a + b) * (c + d)$$

**Energy per operation: 1-3pJ**



Control Locality

# Single Instruction Multiple Data/Threads (SIMD - Vector CPU, SIMT - GPU)



[1]: Marc Horowitz, Computing's Energy Problem (and what we can do about it), ISSC 2014, plenary

# High-performance Computing (Supercomputing)

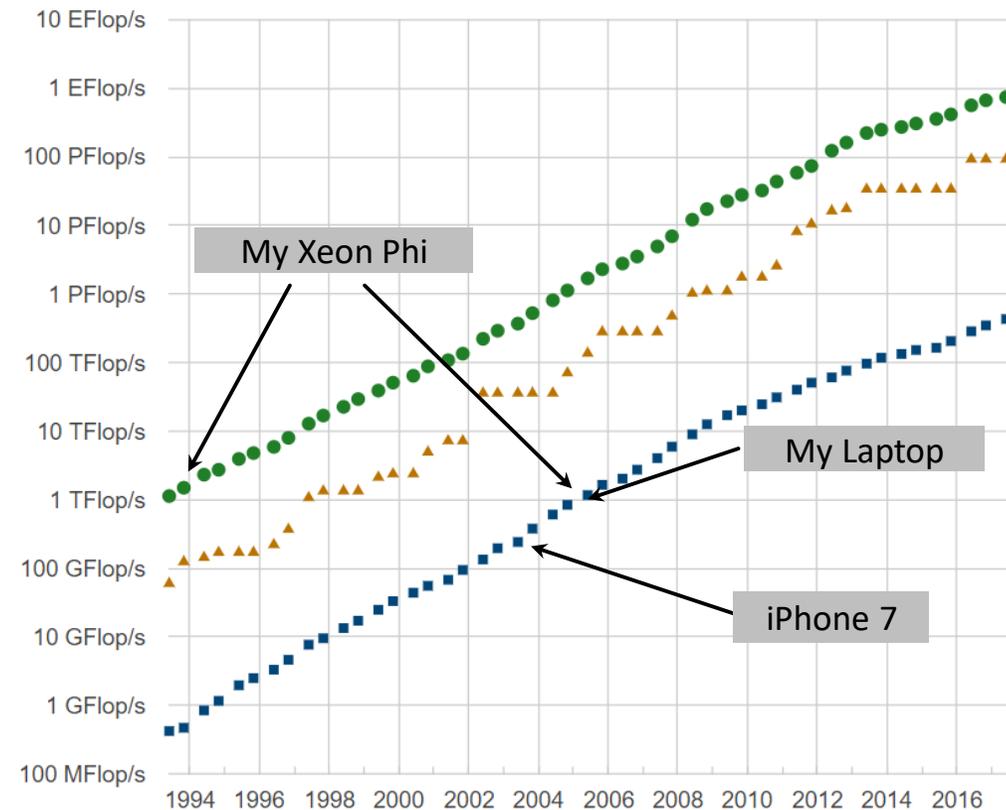


.... next: specialized & reconfigurable computing

# Top 500



- **A benchmark, solve  $Ax=b$** 
  - As fast as possible! → as big as possible 😊
  - Reflects **some** applications, not all, not even many
  - Very good historic data!
- **Speed comparison for computing centers, states, countries, nations, continents 😞**
  - Politicized (sometimes good, sometimes bad)
  - Yet, fun to watch



Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	<b>Summit</b> - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM DOE/SC/Oak Ridge National Laboratory United States	2,397,824	143,500.0	200,794.9	9,783
2	<b>Sierra</b> - IBM Power System S922LC, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM / NVIDIA / Mellanox DOE/NNSA/LLNL United States	1,572,480	94,640.0	125,712.0	7,438
3	<b>Sunway TaihuLight</b> - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway , NRCPC National Supercomputing Center in Wuxi China	10,649,600	93,014.6	125,435.9	15,371
4	<b>Tianhe-2A</b> - TH-IVB-FEP Cluster, Intel Xeon E5-2692v2 12C 2.2GHz, TH Express-2, Matrix-2000 , NUDT National Super Computer Center in Guangzhou China	4,981,760	61,444.5	100,678.7	18,482
5	<b>Piz Daint</b> - Cray XC50, Xeon E5-2690v3 12C 2.6GHz, Aries interconnect , NVIDIA Tesla P100 , Cray Inc. Swiss National Supercomputing Centre (CSCS) Switzerland	387,872	21,230.0	27,154.3	2,384
6	<b>Trinity</b> - Cray XC40, Xeon E5-2698v3 16C 2.3GHz, Intel Xeon Phi 7250 68C 1.4GHz, Aries interconnect , Cray Inc. DOE/NNSA/LANL/SNL United States	979,072	20,158.7	41,461.2	7,578
7	<b>AI Bridging Cloud Infrastructure (ABCI)</b> - PRIMERGY CX2570 M4, Xeon Gold 6148 20C 2.4GHz, NVIDIA Tesla V100 SXM2, Infiniband EDR , Fujitsu National Institute of Advanced Industrial Science and Technology (AIST) Japan	391,680	19,880.0	32,576.6	1,649

## The November 2018 List



Want to run on that system?

# Computing Pi on a supercomputer!

```
int main( int argc, char *argv[] ) {
    // definitions ...
    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD, &numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD, &myid);

    double t = -MPI_Wtime();
    for (j=0; j<n; ++j) {
        h = 1.0 / (double) n;
        sum = 0.0;
        for (i = myid + 1; i <= n; i += numprocs) { x = h * ((double)i - 0.5); sum += (4.0 / i); }
        mypi = h * sum;
        MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);
    }
    t+=MPI_Wtime();

    if (!myid) {
        printf("pi is approximately %.16f, Error is %.16f\n", pi, fabs(pi - PI25DT));
        printf("time: %f\n", t);
    }

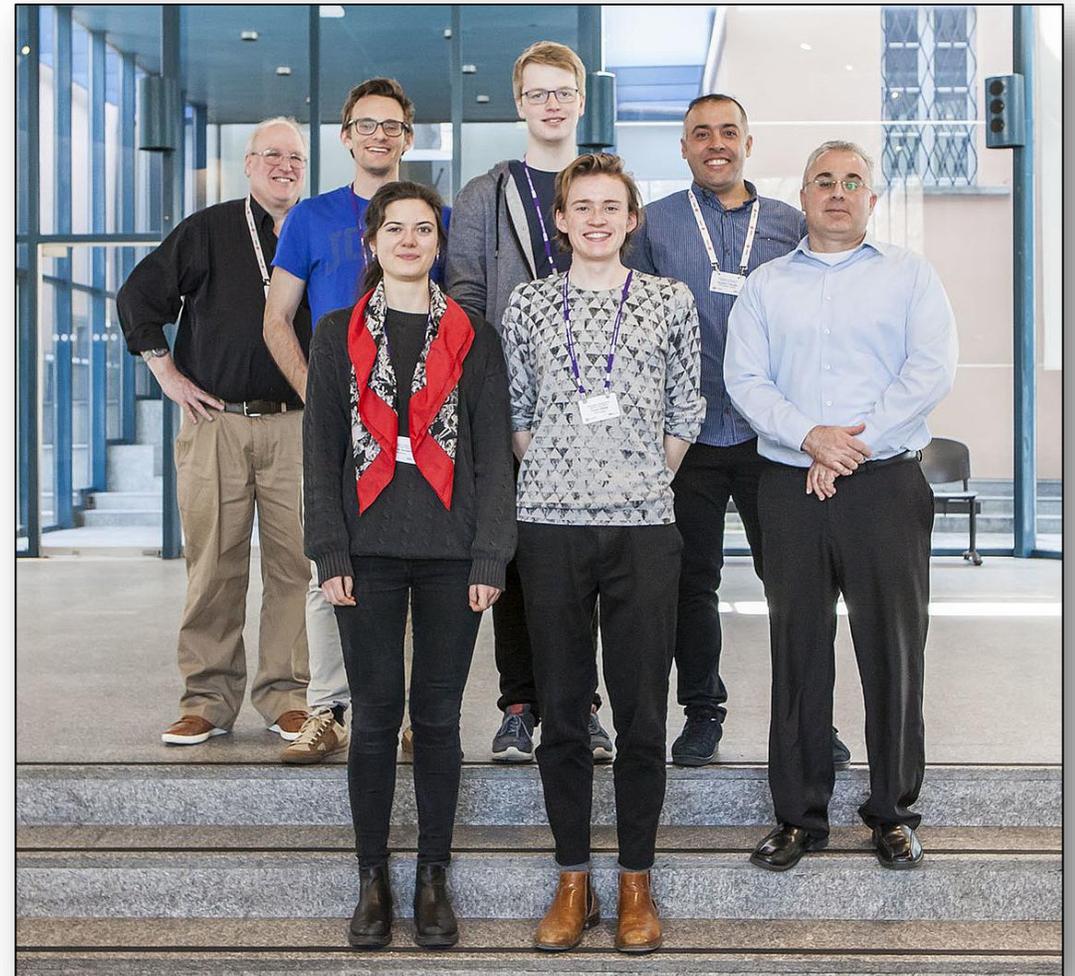
    MPI_Finalize();
}
```

```
htor@hassi:~
htor@daint104:~> salloc --partition debug -N 4 -C mc -t 10
salloc: Pending job allocation 7815988
salloc: job 7815988 queued and waiting for resources
salloc: job 7815988 has been allocated resources
salloc: Granted job allocation 7815988
salloc: Waiting for resource configuration
salloc: Nodes nid000[08-11] are ready for job
htor@daint104:~> srun -n 1 ./a.out
srun: Warning: can't run 1 processes on 4 nodes, setting nnodes to 1
pi is approximately 3.1415926535981167, Error is 0.0000000000083236
time: 13.022794
htor@daint104:~> srun -n 4 ./a.out
pi is approximately 3.1415926535981260, Error is 0.0000000000083329
time: 3.598728
htor@daint104:~> srun -n 8 ./a.out
pi is approximately 3.1415926535981251, Error is 0.0000000000083320
time: 2.120363
htor@daint104:~> srun -n 16 ./a.out
pi is approximately 3.1415926535981269, Error is 0.0000000000083338
time: 1.366739
htor@daint104:~> srun -n 32 ./a.out
pi is approximately 3.1415926535981265, Error is 0.0000000000083333
time: 1.034170
htor@daint104:~> srun -n 64 ./a.out
pi is approximately 3.1415926535981269, Error is 0.0000000000083338
time: 0.859992
htor@daint104:~> srun -n 128 ./a.out
pi is approximately 3.1415926535981269, Error is 0.0000000000083338
time: 0.740548
htor@daint104:~> srun -n 256 ./a.out
pi is approximately 3.1415926535981269, Error is 0.0000000000083338
time: 0.953909
htor@daint104:~>
```

# Student Cluster Competition

- **6 undergrads, 1 advisor, 1 cluster, 2x13 amps**
  - 20 teams, most continents @SC or @ISC
  - 48 hours, five applications, non-stop!
  - top-class conference (>13,000 attendees)
- **Lots of fun**
  - Even more experience!
- **Introducing team Racklette**
  - <https://racklette.ethz.ch/>
  - Search for “Student Cluster Challenge”
  - HPC-CH/CSCS is helping
- **Let me know, my assistants are happy to help!**
  - If we have a full team

Want to become an expert  
in HPC?



# Finito

- **Thanks for being such fun to teach 😊**
  - Comments (also anonymous) are always appreciated!
- **If you are interested in parallel computing research, talk to me or my assistants!**
  - Large-scale (datacenter) systems
  - Next-generation parallel programming (e.g., FPGAs)
  - Parallel computing (SMP and MPI)
  - GPUs (CUDA), FPGAs, Manycore ...
  - ... spcl-friends mailing list (subscribe on webpage)
  - ... on twitter: [@spcl\\_eth](https://twitter.com/spcl_eth) 😊
- Hope to see you again!  
*Maybe in Design of Parallel and High-Performance Computing in the Masters 😊*
- Or for theses/research projects:  
<http://spcl.inf.ethz.ch/SeMa/>

