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DPHPC, Network models

Recitation session, 12.12.2019

Admin

- **Presentations starting next Monday!**

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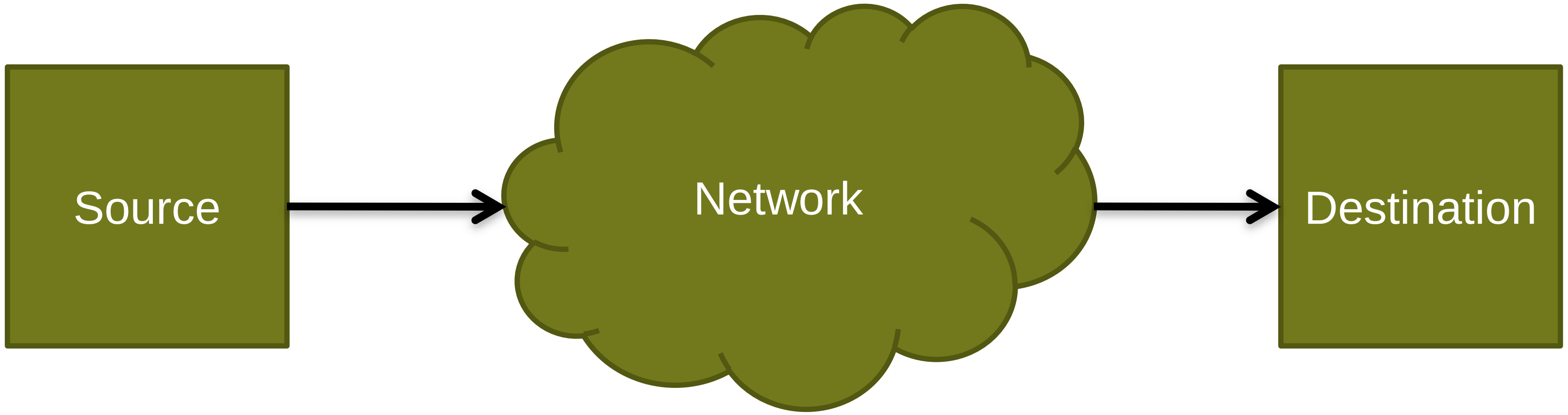
- **How to give good presentations?**
  - Make multiple dry-runs!
  - Short and clear introduction & motivation
  - Plots should be self-explanatory.
  - Avoid long bullet-point lists and walls of texts.
  - PowerPoint/Impress make it quite easy and fast to create diagrams.

HPC Networking Basics

- **Familiar (non-HPC) network: Internet TCP/IP**
  - Common model:



- **Class Question: What parameters are needed to model the performance (including pipelining)?**

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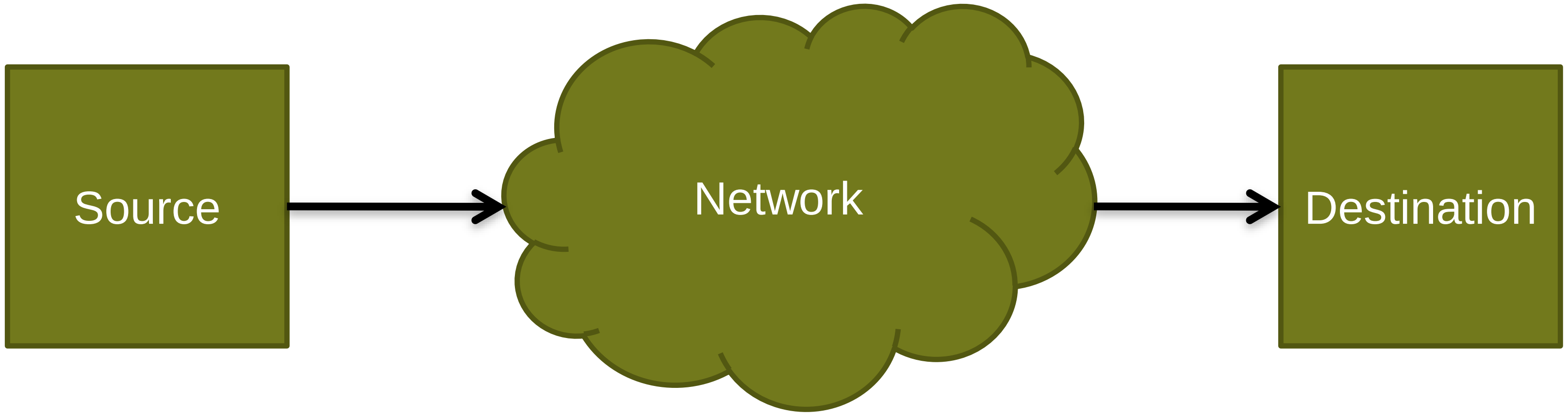


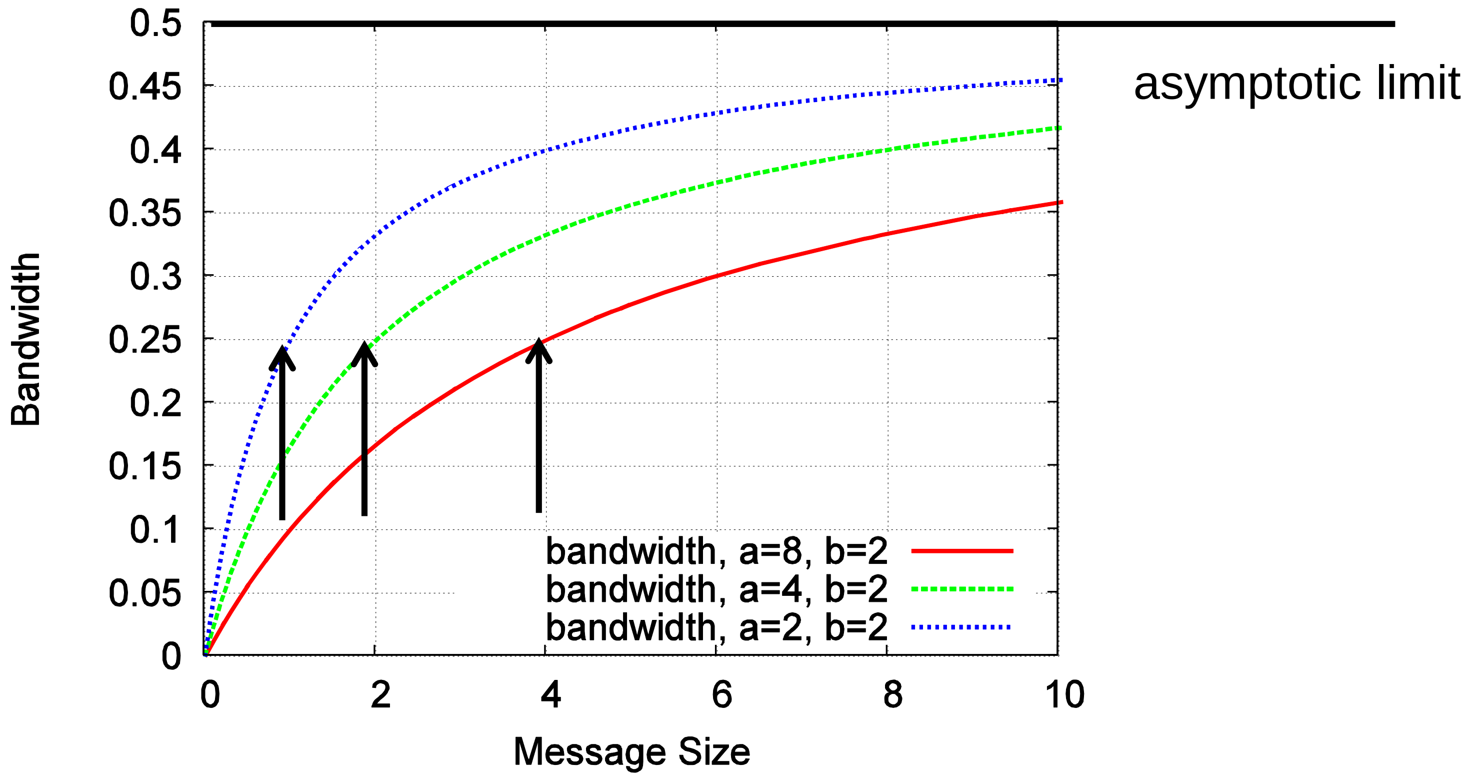
- **Class Question: What parameters are needed to model the performance (including pipelining)?**
  - Latency, Bandwidth, Injection Rate, Host Overhead

A Simple Model for Communication

- **Transfer time $T(s) = \alpha+\beta s$**
  - α = startup time (latency)
  - β = cost per byte (bandwidth=$1/\beta$)
- **As s increases, bandwidth approaches $1/\beta$ asymptotically**
  - Convergence rate depends on α
  - $s_{1/2} = \alpha/\beta$
- **Assuming no pipelining (new messages can only be issued from a process after all arrived)**

Bandwidth vs. Latency

- **$s_{1/2}$ = α/β often used to distinguish bandwidth- and latency-bound messages**
  - $s_{1/2}$ is in the order of kilobytes on real systems



Broadcast

- **Simplest linear broadcast**
  - One process has a data item to be distributed to all processes
- **Broadcasting s bytes among P processes:**
  - T(s) = $(P - 1) \times (\alpha + \beta s) = \mathcal{O}(\mathrm{P})$

- **Class question: Do we know a faster method to accomplish the same?**

k-ary tree broadcast

- Origin process is the root of the tree, passes messages to k neighbors which pass them on.

- What is the broadcast time in the simple latency/bandwidth model?

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    # of messages on each level

    # tree levels

- **What is the optimal k?**

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tree levels

- **What is the optimal k?**
  - $0 = \frac{ln(P) \cdot k}{ln(k)} \frac{d}{dk} = \frac{ln(P)ln(k) - ln(P)}{ln^2(k)} \rightarrow k = e = 2.71...$
  - Independent of P, α, β, s!

Better tree broadcast

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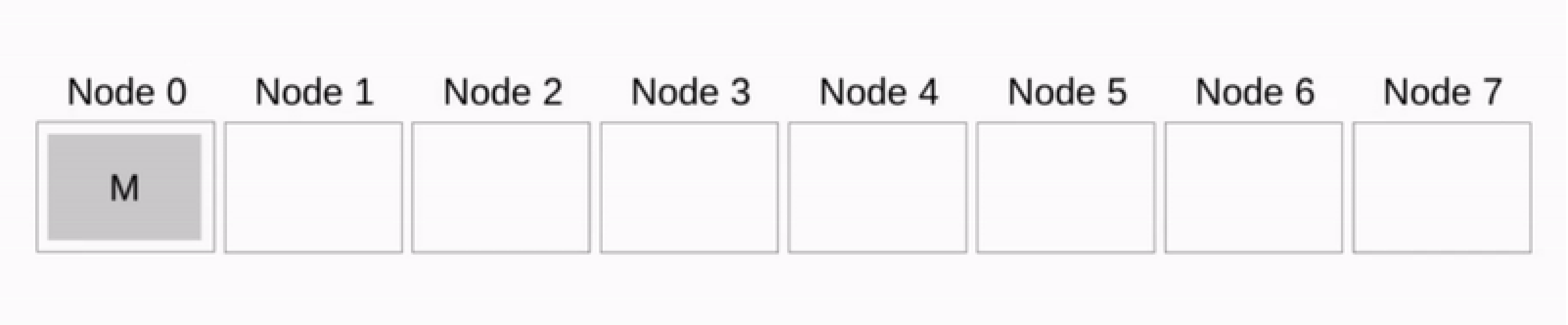

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- **Class Question: Can we broadcast faster than in a k-nomial tree?**
  - $\mathcal{O}(log(P))$ is asymptotically optimal for s=1!
  - But what about large s?

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Very Large Message Broadcast

- **Extreme case (P small, s large): simple pipeline**
  - Split message into segments of size z
  - Send segments from PE i to PE i+1

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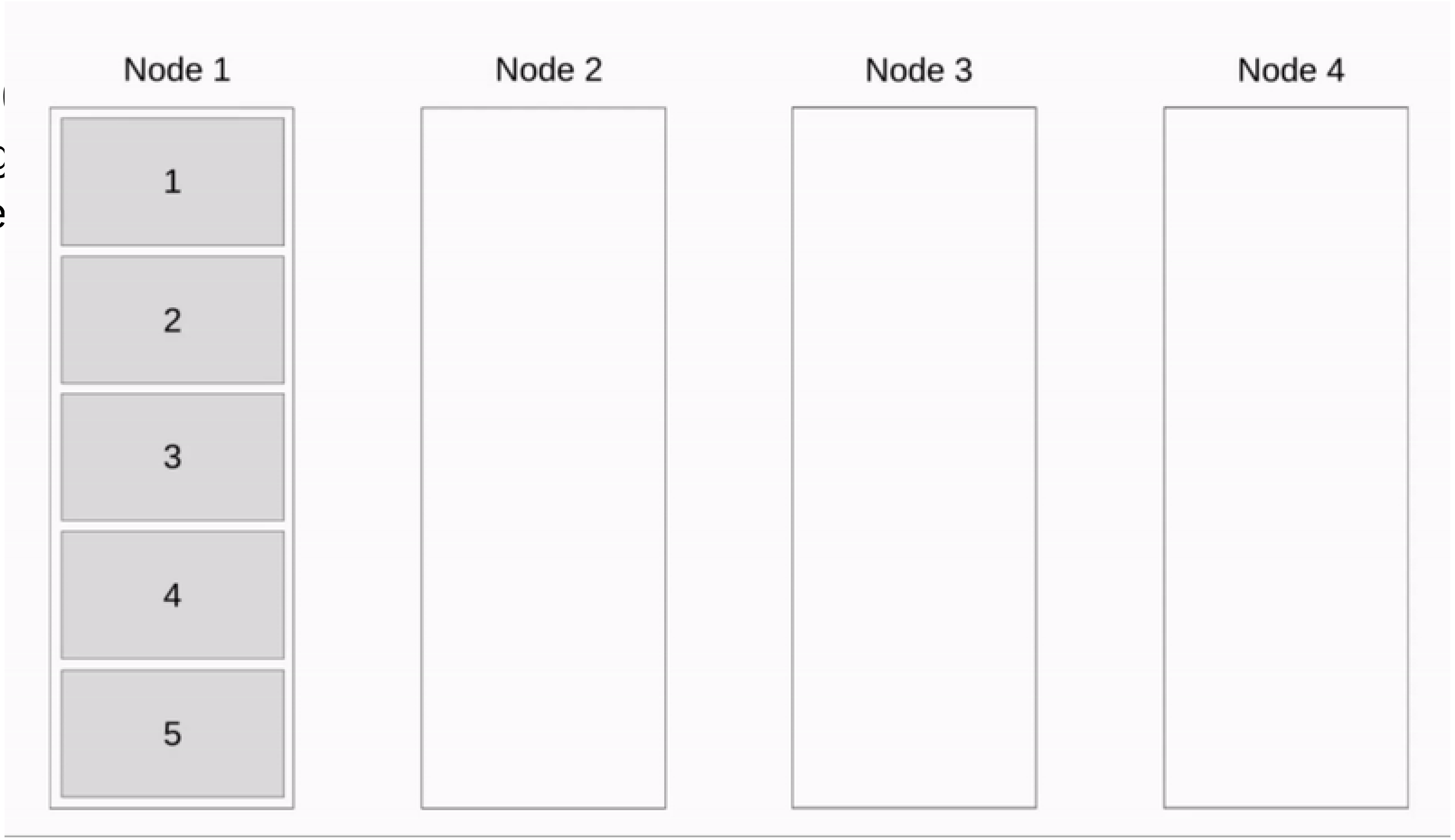

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  - 2,000,020 vs. 1,200,120
- **Class Question: Can we do better for given α, β, P, s?**
  - Derive optimal z
- **What is the time for simple pipeline for α=10, β=1, P=4, s=10^6, z_{opt}?**
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Lower Bounds

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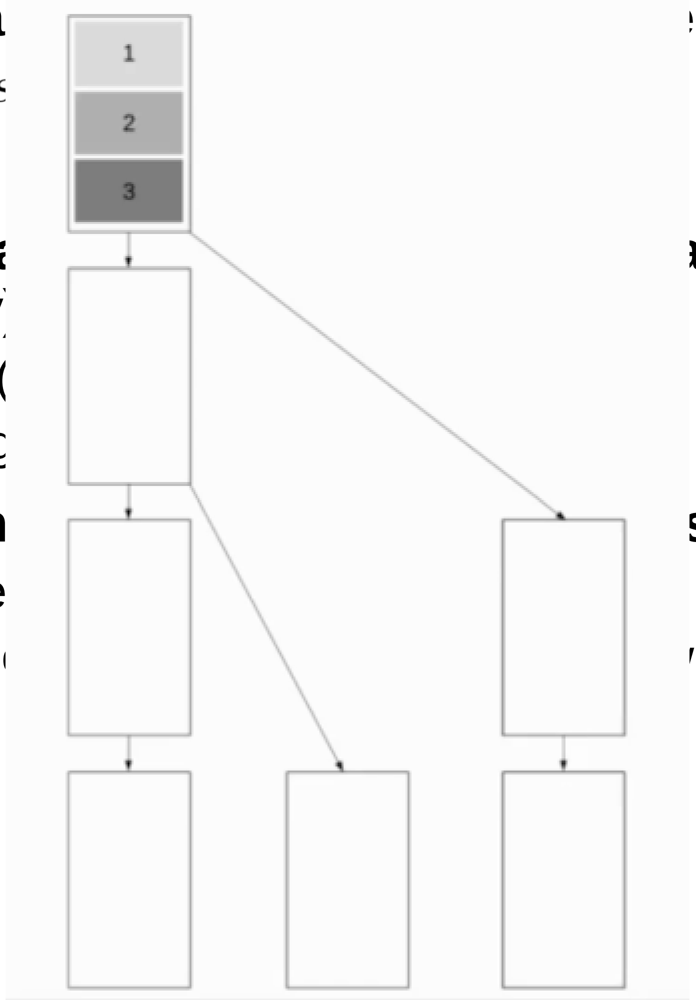

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- **Class Question: What is the optimal z?**
  - $z_{opt} = \sqrt{\frac{\alpha s}{\beta(\lceil \log_2 P \rceil - 2)}}$

Towards an Optimal Algorithm

- **What is the complexity of the pipelined tree with z_{opt} for small s, large P and for large s, constant P?**
  - Small messages, large P: s=1; z=1 (s≤z), will give O(log P)
  - Large messages, constant P: assume α, β, P constant, will give asymptotically O(sβ)

    Asymptotically optimal for large P and s but bandwidth is off by a factor of 2! Why?

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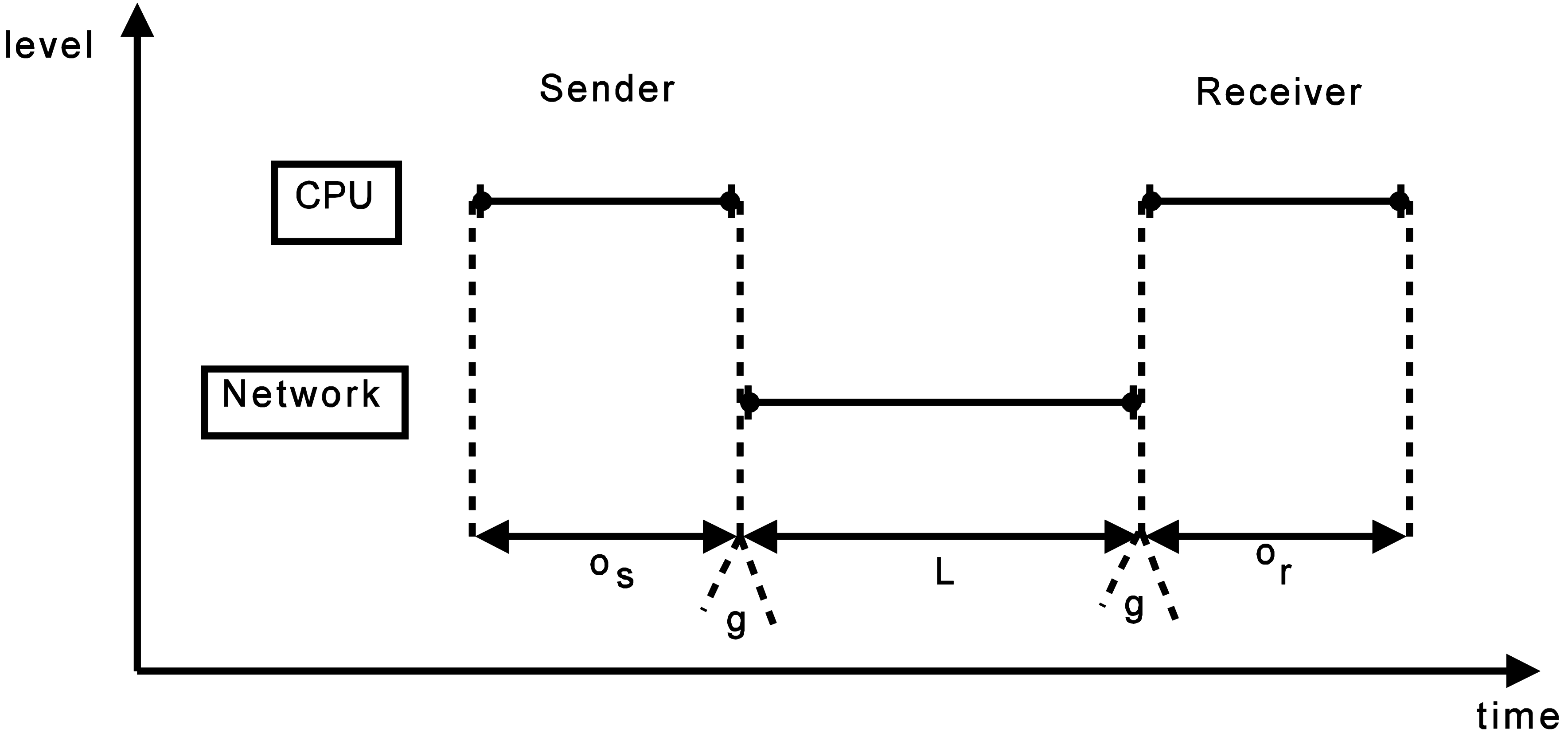
- **Bandwidth-optimal algorithms exist, e.g., Sanders et al. *"Full Bandwidth Broadcast, Reduction and Scan with Only Two Trees"*. 2007**
  - Intuition: in binomial tree, all leaves (P/2) only receive data and never send → wasted bandwidth
  - Send along two simultaneous binary trees where the leafs of one tree are inner nodes of the other
  - Construction needs to avoid endpoint congestion (makes it complex)

The LogP Model

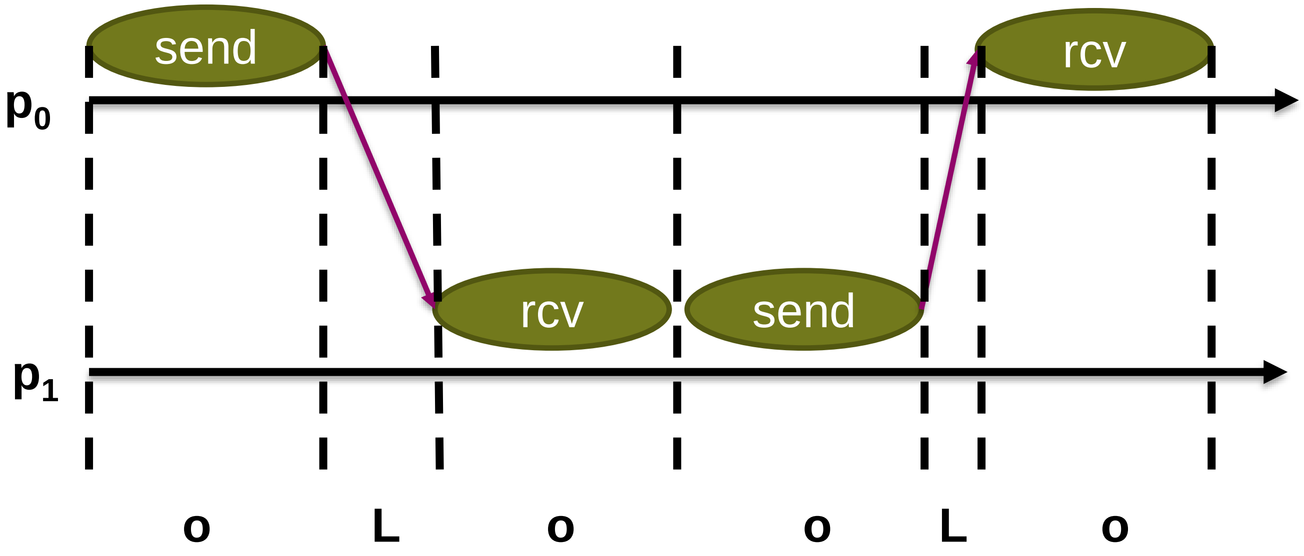
- **Defined by four parameters:**
  - L: an upper bound on the latency, or delay, incurred in communicating a message containing a word (or small number of words) from its source module to its target module.
  - o: the overhead, defined as the length of time that a processor is engaged in the transmission or reception of each message; during this time, the processor cannot perform other operations.
  - g: the gap, defined as the minimum time interval between consecutive message transmissions or consecutive message receptions at a processor. The reciprocal of g corresponds to the available per-processor communication bandwidth.
  - P: the number of processor/memory modules. We assume unit time for local operations and call it a cycle.

The LogP Model

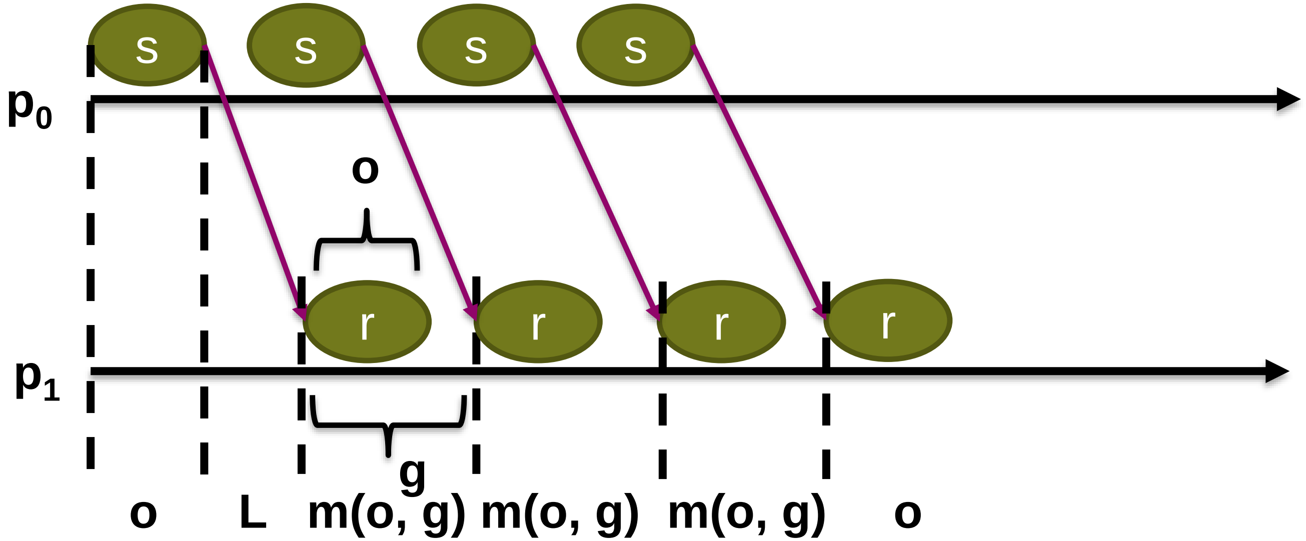


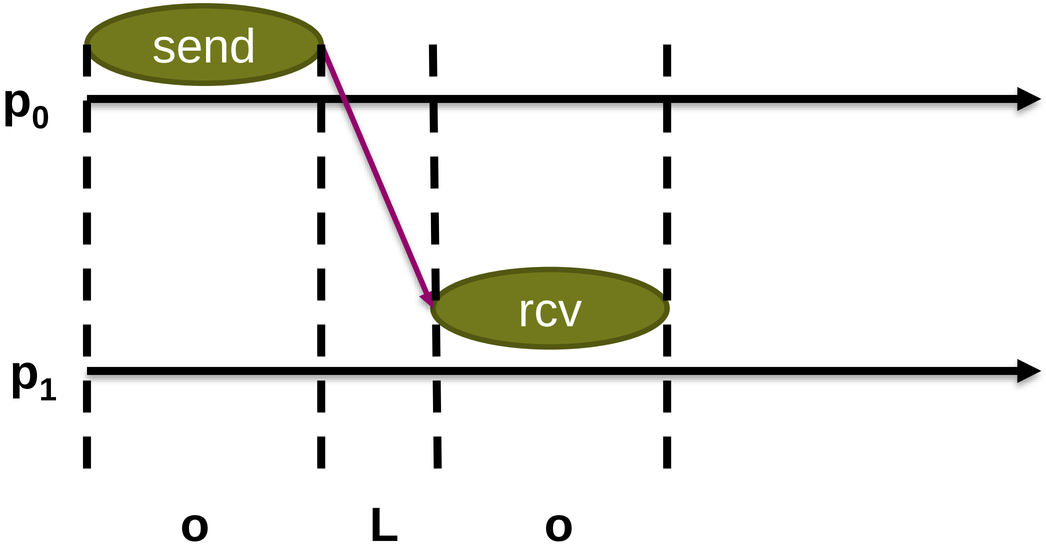
Simple Examples

- **Sending a single message**
  - $T = 2o + L$



- **Ping-Pong Round-Trip**
  - $T = 4o + 2L$
- **Transmitting n messages**
  - $T = L + (n - 1) * \max(g, o) + 2o$





Simplifications

- **o is bigger than g on some machines**
  - g can be ignored (eliminates max() terms)
  - be careful with multicore!
- **Offloading networks might have very low o**
  - Can be ignored (not yet but hopefully soon)
- **L might be ignored for long message streams**
  - If they are pipelined
- **Account g also for the first message**
  - Eliminates “-1”

Benefits

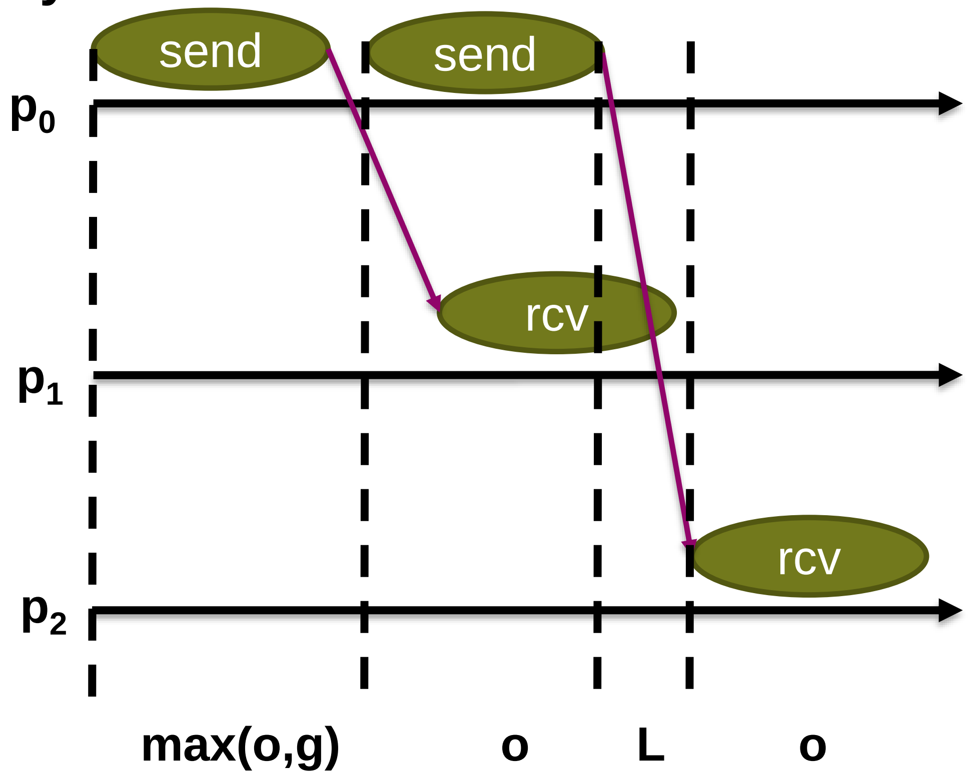
- **Models pipelining**
  - How to model N incoming messages in alfa-beta model?
  - Finite capacity of network L/g messages can be "in flight"
  - Captures state of the art (cf. TCP windows)
- **Models computation/communication overlap**
  - CPU and NIC operations are not necessarily serialized.
  - Asynchronous algorithms
- **Models endpoint congestion/overload**
  - Can the CPU/NIC process a sequence of incoming packages?
  - Benefits balanced algorithms

Example: Broadcast

- **Class Question: What is the LogP running time for a linear broadcast of a single packet?**
  - $T = L + (P - 2) * \max(o, g) + 2o$

- **Class Question: Approximate the LogP runtime for a binary-tree broadcast of a single packet?**
  - $T \leq \log_2 P * (L + \max(o, g) + 2o)$

- **Class Question: Approximate the LogP runtime for an k-ary-tree broadcast of a single packet?**
  - $T \leq \log_k P * (L + (k - 1) * \max(o, g) + 2o)$



Example: Broadcast

- **Class Question: Approximate the LogP runtime for a binomial tree broadcast of a single packet (assume L > g)?**
  - $T \leq \log_2 P * (L + 2o)$

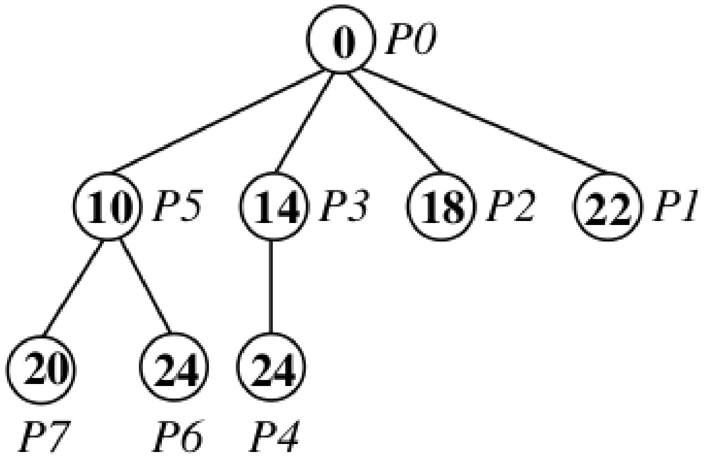
- **Class Question: Approximate the LogP runtime for a k-nomial tree broadcast of a single packet?**
  - $T \leq \log_k P * (L + (k-2) * \max(o, g) + 2o)$

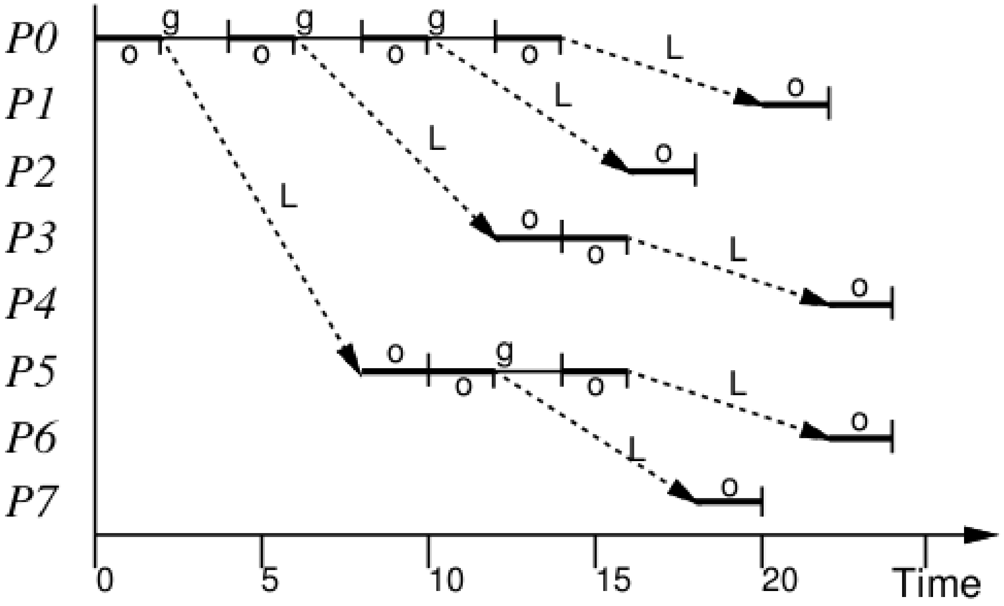
Broadcast: can we do better?

- **Can we do better than k_{opt}-ary binomial broadcast?**
  - Problem: fixed k in all stages might not be optimal
  - We can construct a schedule for the optimal broadcast in practical settings
  - First proposed by Karp et al. in “Optimal Broadcast and Summation in the LogP Model”

Optimal broadcast

- **Broadcast to P-1 processes**
  - Each process who received the value sends it on; each process receives exactly once





Source: Culler et al., "LogP: towards a realistic model of parallel computation."

Optimal broadcast runtime

- **This determines the maximum number of PEs (P(t)) that can be reached in time t**
- **P(t) can be computed with a generalized Fibonacci recurrence (assuming o> g):**

$$P(t) = \begin{cases} 1 : & t < 2o + L \\ P(t-o) + P(t-L-2o) : & \text{otherwise.} \end{cases} \qquad (1)$$

How many processors could receive message sent L + 2o time ago?

- **Which can be bounded by (see [1]):**

$$2^{\left\lfloor \frac{t}{L+2o} \right\rfloor} \leq P(t) \leq 2^{\left\lfloor \frac{t}{o} \right\rfloor}$$

How many processors are processing receive from o time ago?

[1]: Hoefler et al.: “Scalable Communication Protocols for Dynamic Sparse Data Exchange” (Lemma 1)

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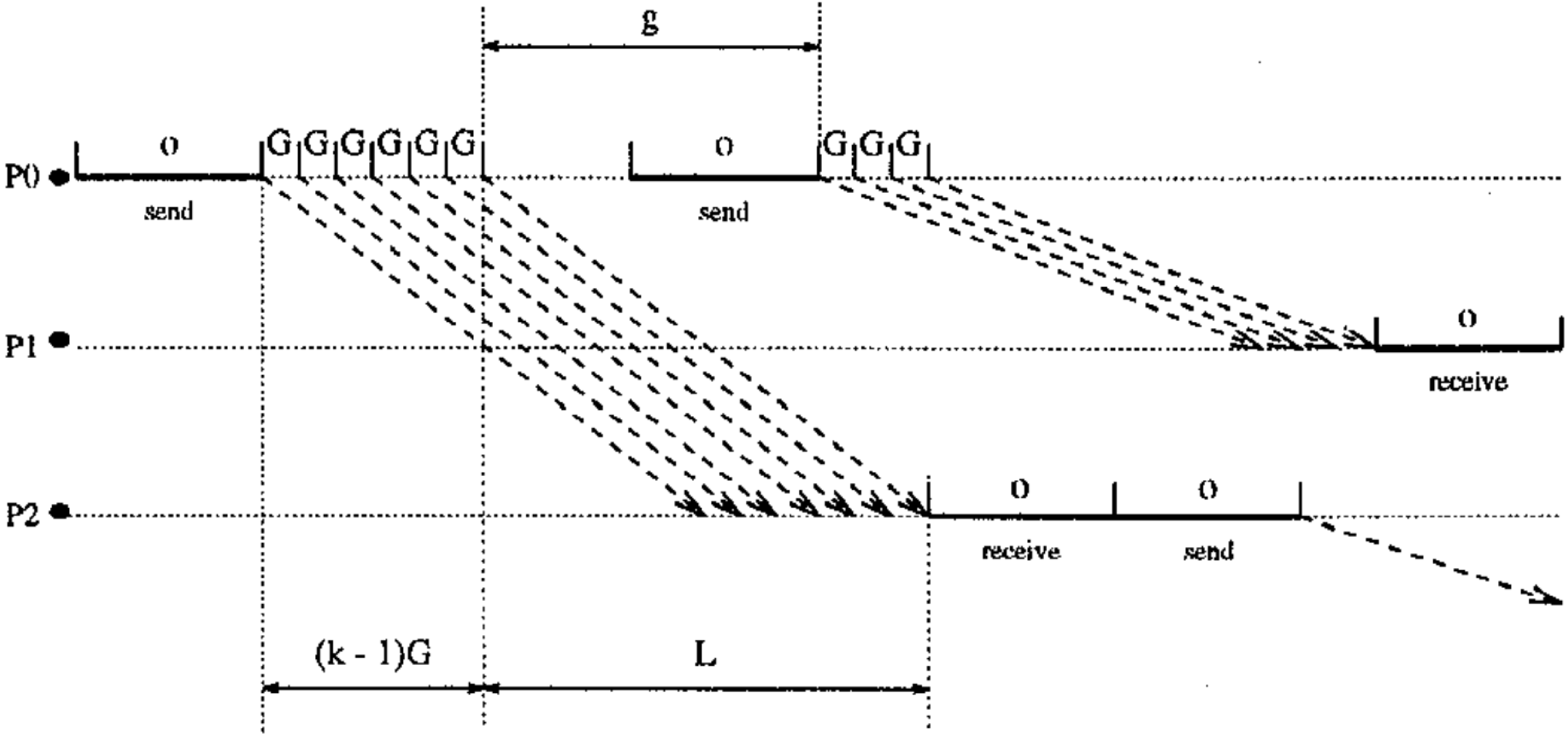
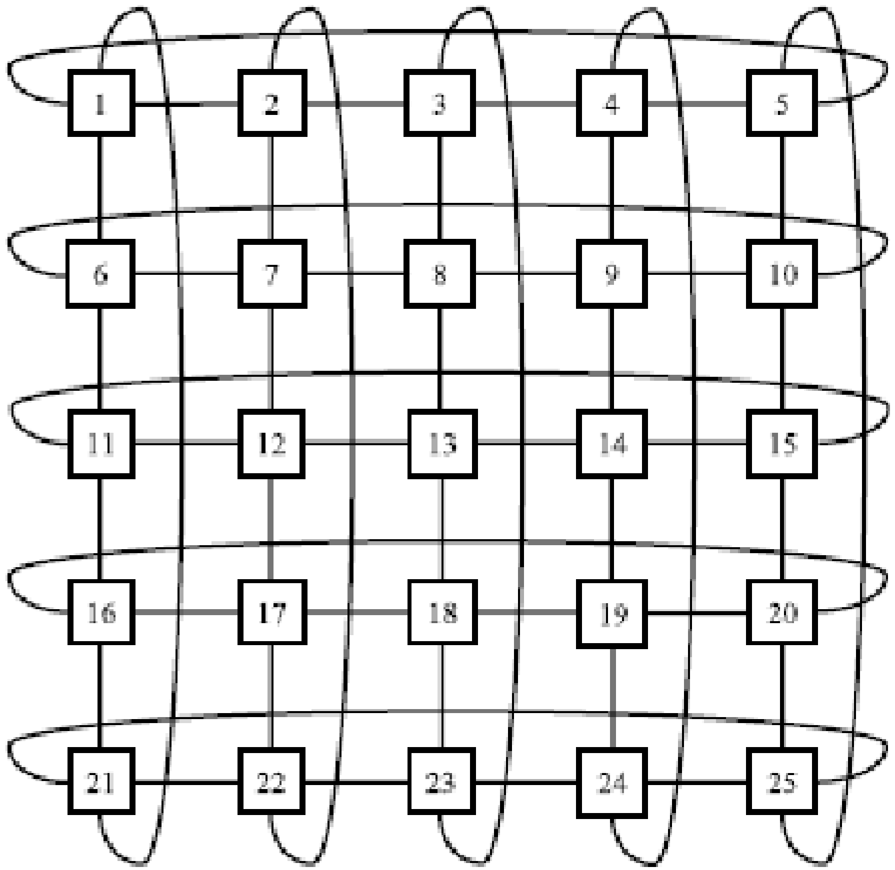
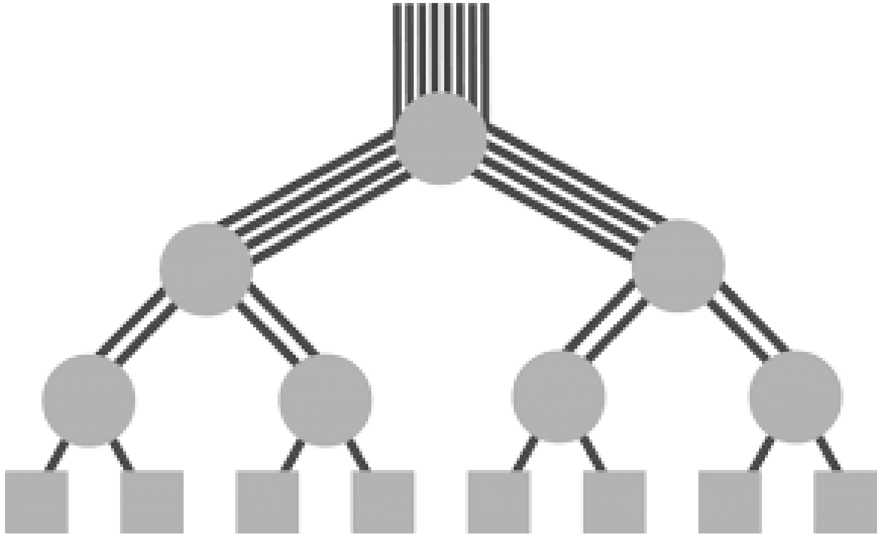


Network topologies



2D Torus



Fat tree

Graph Metrics

- **Diameter**
  - What is the maximum distance between any two nodes?

Topology	Symbol	Example System	Diameter
3-dimensional torus [3]	T3D	Cray Gemini [3]	$\lceil 3/2 \sqrt[3]{N_r} \rceil$
5-dimensional torus [9]	T5D	IBM BlueGene/Q [8]	$\lceil 5/2 \sqrt[5]{N_r} \rceil$
Hypercube [42]	HC	NASA Pleiades [42]	$\lceil \log_2 N_r \rceil$
3-level fat tree [30]	FT-3	Tianhe-2 [15]	4
3-level Flat. Butterfly [27]	FBF-3	-	3
Dragonfly topologies [28]	DF	IBM PERCS [4]	3
Random topologies [29]	DLN	-	3–10
Long Hop topologies [39]	LH-HC	Infinetics Systems [39]	4–6
Slim Fly MMS	**SF**	-	**2**

TABLE II: Topologies compared in the paper, their diameters (§ III-A), and example existing HPC systems that use respective topologies.

Besta et al.: “Slim Fly: A Cost Effective Low-Diameter Network Topology”

Graph Metrics

- **Average distance**

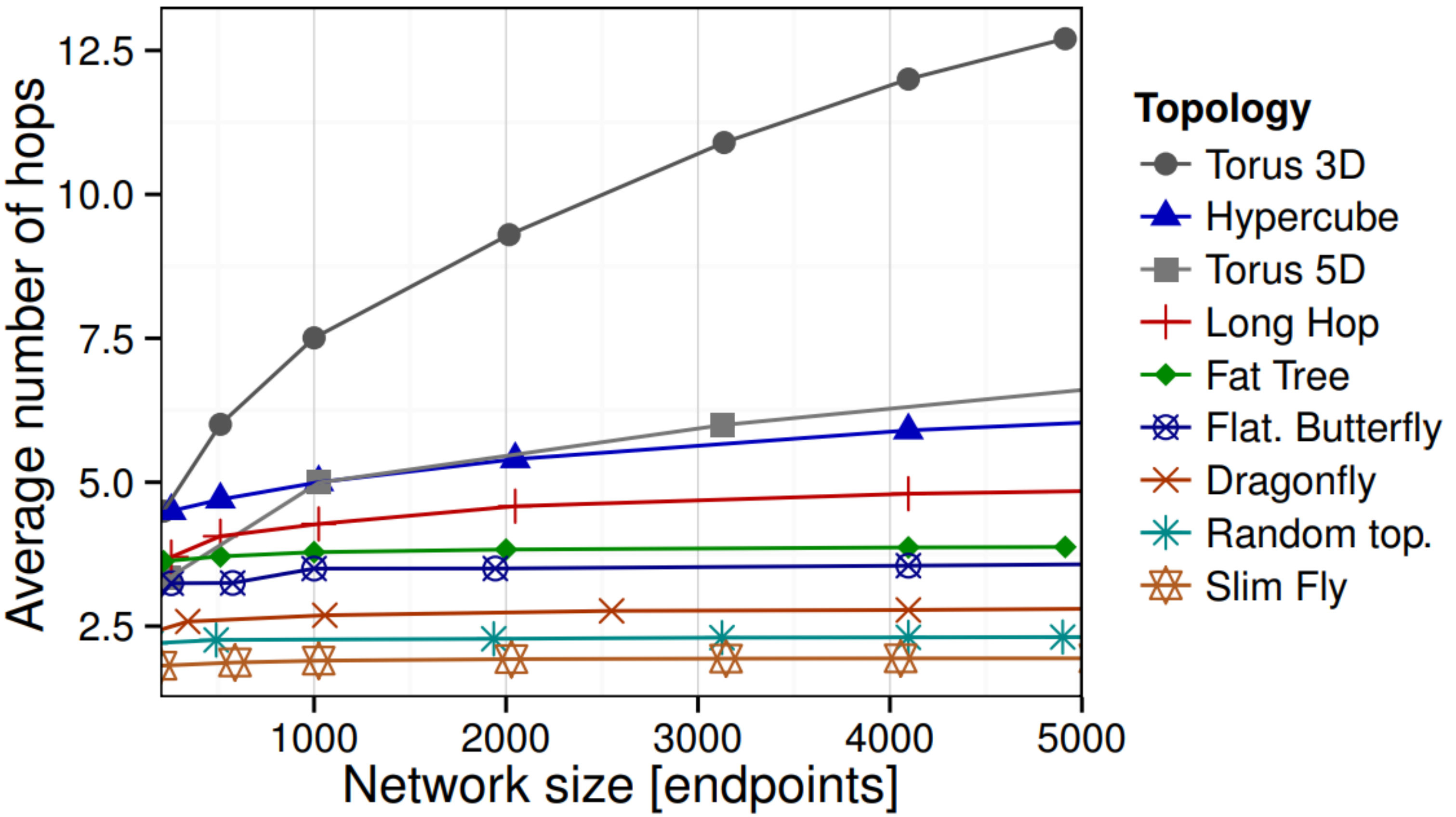
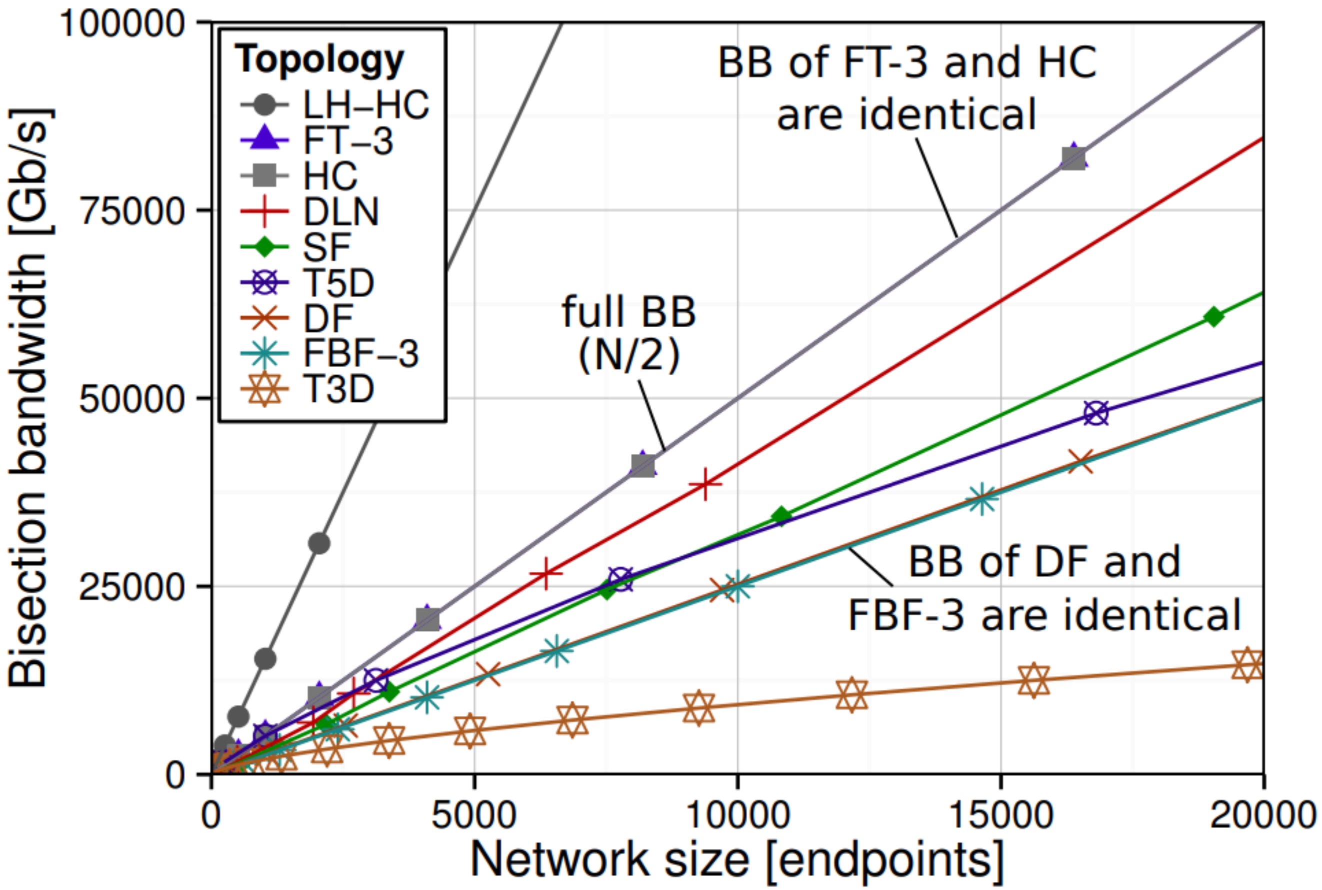


Fig. 1: Comparison of the average number of hops (uniform traffic) in Slim Fly and other networks. Topologies are in balanced or close to balanced configurations (explained in Section III), allowing for highest global bandwidth.[1]

Besta et al.: "Slim Fly: A Cost Effective Low-Diameter Network Topology"

Graph Metrics

- **Bisection bandwidth**
  - If we cut a graph into two partitions, what's the bandwidth between them? Find the minimum!
  - Reveals true bandwidth of the network – potential bottleneck.



(c) Bisection bandwidth (BB) comparison (§ III-C).

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Good luck!

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