MARCIN COPIK <MARCIN.COPIK@INF.ETHZ.CH>

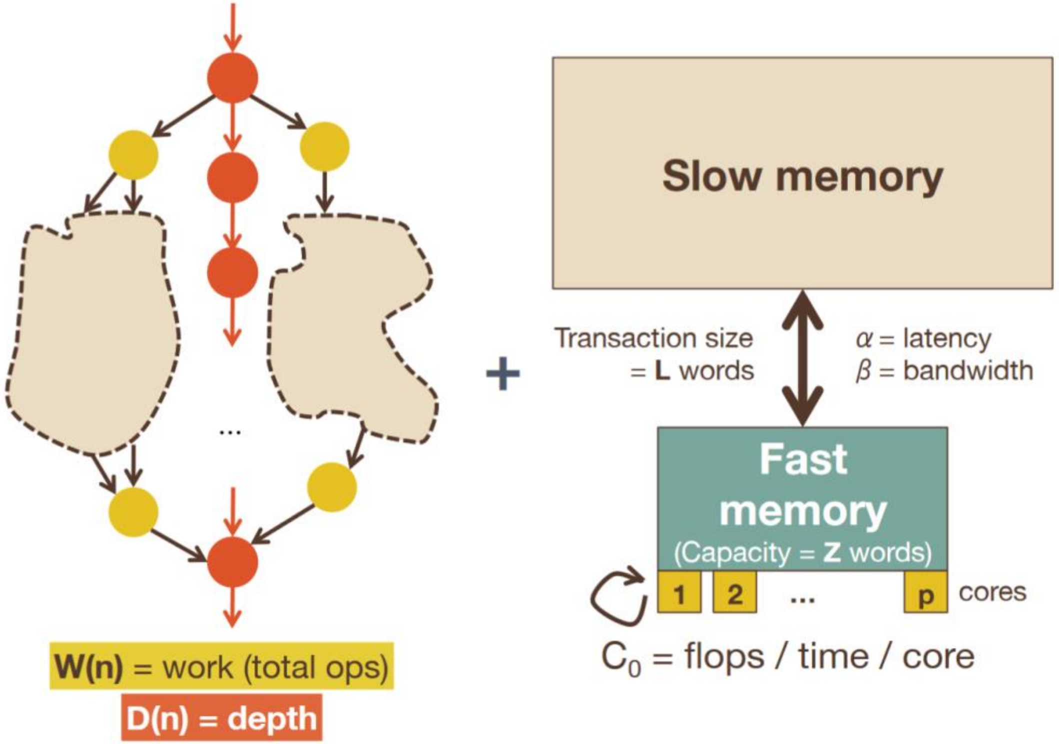
DPHPC: Balance Principle, SIMD

Recitation session, 28.11.2019

Deriving a balance principle

- **Concept of balance:** a computation running on some machine is efficient if the compute-time dominates the I/O time. *[Kung, 1986]*

- **Deriving a balance principle:**
  - Algorithmically analyze the parallelism
  - Algorithmically analyze the I/O behavior (i.e., number of memory transfers)
  - Combine these two analyses with a cost model for an abstract machine

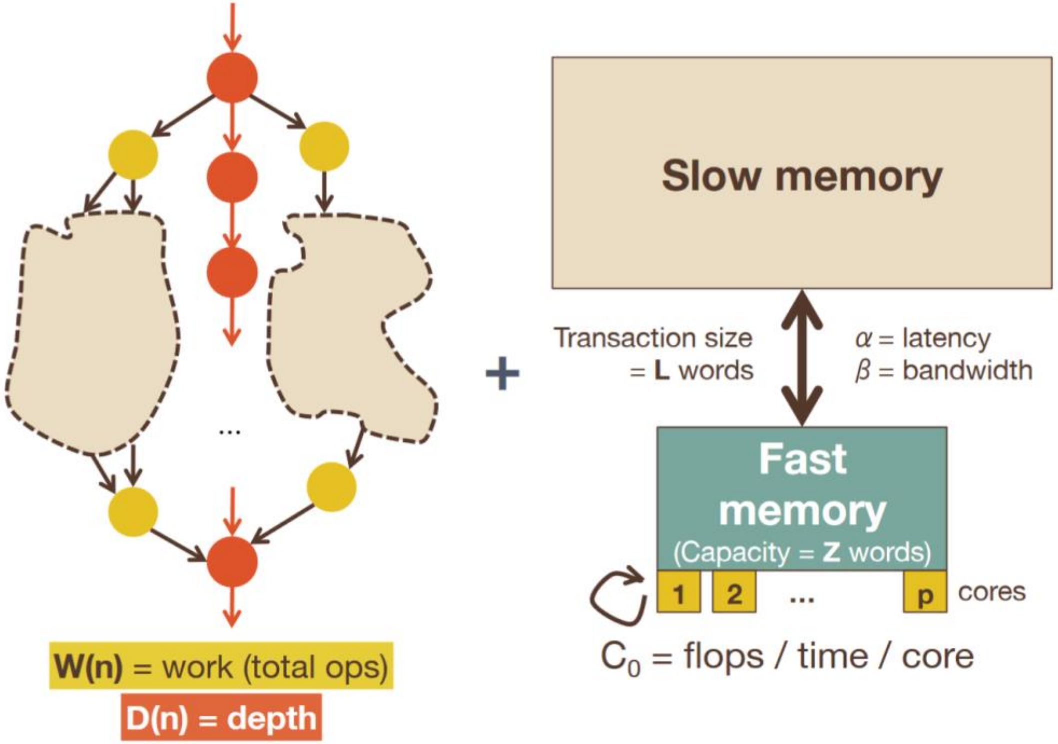


- **Goal: say precisely and analytically how**
  - Changes to the architecture might affect the scaling of a computation
  - Identify what classes of computation might execute efficiently on a given architecture

Czechowski, Kent, et al. "Balance Principles for Algorithm-Architecture Co-Design." HotPar 11 (2011)

Analyzing I/Os

- **We use the classical external memory model**
- **Two level memory**
  - One large&slow
  - The other small&fast (capacity: Z words)
    It can be an automatic cache or
    a software-controlled scratchpad
- **Work operations can be performed only on data in fast memory**
- **Slow <-> Fast memory transfers occur in blocks of L words**
- **$Q_{Z,L}(n)$ is the number of L-sized transfers between slow and fast memory for an input of size n**

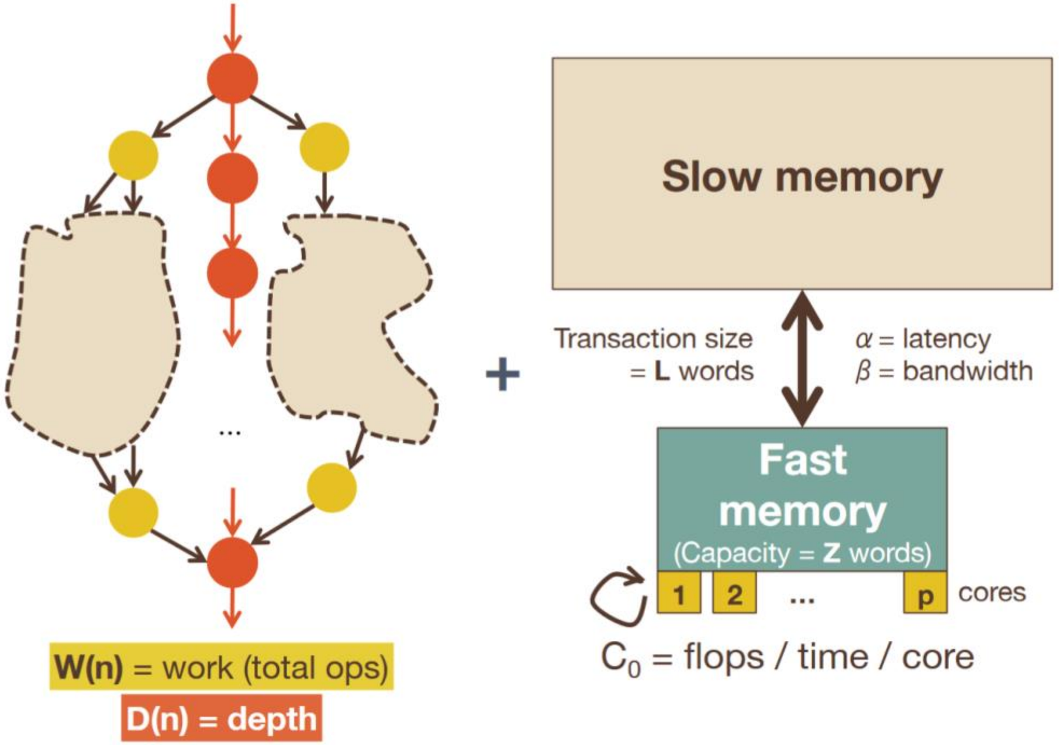


Goal is to optimize the computational intensity: $\frac{W(n)}{Q_{Z,L}(n) \times L}$

Czechowski, Kent, et al. "Balance Principles for Algorithm-Architecture Co-Design." HotPar 11 (2011)

Architecture-Specific Cost Model

- **We need to introduce the time**
  - This depends on the specific architecture
- **p cores**
- **Each core can deliver C_0 operations per time**
- **The time to transfer $m \cdot L$ words is:**
  - $\alpha + m \times L/\beta$
  - α is the latency
  - β is the bandwidth in units of words per time

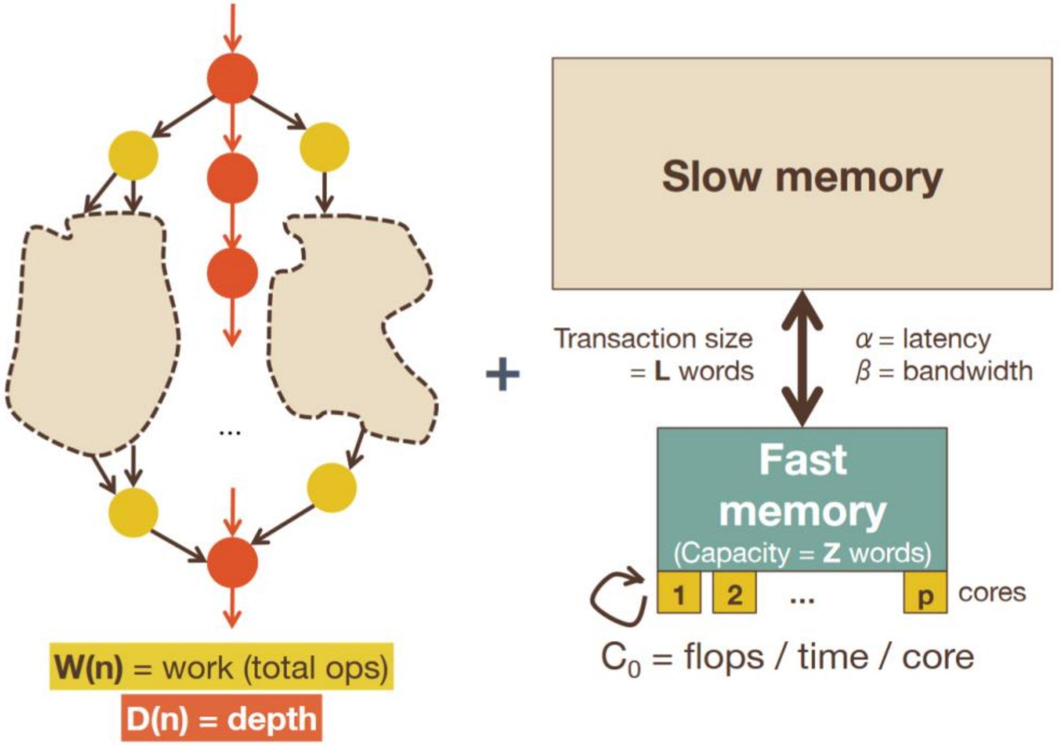


- **The best possible compute time is (Brent's theorem):**

$$T_{comp}(n;p,C_0) = \left(D(n) + \frac{W(n)}{p}\right) \times \frac{1}{C_0}$$

Architecture-Specific Cost Model

- **$Q_{Z,L}(n)$ is for the sequential case**
- **We need to move to the parallel case $Q_{p;Z,L}(n)$**
  - We can bound $Q_{p;Z,L}(n)$ in terms of $Q_{Z,L}(n)$
    Need to select a specific scheduler!
  - Compute it directly
- **Assumptions:**
  - the latency is accounted for each node in the critical path
  - all the $Q_{p;Z,L}(n)$ are aggregated and pipelined by the memory system
    Hence they are delivered at the peak bandwidth



- **We can estimate the memory cost as:**

$$T_{mem}(n; p, Z, L, \alpha, \beta) = \alpha \times D(n) + \frac{Q_{p;Z,L}(n) \times L}{\beta}$$

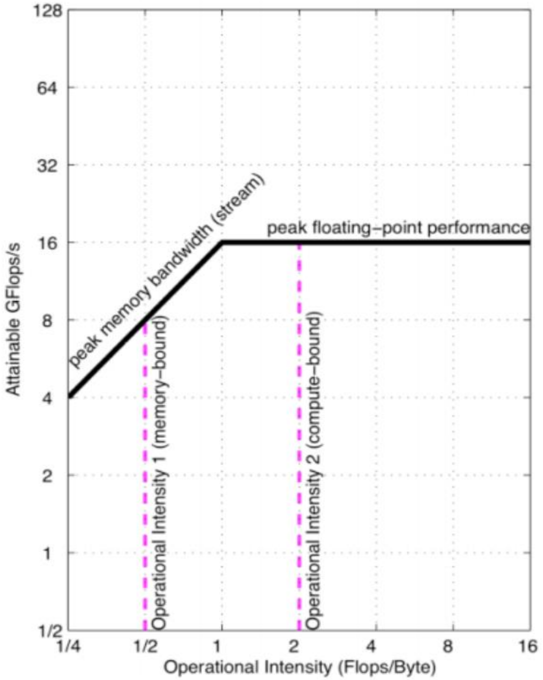
Blelloch et al., „Low depth cache-oblivious algorithms." SPAA 2010

The Balance Principle

- **The balance principle follows by imposing $\boldsymbol{T_{mem} \leq T_{comp}}$**

$$T_{mem}(n; p, Z, L, \alpha, \beta) = \alpha \times D(n) + \frac{Q_{p;Z,L}(n) \times L}{\beta} \quad T_{comp}(n; p, C_0) = \left(D(n) + \frac{W(n)}{p}\right) \times \frac{1}{C_0}$$

$$\underbrace{\frac{pC_0}{\beta}}_{\text{balance}} \left(1 + \underbrace{\frac{\alpha\beta/L}{Q/D}}_{\text{Little's}}\right) \leq \underbrace{\frac{W}{QL}}_{\text{intensity}} \left(1 + \underbrace{\frac{p}{W/D}}_{\text{Amdahl's}}\right)$$



Czechowski, Kent, et al. "Balance Principles for Algorithm-Architecture Co-Design." HotPar 11 (2011)

The Balance Principle: matrix-matrix product

- **Clearly, $\boldsymbol{W(n) > D(n)}$ and**

$$Q_{p;Z,L}(n) \geq \frac{W(n)}{\sqrt{2 \times L \times \sqrt{Z/p}}}$$

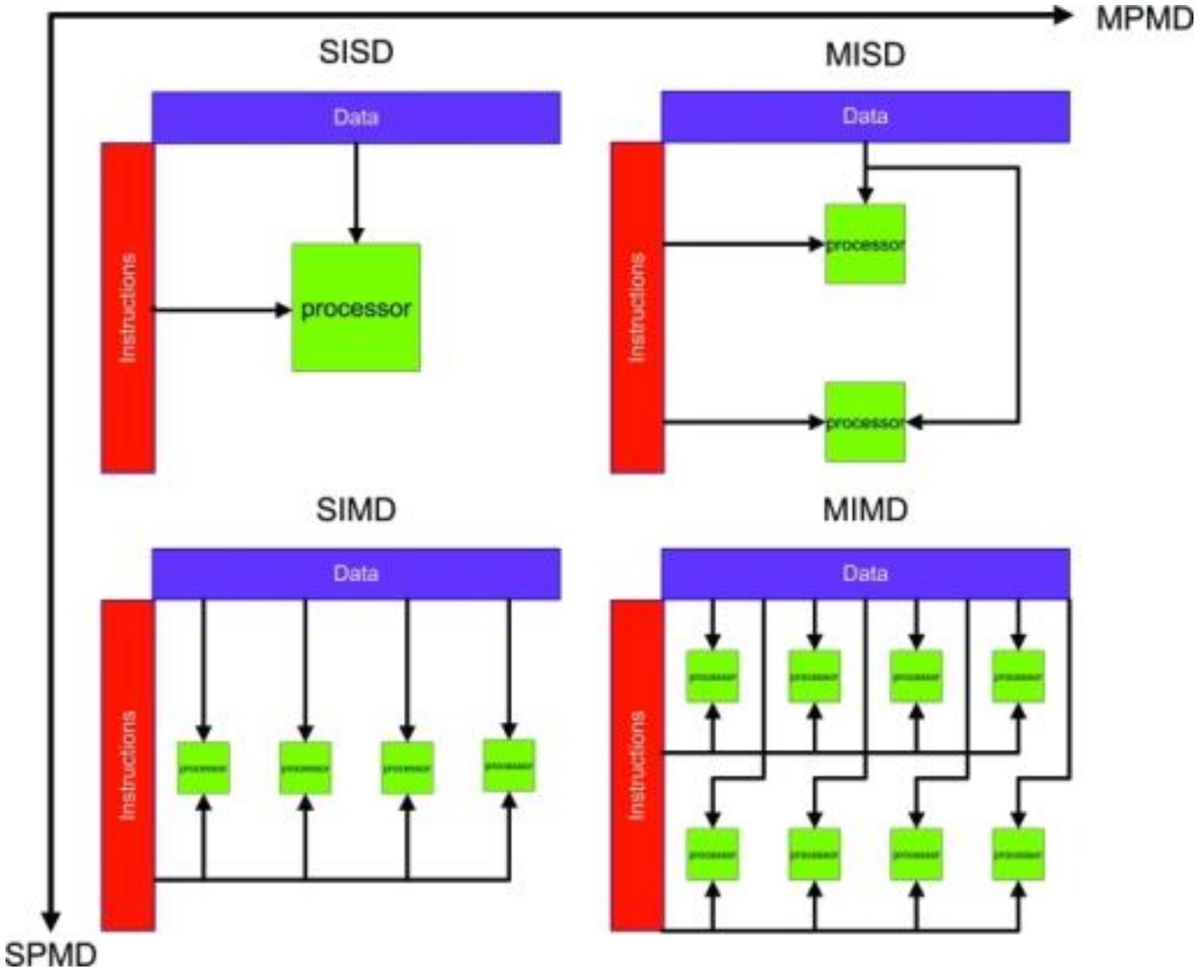
- **We get the following**

$$\frac{p \times C_0}{\beta} \leq \sqrt{\frac{Z}{p}}$$

- **What if we double the number of cores?**

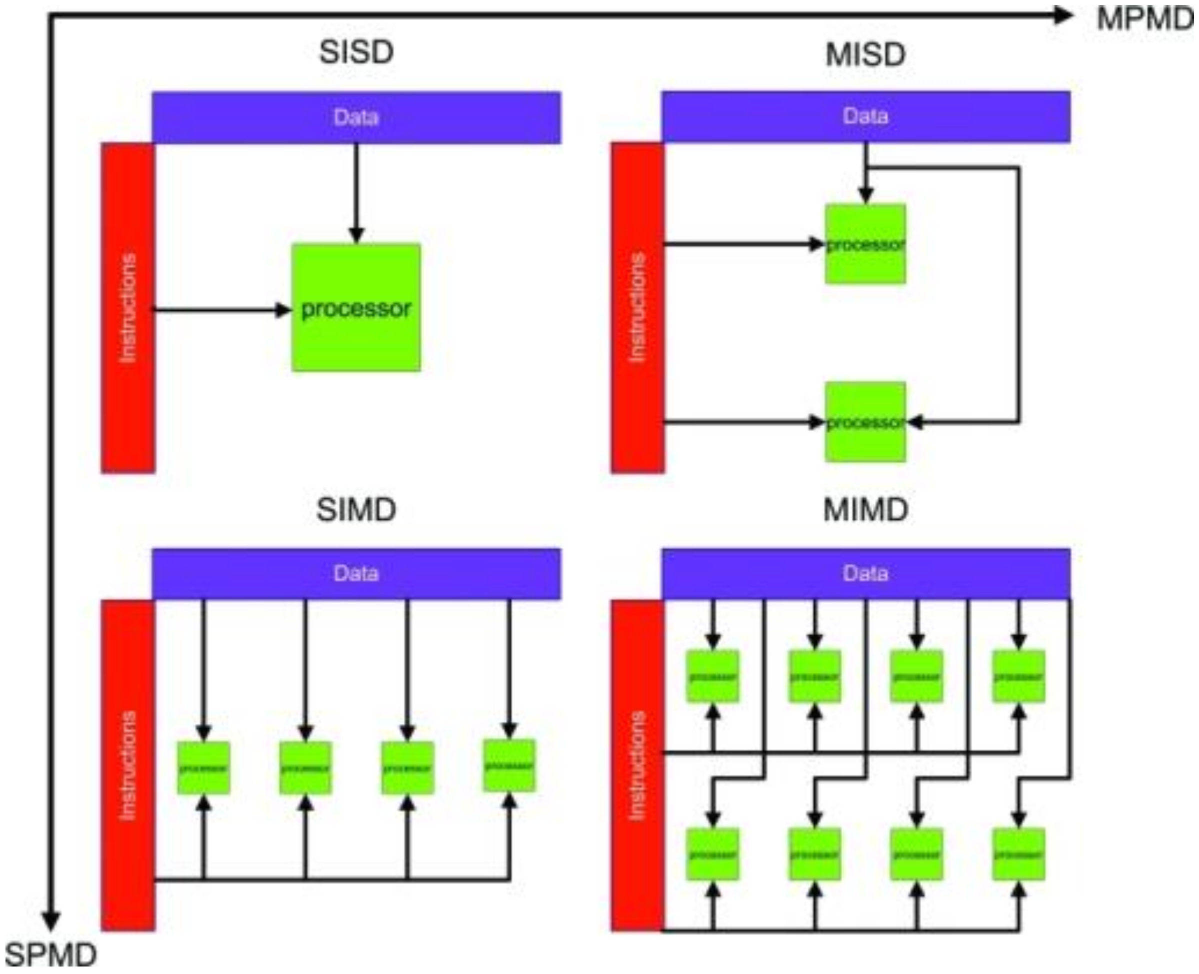
Czechowski, Kent, et al. "Balance Principles for Algorithm-Architecture Co-Design." HotPar 11 (2011)

Flynn's Taxonomy



https://openi.nlm.nih.gov/detailedresult?img=PMC2703572_d-65-00659-fig2&req=4

Flynn’s Taxonomy



Vectors, GPUs

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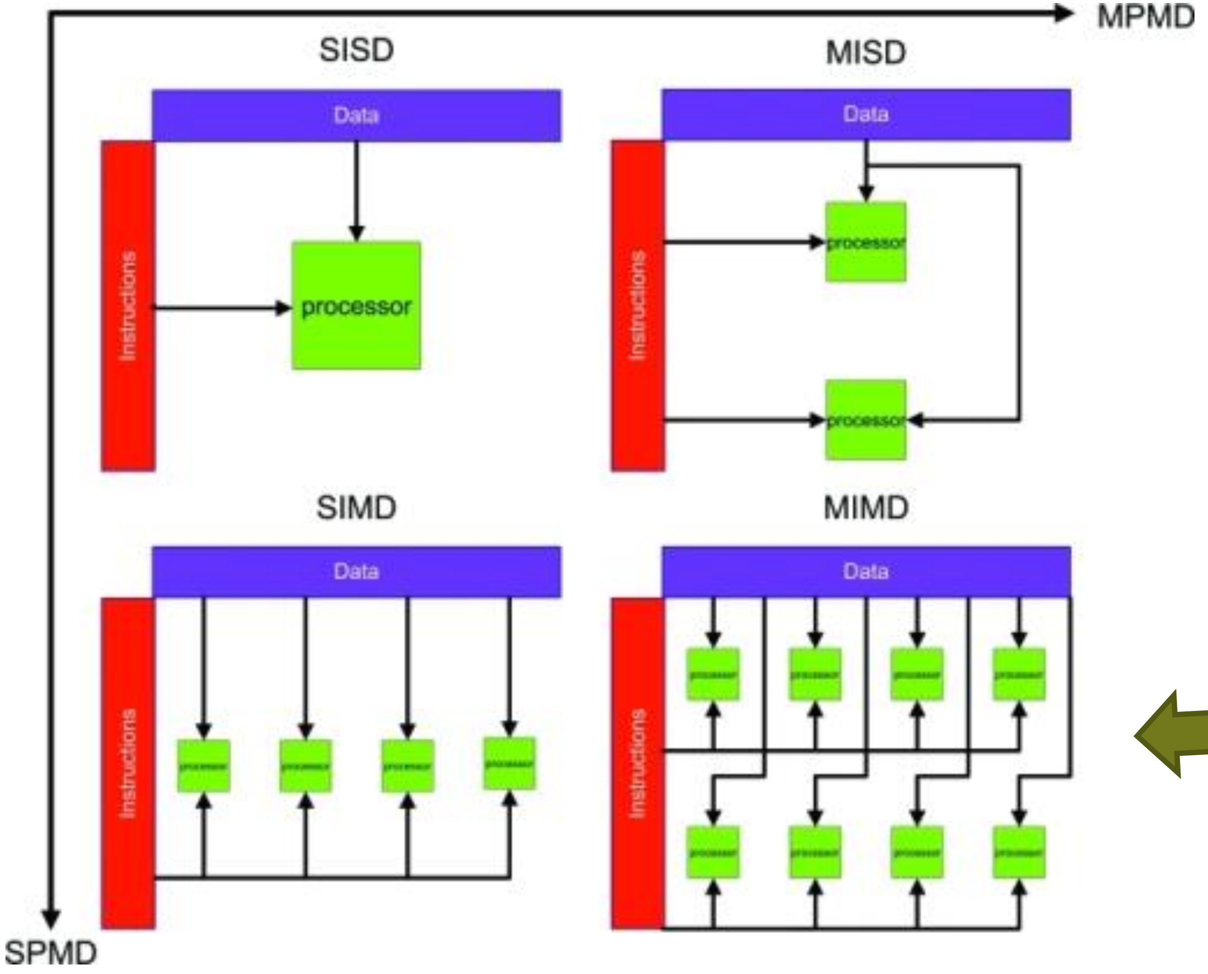

Vectors, GPUs

Multiprocessing

https://openi.nlm.nih.gov/detailedresult?img=PMC2703572_d-65-00659-fig2&req=4

Libraries

```
using Vec3D = std::array<float, 3>;
float scalar_product(Vec3D a, Vec3D b) {
   return a[0] * b[0] + a[1] * b[1] + a[2] * b[2];
}
```

```
using Vec3D = std::array<__m128, 3>;
__m128 scalar_product(Vec3D a, Vec3D b) {
   return _mm_add_ps(
      _mm_add_ps(
         _mm_mul_ps(a[0], b[0]), _mm_mul_ps(a[1], b[1])
      ),
      _mm_mul_ps(a[2], b[2])
   );
}
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    _mm_mul_ps(a[2], b[2])
  );
}
```

```
using Vc::float_v;
using Vec3D = std::array<float_v, 3>;
float_v scalar_product(Vec3D a, Vec3D b) {
  return a[0] * b[0] + a[1] * b[1] + a[2] * b[2];
}
```

Libraries

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using Vec3D = std::array<float, 3>;
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}
```

VcDevel/Vc

Compiler auto-vectorization I

```
#include <vector>
void foo( std::vector<unsigned>& lhs, std::vector<unsigned>& rhs ) {
    for( unsigned i = 0; i < lhs.size(); i++ ) {
        lhs[i] = ( rhs[i] + 1 ) >> 1;
    }
}
```

godbolt.org/z/fTCD_e

Compiler auto-vectorization II - reports

```
#include <vector>
void foo( std::vector<unsigned>& lhs, std::vector<unsigned>& rhs ) {
    for( unsigned i = 0; i < lhs.size(); i++ ) {
        lhs[i] = ( rhs[i] + 1 ) >> 1;
    }
}
```

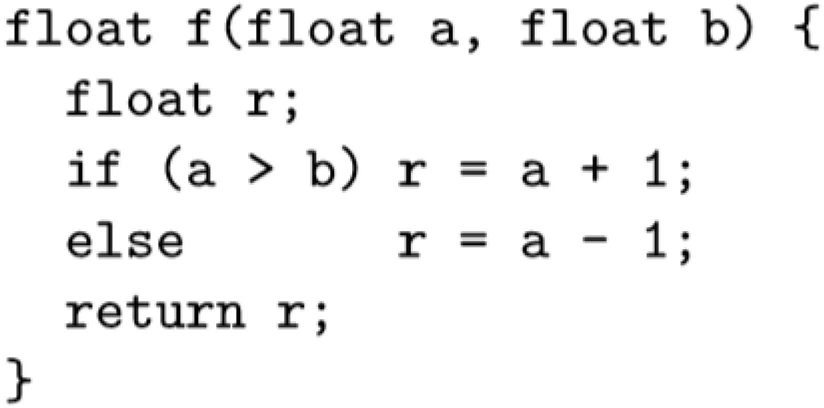
godbolt.org/z/Sk_8Tq

Compiler auto-vectorization III – controlling width

```
#include <vector>
void foo( std::vector<unsigned>& lhs, std::vector<unsigned>& rhs ) {
    for( unsigned i = 0; i < lhs.size(); i++ ) {
        lhs[i] = ( rhs[i] + 1 ) >> 1;
    }
}
```

godbolt.org/z/yap5-_

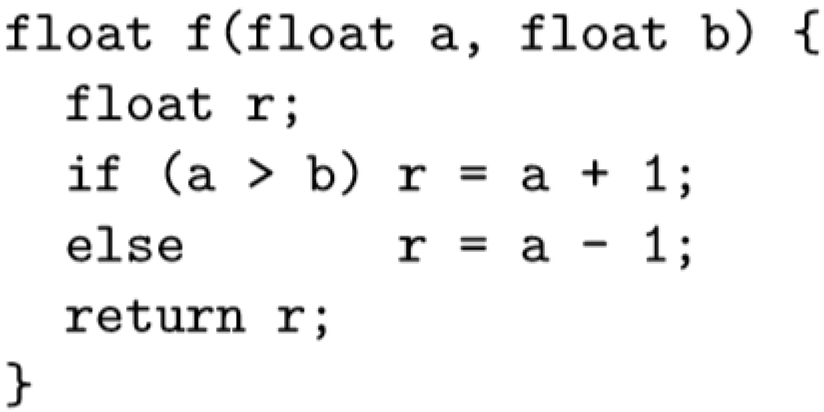
Vectorization with dataflow



```
float f(float a, float b) {
  float r;
  if (a > b) r = a + 1;
  else       r = a - 1;
  return r;
}
```

Karrenberg et al., “Whole-Function Vectorization”

Vectorization with dataflow



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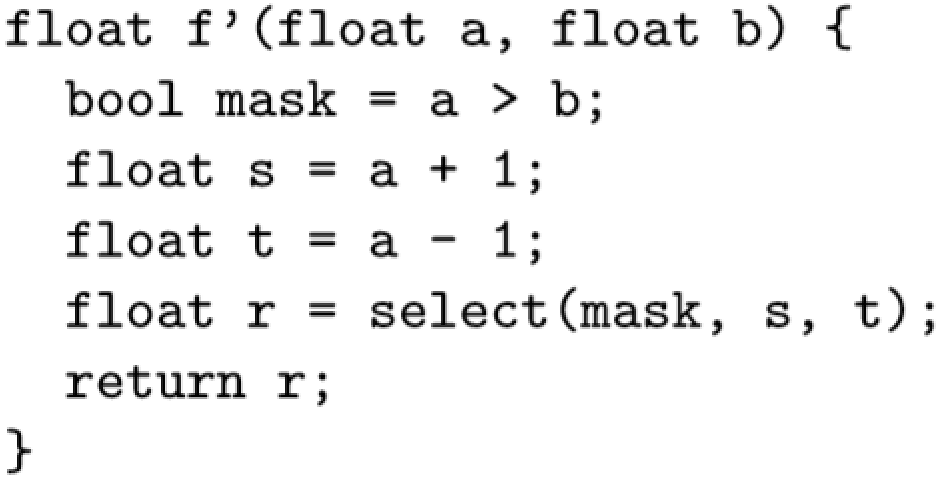
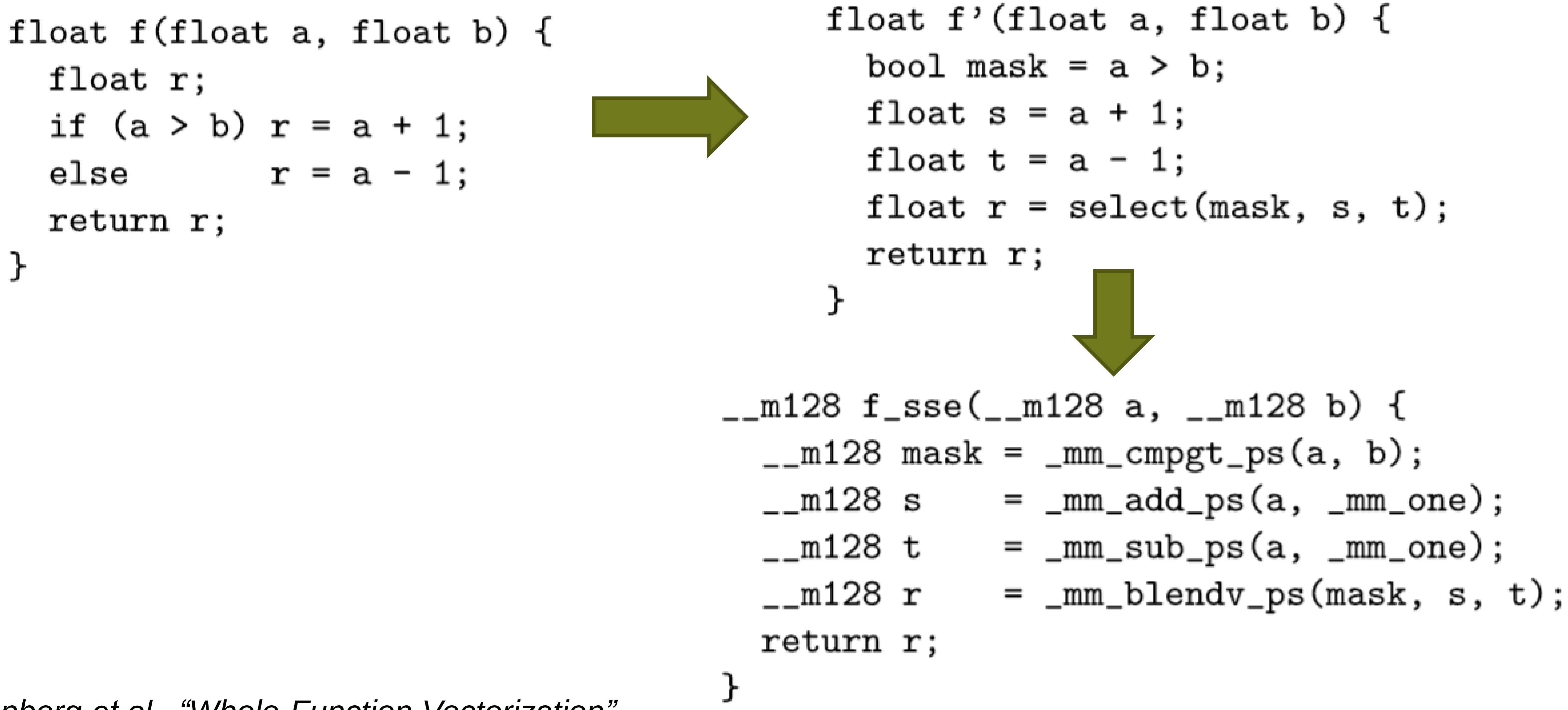


```
float f'(float a, float b) {
  bool mask = a > b;
  float s = a + 1;
  float t = a - 1;
  float r = select(mask, s, t);
  return r;
}
```

Karrenberg et al., “Whole-Function Vectorization”

Vectorization with dataflow



```
float f(float a, float b) {
  float r;
  if (a > b) r = a + 1;
  else       r = a - 1;
  return r;
}
```

```
float f'(float a, float b) {
  bool mask = a > b;
  float s = a + 1;
  float t = a - 1;
  float r = select(mask, s, t);
  return r;
}
```

```
__m128 f_sse(__m128 a, __m128 b) {
  __m128 mask = _mm_cmpgt_ps(a, b);
  __m128 s    = _mm_add_ps(a, _mm_one);
  __m128 t    = _mm_sub_ps(a, _mm_one);
  __m128 r    = _mm_blendv_ps(mask, s, t);
  return r;
}
```

Karrenberg et al., “Whole-Function Vectorization”

Vectorization pitfalls

```
void bar(float *A, float* B, float K, int n) {
    for (int i = 0; i < n; ++i)
        A[i] *= B[i] + K;
}
```

Vectorization pitfalls

```
void bar(float *A, float* B, float K, int n) {
    for (int i = 0; i < n; ++i)
        A[i] *= B[i] + K;
}
```

What if **bar(ptr, ptr + 1, 0.0, 10)**? Runtime check is needed!

https://godbolt.org/z/u2-6Vd

Tips

- Prefer known trip counts (no unpredictable exits)
- Write simple loops (no complex control-flow)
- Branching code? Masking might help.
- Avoid loop-carried dependencies (scan maybe?)
- Use loop index to index arrays and avoid pointer arithmetic
- Prefer consecutive memory access, avoid indirection.
- Check output of compiler and use libraries, flags and directives to help.

Tutorial: **https://easyperf.net/blog/2017/10/24/Vectorization_part1**

Docs: **https://llvm.org/docs/Vectorizers.html**