





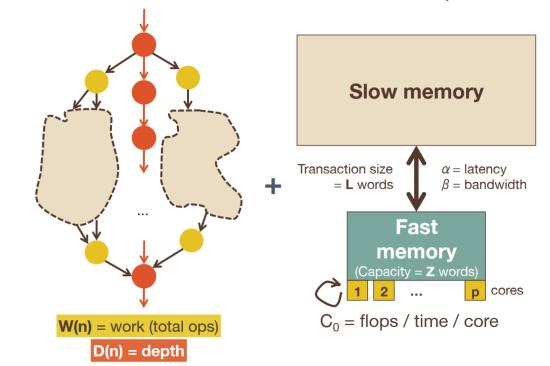
Deriving a Balance Principle

Concept of balance: a computation running on some machine is efficient if the compute-time

dominates the I/O time. [Kung, 1986]

Deriving a balance principle:

- Algorithmically analyze the parallelism
- Algorithmically analyze the I/O behavior (i.e., number of memory transfers)
- Combine these two analyses with a cost model for an abstract machine.



- Goal: say precisely and analytically how
 - Changes to the architecture might affect the scaling of a computation
 - Identify what classes of computation might execute efficiently on a given architecture



- b) Assume a single-core system with an LRU data cache, a peak performance of $\pi = 4$ single precision floating point operations/cycle, and a memory bandwidth of $\beta = 8$ bytes/cycle.
 - What is the ridge point in the roofline point of the above described system? (2pt)

• Consider the following function operating on a matrix A of n^2 floats. A is stored in row-major order. Assume that the cache size γ is much smaller than n ($\gamma \ll n$) and that a cache block has size equal to 8 floats (a float is 8 bytes). No elements of A are initially in cache (i.e., cold cache). What is the operational intensity of the following code? Is it compute or memory bound on this system? Justify your answer. (4pt)

```
void foo(float A[n][n]) {
  for (int j=0; j<n; j++) {
    for (int i=1; i<n; i++) {
        A[0][j] = A[0][j] + A[i][j];
    }
}</pre>
```





c) Assume a program with an operational intensity of $I = \Theta(\sqrt{\gamma})$ that is balanced with respect to a given architecture (single-core). If the peak performance (π) doubles every 2 years and the memory bandwidth (β) doubles every 4 years, with which yearly rate does the cache size need to increase in order to keep the balance? (4pt)







Vectorizing the Vandermonde Matrix Determinant Computation

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_N \\ x_1^2 & x_2^2 & \cdots & x_N^2 \\ & & \cdots & & \\ x_1^{N-1} & x_2^{N-1} & \cdots & x_N^{N-1} \end{bmatrix}$$

$$V_n = \prod_{1 \, \leq \, i \, < \, j \, \leq \, n} \left(x_j - x_i
ight)$$