Design of Parallel and High-Performance Computing

Fall 2016

Lecture: Lock-Free and distributed memory

Motivational video: https://www.youtube.com/watch?v=PuCx50FdSic

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Administrivia

- **Final project presentation: Monday 12/19 (two weeks)**
  - Should have (pretty much) final results
  - Show us how great your project is
  - Some more ideas what to talk about:

    Which architecture(s) did you test on?

    How did you verify correctness of the parallelization?

    Use bounds models for comparisons!

    (Somewhat) realistic use-cases and input sets?

    Emphasize on the key concepts (may relate to theory of lecture)!

    What are remaining issues/limitations?

- **Report will be due in January!**
  - Still, starting to write early is very helpful --- write – rewrite – rewrite (no joke!)
  - Last unit today: Entertainment with bogus results!

Review of last lecture

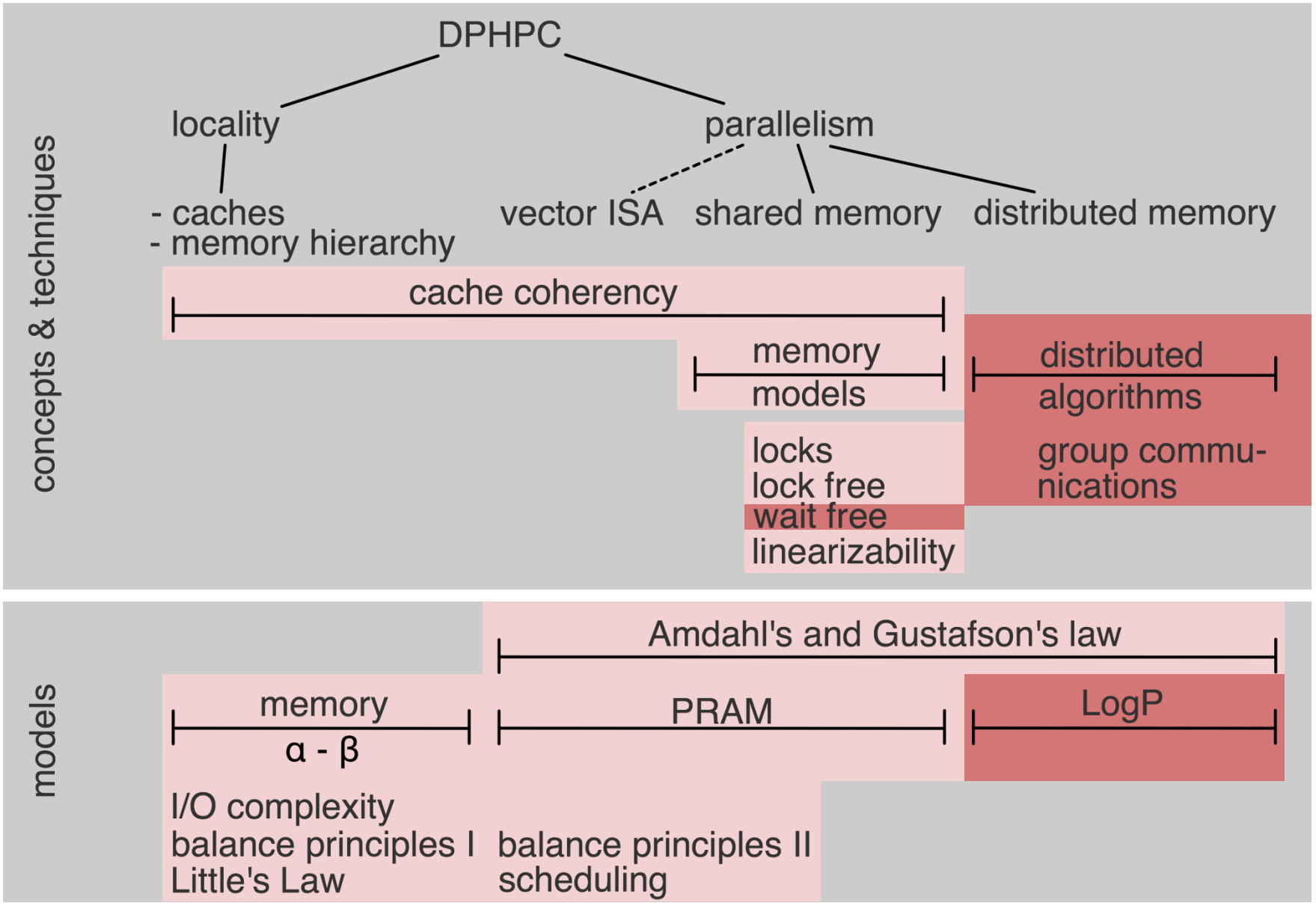
- **Various multi-process locks**
  - Bakery
  - Spinning locks

    Contention issues etc.
  - Queue-based locks

    CLH, MCS

- **MCS – do not forget ☺**
  - RW locks
  - Lock properties/issues (deadlock, priority inversion, blocking vs. spinning)
  - Competitive spinning

DPHPC Overview



Goals of this lecture

- **Locked and Lock-free tricks**
  - (coarse-grained locking)
  - Fine-grained locking
  - RW locking
  - Optimistic synchronization
  - Lazy locking
  - Lock-free (& wait-free)
- **Finish wait-free/lock-free**
  - Consensus hierarchy
  - The promised proof!
- **Maybe: Scientific benchmarking!**
  - Common mistakes!
  - How to improve current practice
  - Important for your project

    Brush up your statistics

Coarse-grained Locking

- **Is the algorithm performing well with many concurrent threads accessing it?**
  - No, access to the whole list is serialized
- **BUT: it's easy to implement and proof correct**
  - Those benefits should **never** be underestimated
  - May be just good enough
  - *"We should forget about small efficiencies, say about 97% of the time:* ***premature optimization is the root of all evil****. Yet we should not pass up our opportunities in that critical 3%. A good programmer will not be lulled into complacency by such reasoning, he will be wise to look carefully at the critical code; but only after that code has been identified"* — Donald Knuth (in *Structured Programming with Goto Statements*)

How to Improve?

- **Will present some "tricks"**
  - Apply to the list example
  - But often generalize to other algorithms
  - Remember the trick, not the example!
- **See them as "concurrent programming patterns" (not literally)**
  - Good toolbox for development of concurrent programs
  - They become successively more complex

Tricks Overview

1. **Fine-grained locking**
   - Split object into “lockable components”
   - Guarantee mutual exclusion for conflicting accesses to same component
2. **Reader/writer locking**
3. **Optimistic synchronization**
4. **Lazy locking**
5. **Lock-free**

Tricks Overview

1. **Fine-grained locking**
2. **Reader/writer locking**
   - Multiple readers hold lock (traversal)
   - contains() only needs read lock
   - Locks may be upgraded during operation

   Must ensure starvation-freedom for writer locks!
3. **Optimistic synchronization**
4. **Lazy locking**
5. **Lock-free**

Tricks Overview

1. **Fine-grained locking**
2. **Reader/writer locking**
3. **Optimistic synchronization**
   - Traverse without locking

     Need to make sure that this is correct!
   - Acquire lock if update necessary

     May need re-start from beginning, tricky
4. **Lazy locking**
5. **Lock-free**

Tricks Overview

1. **Fine-grained locking**
2. **Reader/writer locking**
3. **Optimistic synchronization**
4. **Lazy locking**
   - Postpone hard work to idle periods
   - Mark node deleted

   Delete it physically later
5. **Lock-free**

Tricks Overview

1. **Fine-grained locking**
2. **Reader/writer locking**
3. **Optimistic synchronization**
4. **Lazy locking**
5. **Lock-free**
   - Completely avoid locks
   - Enables wait-freedom
   - Will need atomics (see later why!)
   - Often very complex, sometimes higher overhead

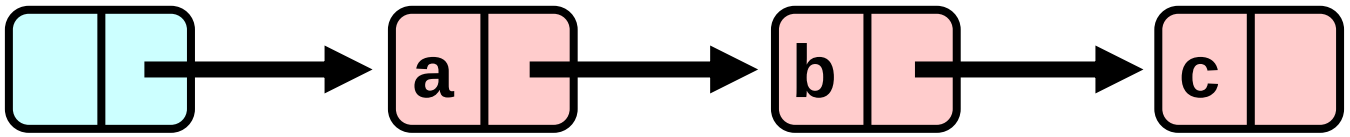
Trick 1: Fine-grained Locking

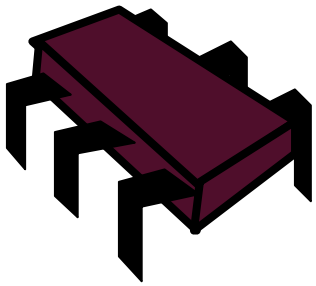
- **Each element can be locked**
  - High memory overhead
  - Threads can traverse list concurrently like a pipeline
- **Tricky to prove correctness**
  - And deadlock-freedom
  - Two-phase locking (acquire, release) often helps
- **Hand-over-hand (coupled locking)**
  - Not safe to release x's lock before acquiring x.next's lock

    will see why in a minute
  - Important to acquire locks in the same order

```
typedef struct {
  int key;
  node *next;
  lock_t lock;
} node;
```

Hand-over-Hand (fine-grained) locking





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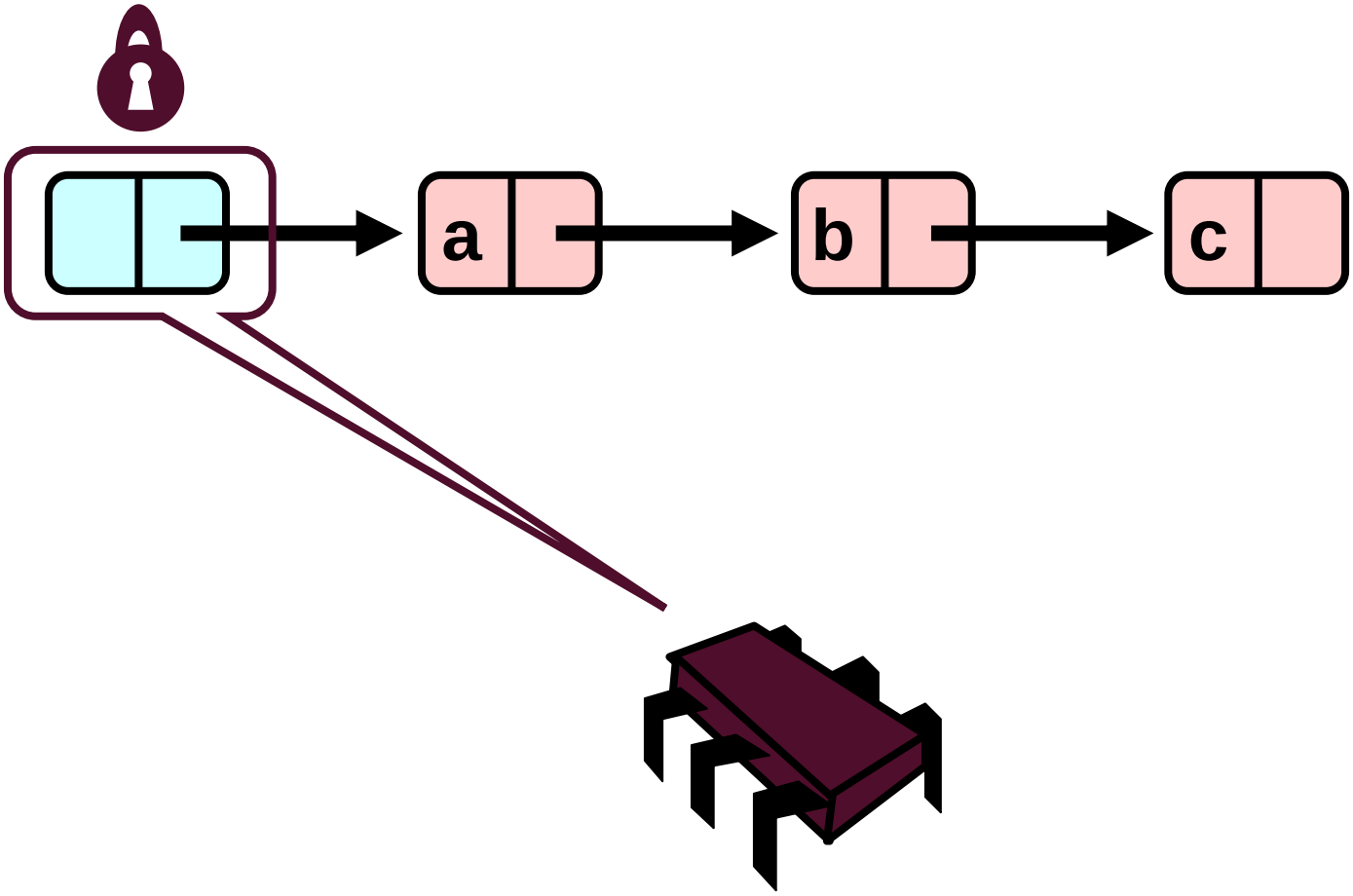

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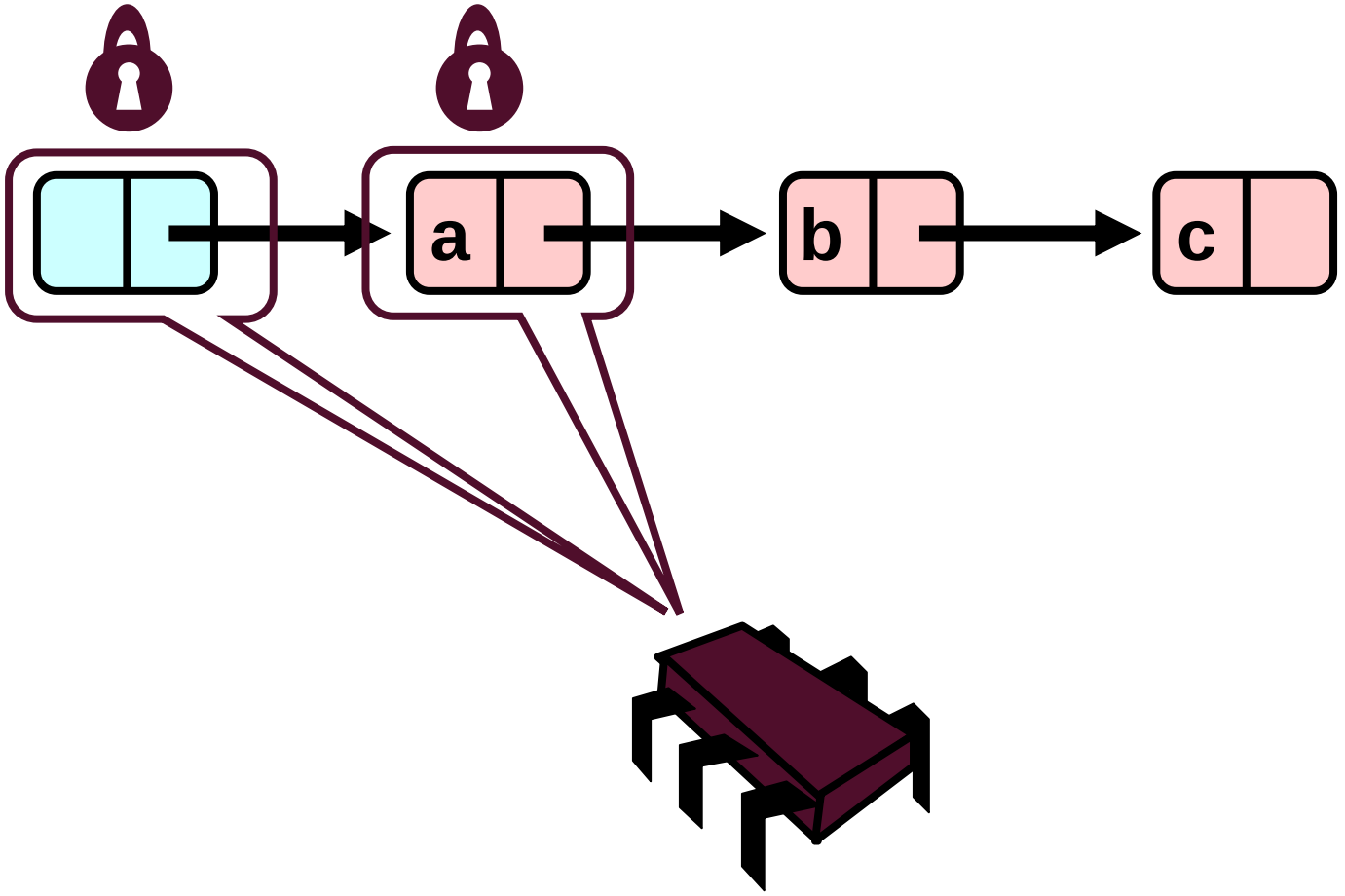

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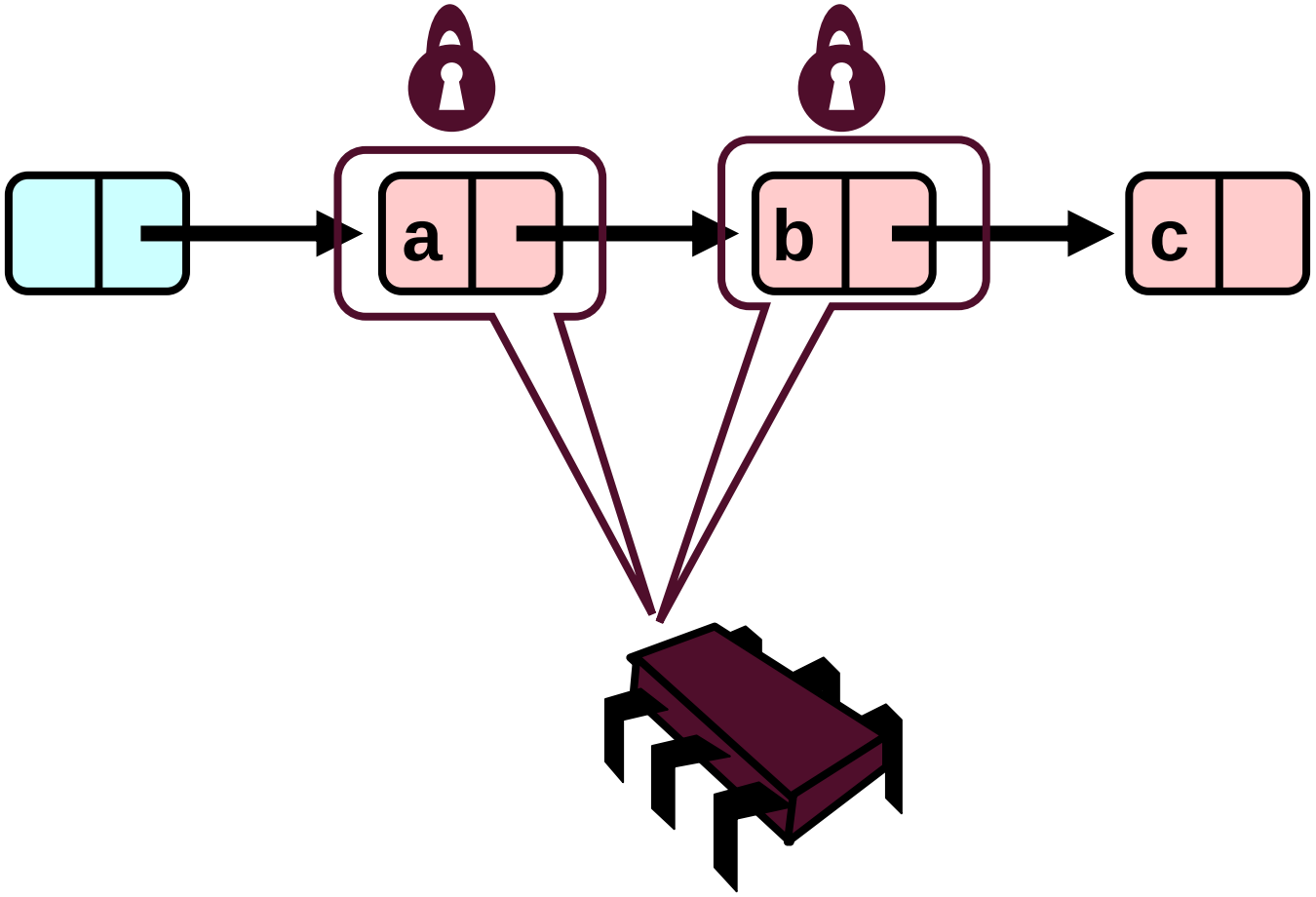

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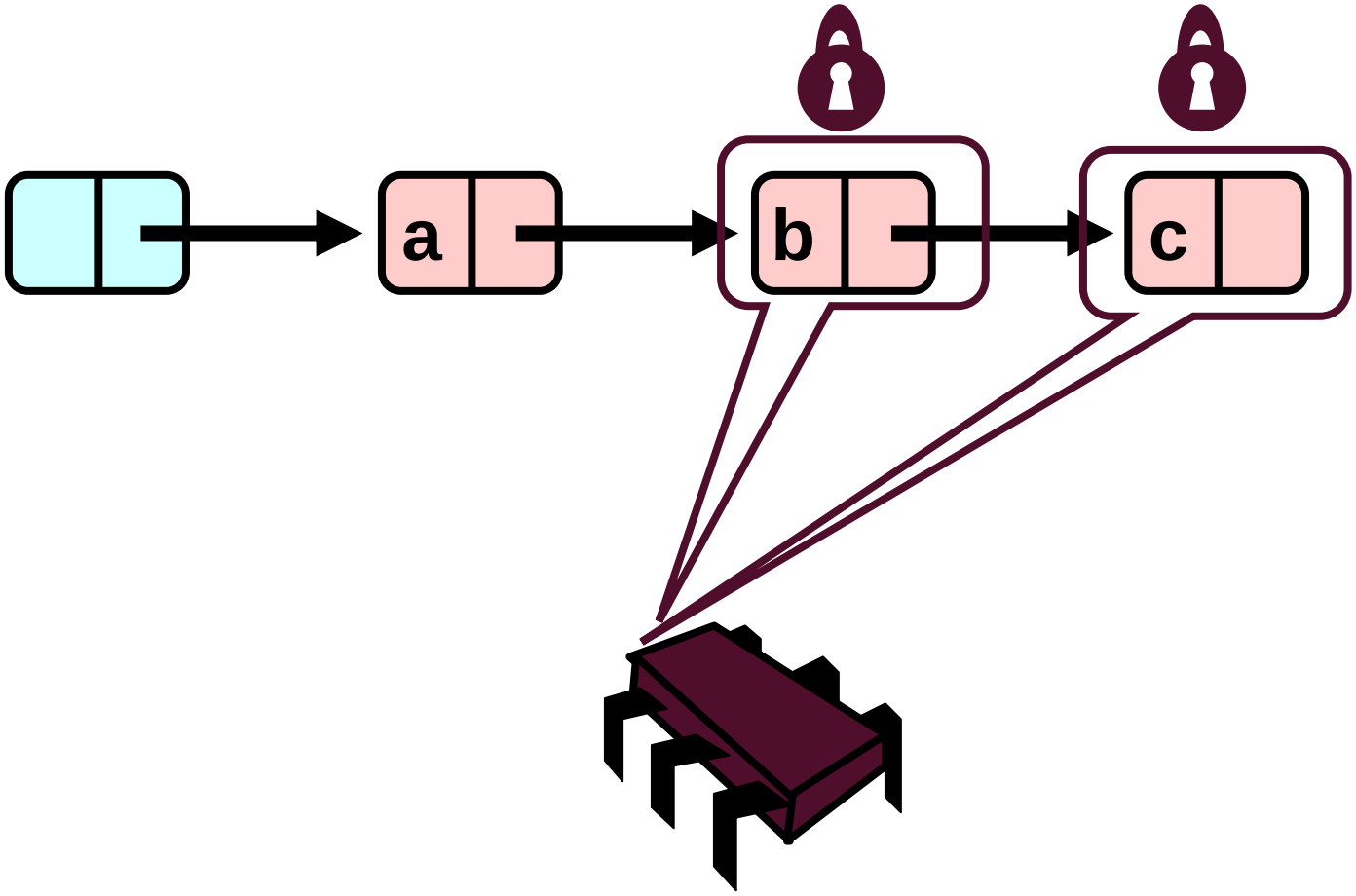
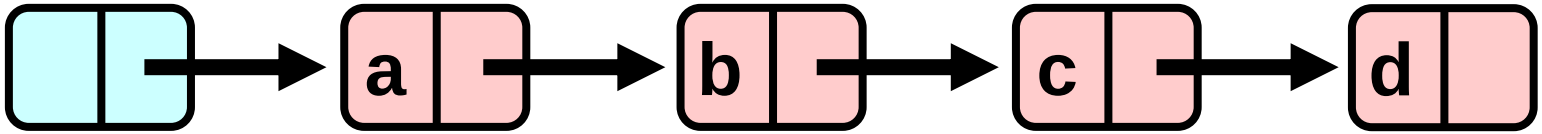
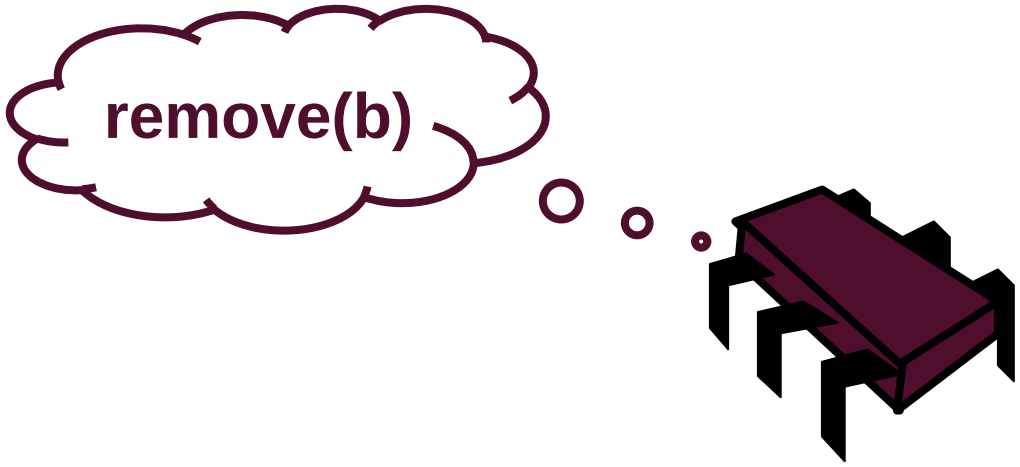


Removing a Node





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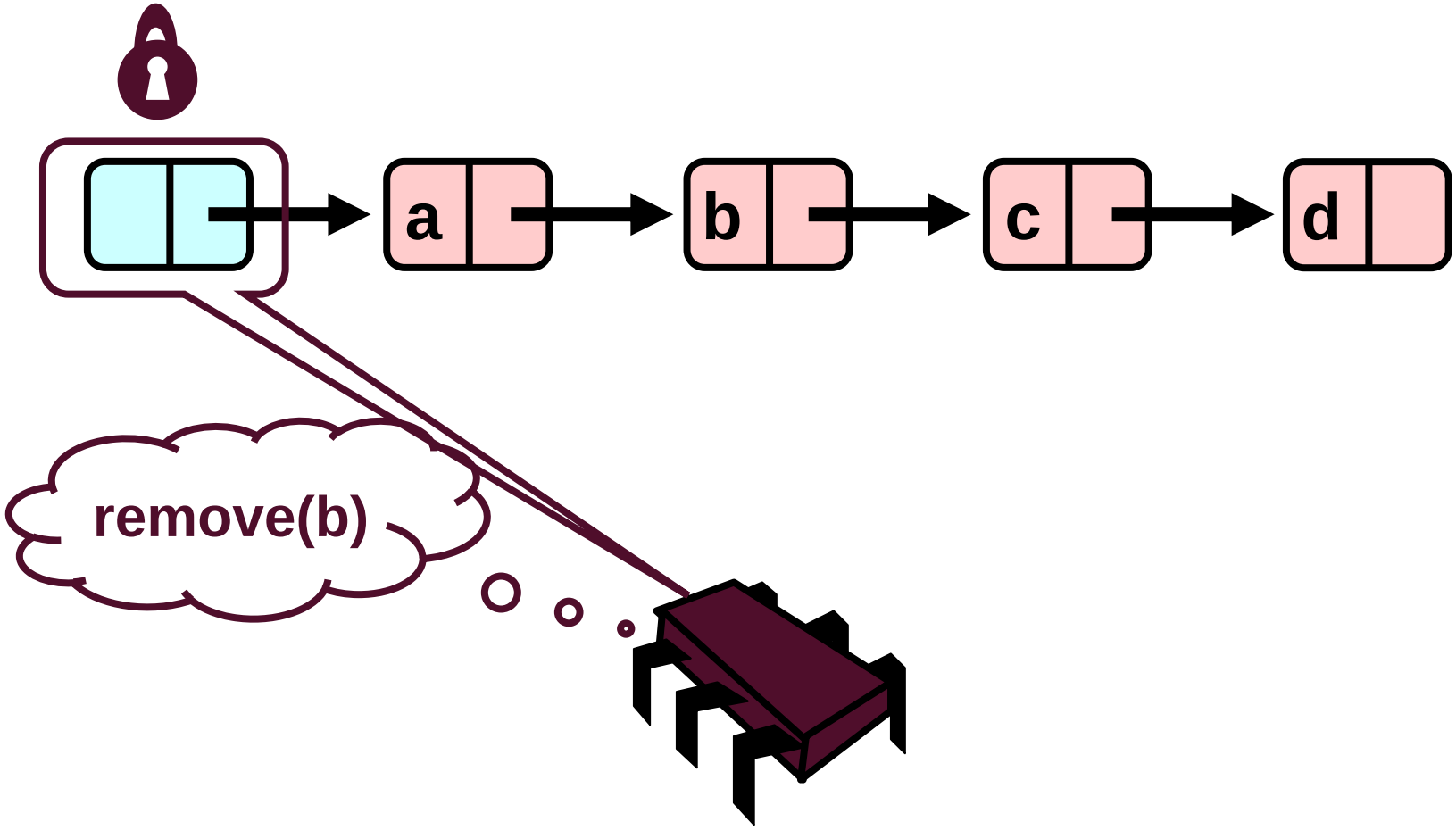

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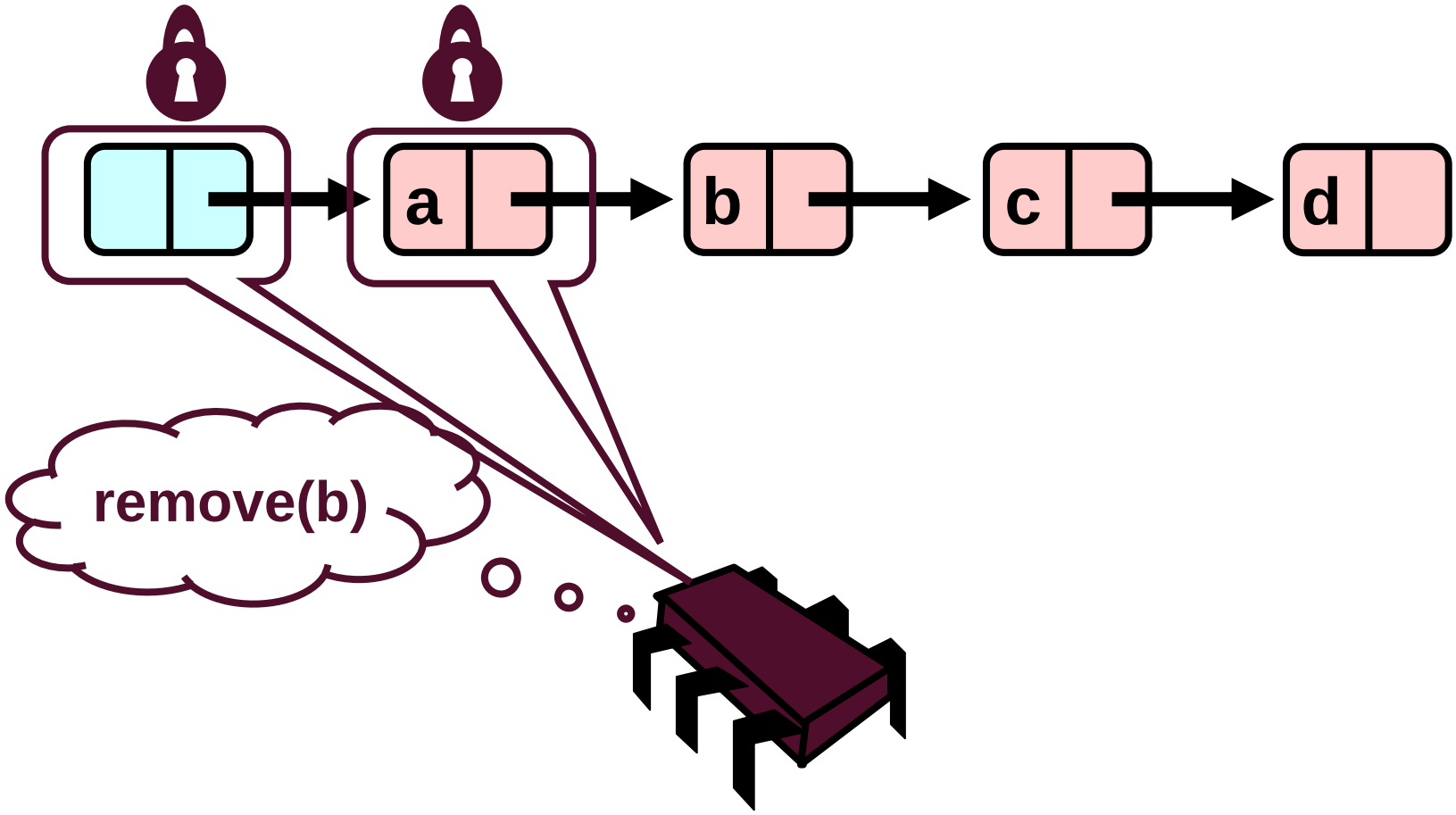

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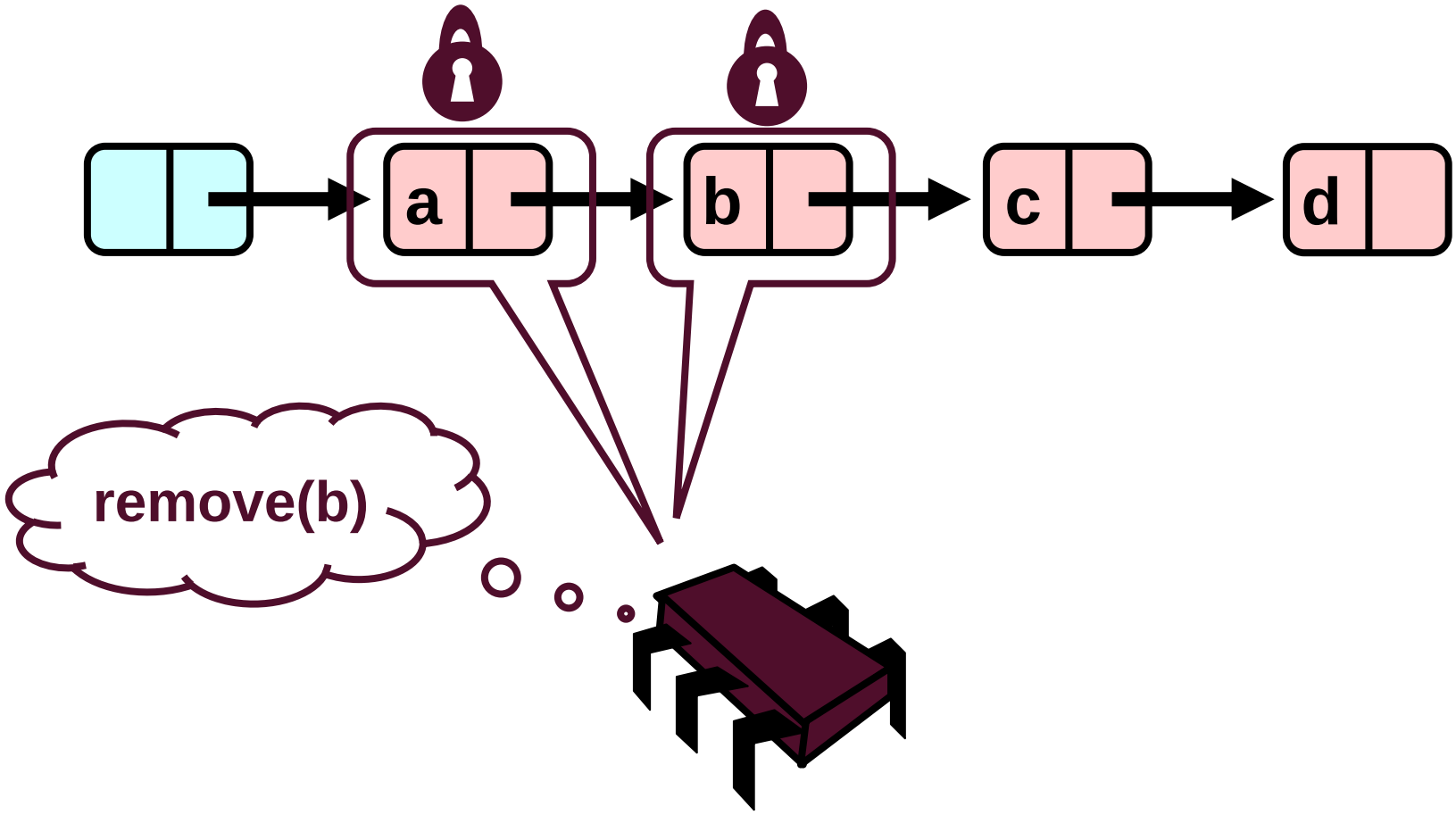

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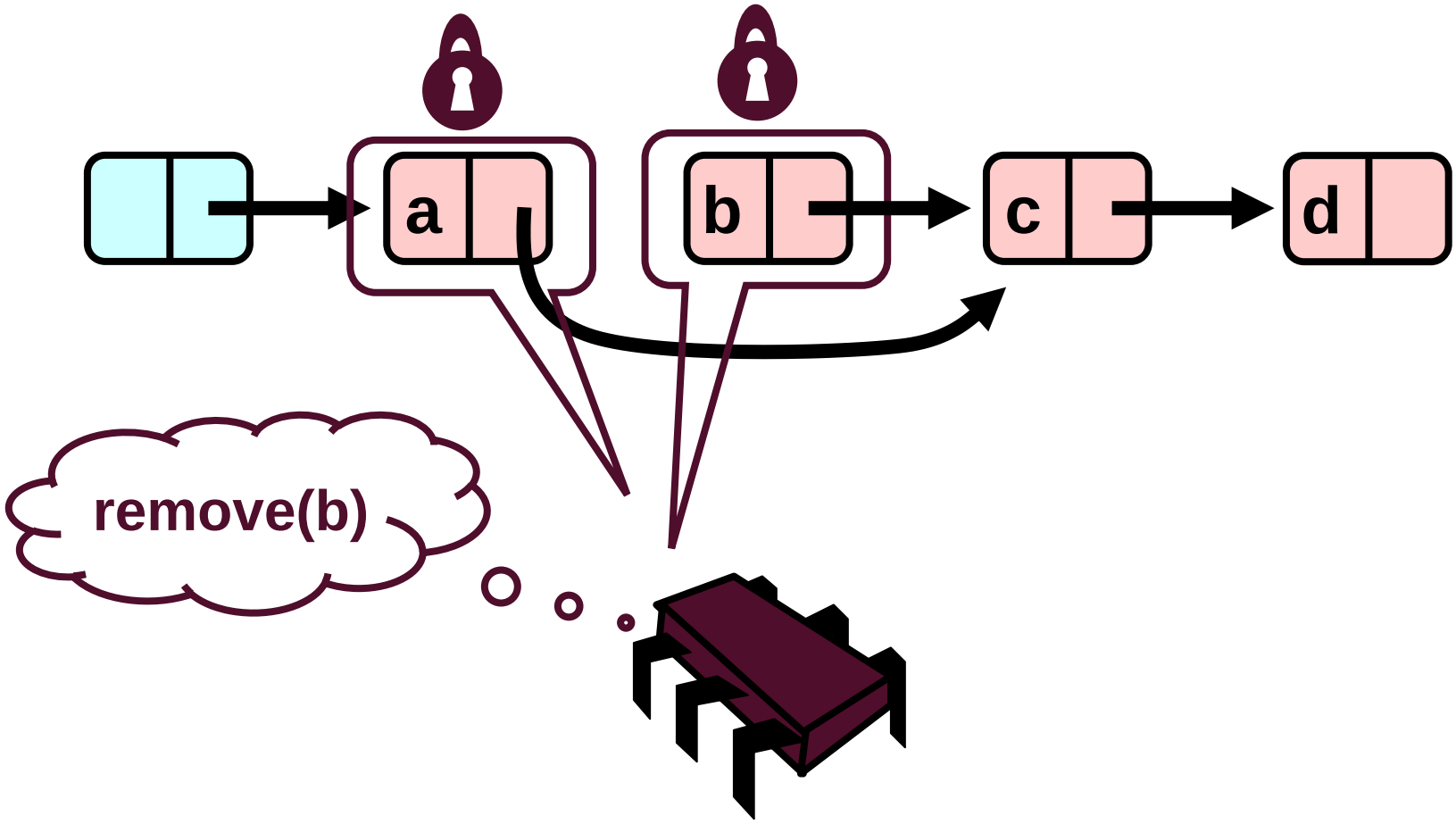

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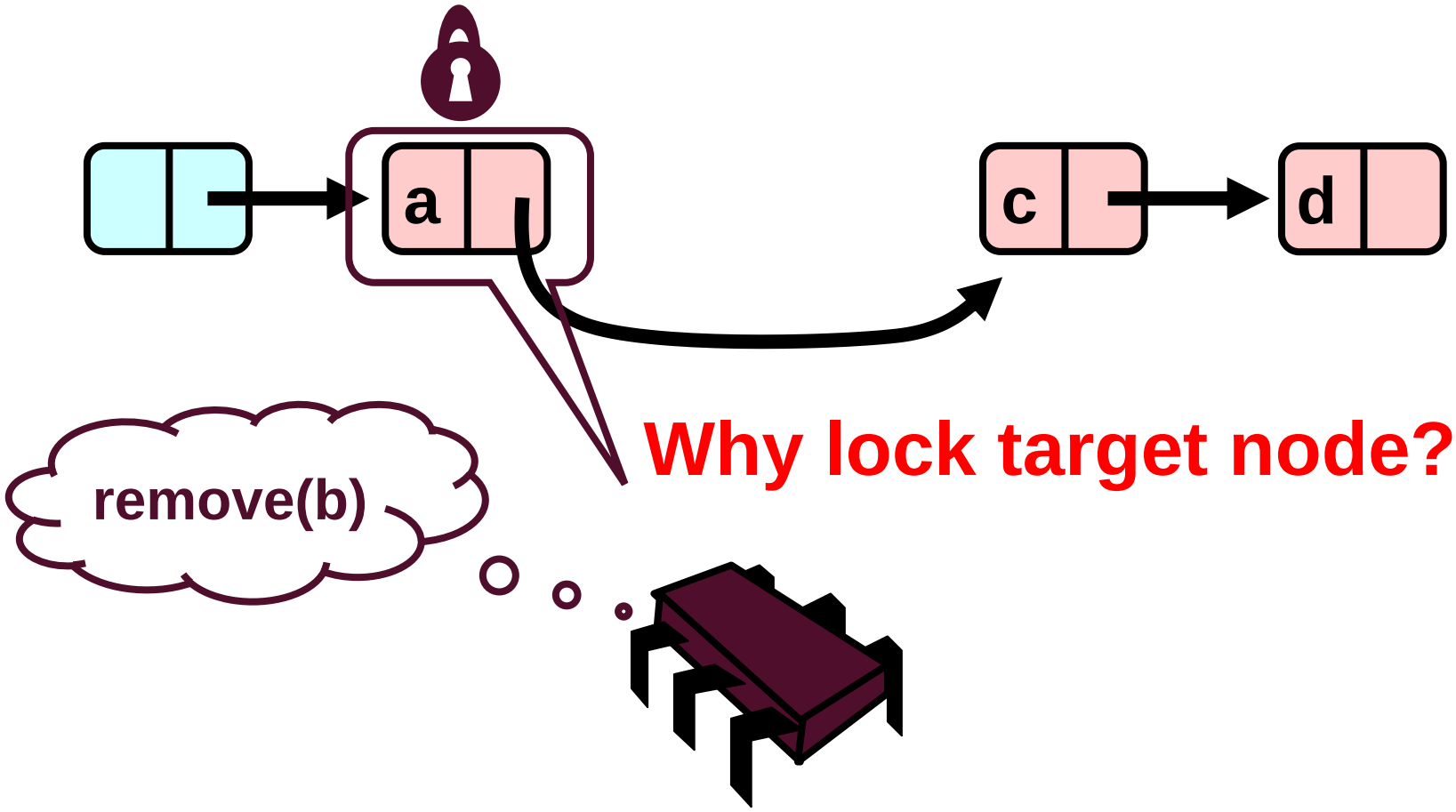
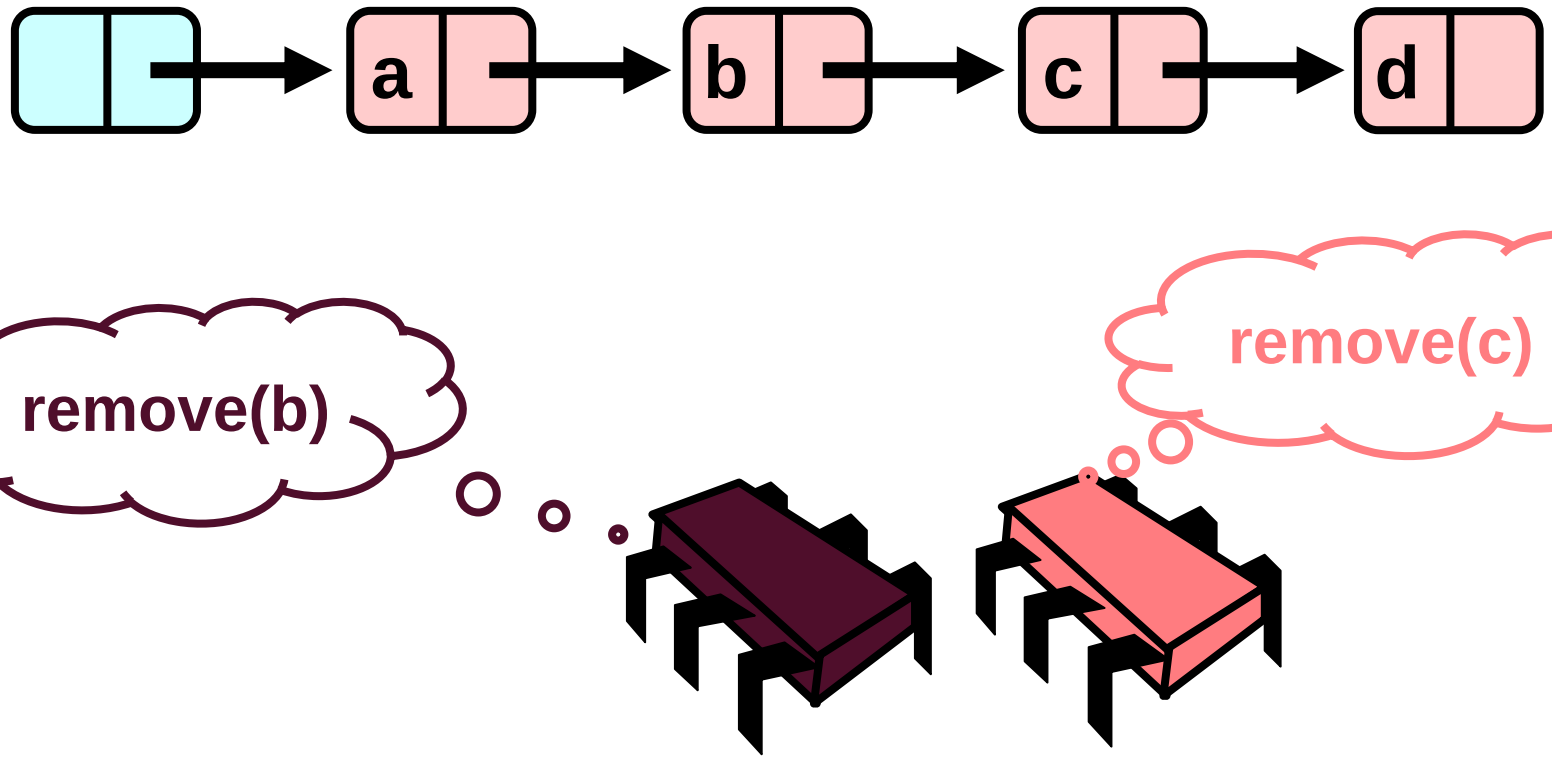


Concurrent Removes



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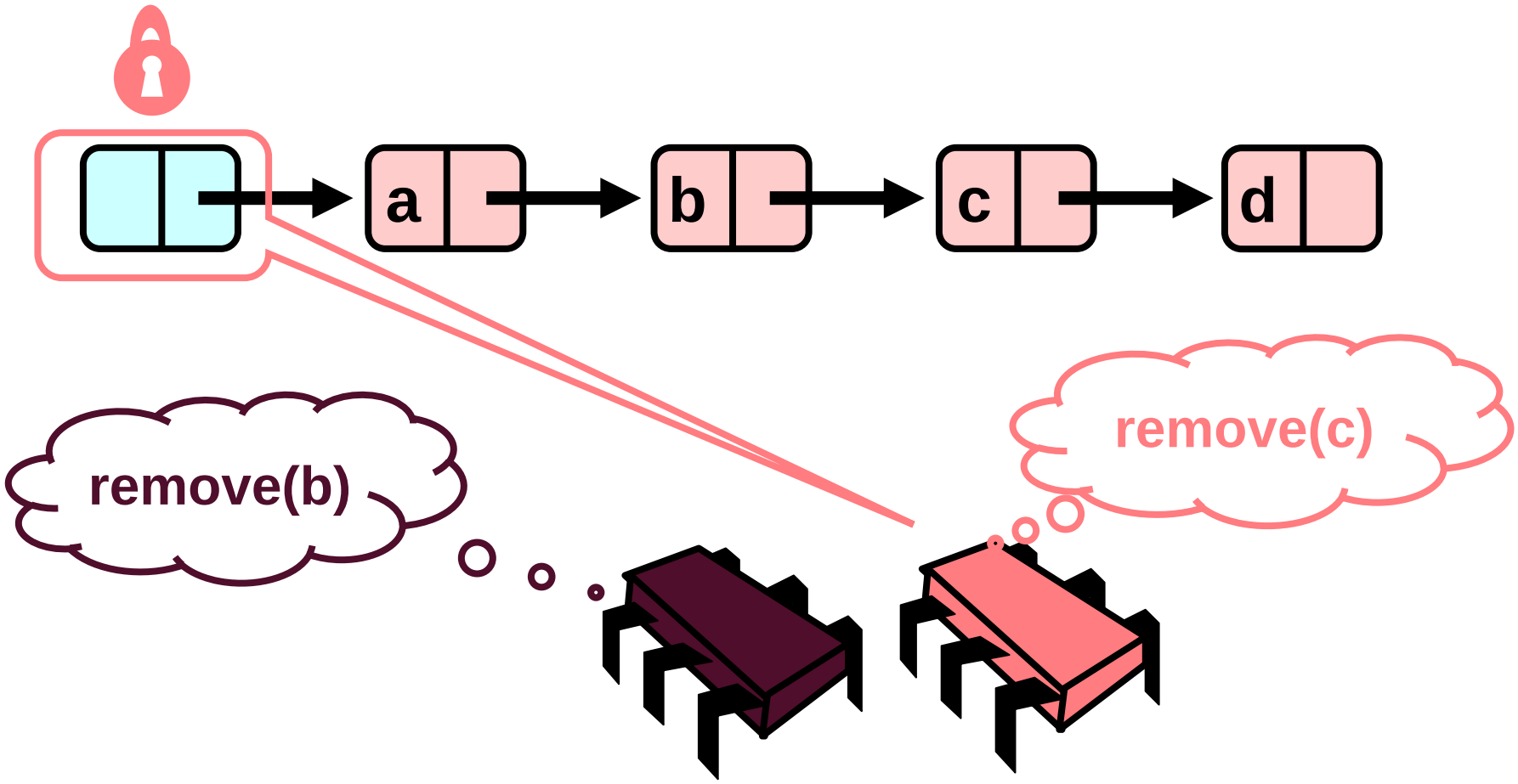

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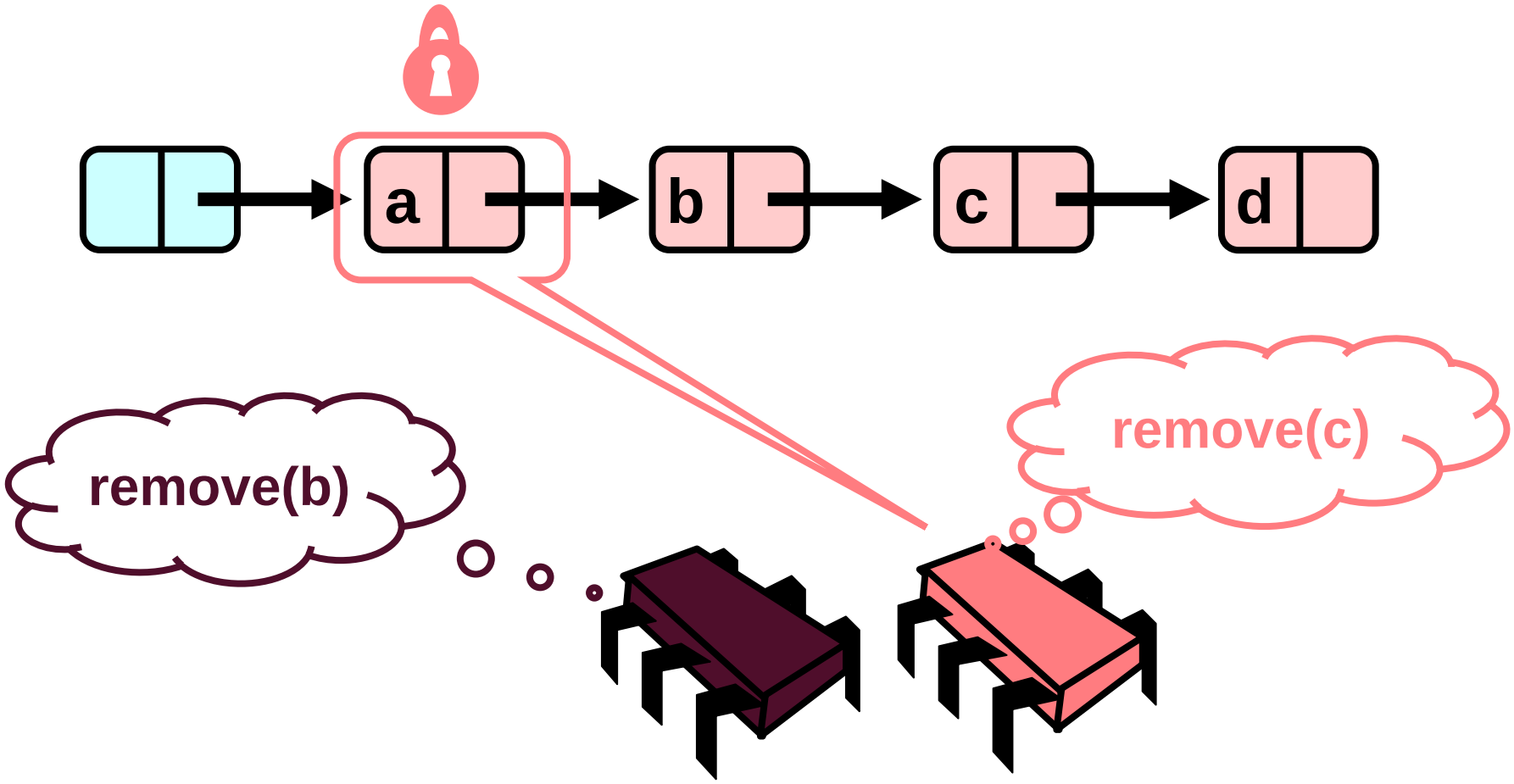

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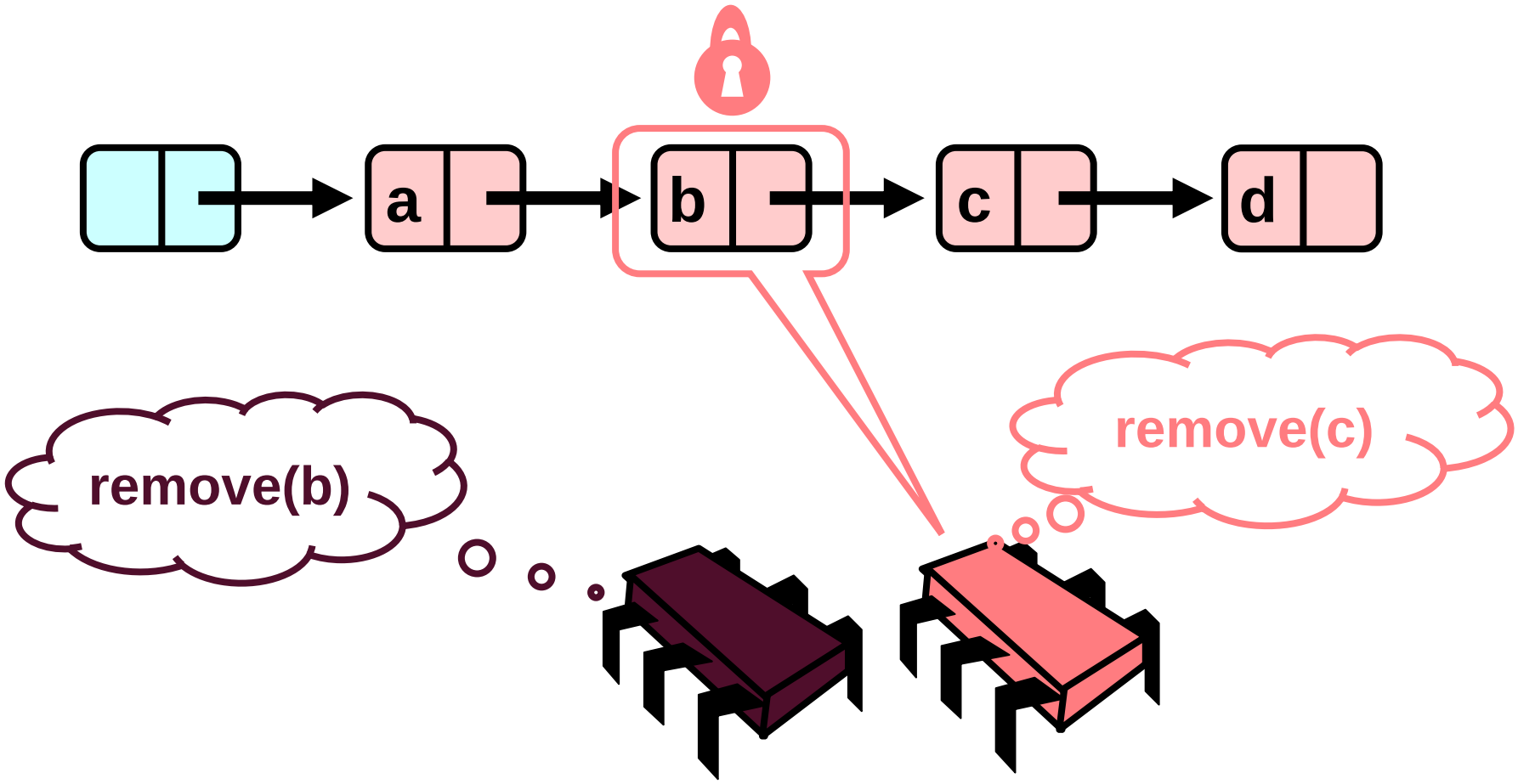

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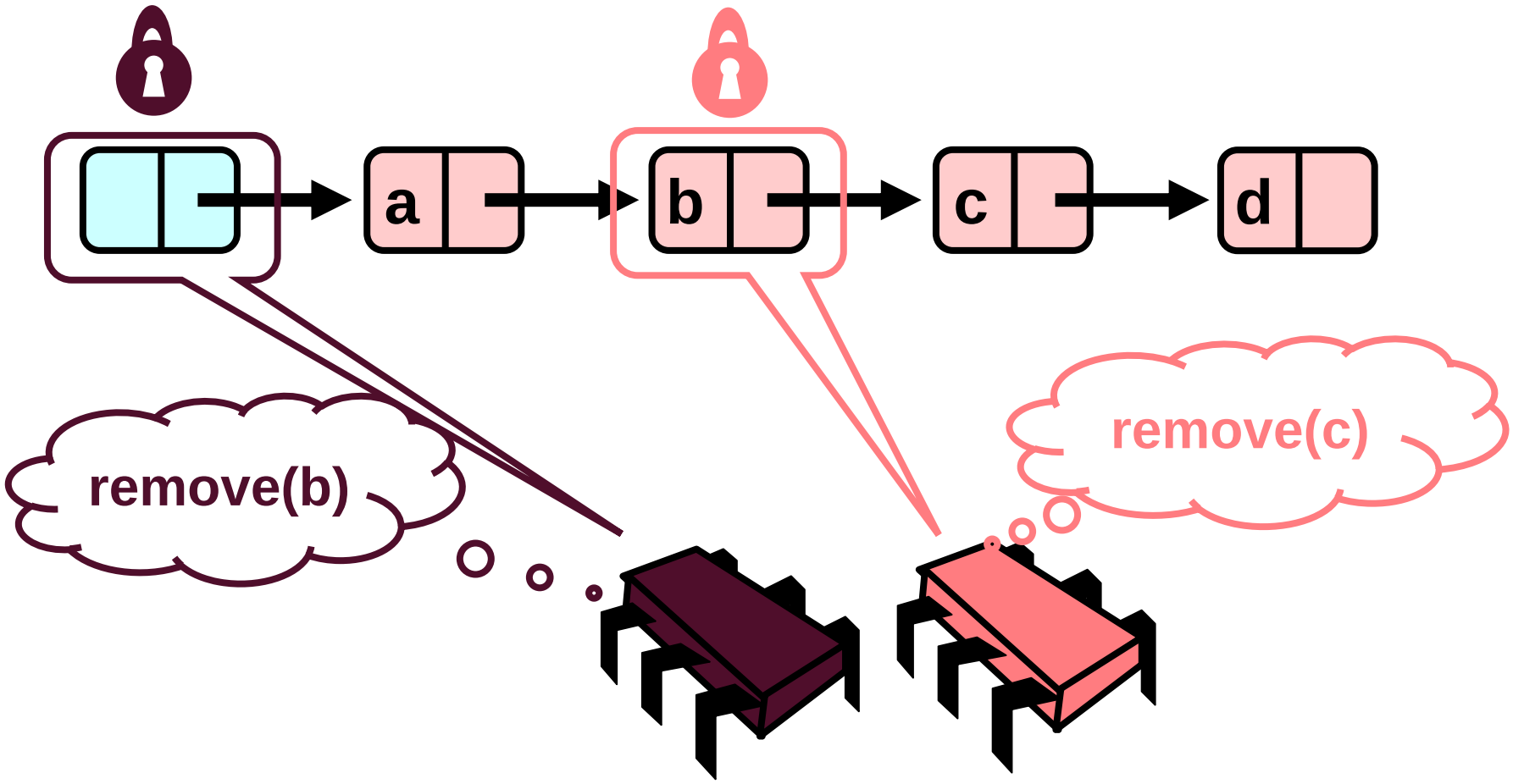

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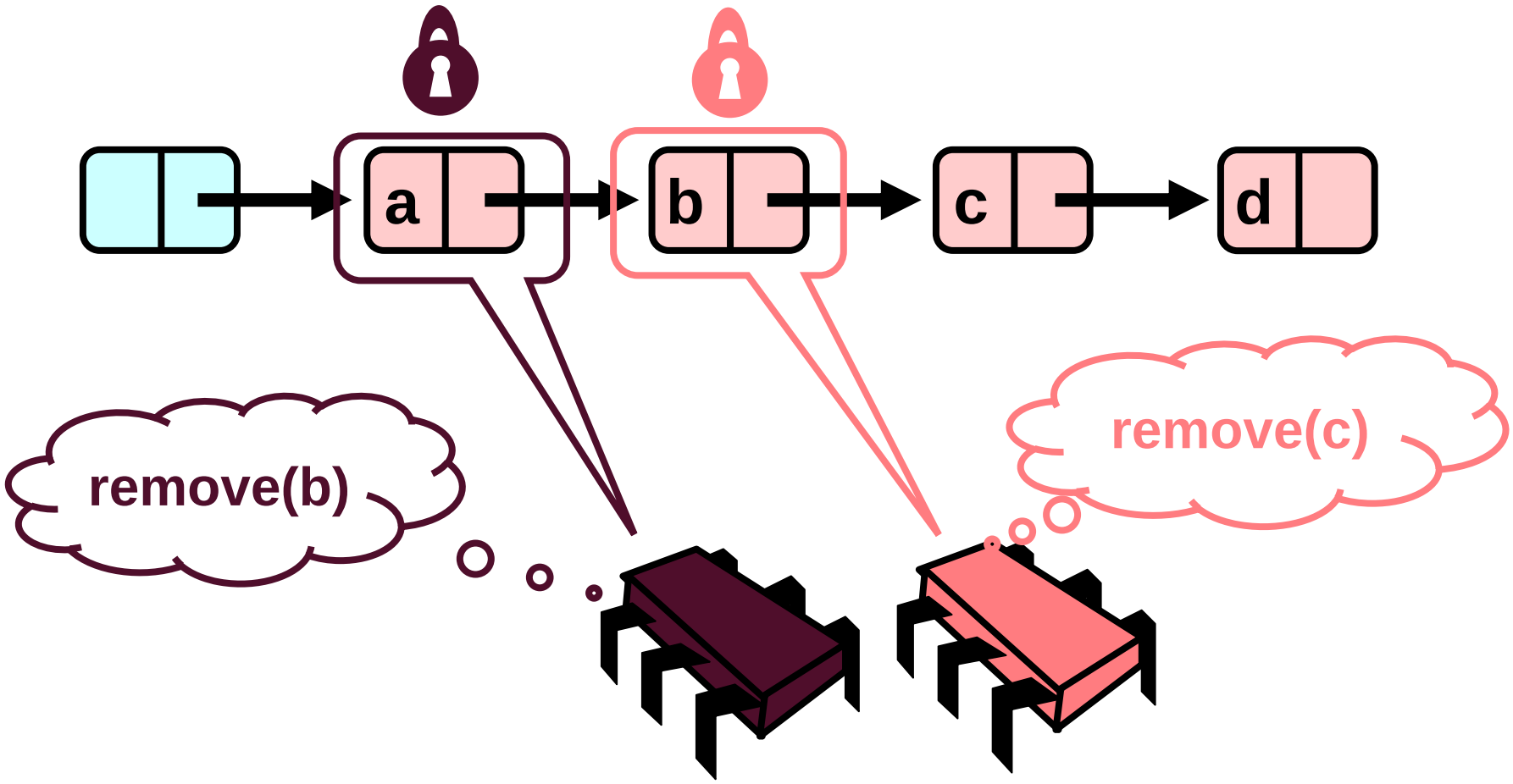

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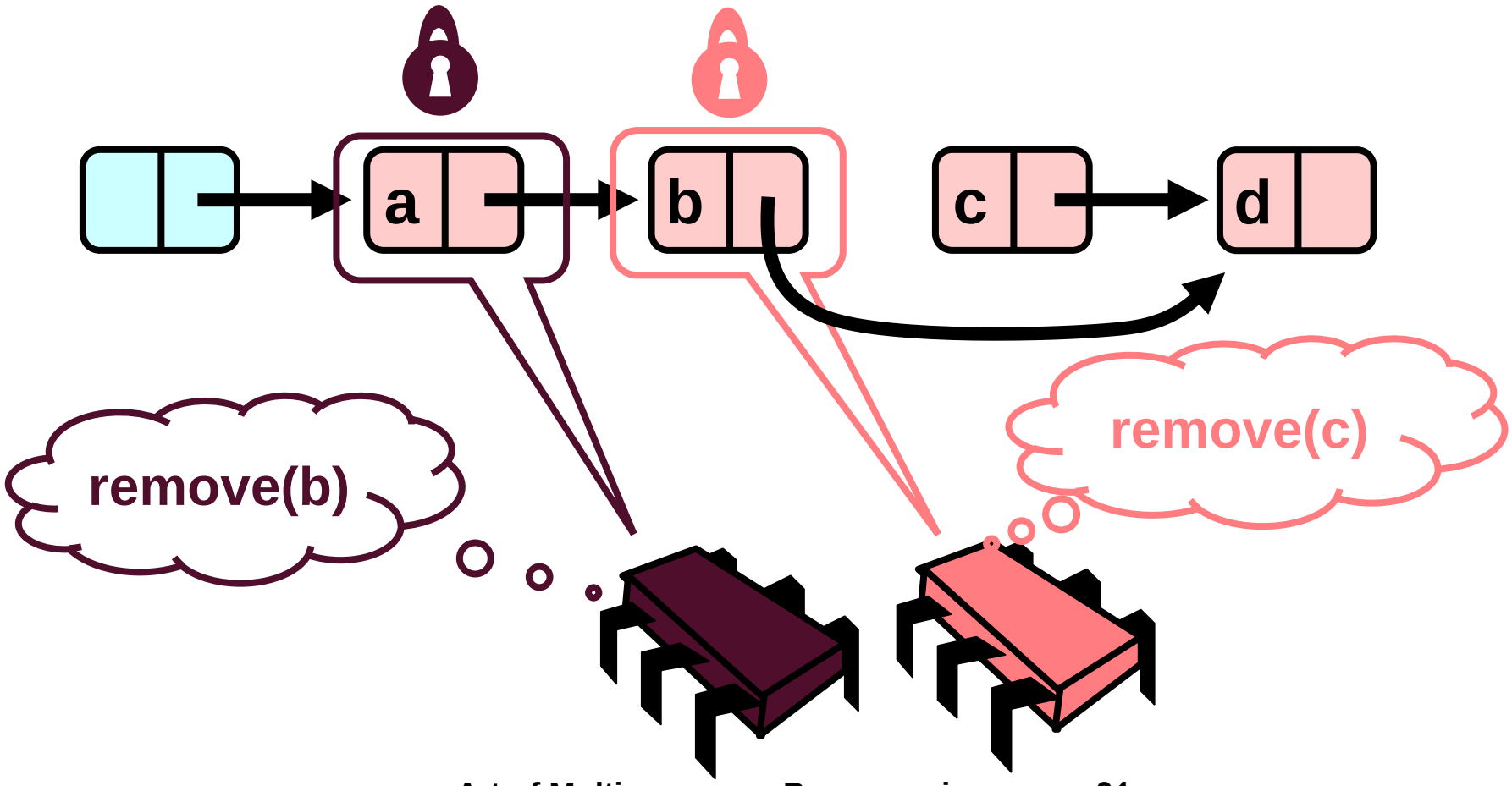

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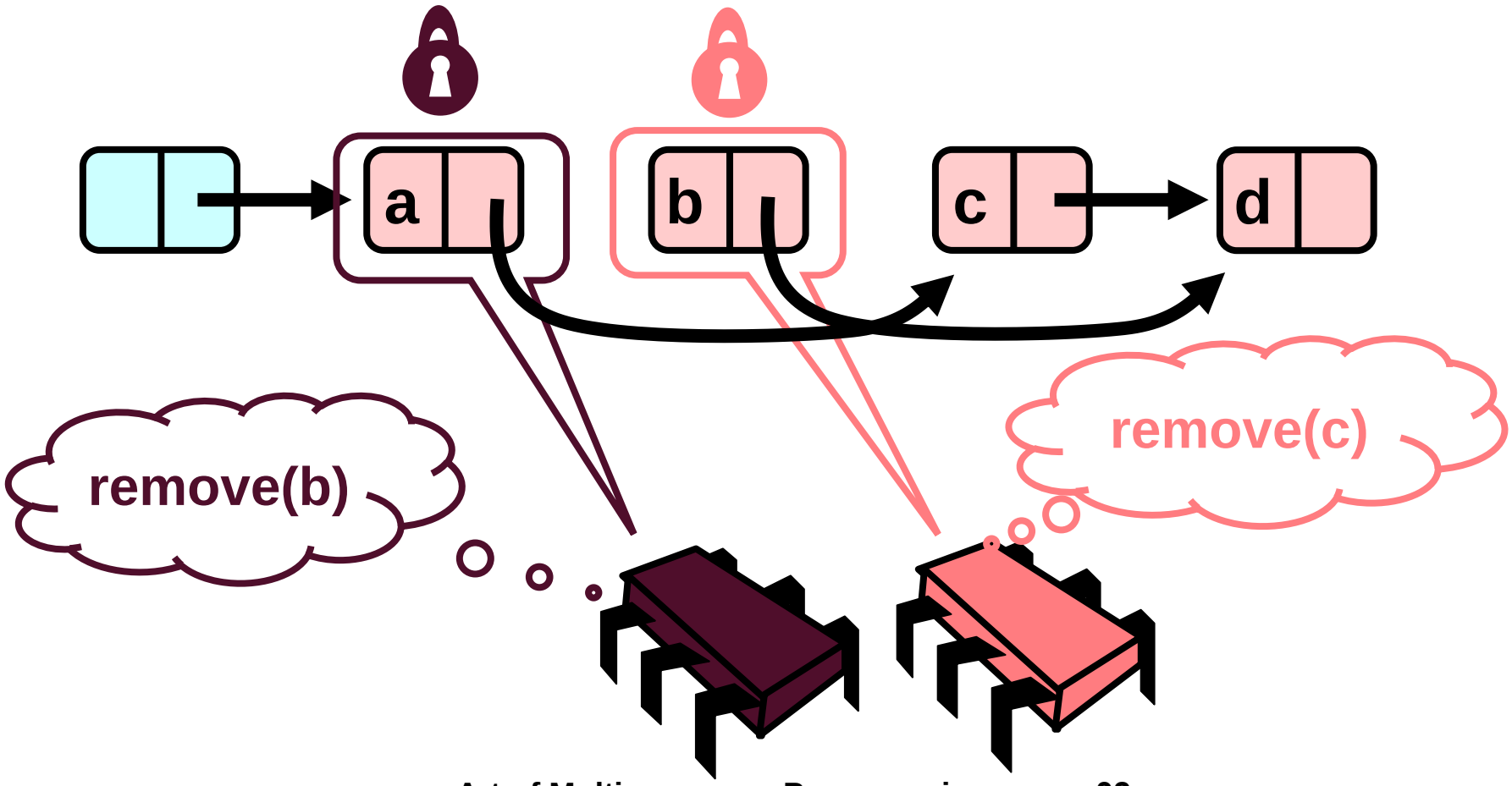
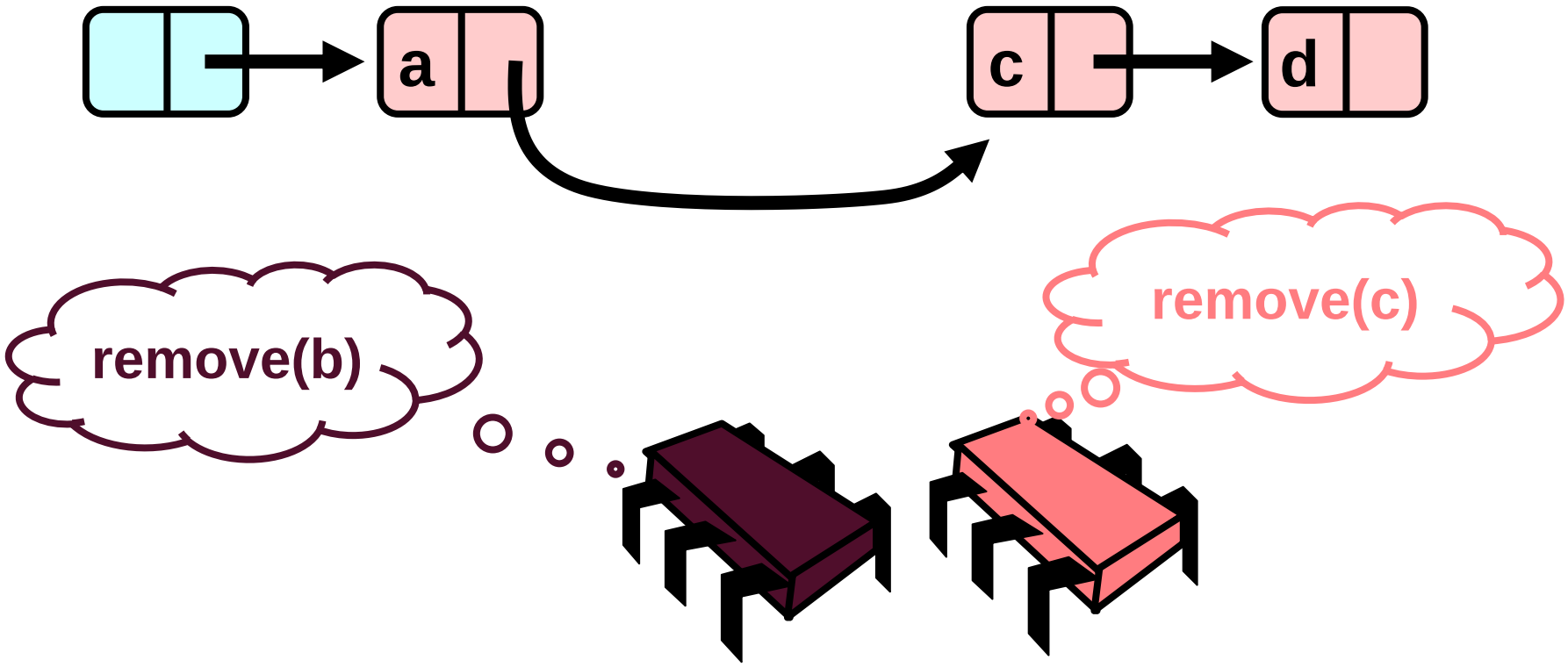


Uh, Oh



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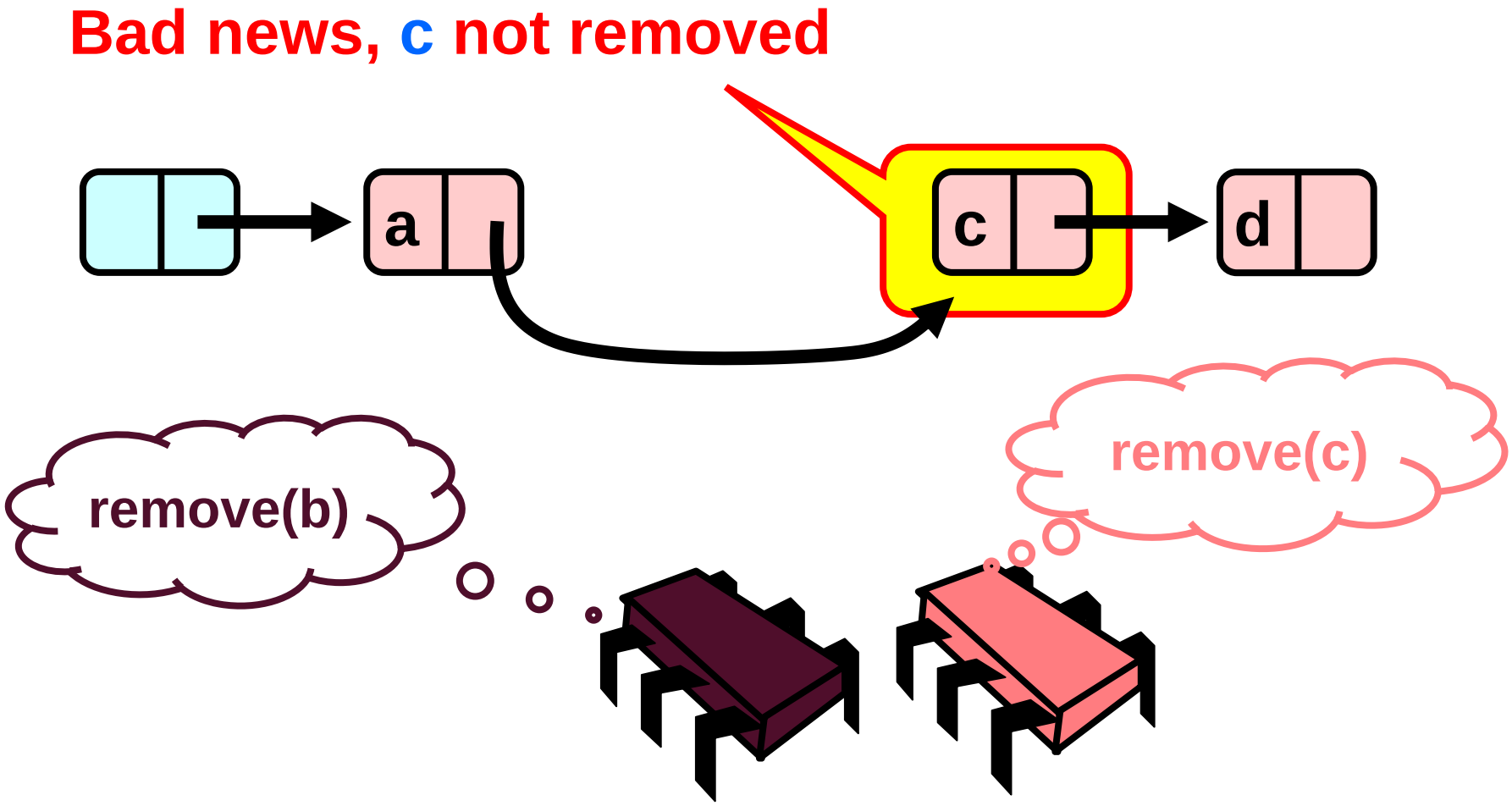
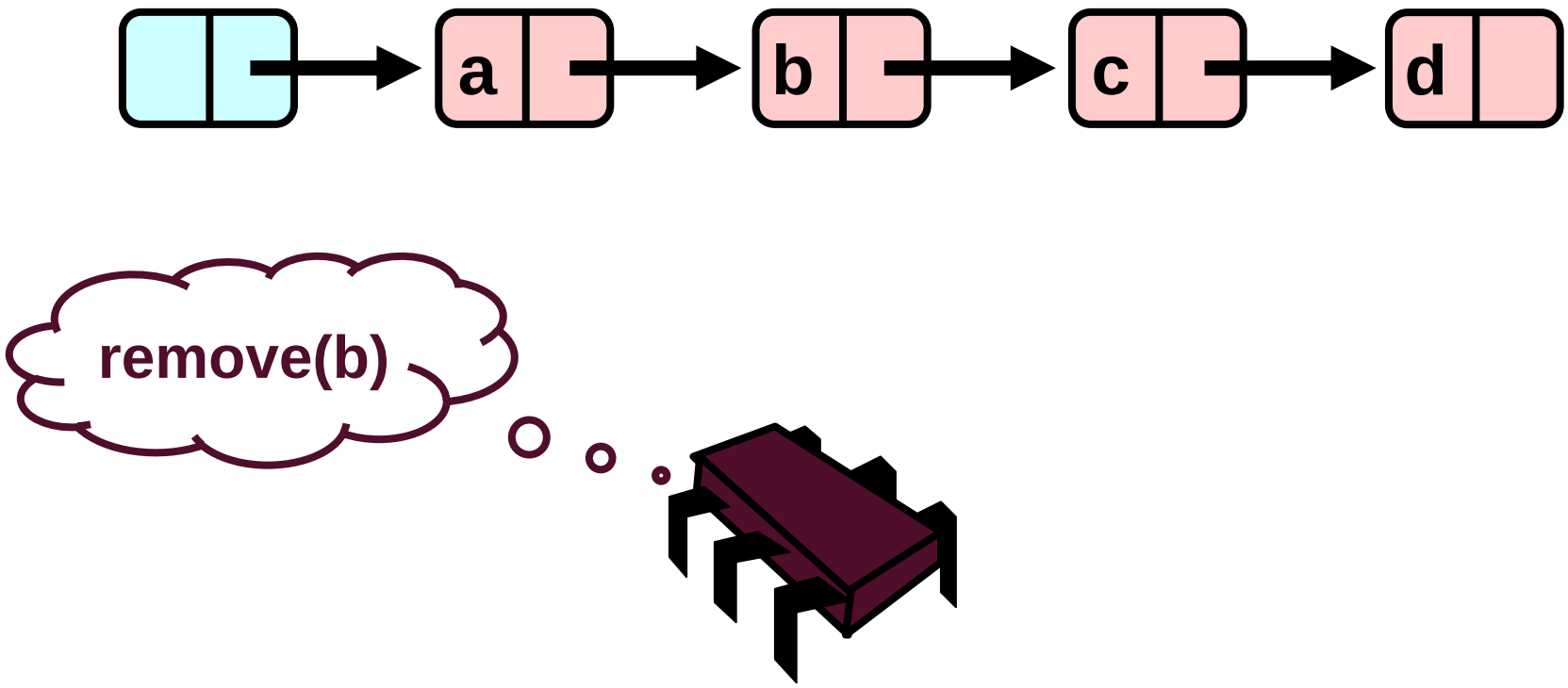


Insight

- **If a node x is locked**
  - Successor of x cannot be deleted!
- **Thus, safe locking is**
  - Lock node to be deleted
  - And its predecessor!
  - → hand-over-hand locking

Hand-Over-Hand Again



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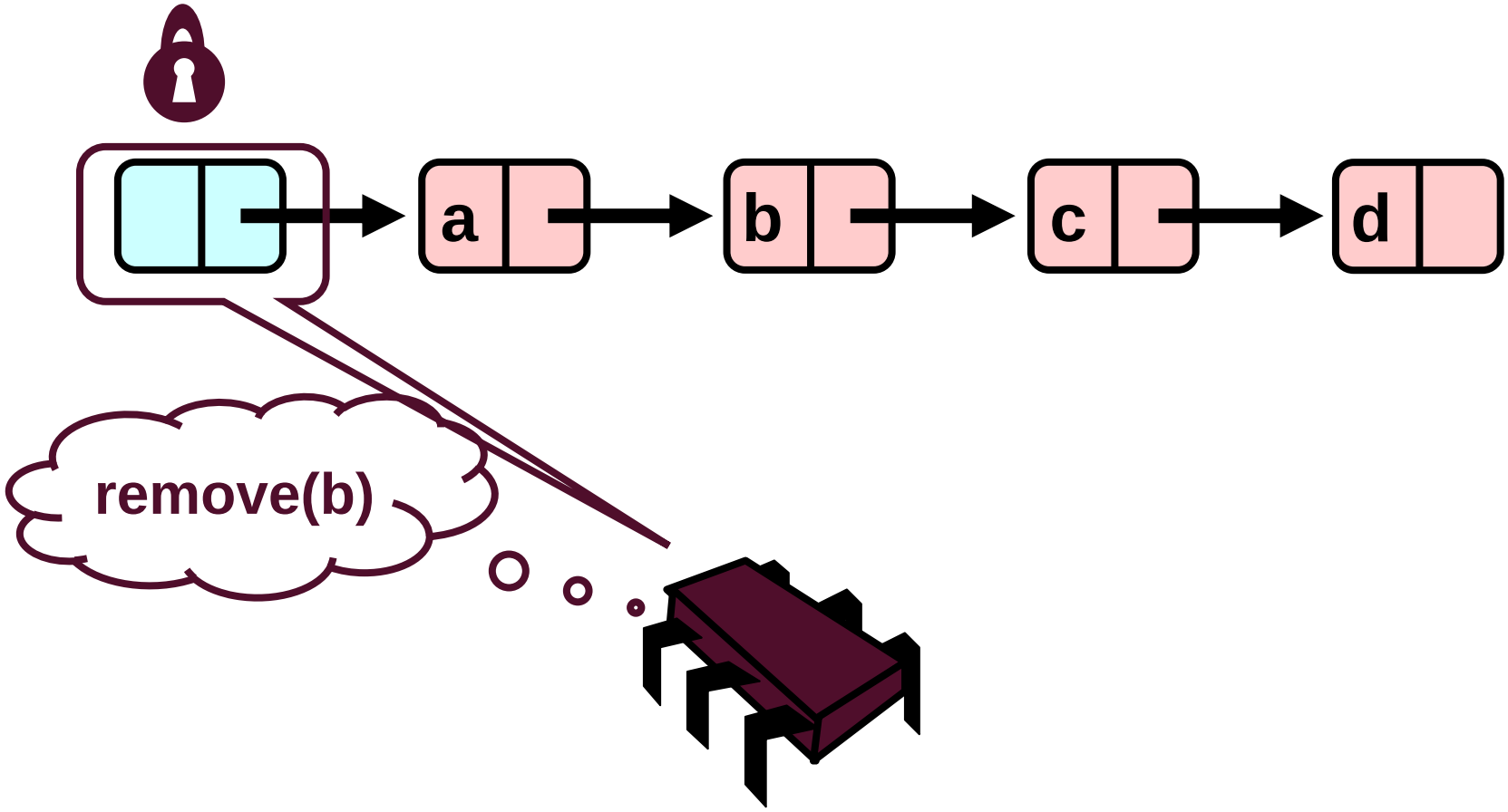

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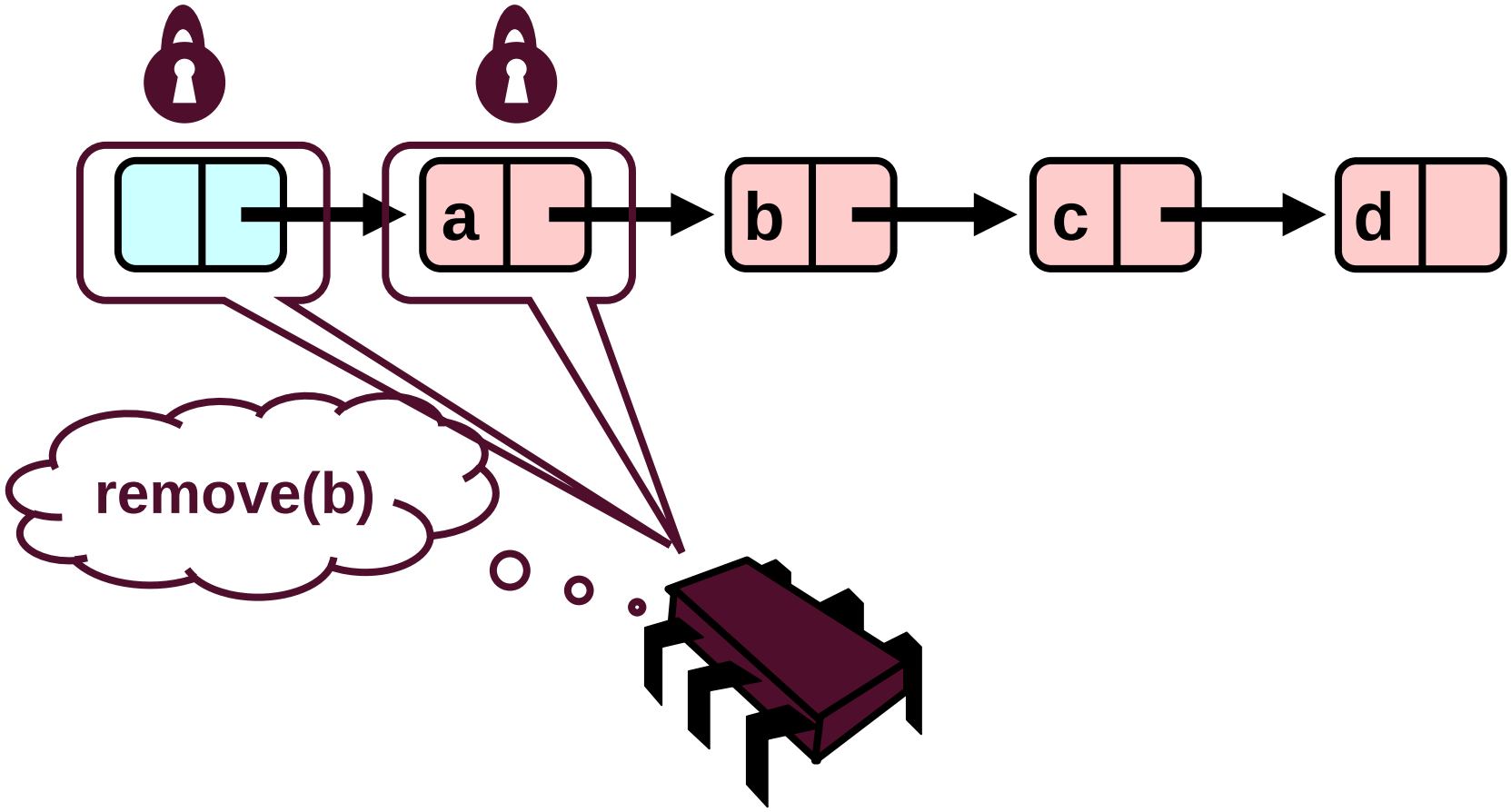

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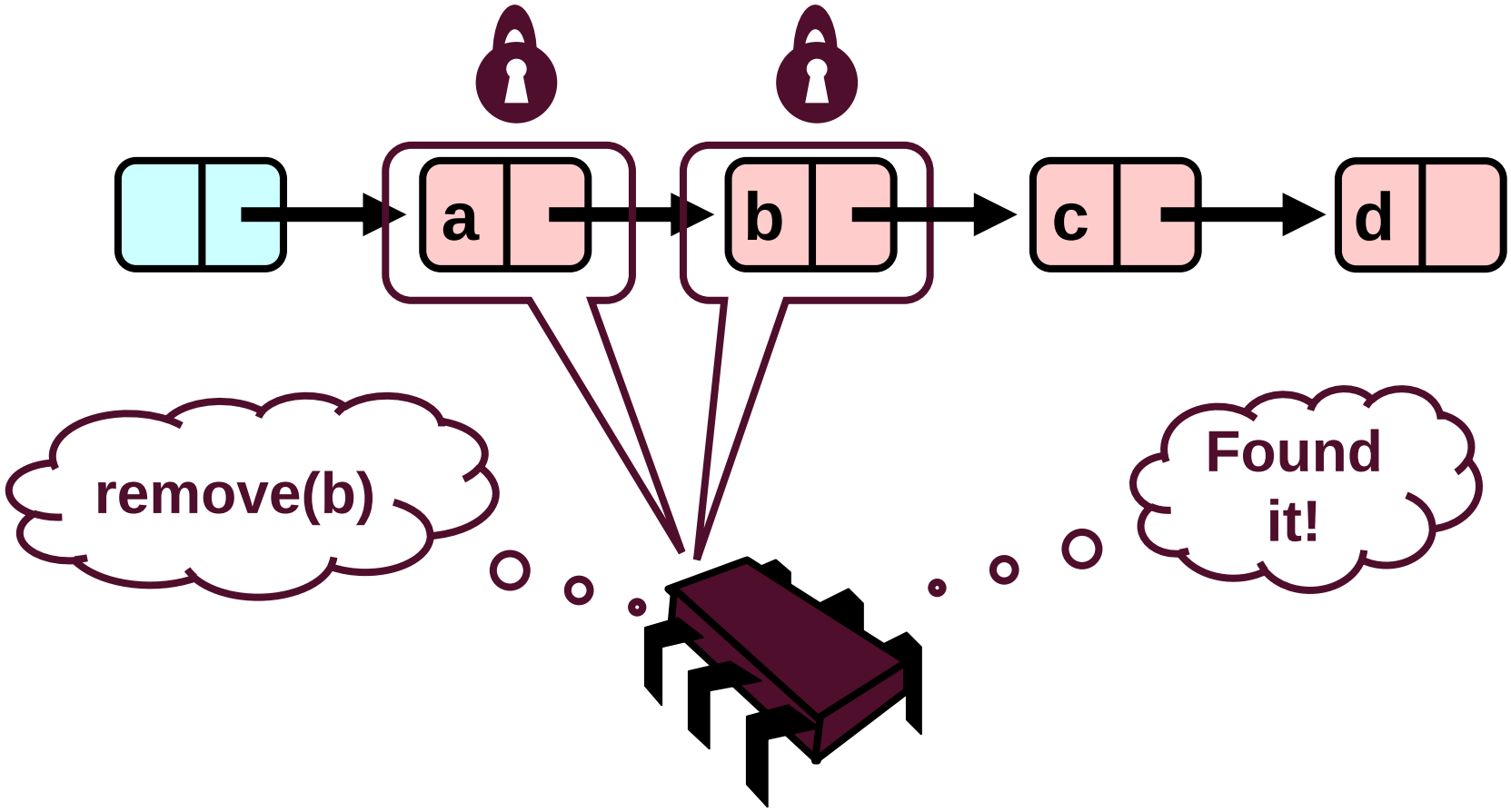

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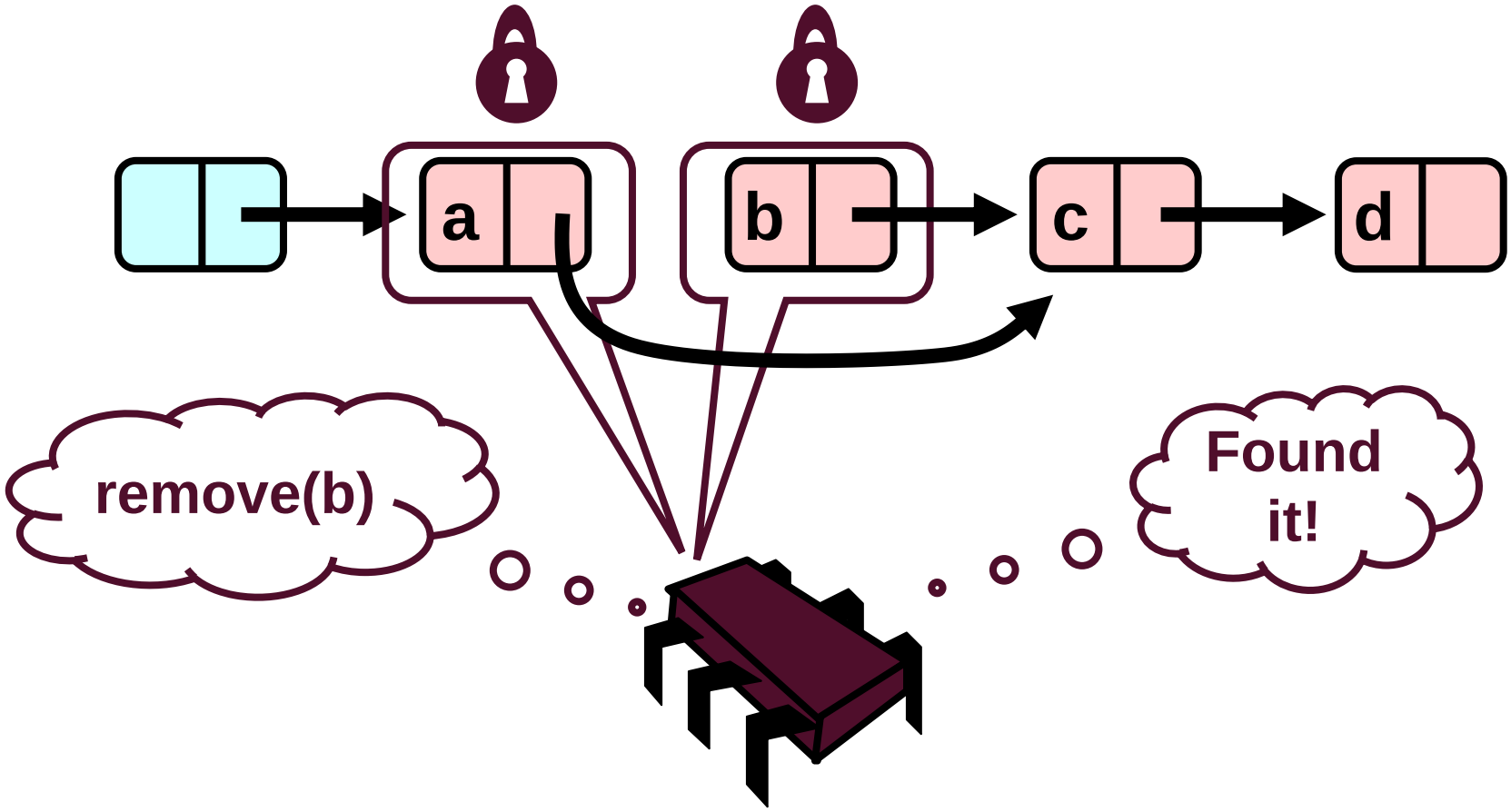

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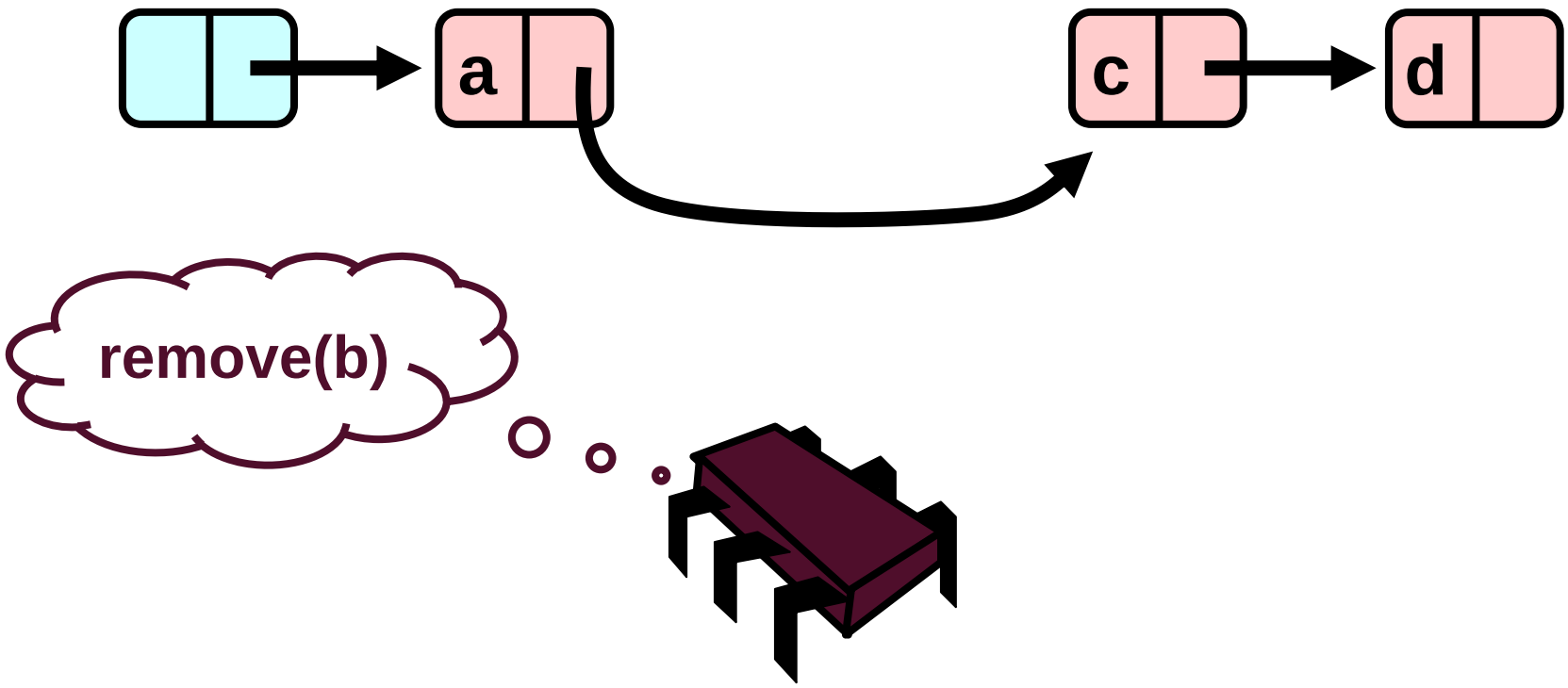
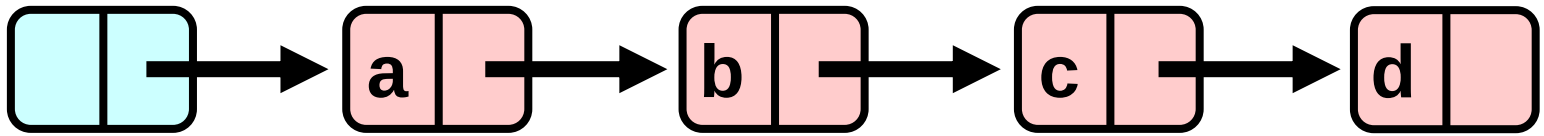
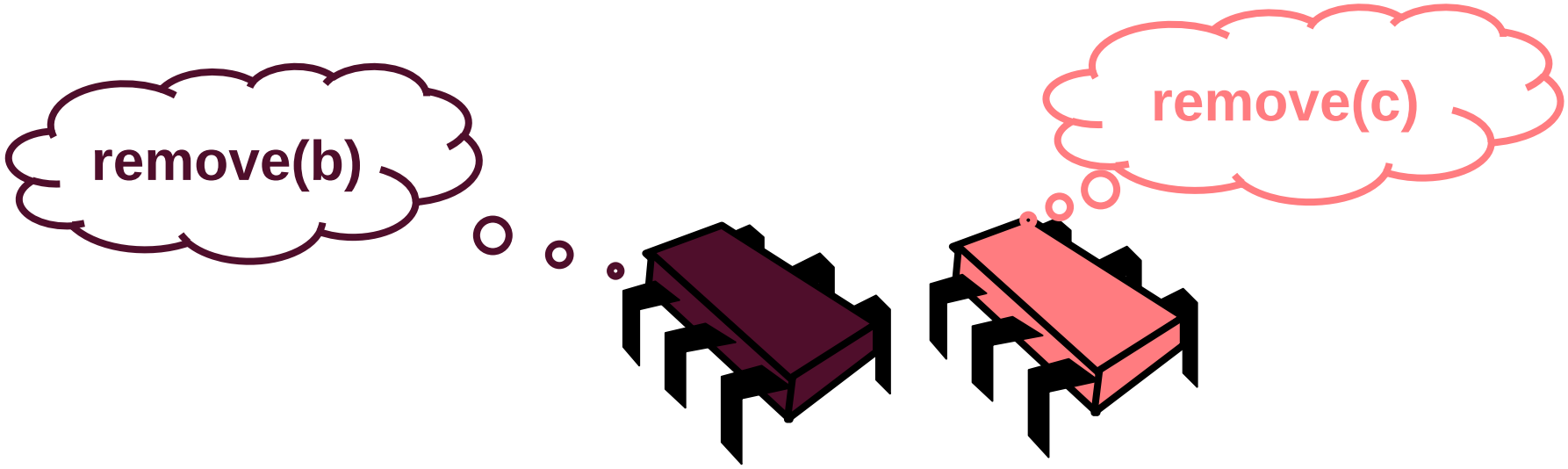


Removing a Node





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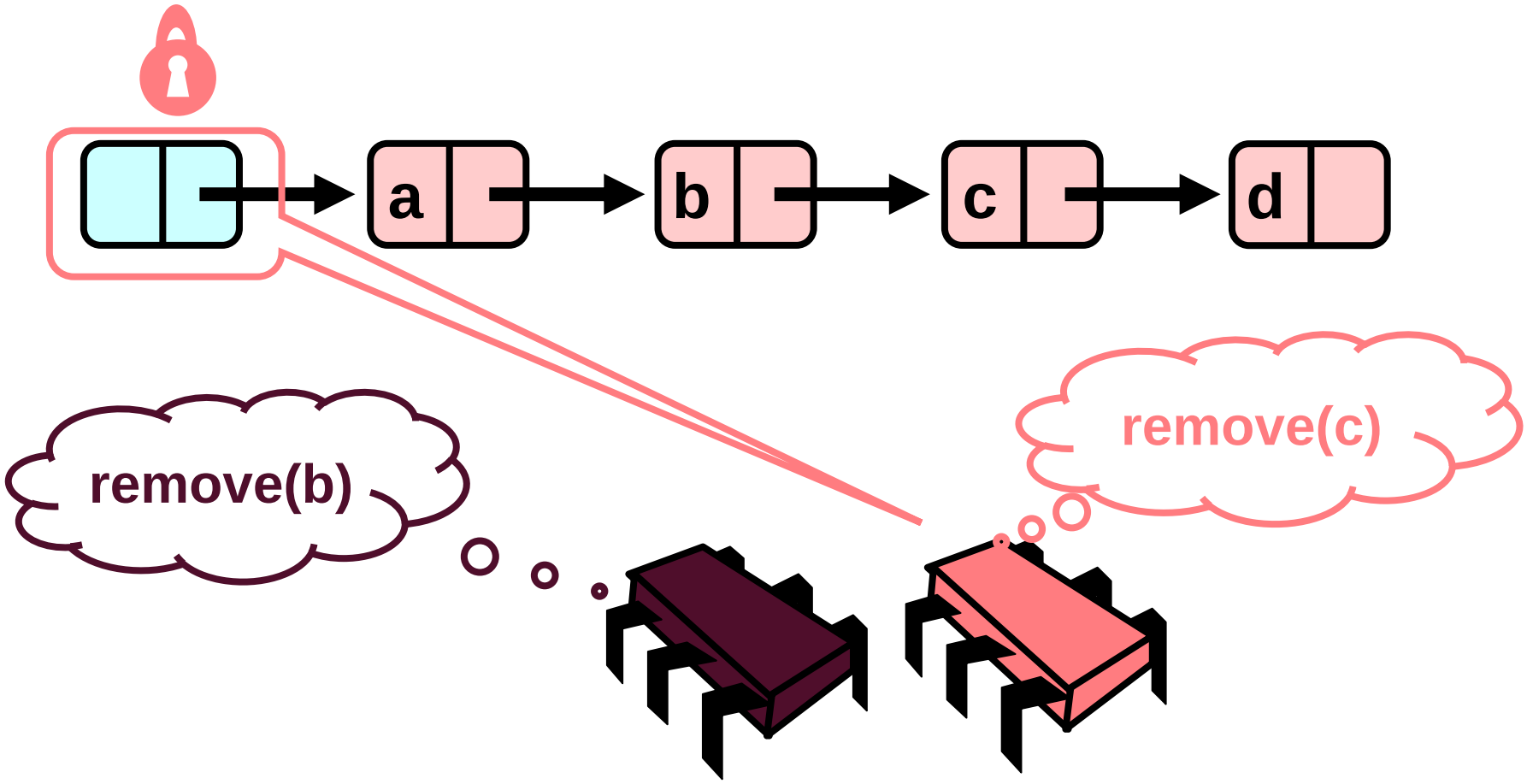

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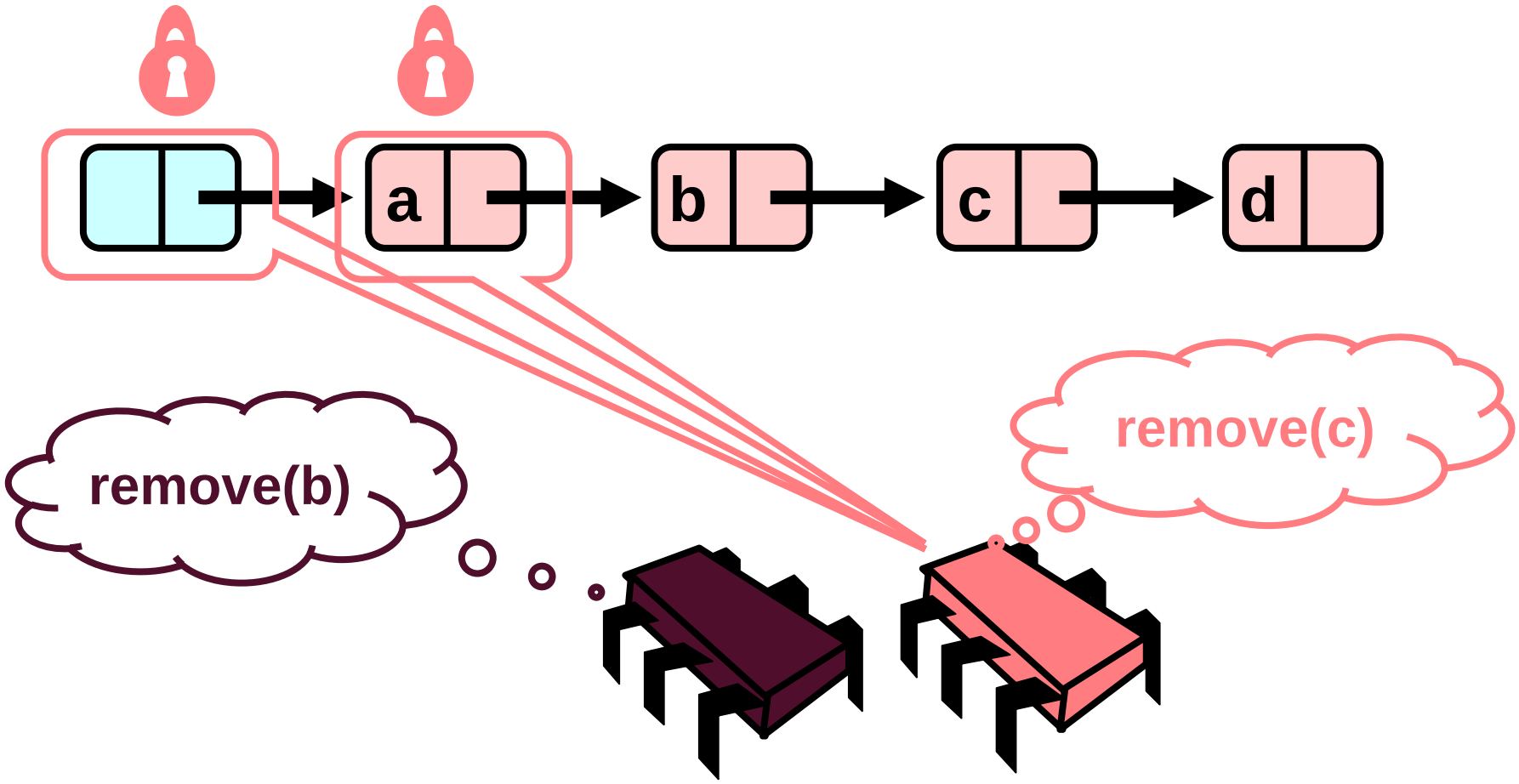

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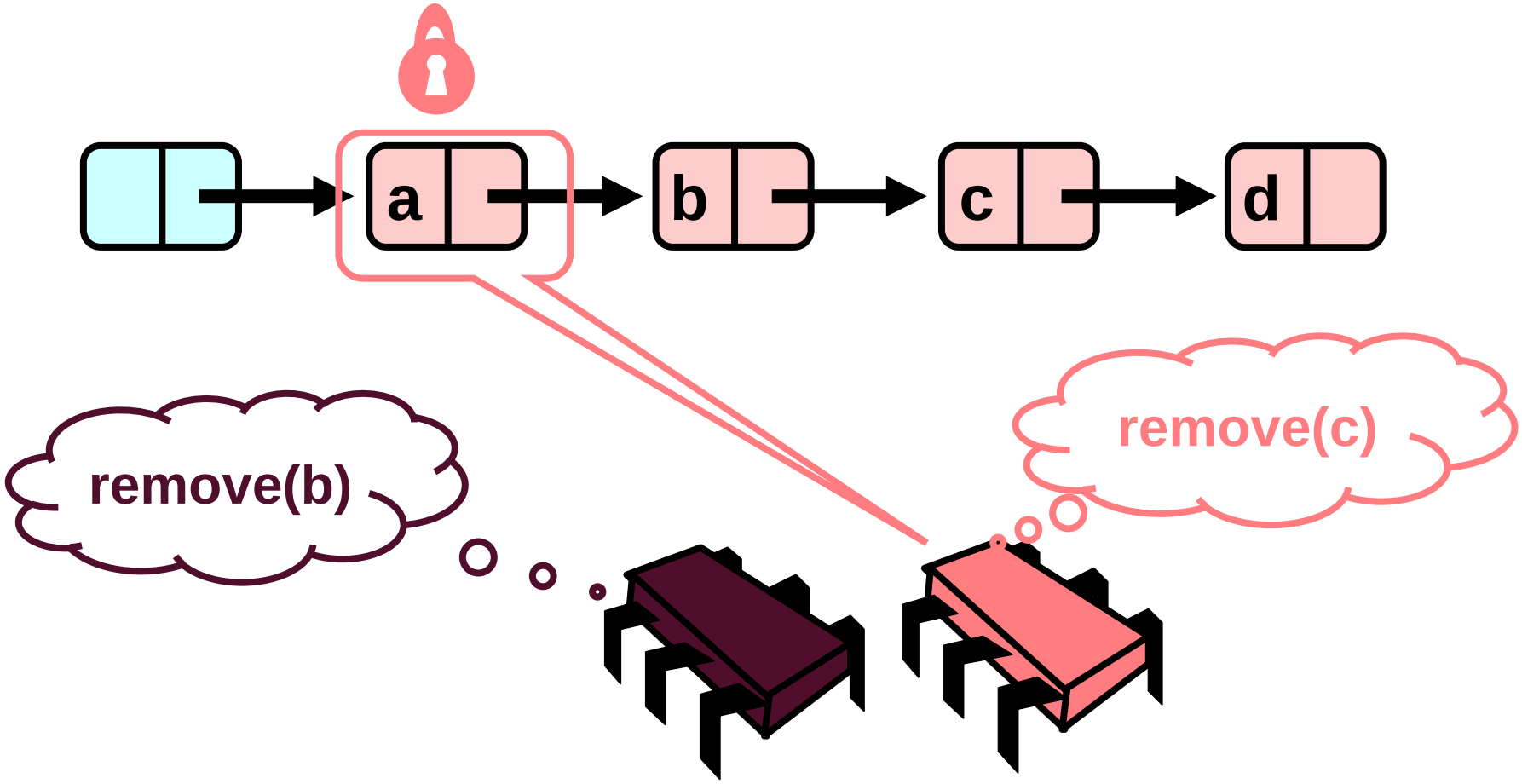

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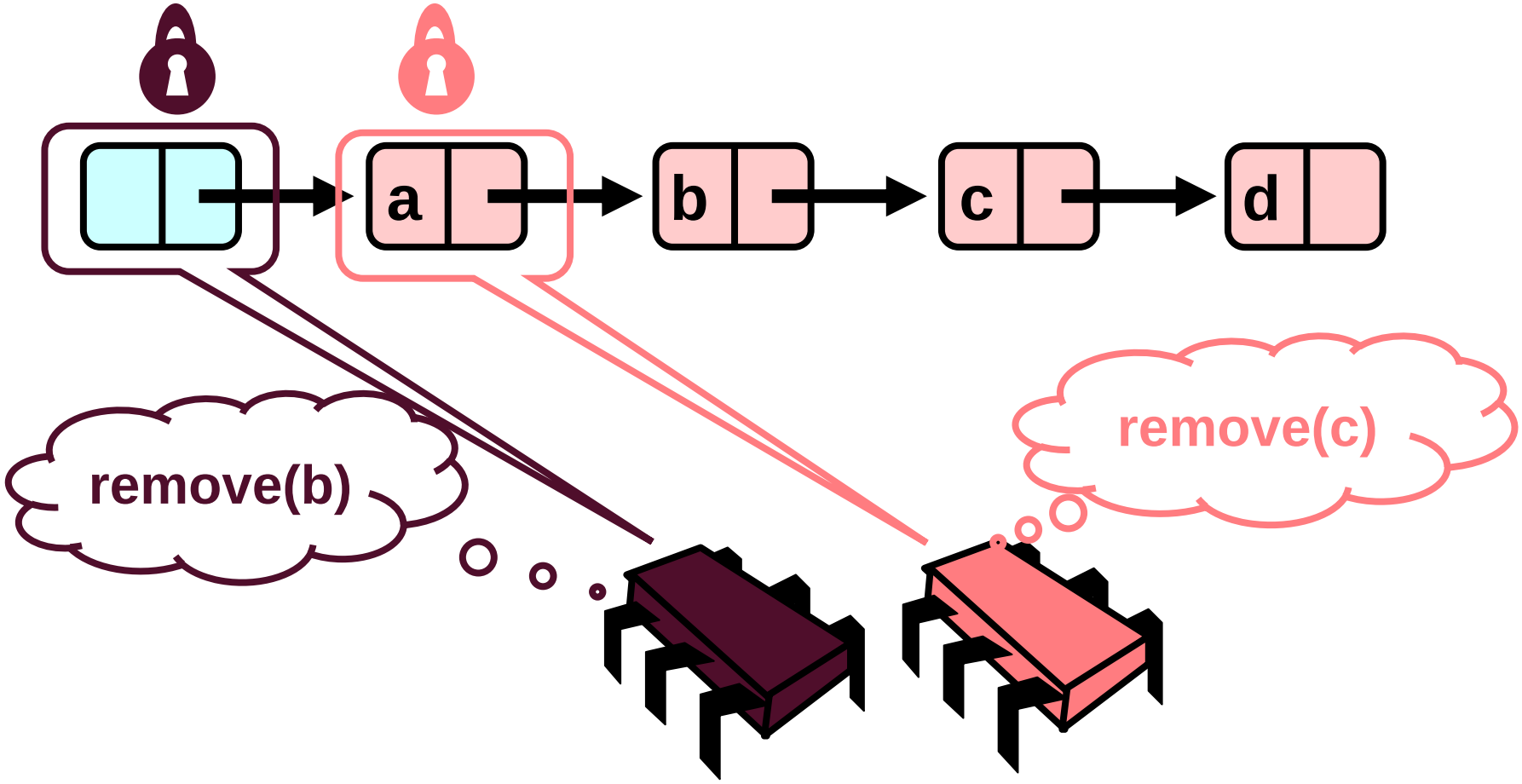

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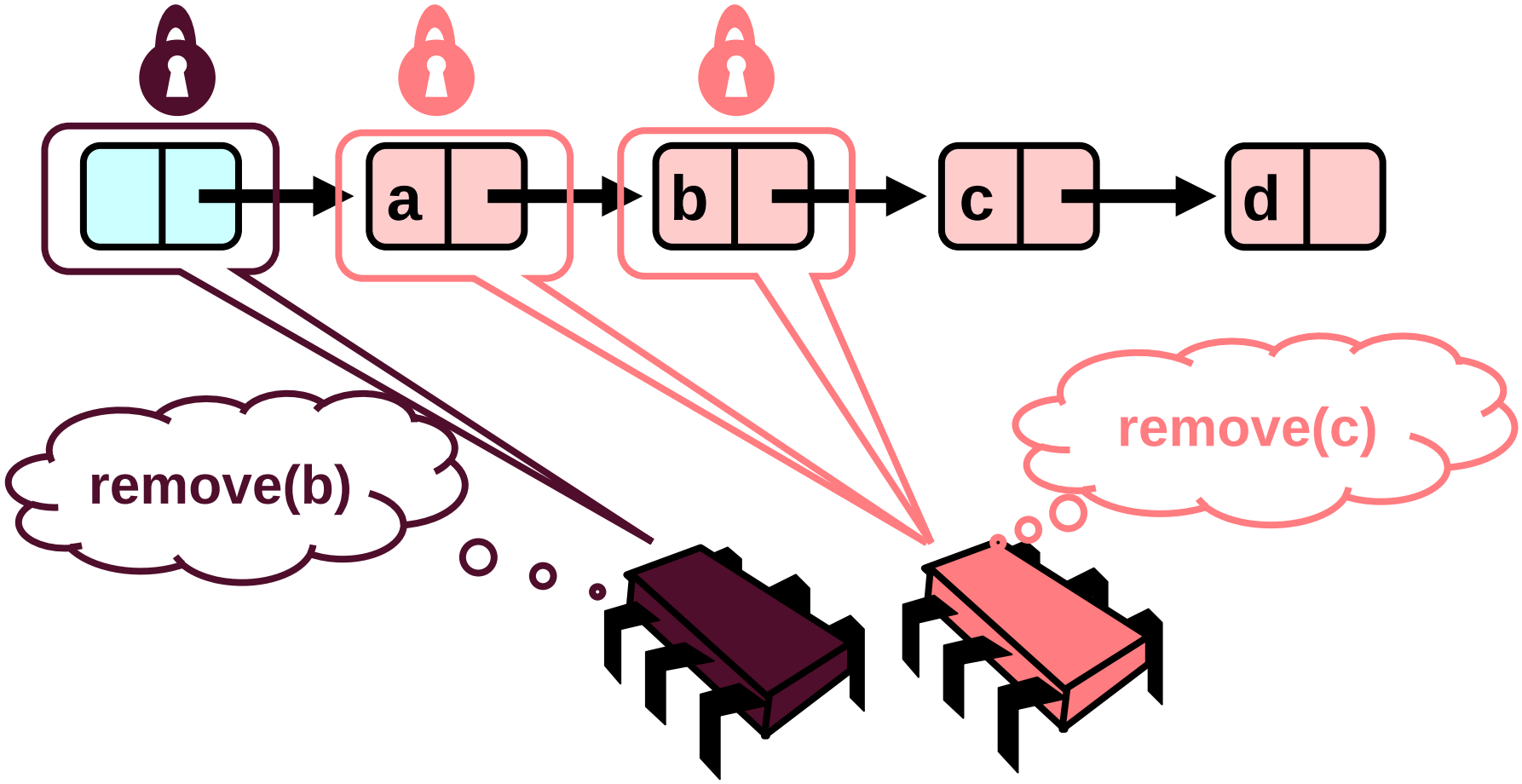

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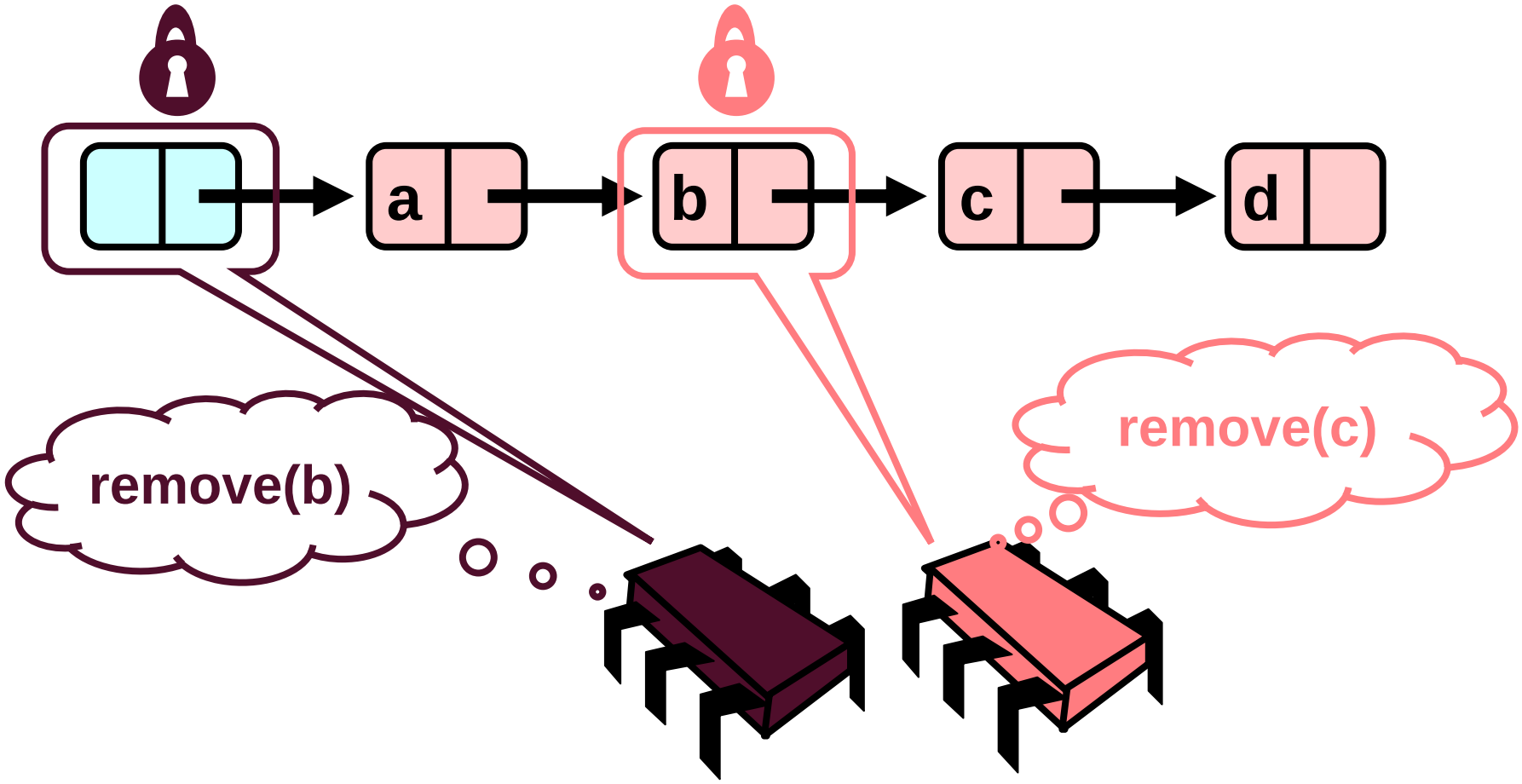

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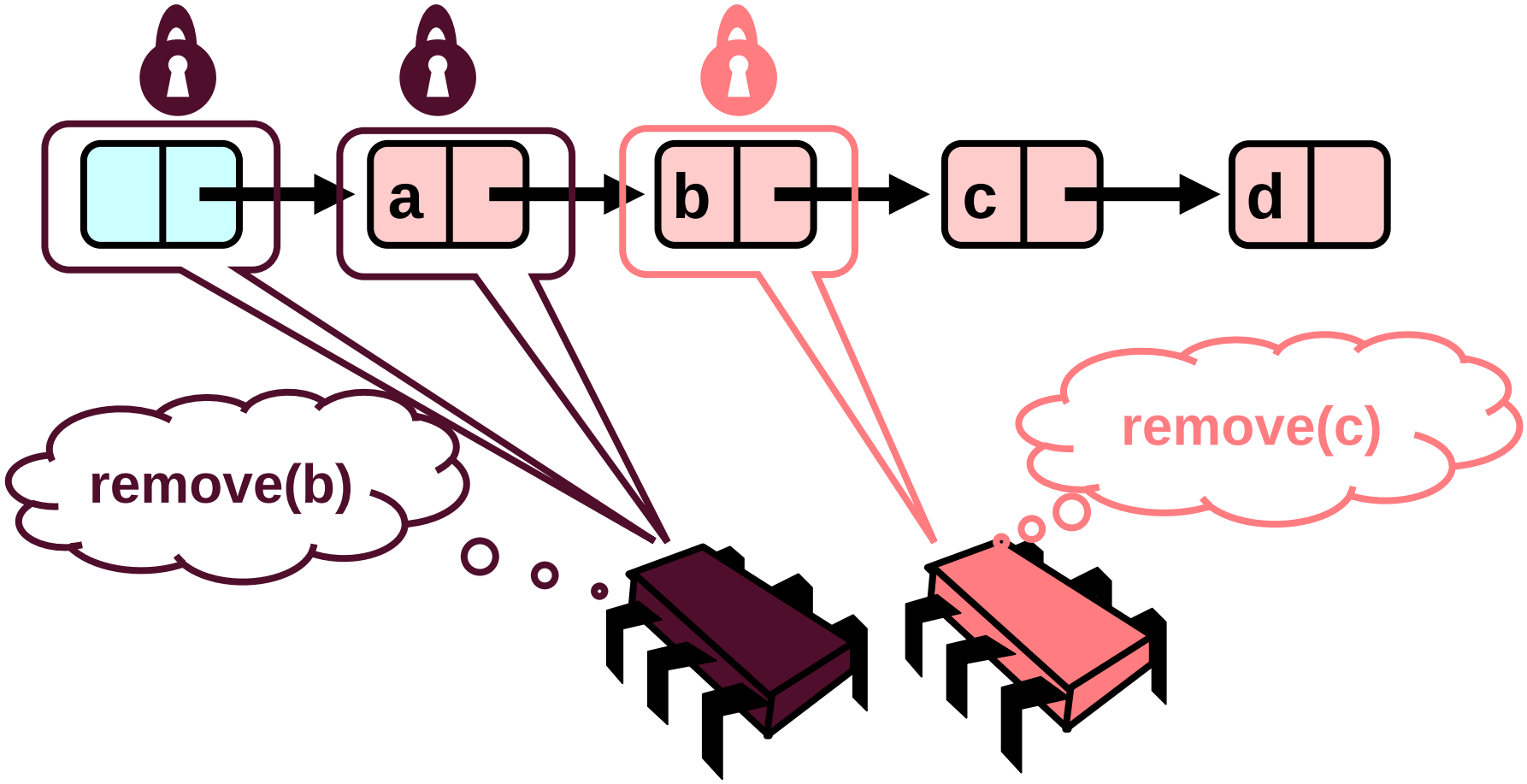

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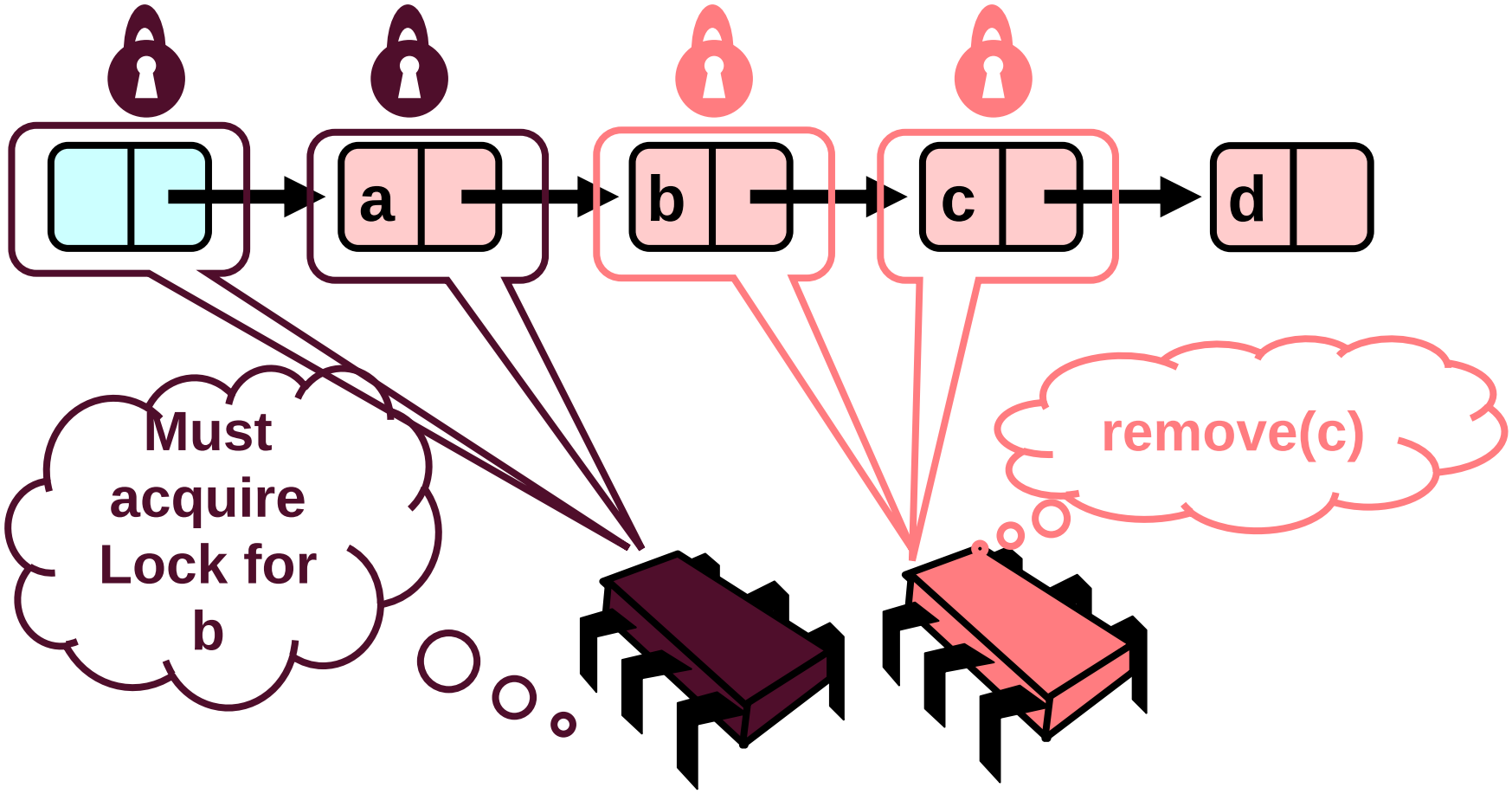

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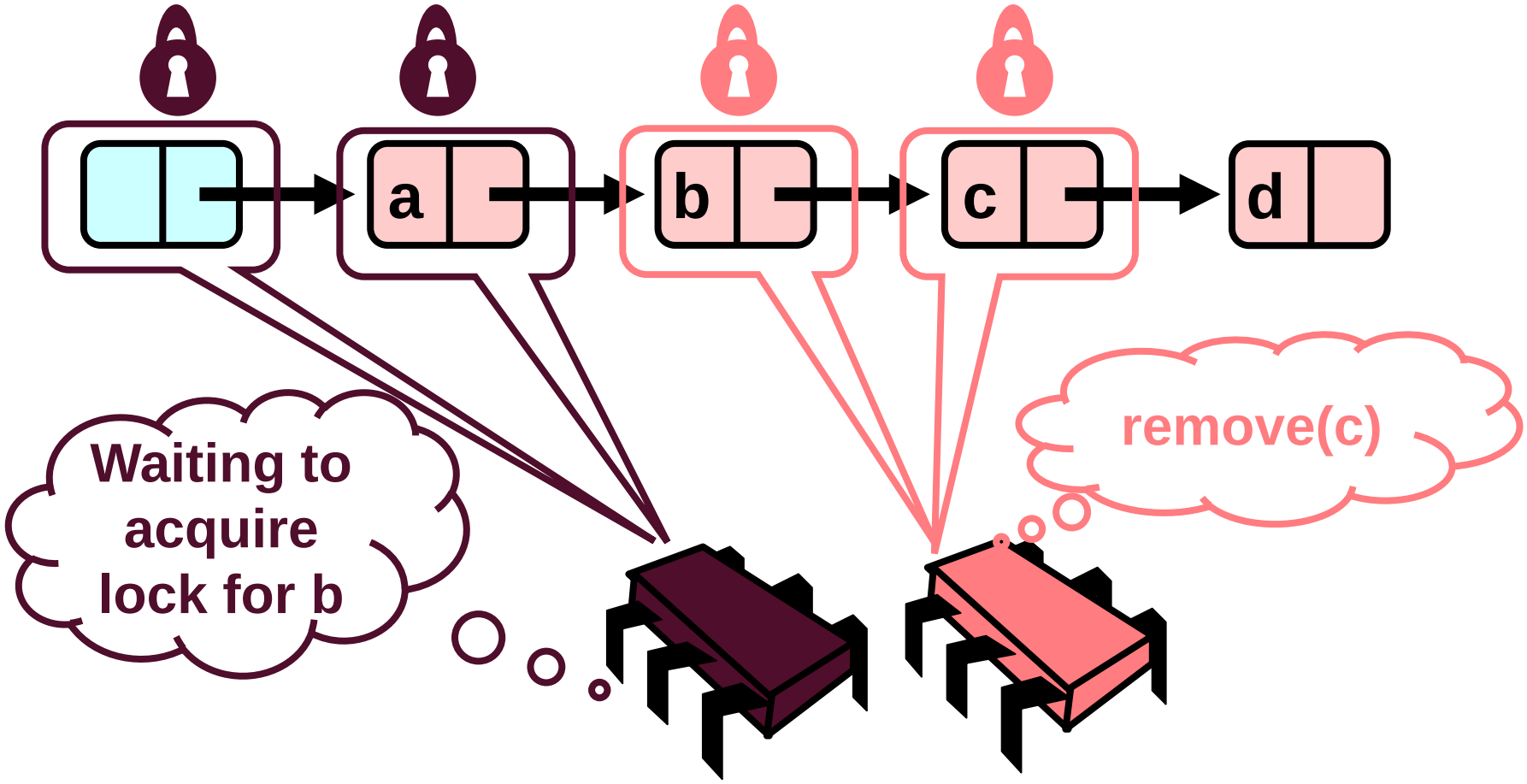

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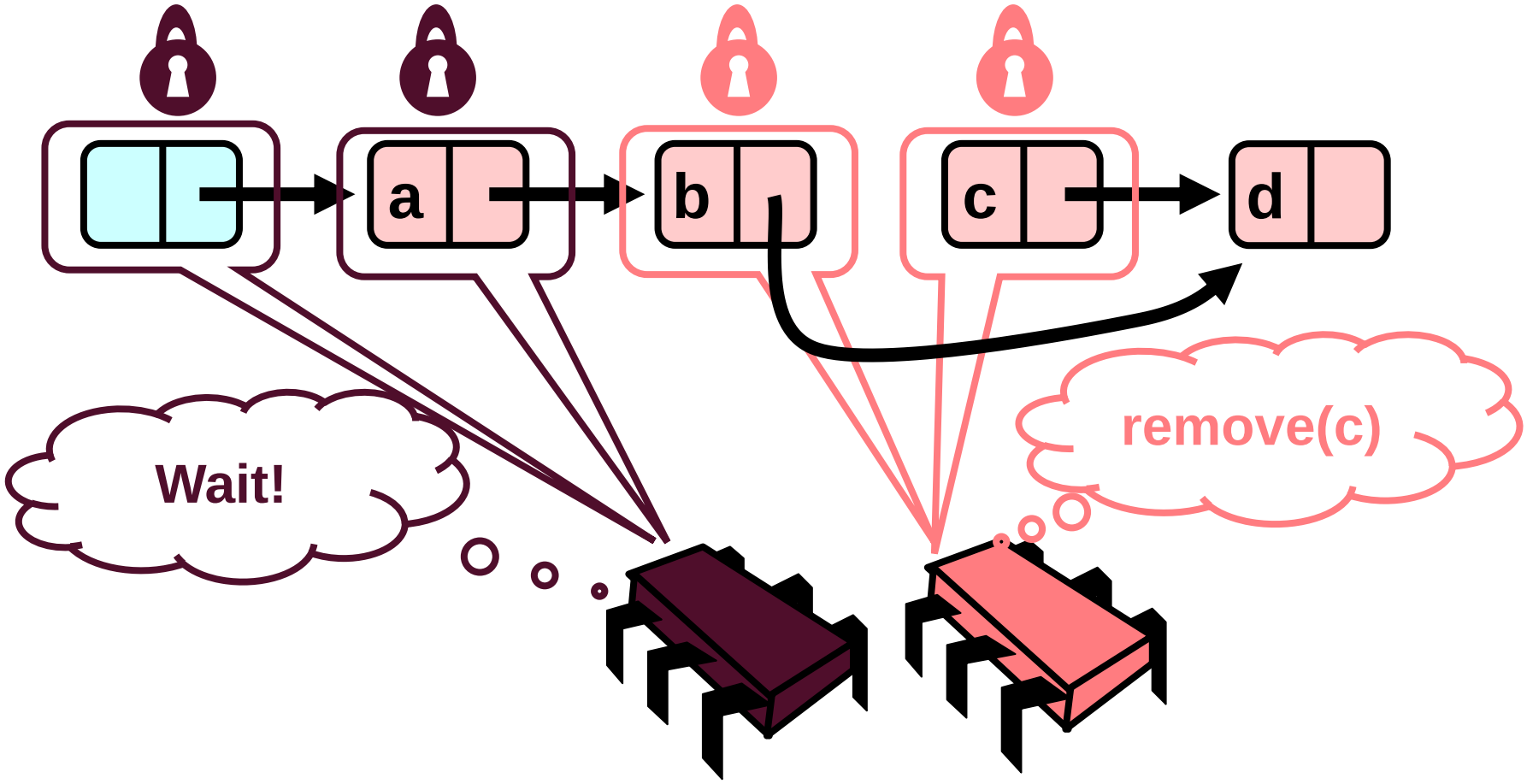

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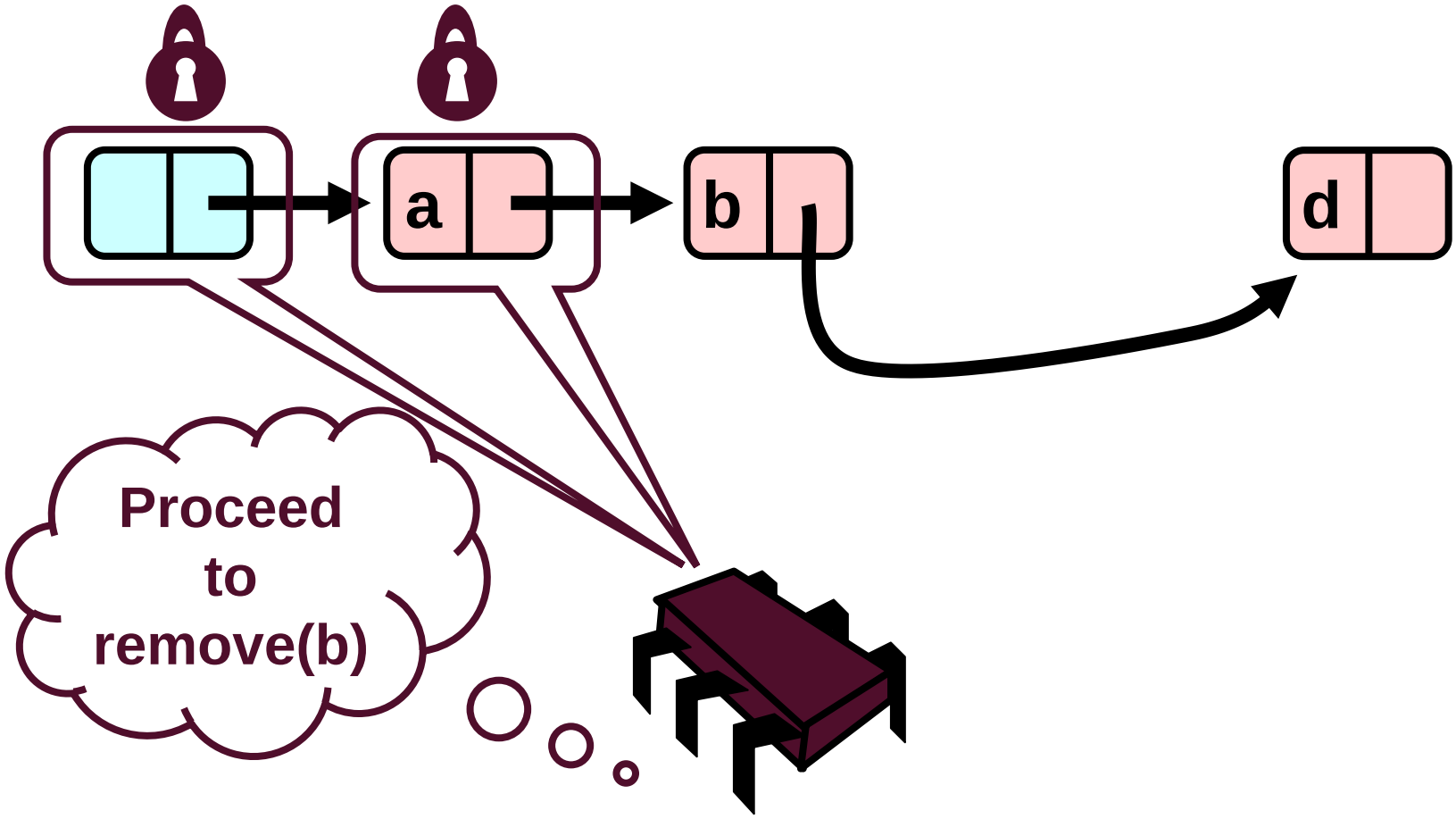

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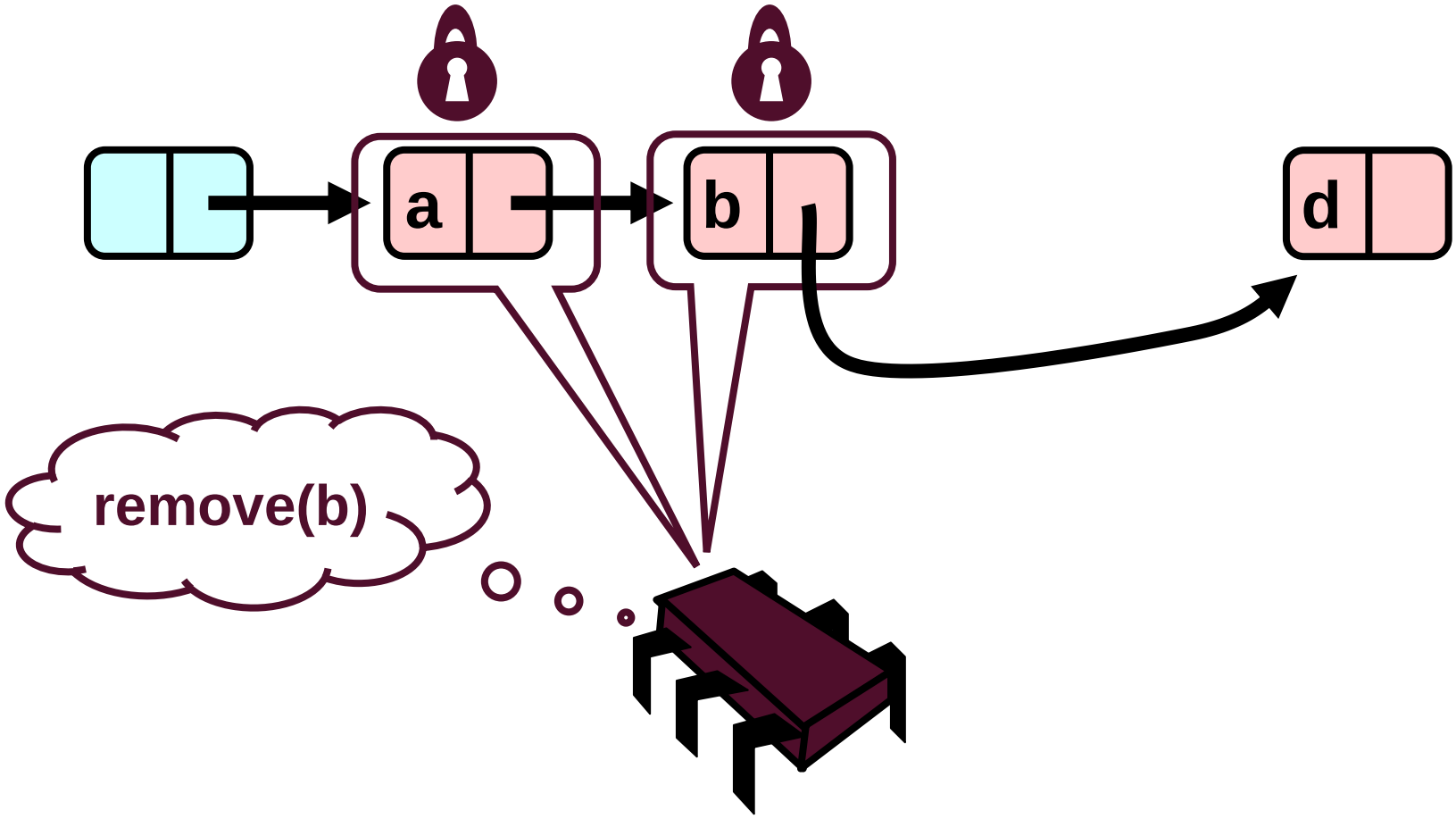

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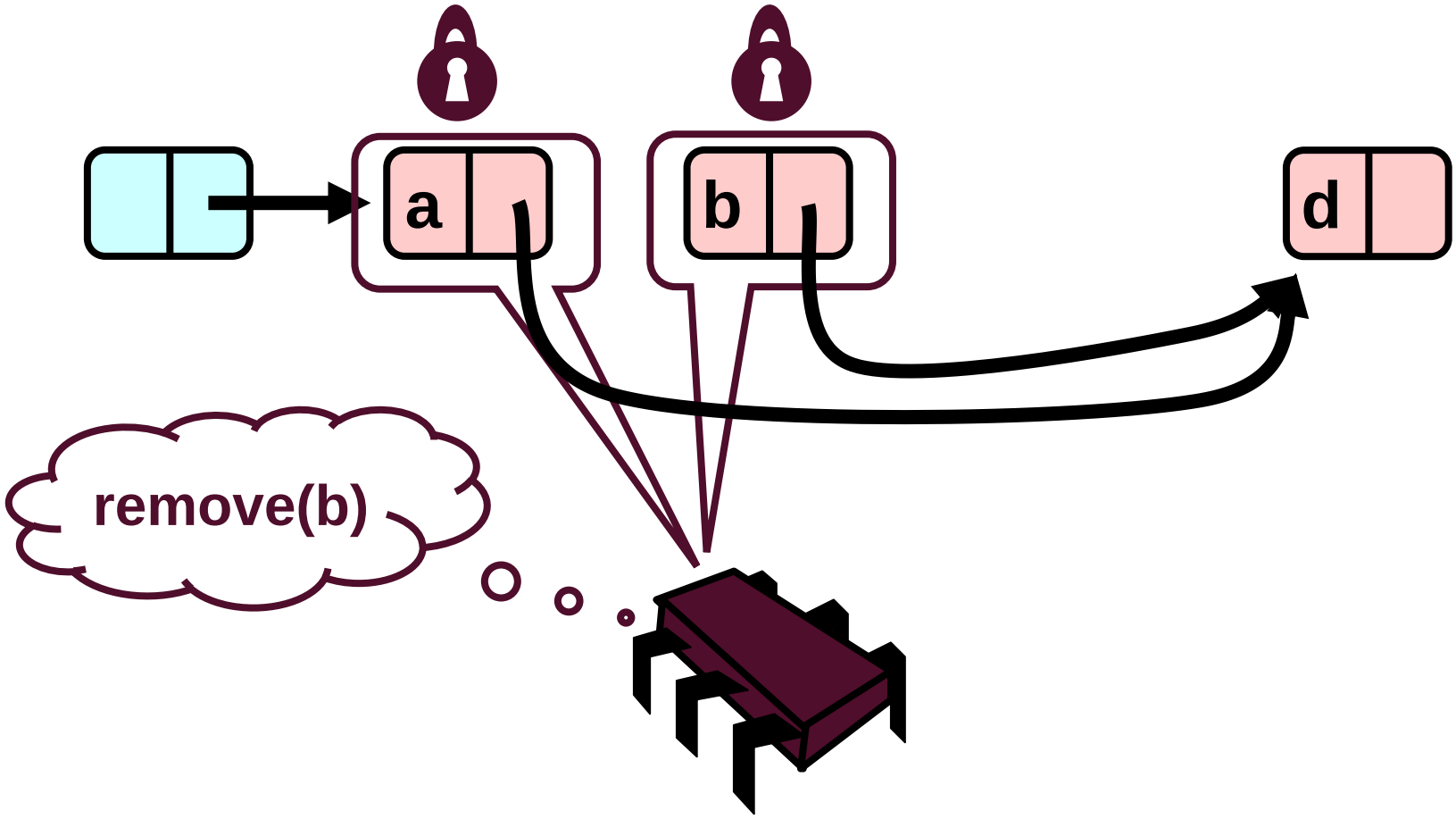

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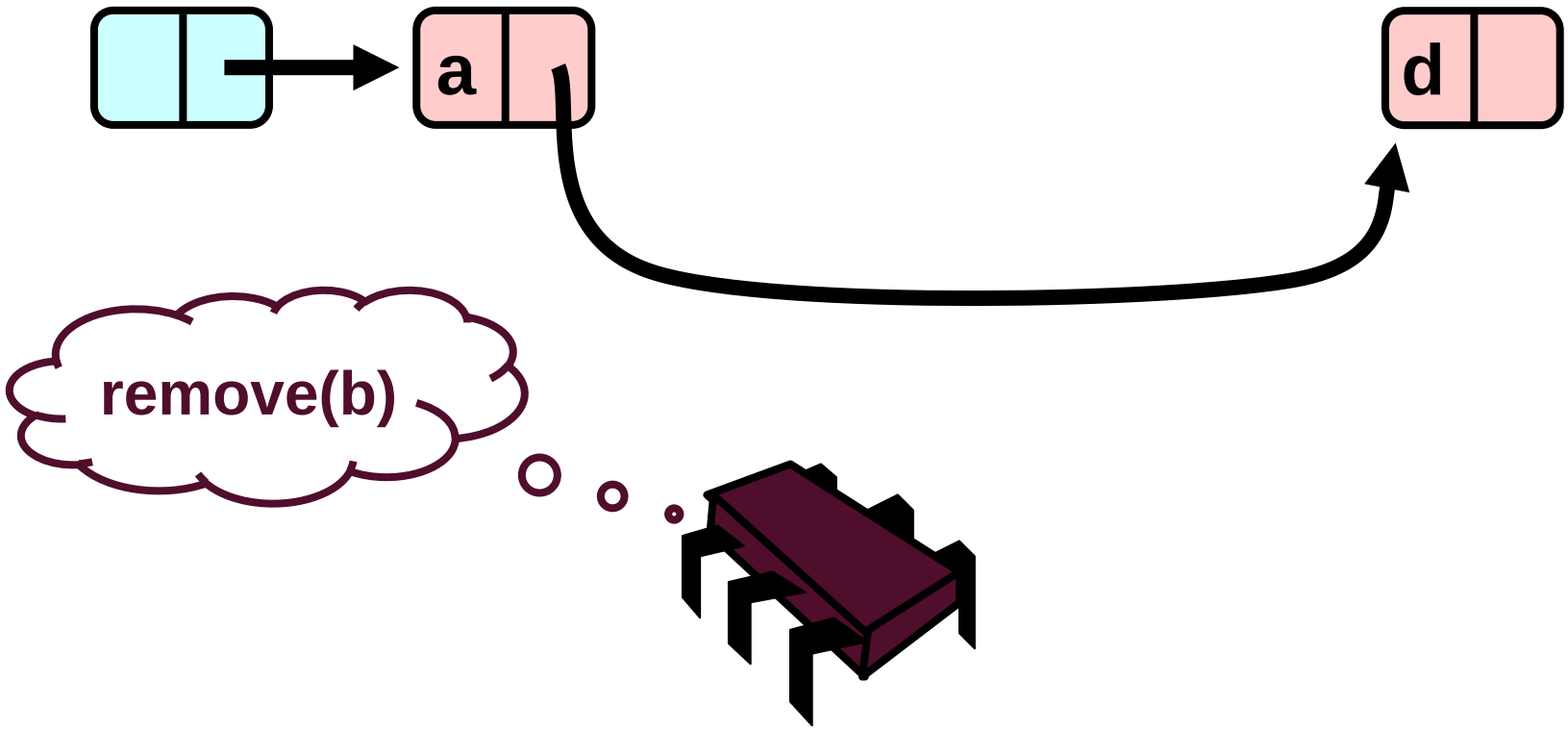


What are the Issues?

- **We have fine-grained locking, will there be contention?**
  - Yes, the list can only be traversed sequentially, a remove of the 3rd item will block all other threads!
  - This is essentially still serialized if the list is short (since threads can only pipeline on list elements)
- **Other problems, ignoring contention?**
  - Must acquire O(|S|) locks

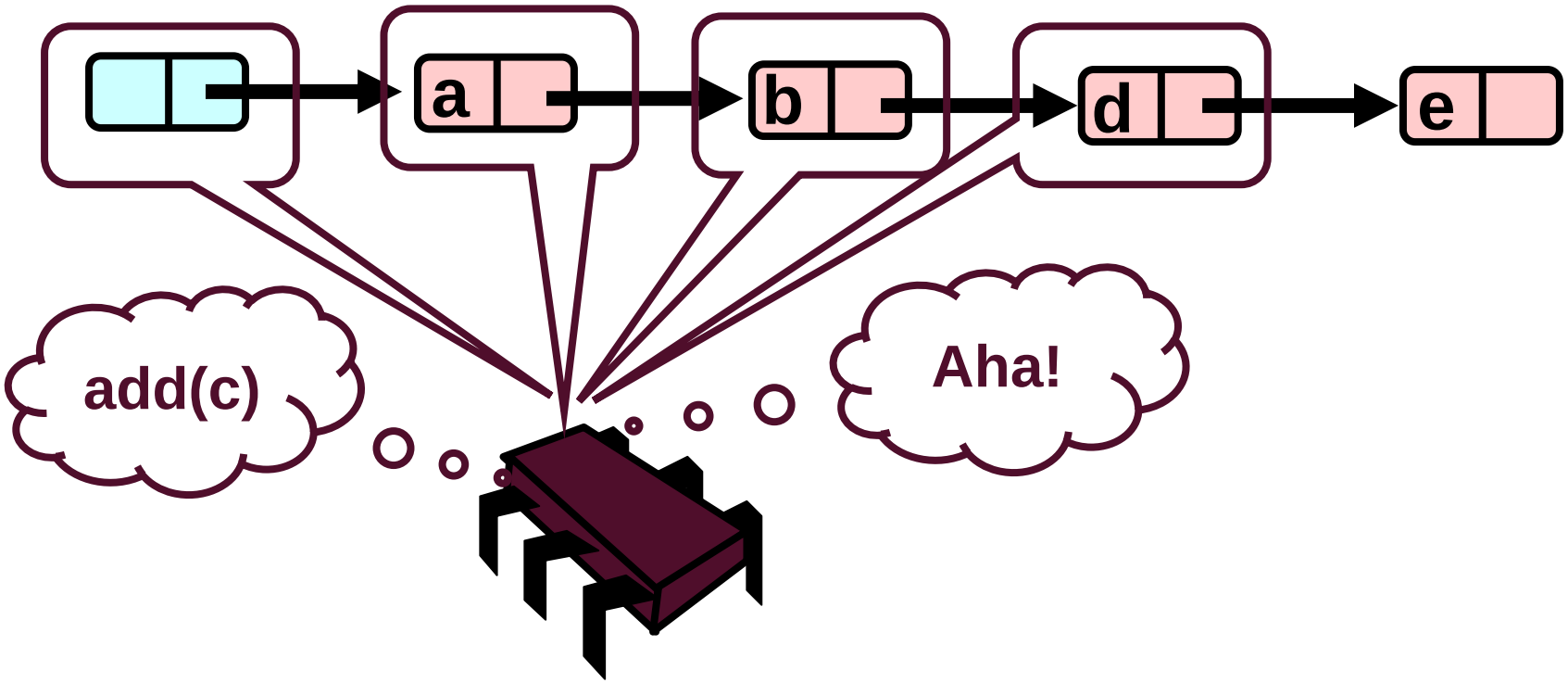
Trick 2: Reader/Writer Locking

- **Same hand-over-hand locking**
  - Traversal uses reader locks
  - Once add finds position or remove finds target node, upgrade **both** locks to writer locks
  - Need to guarantee deadlock and starvation freedom!
- **Allows truly concurrent traversals**
  - Still blocks behind writing threads
  - Still O(|S|) lock/unlock operations

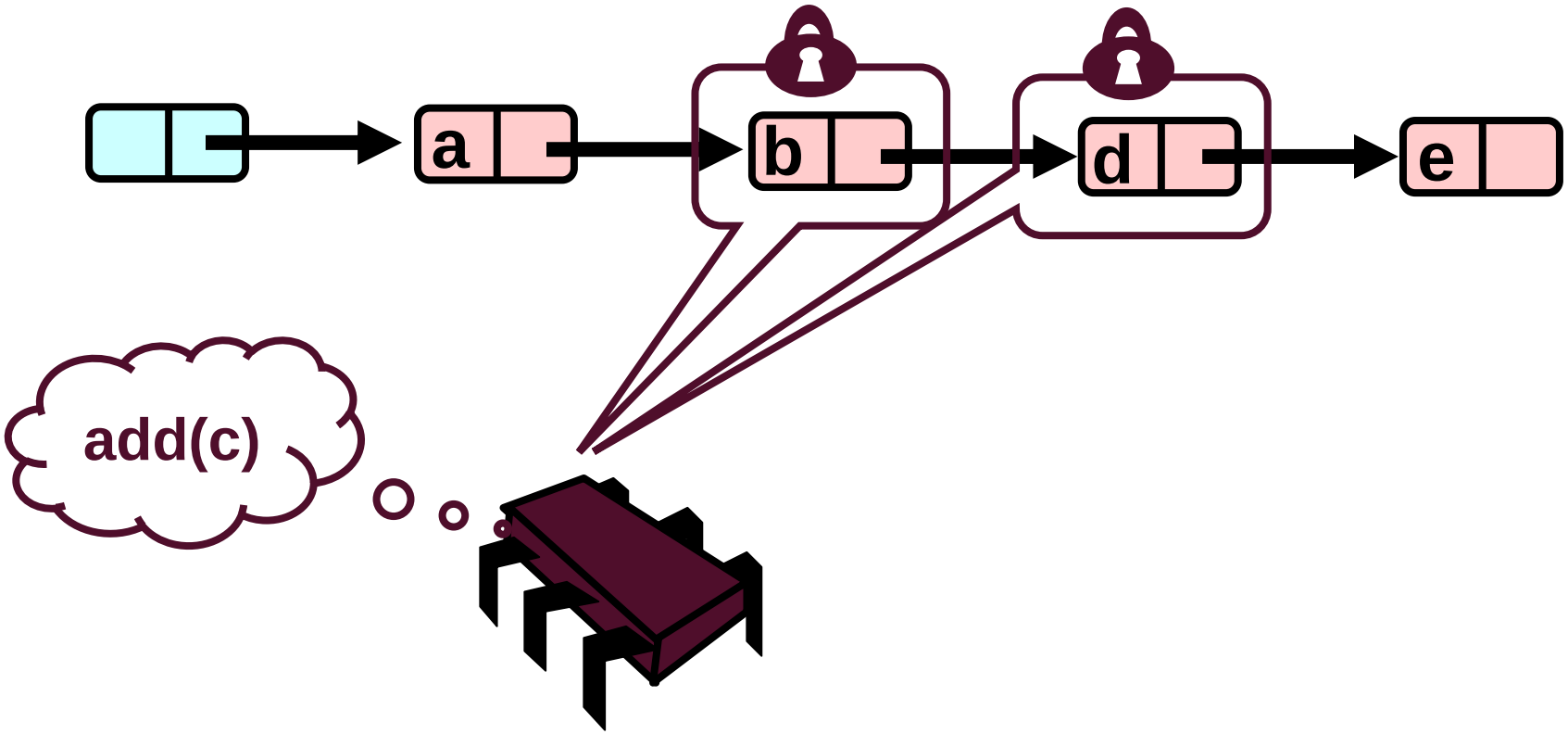
Trick 3: Optimistic synchronization

- **Similar to reader/writer locking but traverse list without locks**
  - Dangerous! Requires additional checks.
- **Harder to proof correct**

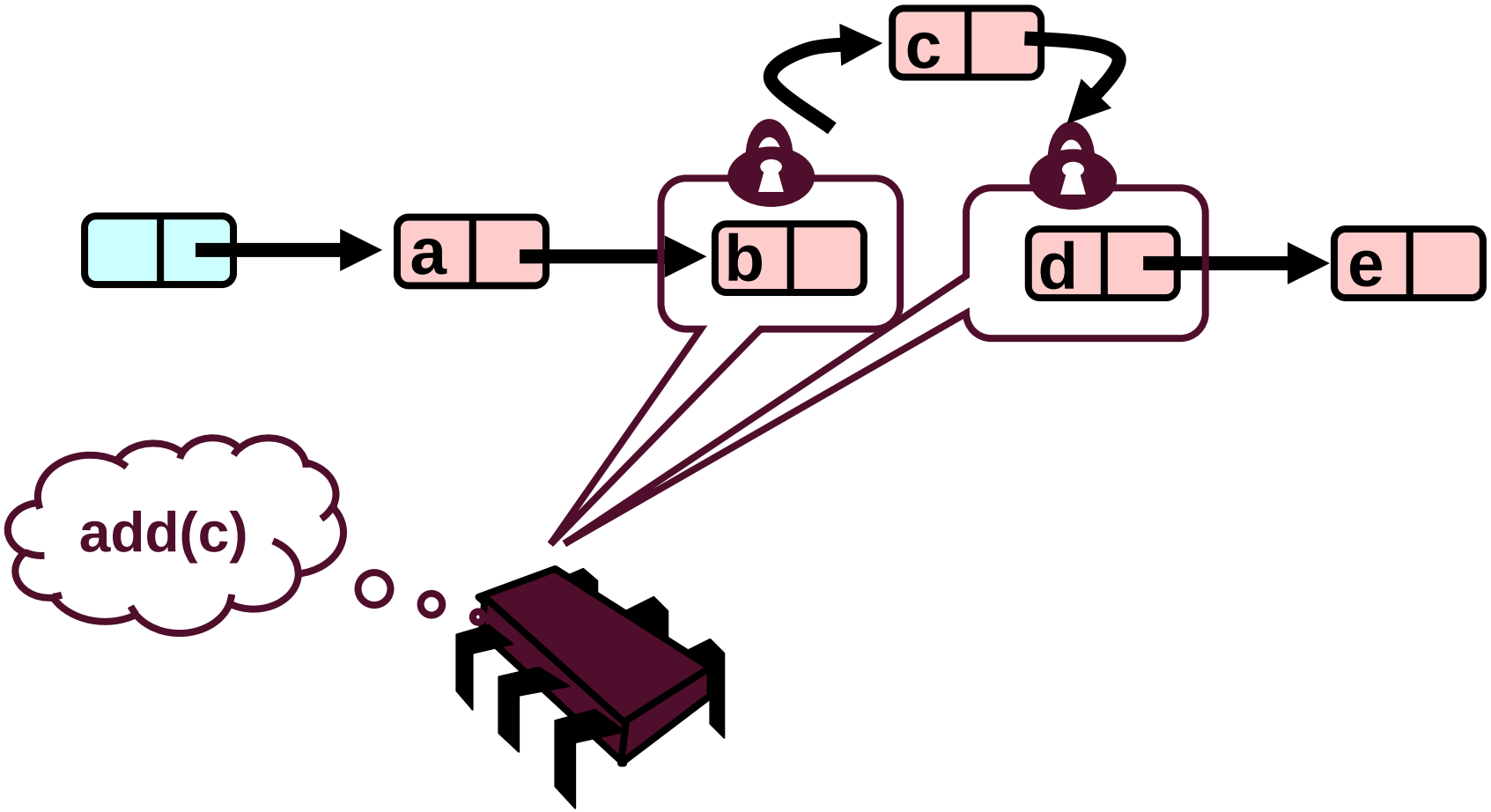
Optimistic: Traverse without Locking



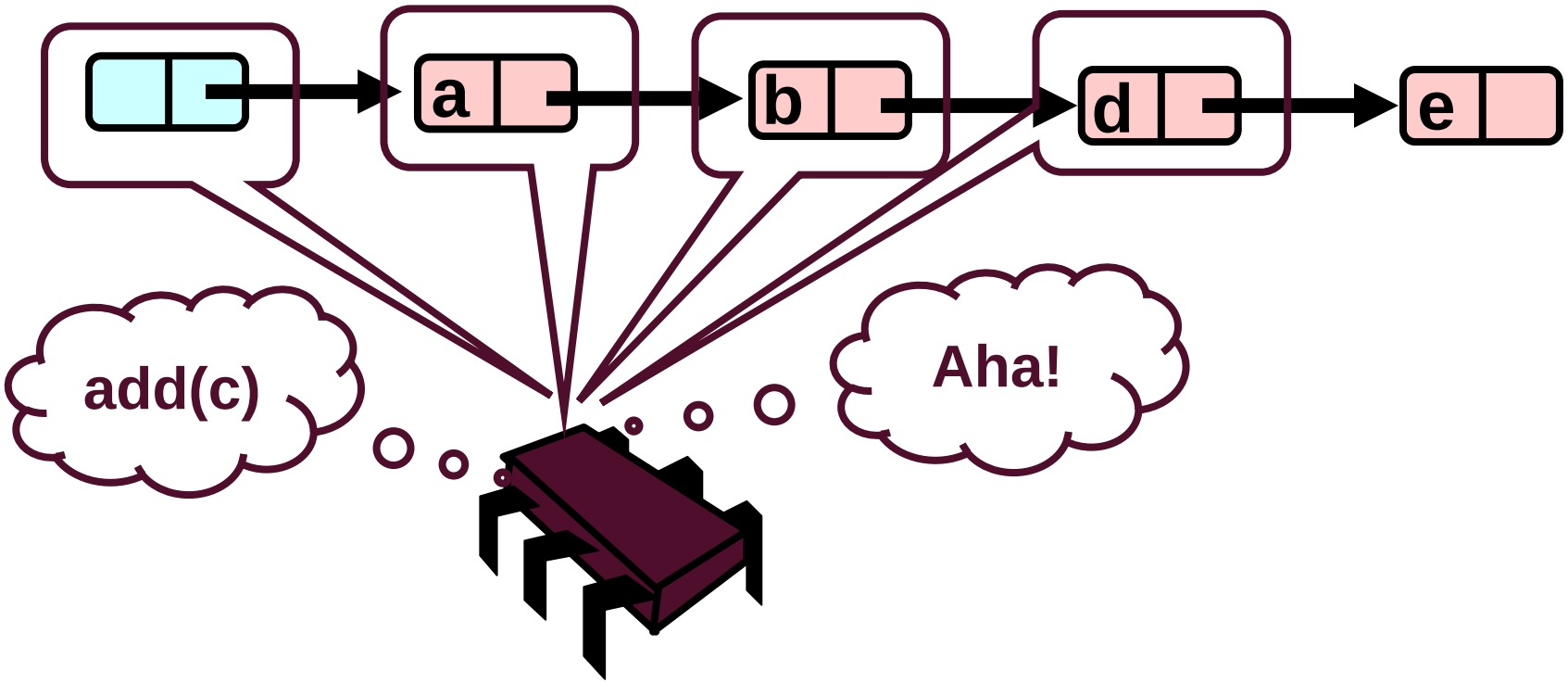
Optimistic: Lock and Load



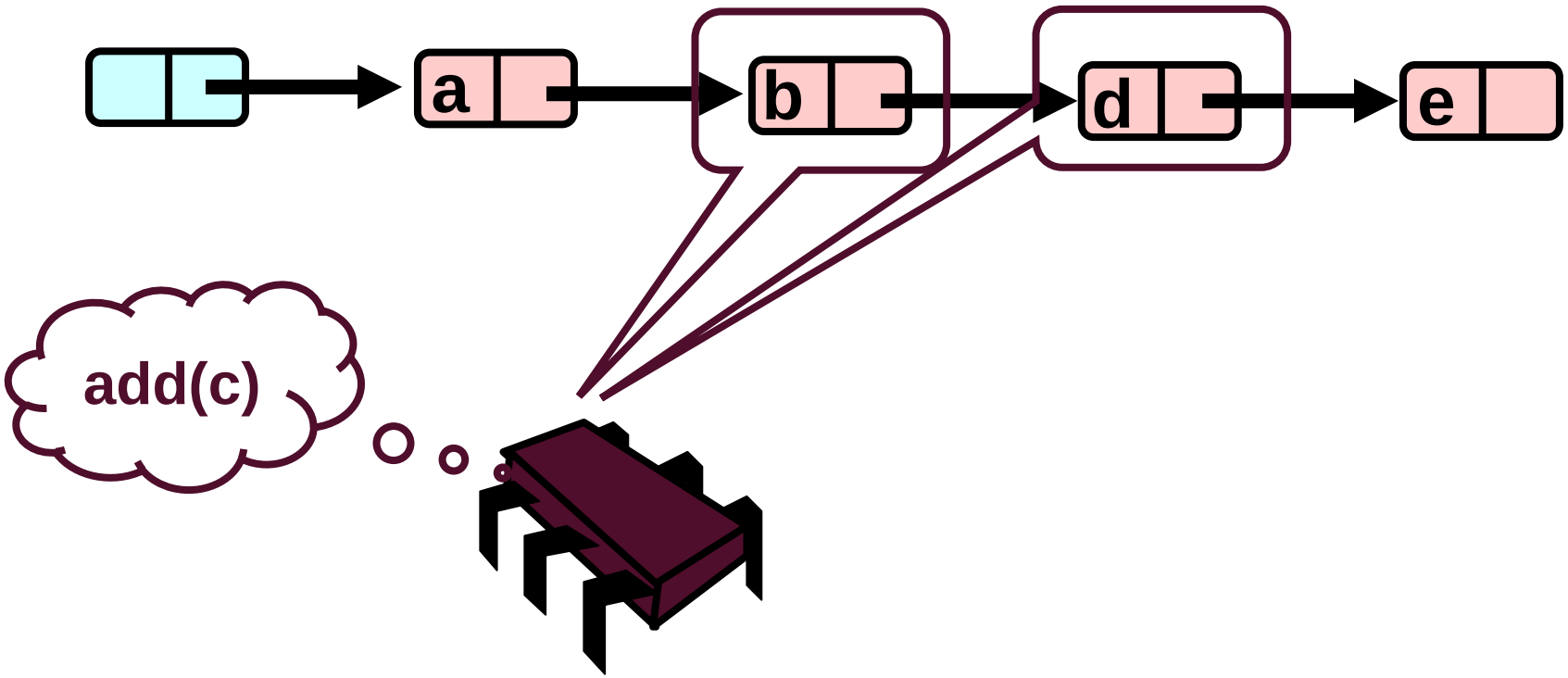
Optimistic: Lock and Load



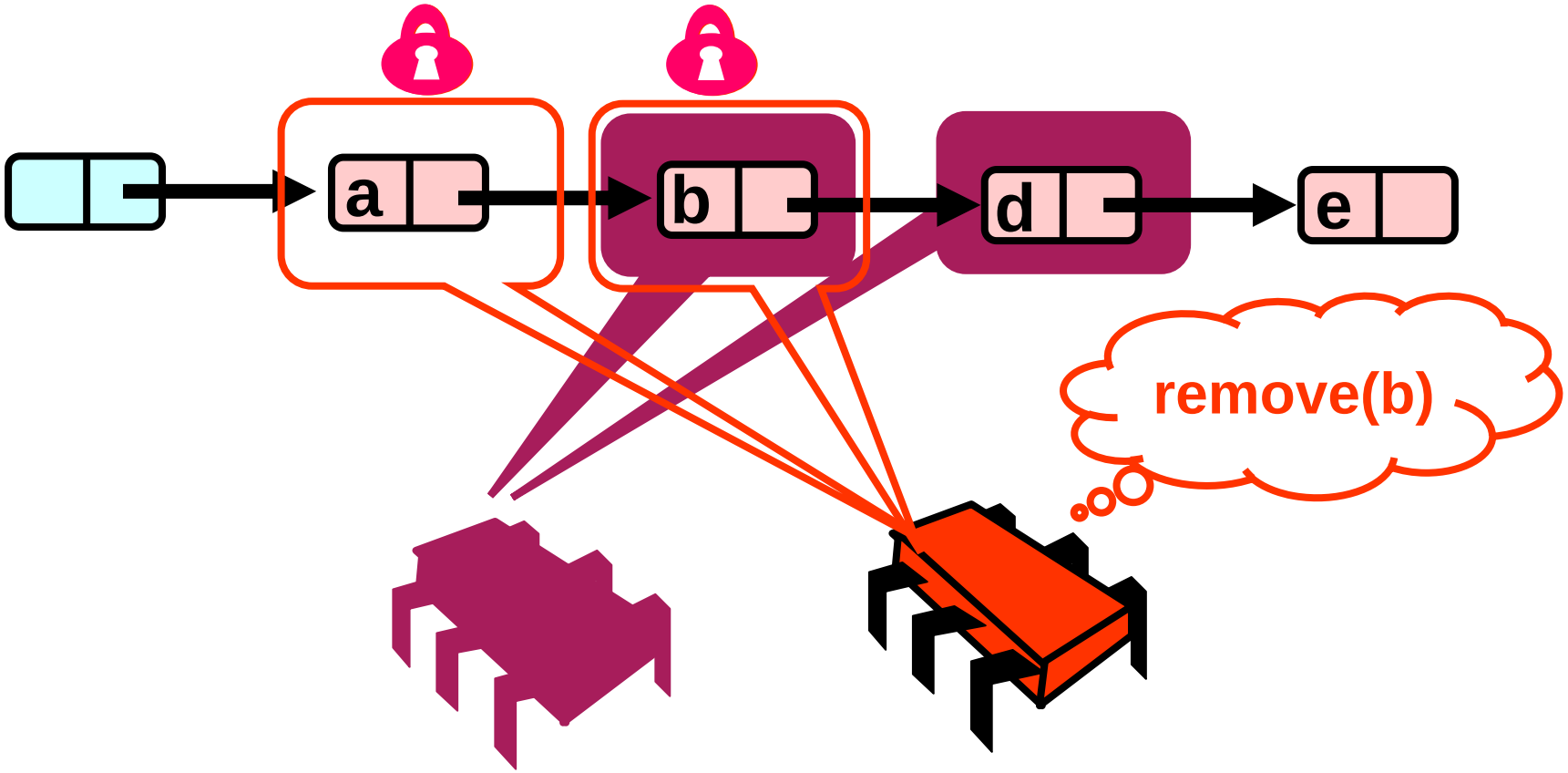
What could go wrong?



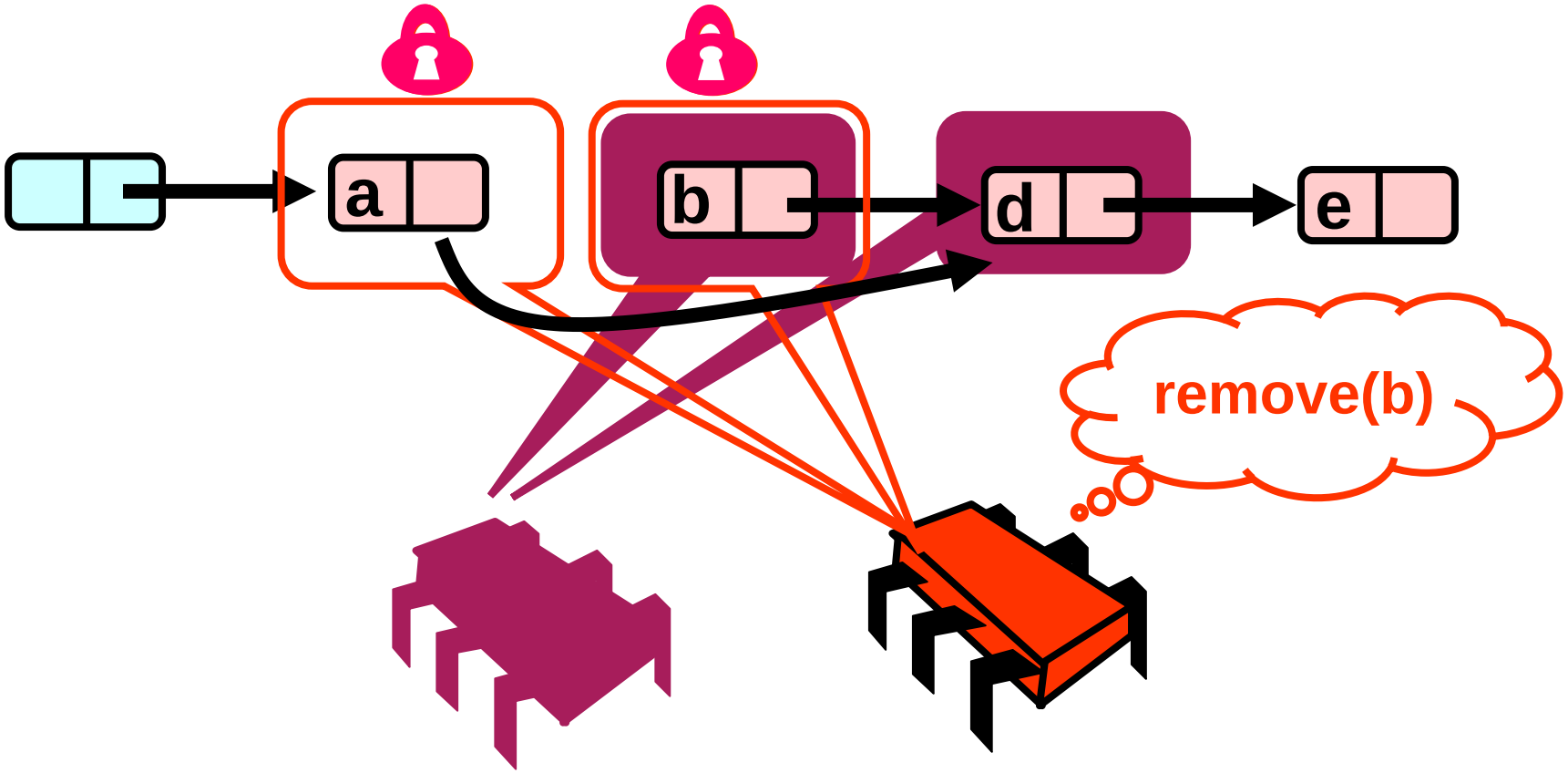
What could go wrong?



What could go wrong?



What could go wrong?



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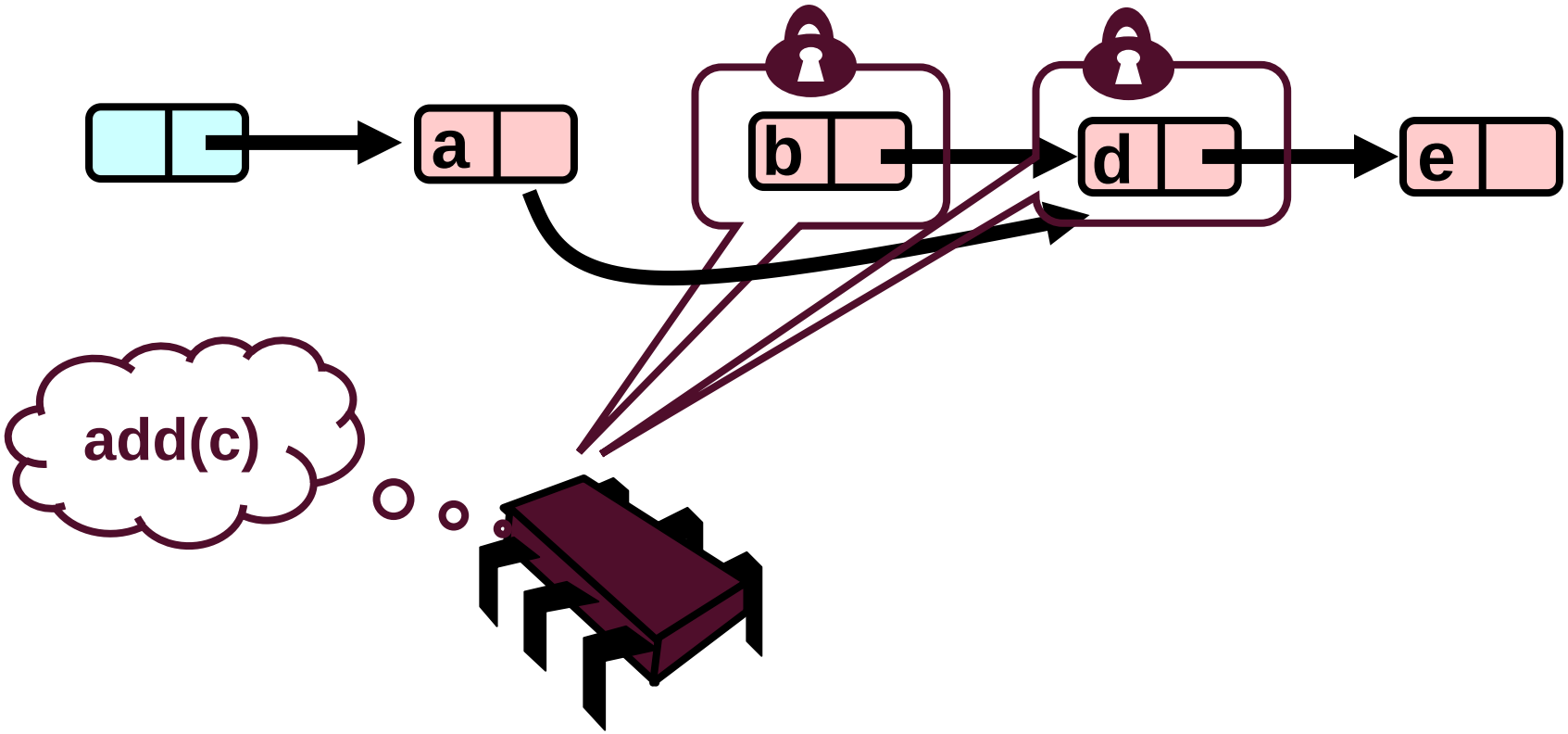
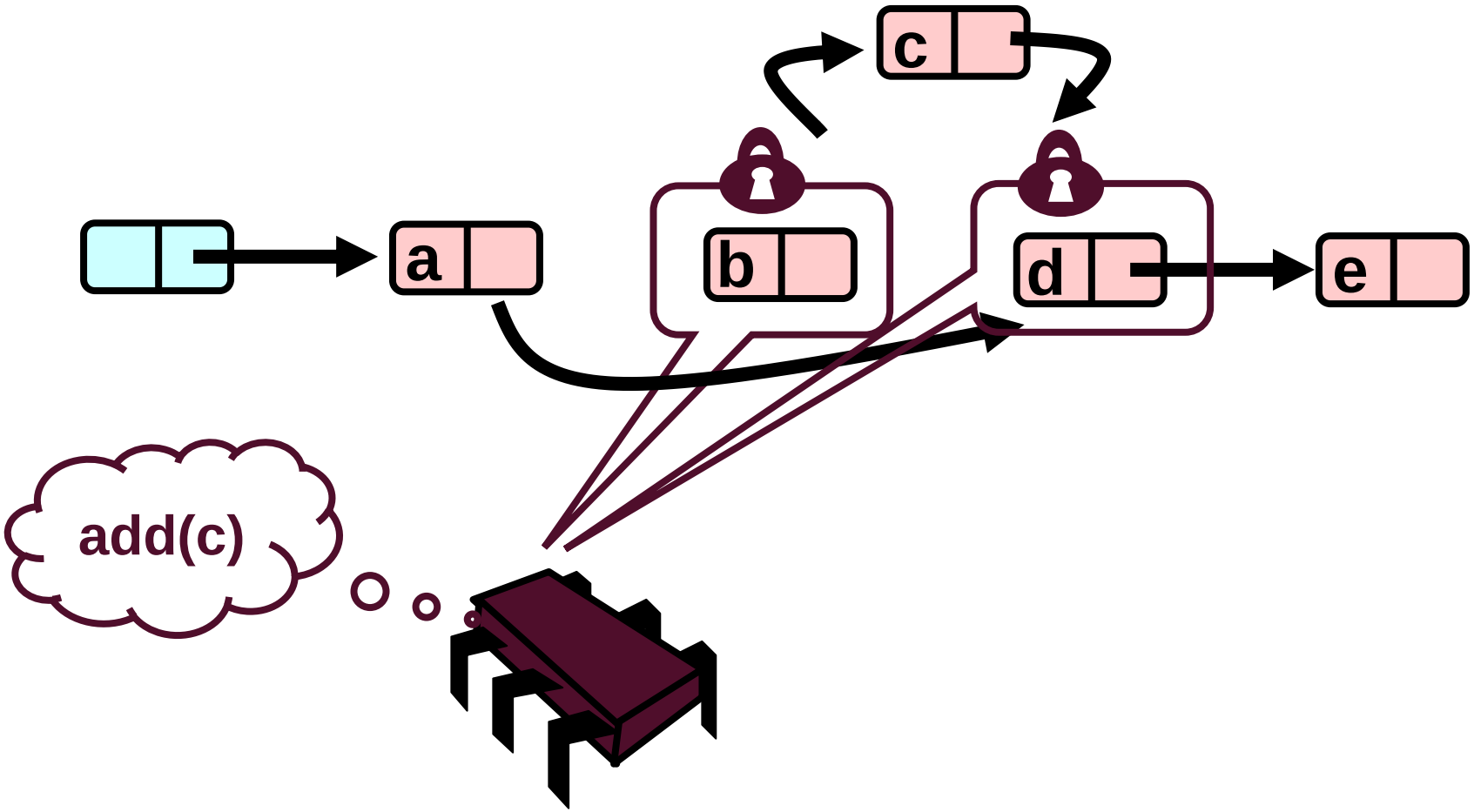
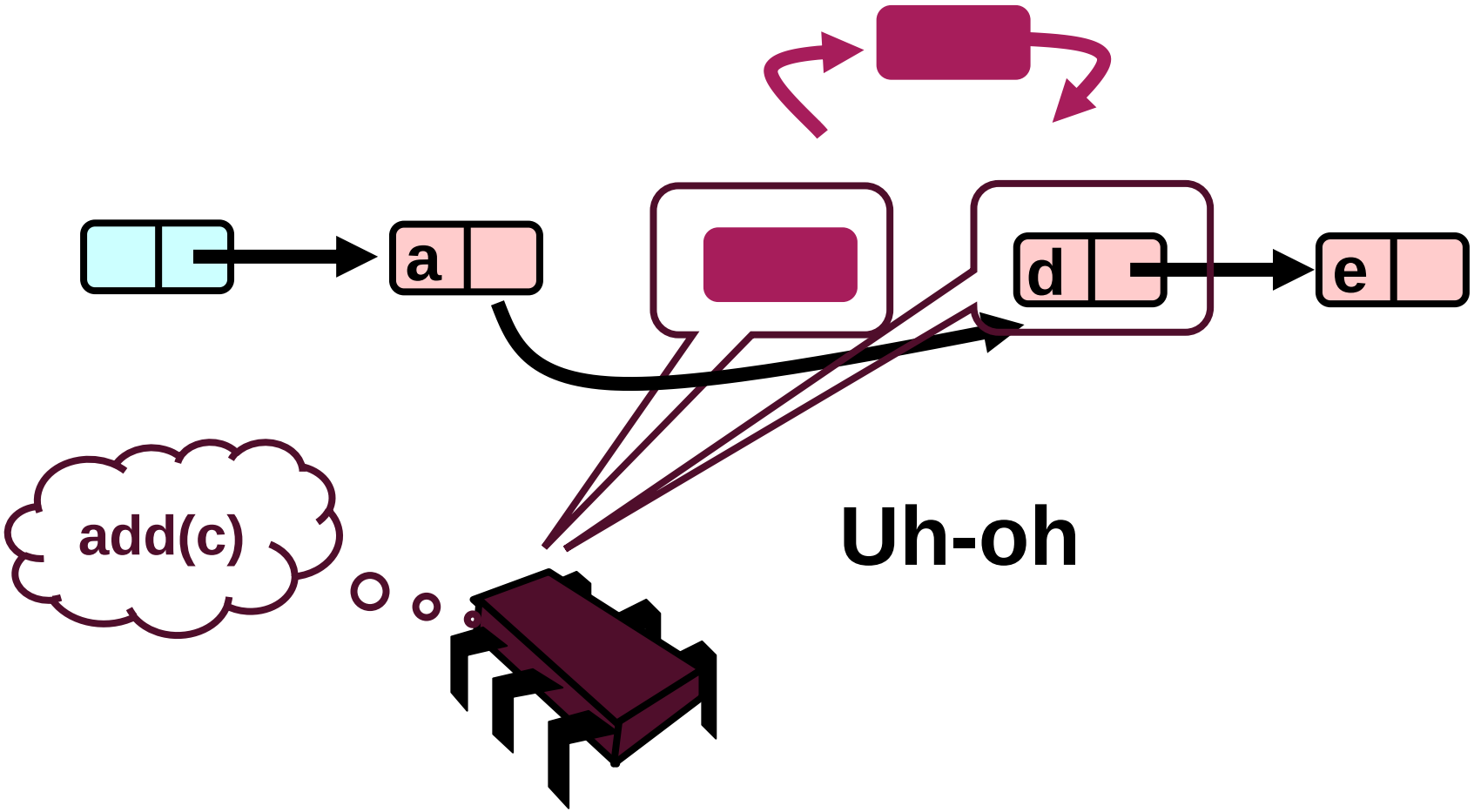


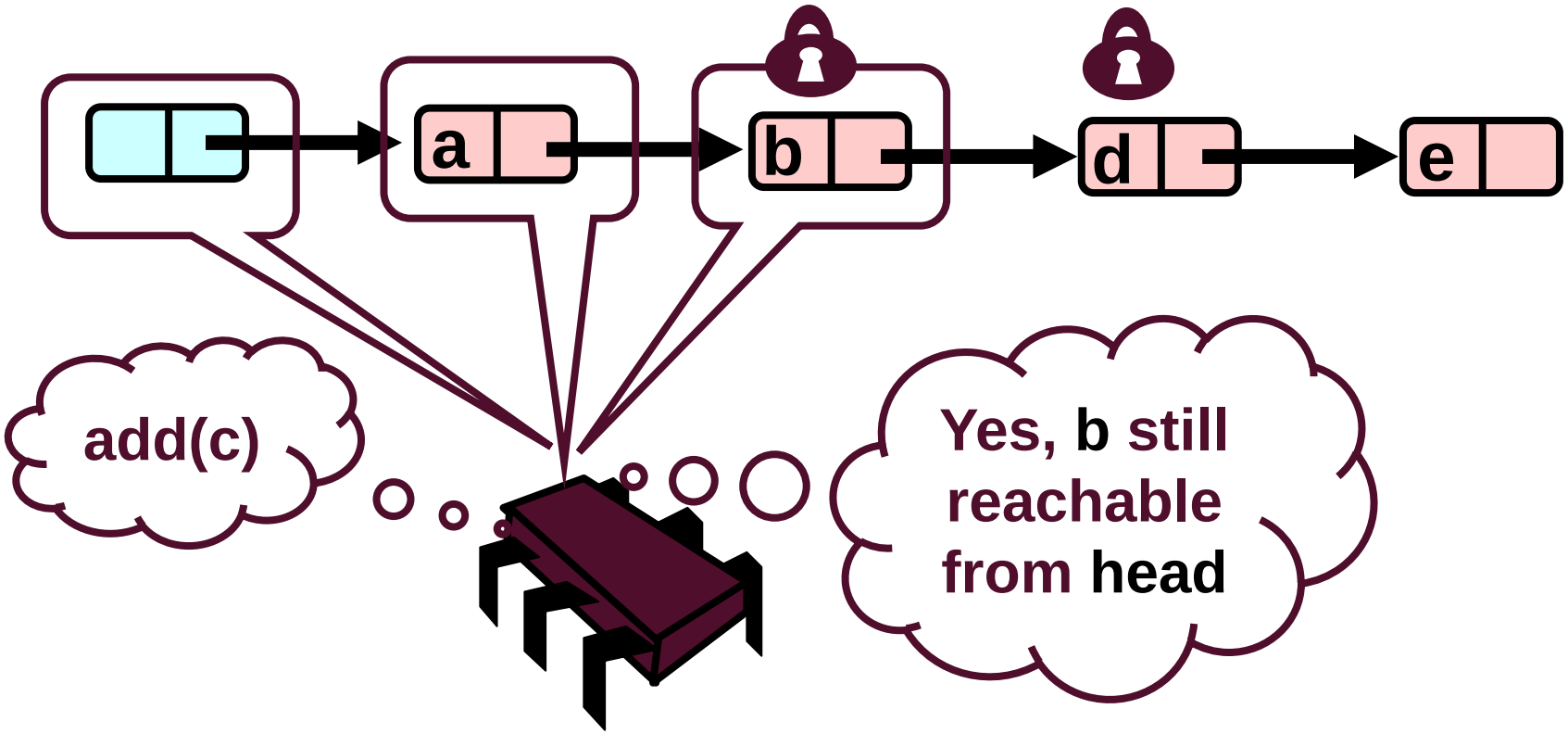
What could go wrong?



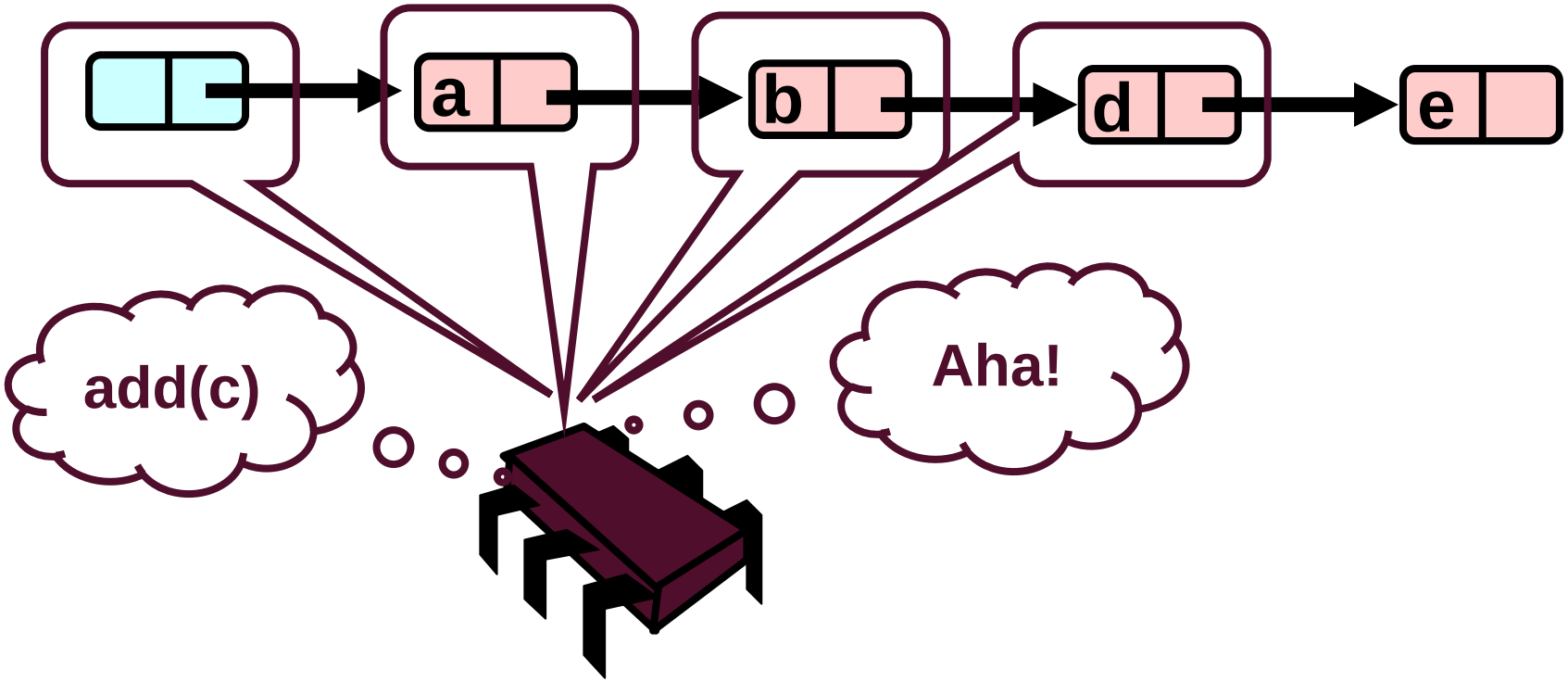
What could go wrong?



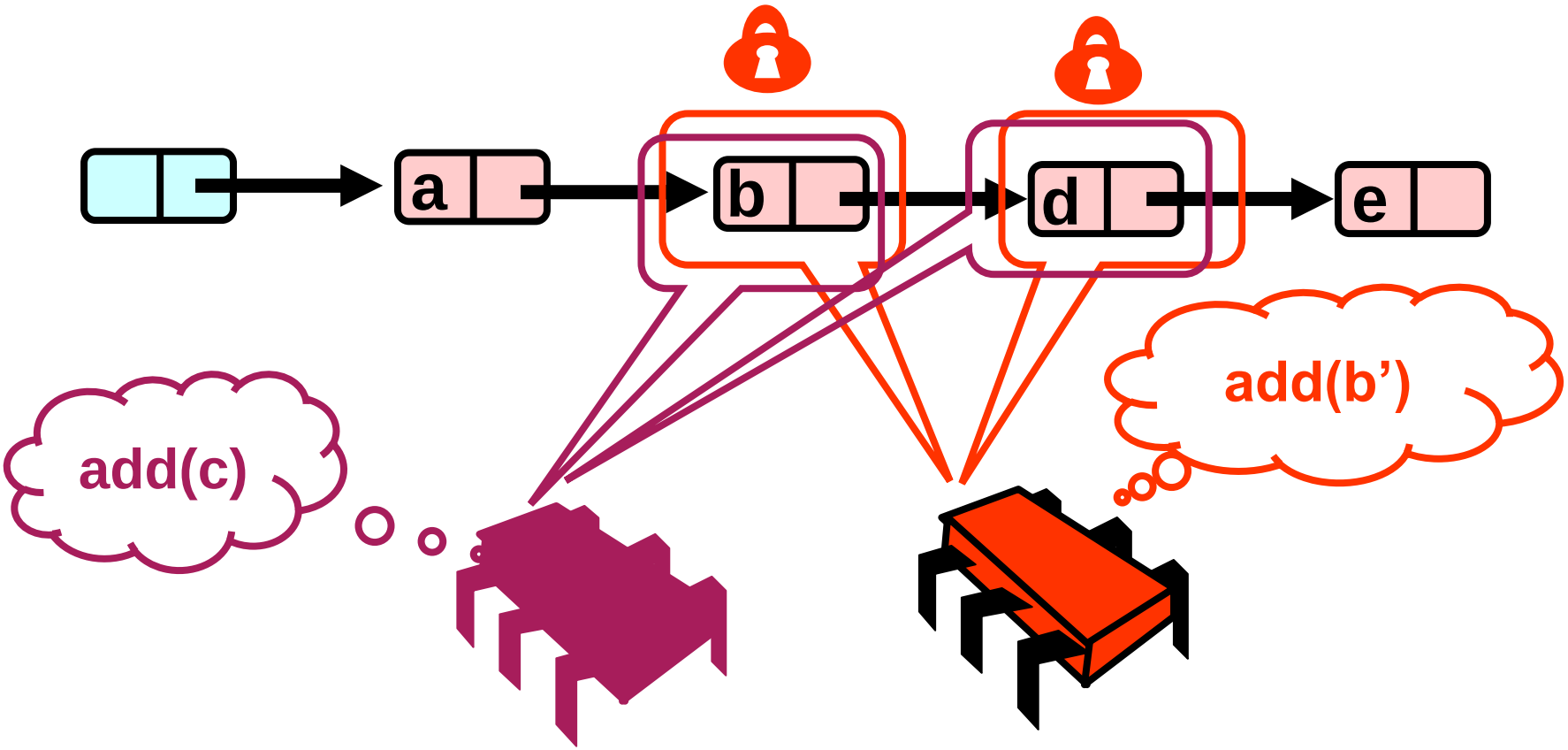
Validate – Part 1



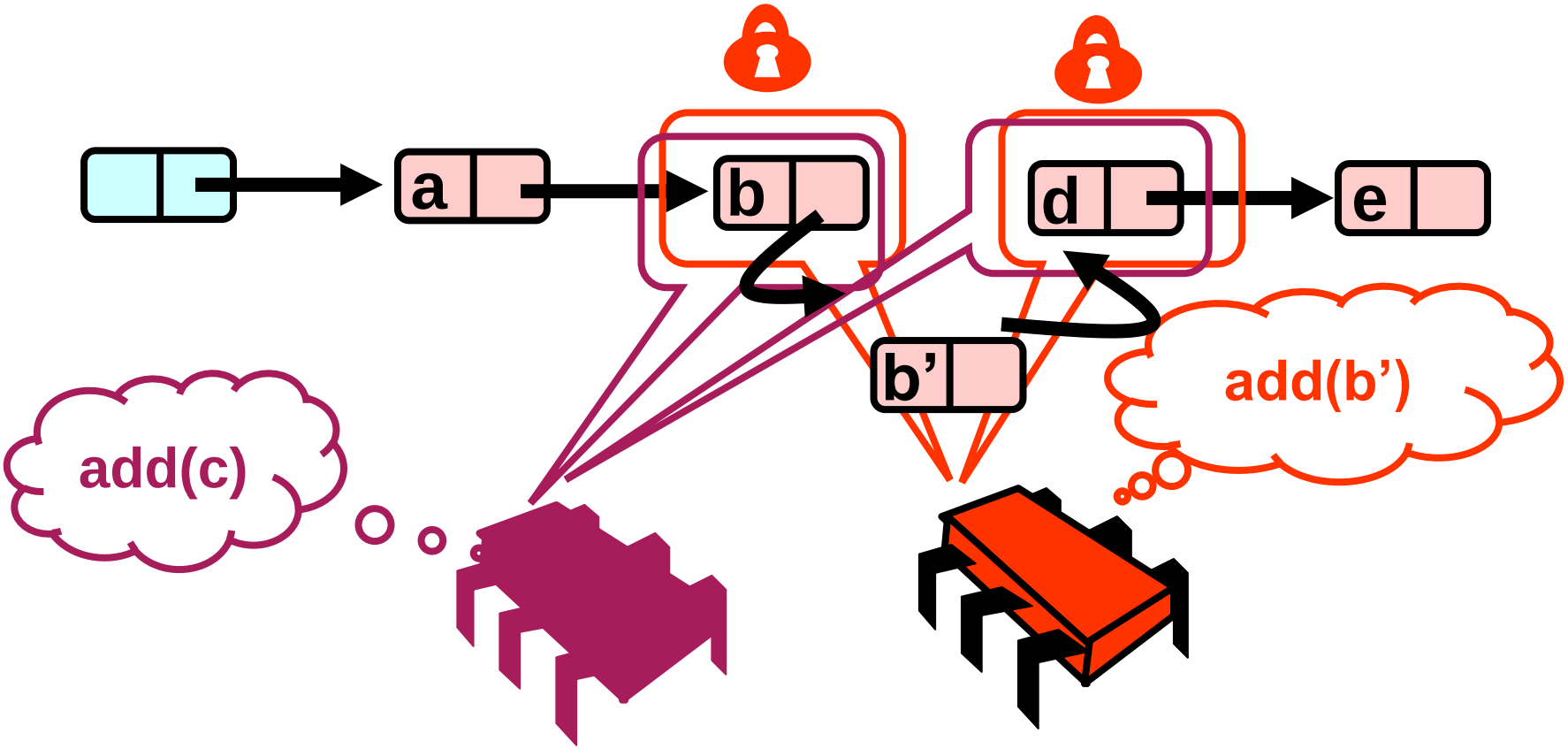
What Else Could Go Wrong?



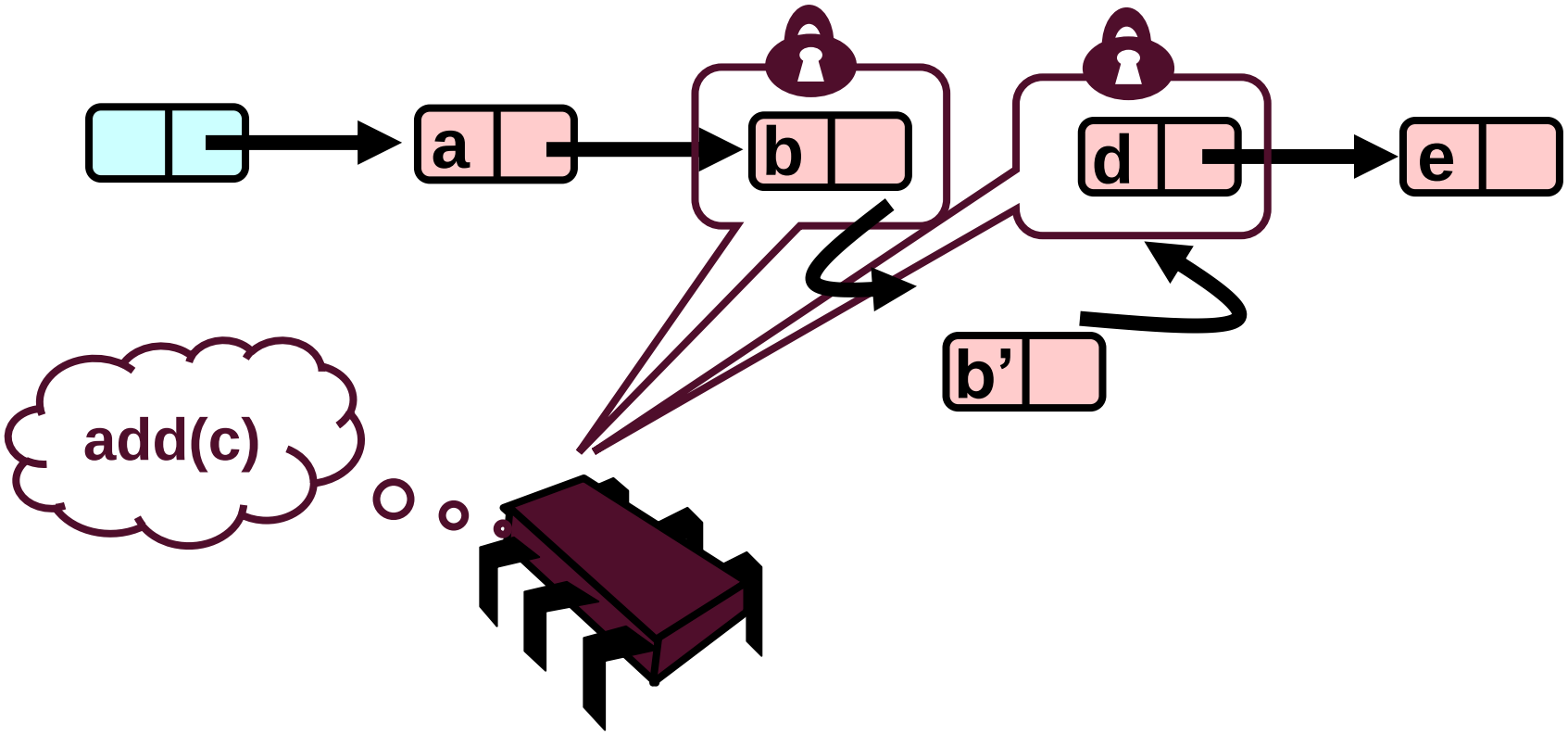
What Else Could Go Wrong?



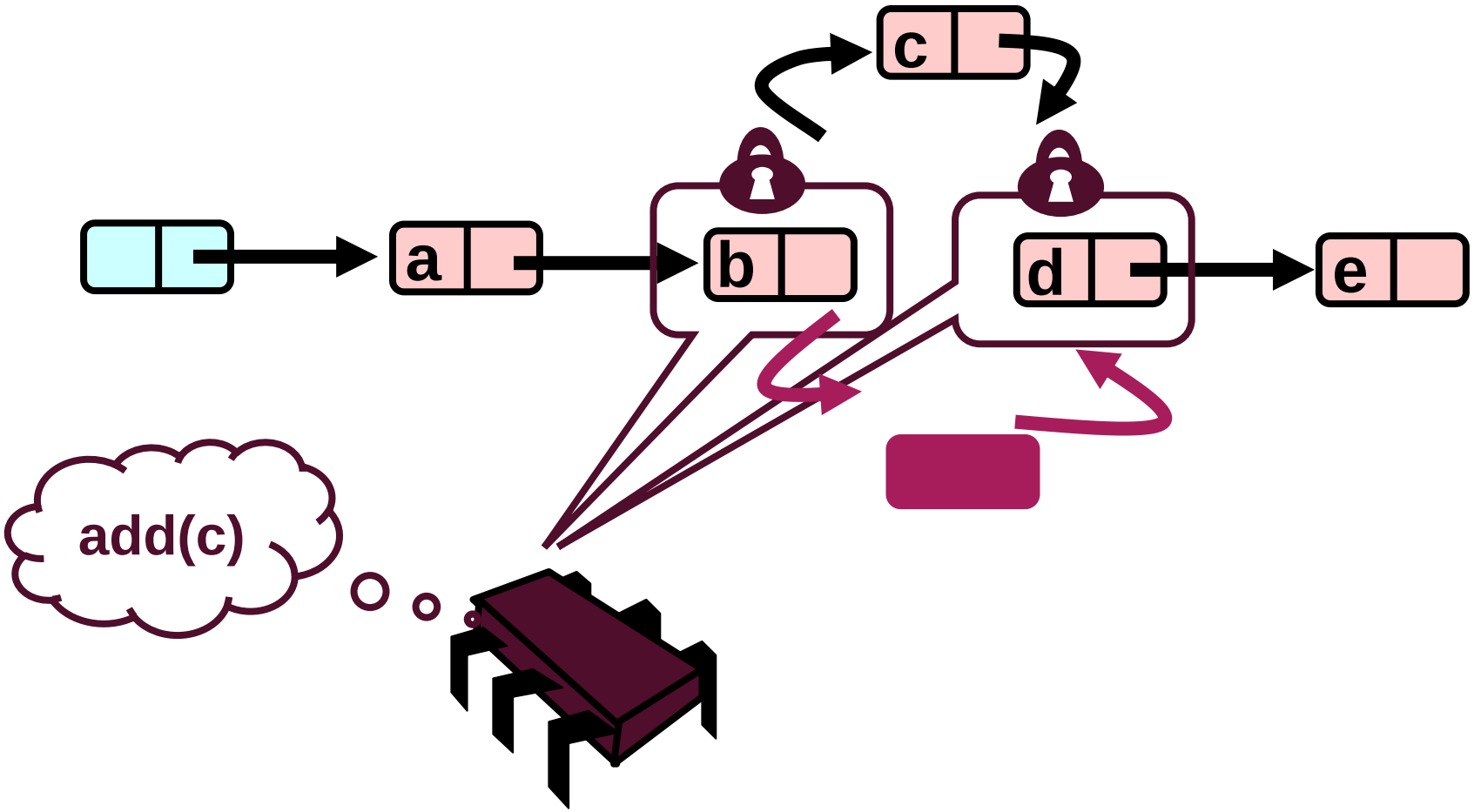
What Else Could Go Wrong?



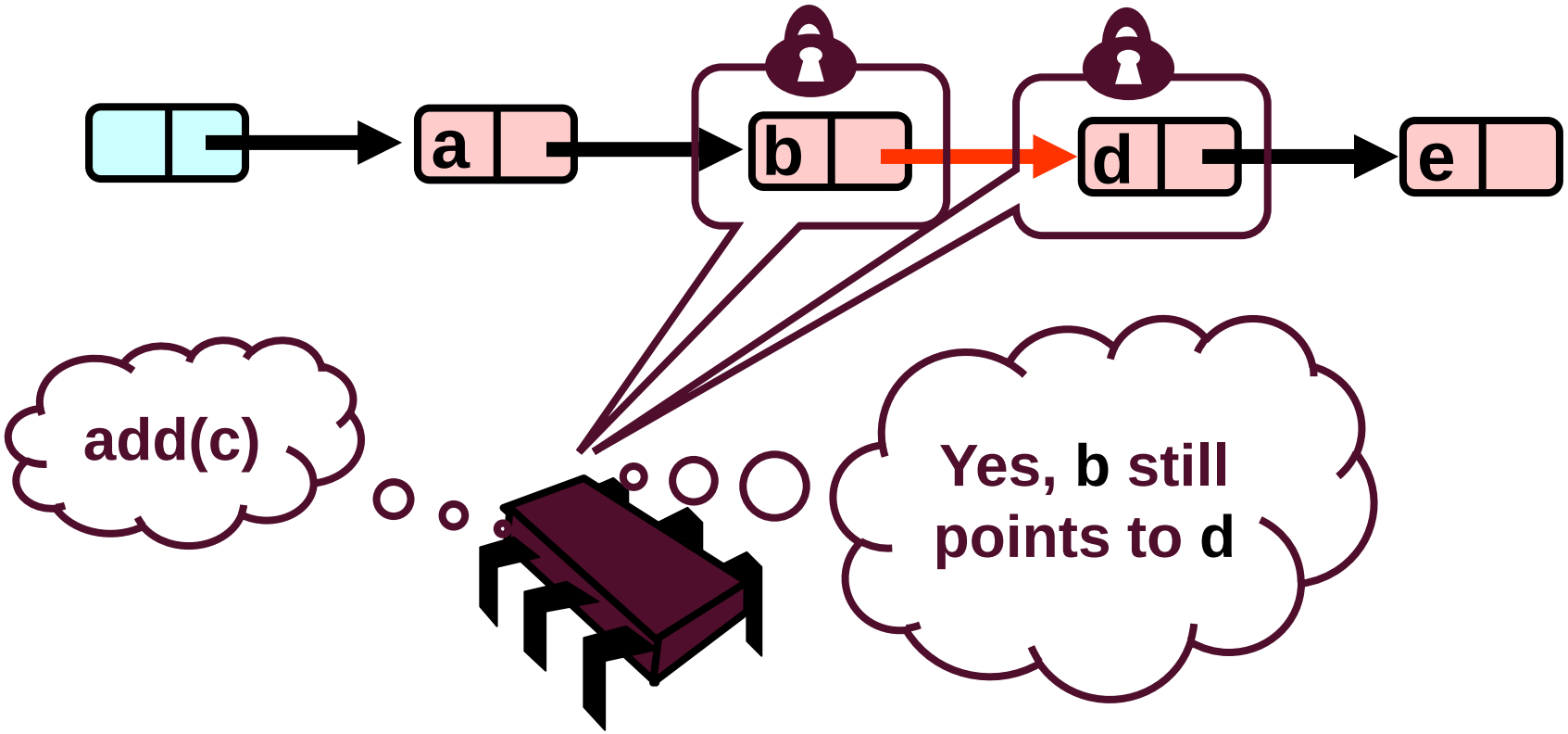
What Else Could Go Wrong?



What Else Could Go Wrong?



Validate Part 2
(while holding locks)



Optimistic synchronization

- **One MUST validate AFTER locking**
  1. Check if the path how we got there is still valid!
  2. Check if locked nodes are still connected
  - If any of those checks fail?

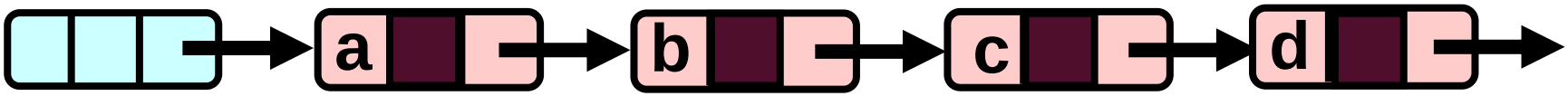
    Start over from the beginning (hopefully rare)
- **Not starvation-free**
  - A thread may need to abort forever if nodes are added/removed
  - Should be rare in practice!
- **Other disadvantages?**
  - All operations require two traversals of the list!
  - Even contains() needs to check if node is still in the list!

Trick 4: Lazy synchronization

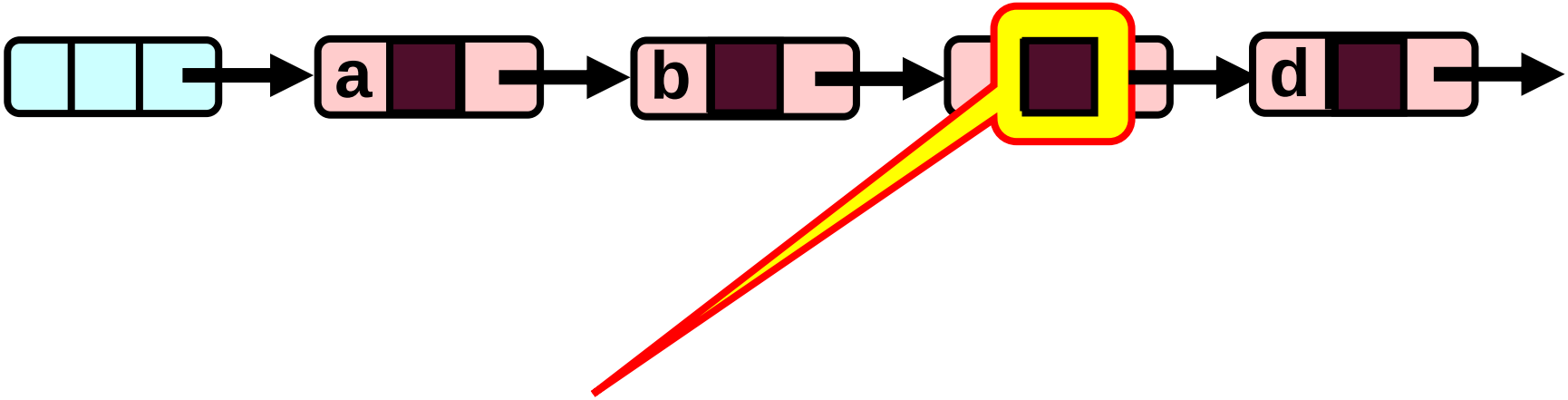
- **We really want one list traversal**
- **Also, contains() should be wait-free**
  - Is probably the most-used operation
- **Lazy locking is similar to optimistic**
  - Key insight: removing is problematic
  - Perform it "lazily"
- **Add a new "valid" field**
  - Indicates if node is still in the set
  - Can remove it without changing list structure!
  - Scan once, contains() never locks!

```
typedef struct {
  int key;
  node *next;
  lock_t lock;
  boolean valid;
} node;
```

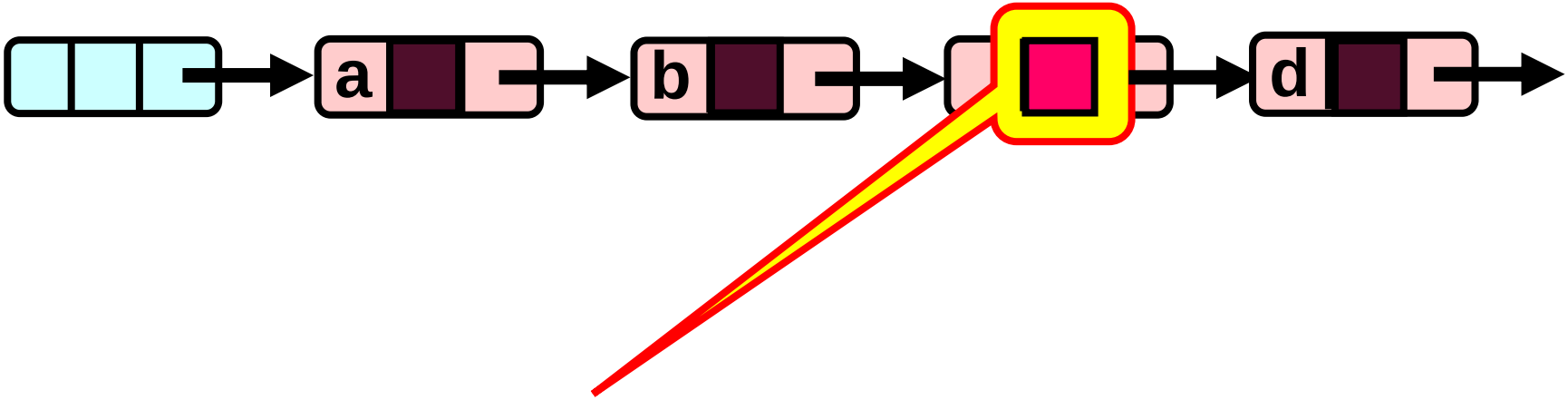
Lazy Removal



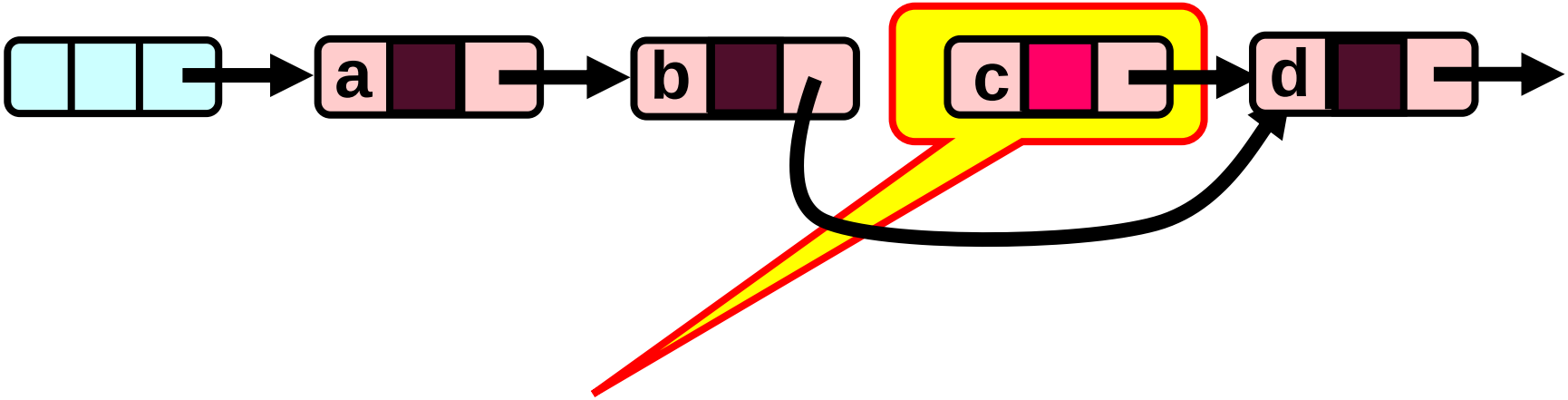
Lazy Removal



Lazy Removal

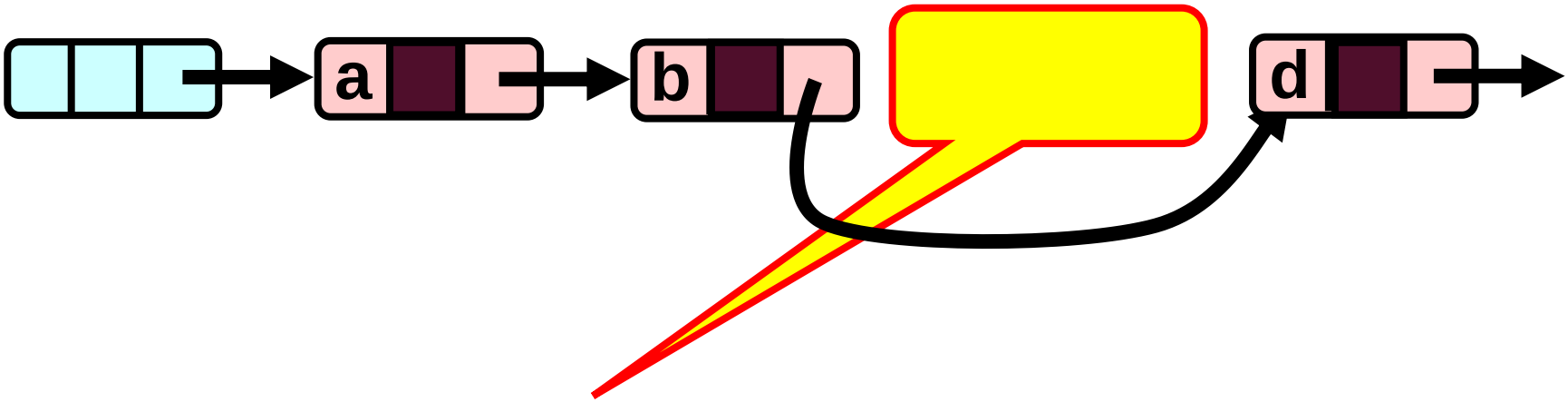


Lazy Removal



Physically deleted

Lazy Removal

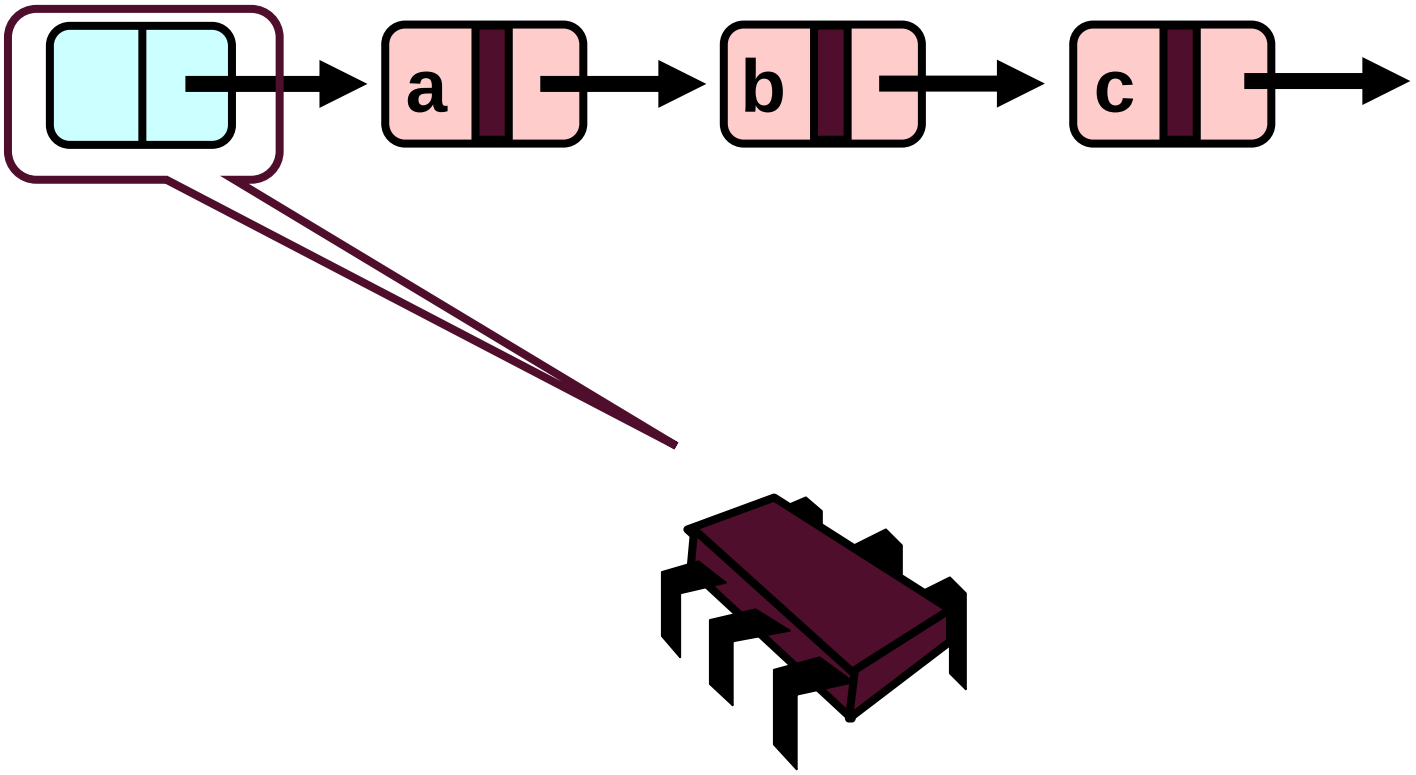


How does it work?

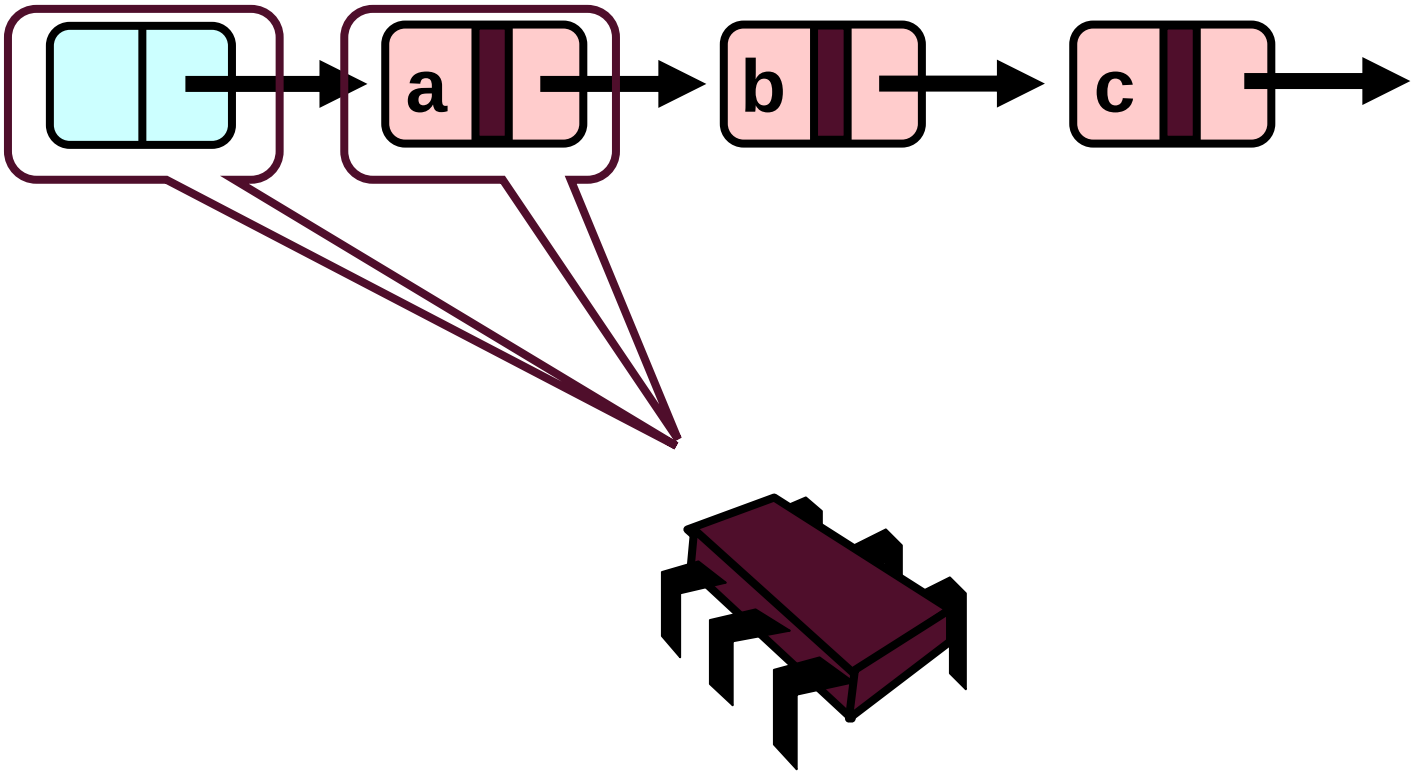
- **Eliminates need to re-scan list for reachability**
  - Maintains invariant that every **unmarked** node is reachable!
- **Contains can now simply traverse the list**
  - Just check marks, not reachability, no locks
- **Remove/Add**
  - Scan through locked and marked nodes
  - Removing does not delay others
  - Must only lock when list structure is updated

    Check if neither pred nor curr are marked, pred.next == curr

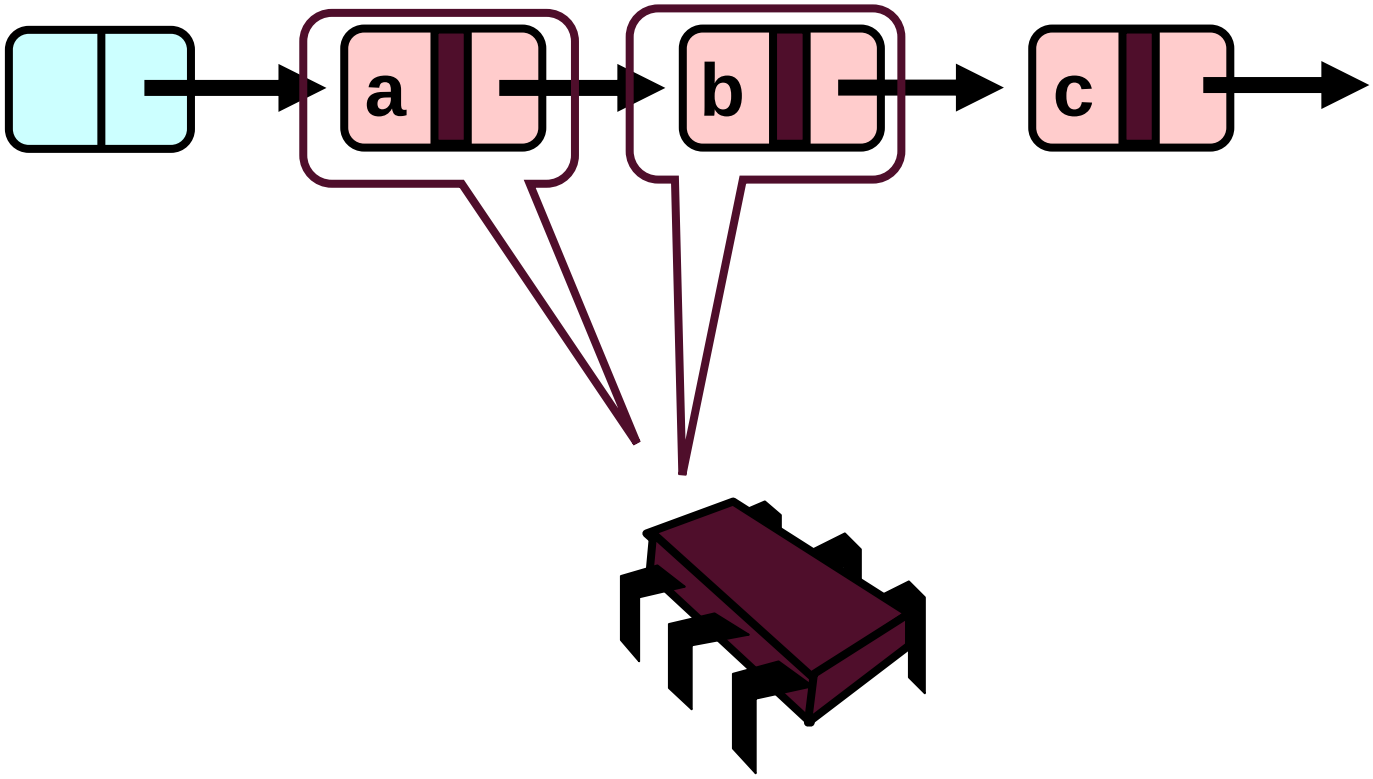
Business as Usual



Business as Usual



Business as Usual



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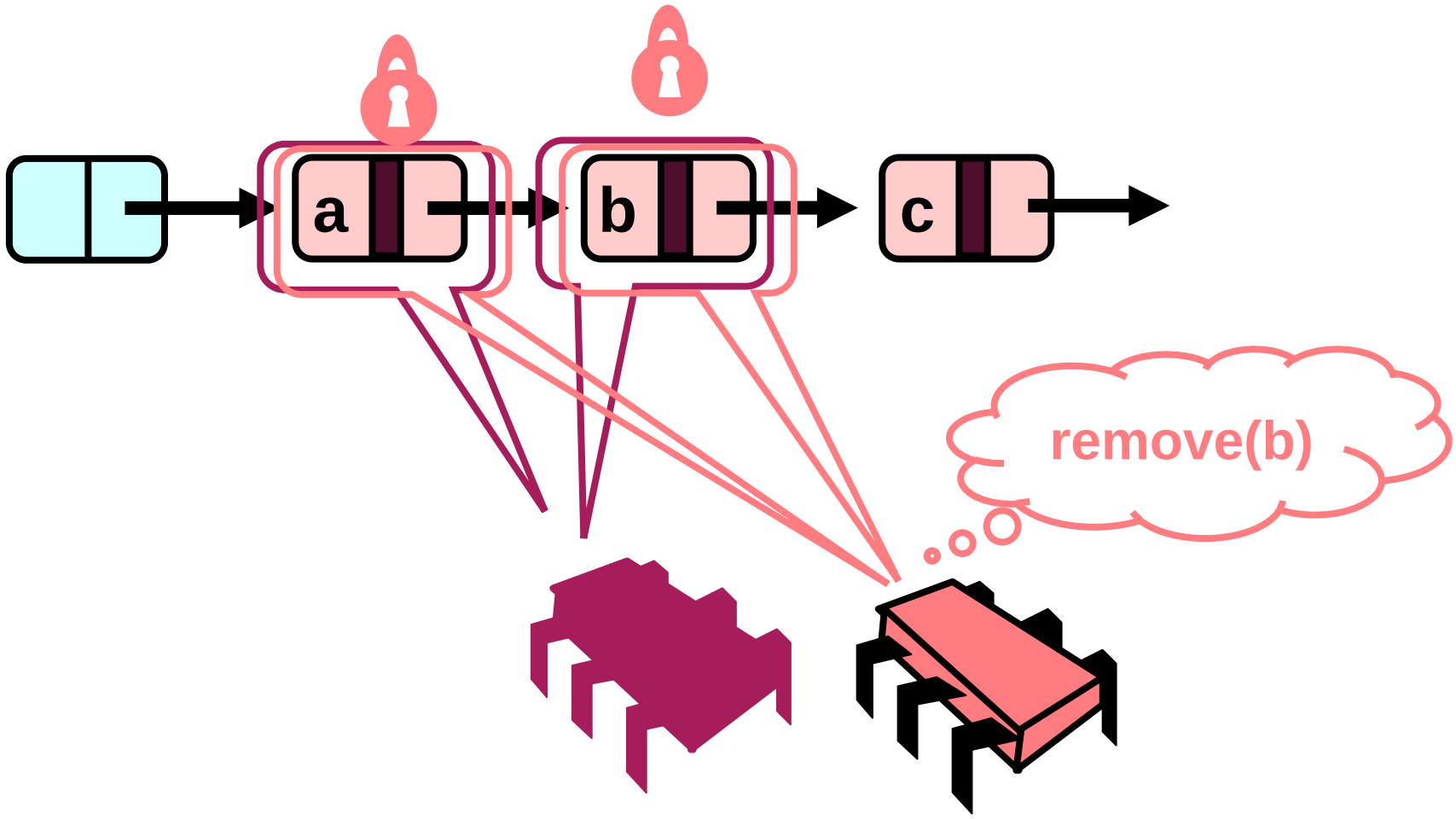

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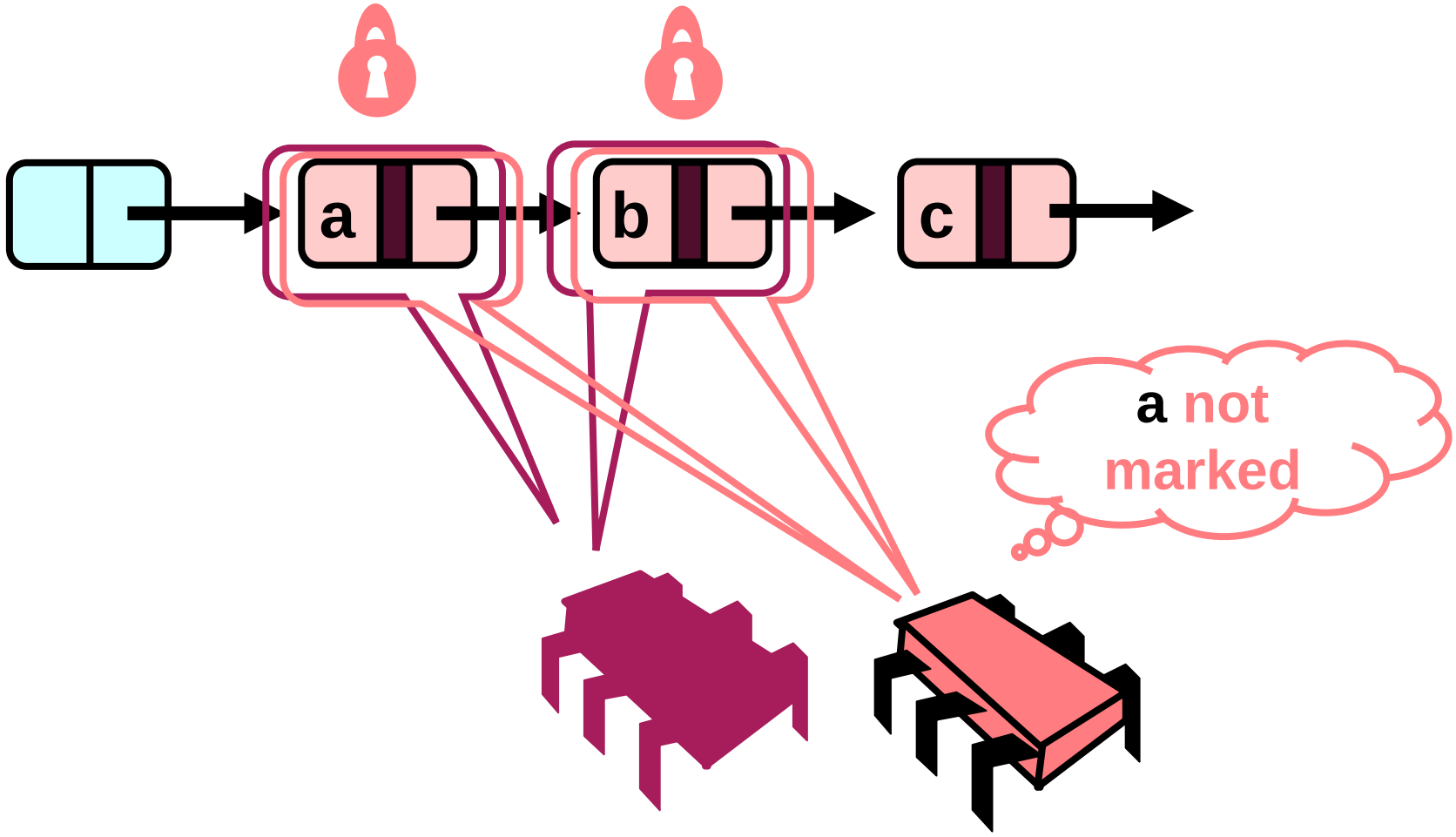

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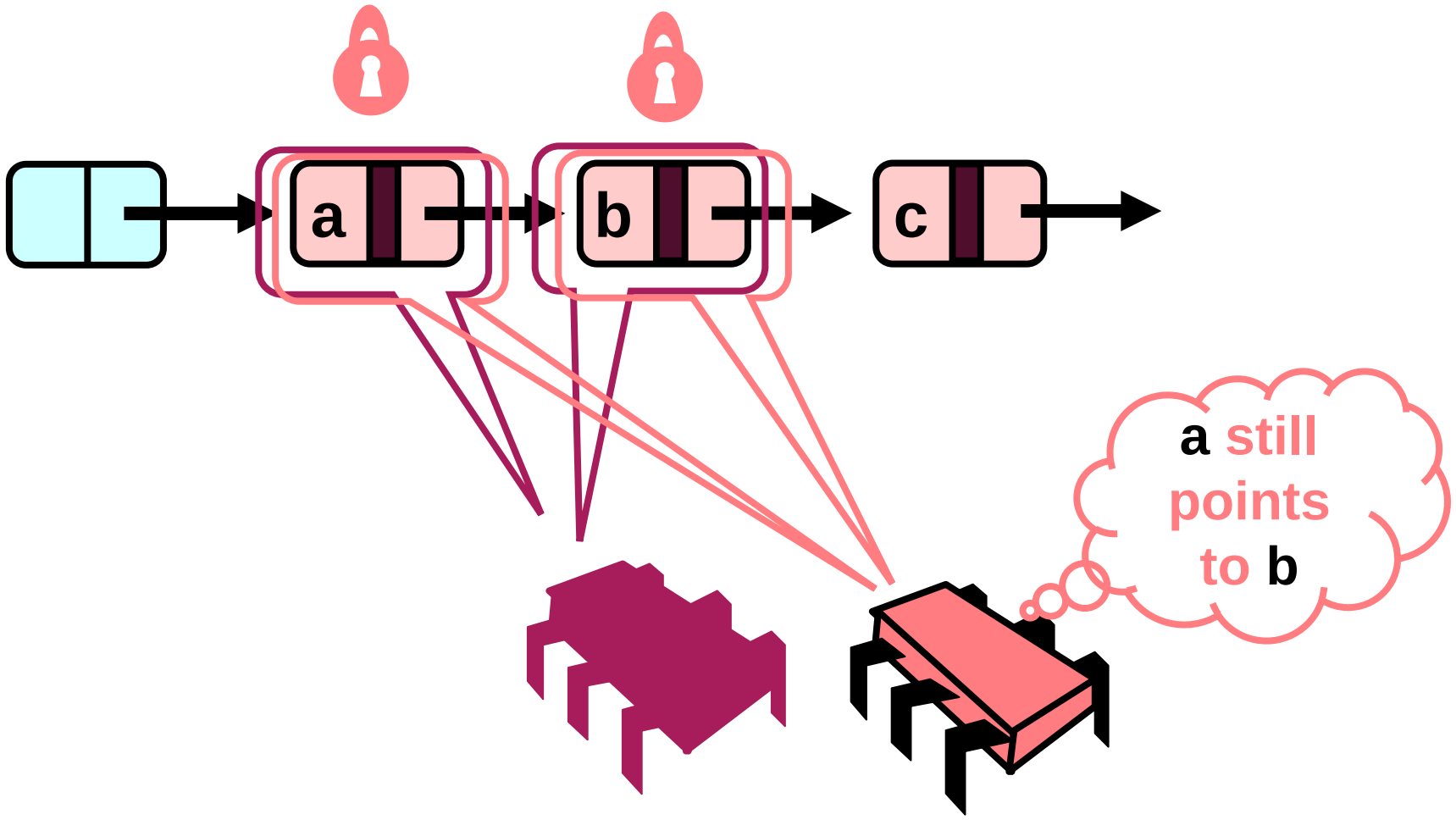

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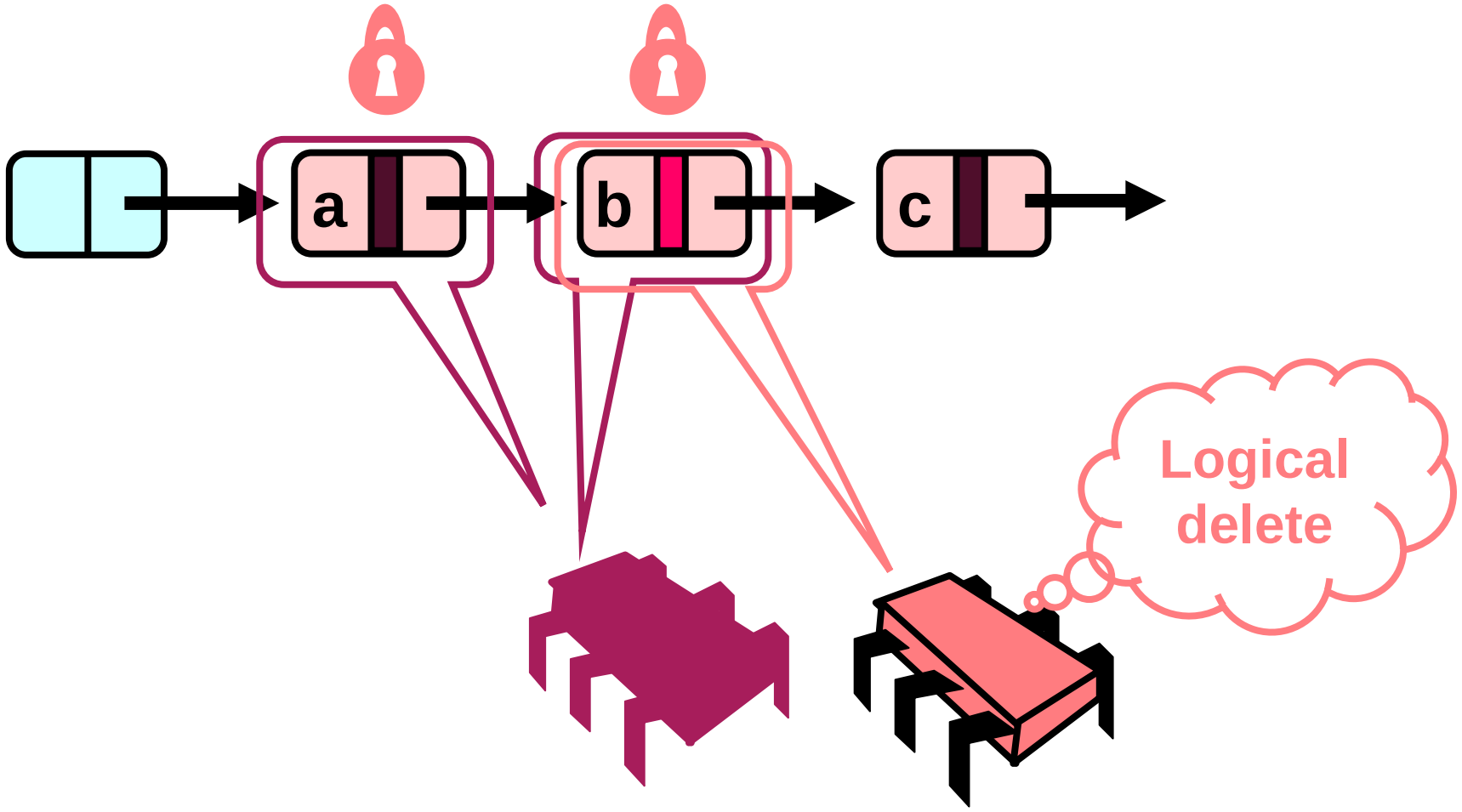

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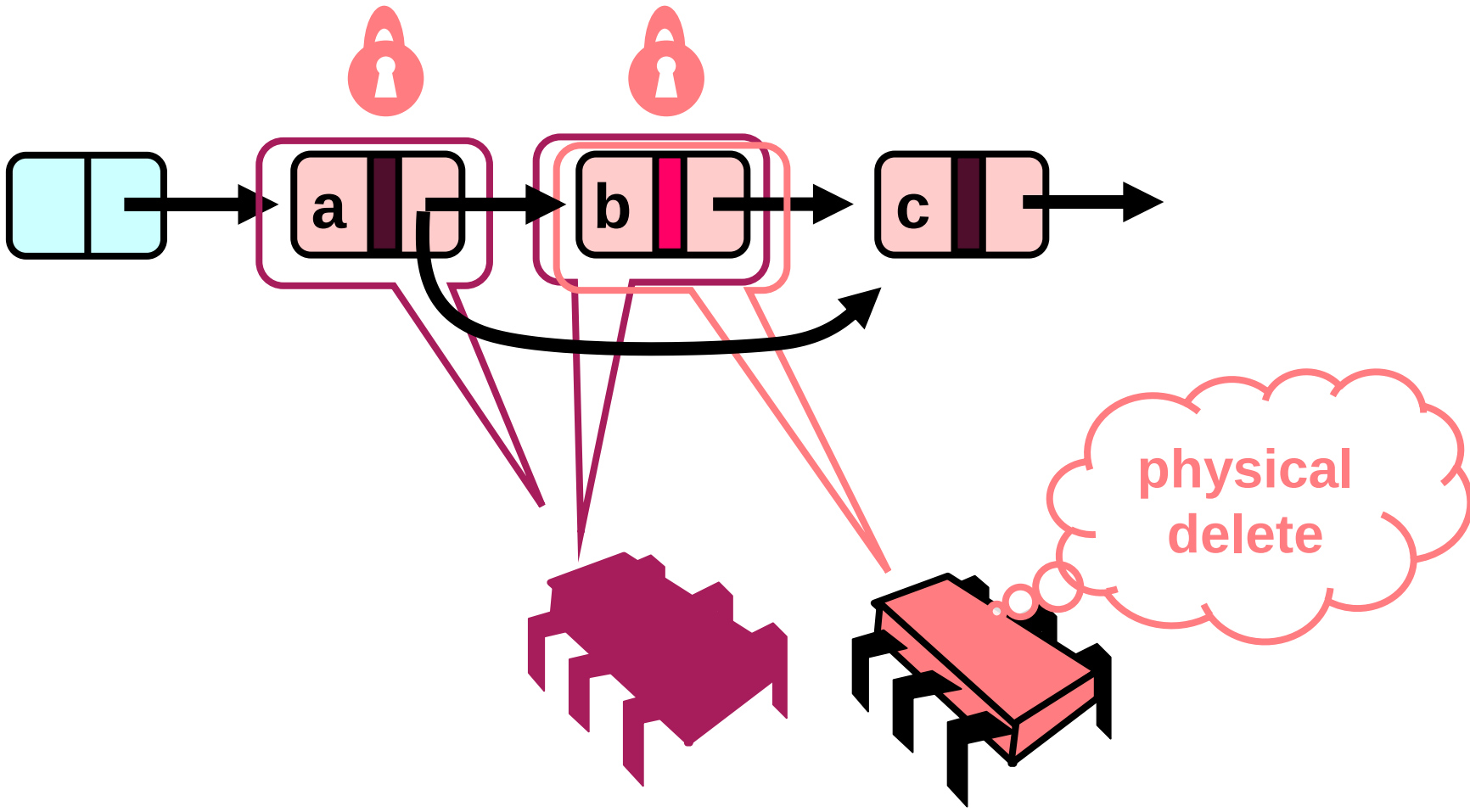
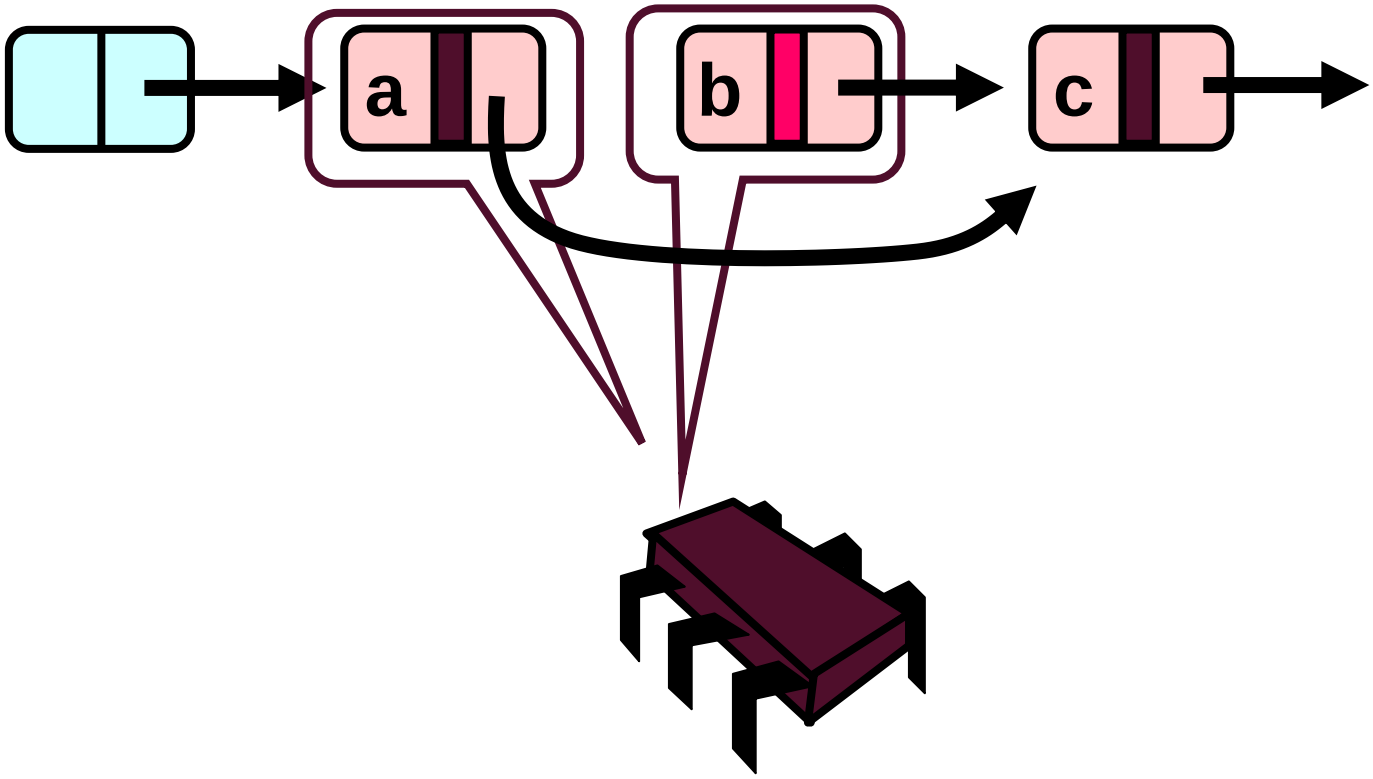
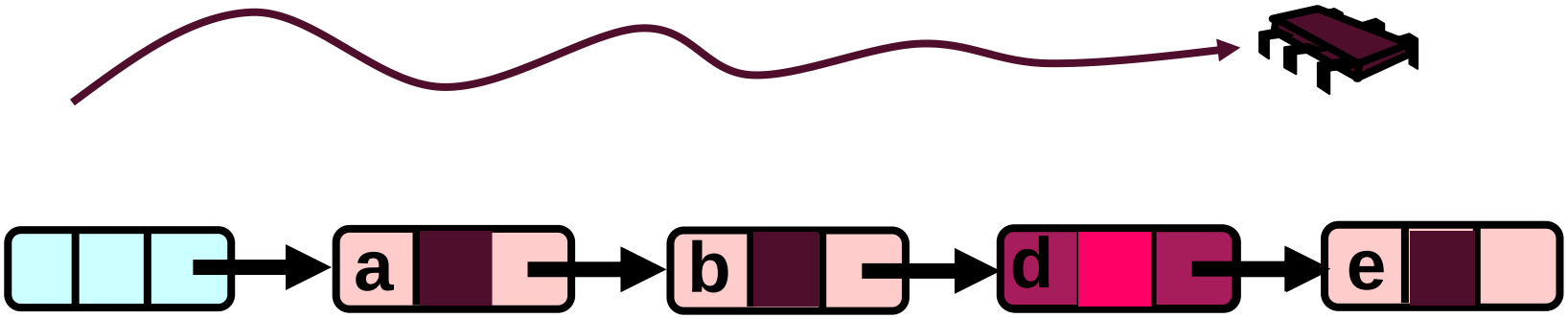


Business as Usual



Summary: Wait-free Contains



Use Mark bit + list ordering

1. **Not marked → in the set**
2. **Marked or missing → not in the set**

Lazy add() and remove() + Wait-free contains()

Problems with Locks

- **What are the fundamental problems with locks?**
- **Blocking**
  - Threads wait, fault tolerance
  - Especially when things like page faults occur in CR
- **Overheads**
  - Even when not contended
  - Also memory/state overhead
- **Synchronization is tricky**
  - Deadlock, other effects are hard to debug
- **Not easily composable**

Lock-free Methods

- **No matter what:**
  - Guarantee minimal progress

    I.e., some thread will advance
  - Threads may halt at bad times (no CRs! No exclusion!)

    I.e., cannot use locks!
  - Needs other forms of synchronization

    E.g., atomics (discussed before for the implementation of locks)

    Techniques are astonishingly similar to guaranteeing mutual exclusion

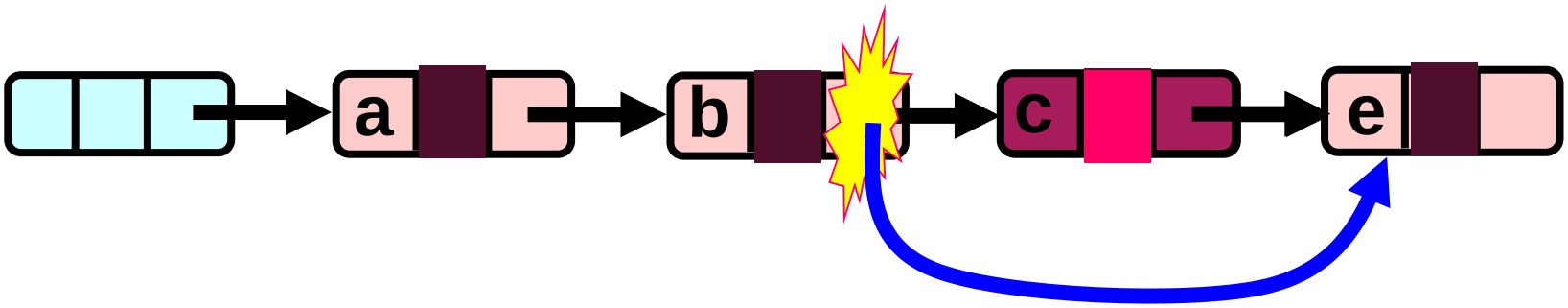
Trick 5: No Locking

- **Make list lock-free**
- **Logical succession**
  - We have wait-free contains
  - Make add() and remove() lock-free!

    Keep logical vs. physical removal
- **Simple idea:**
  - Use CAS to verify that pointer is correct before moving it

Lock-free Lists

(1) Logical Removal

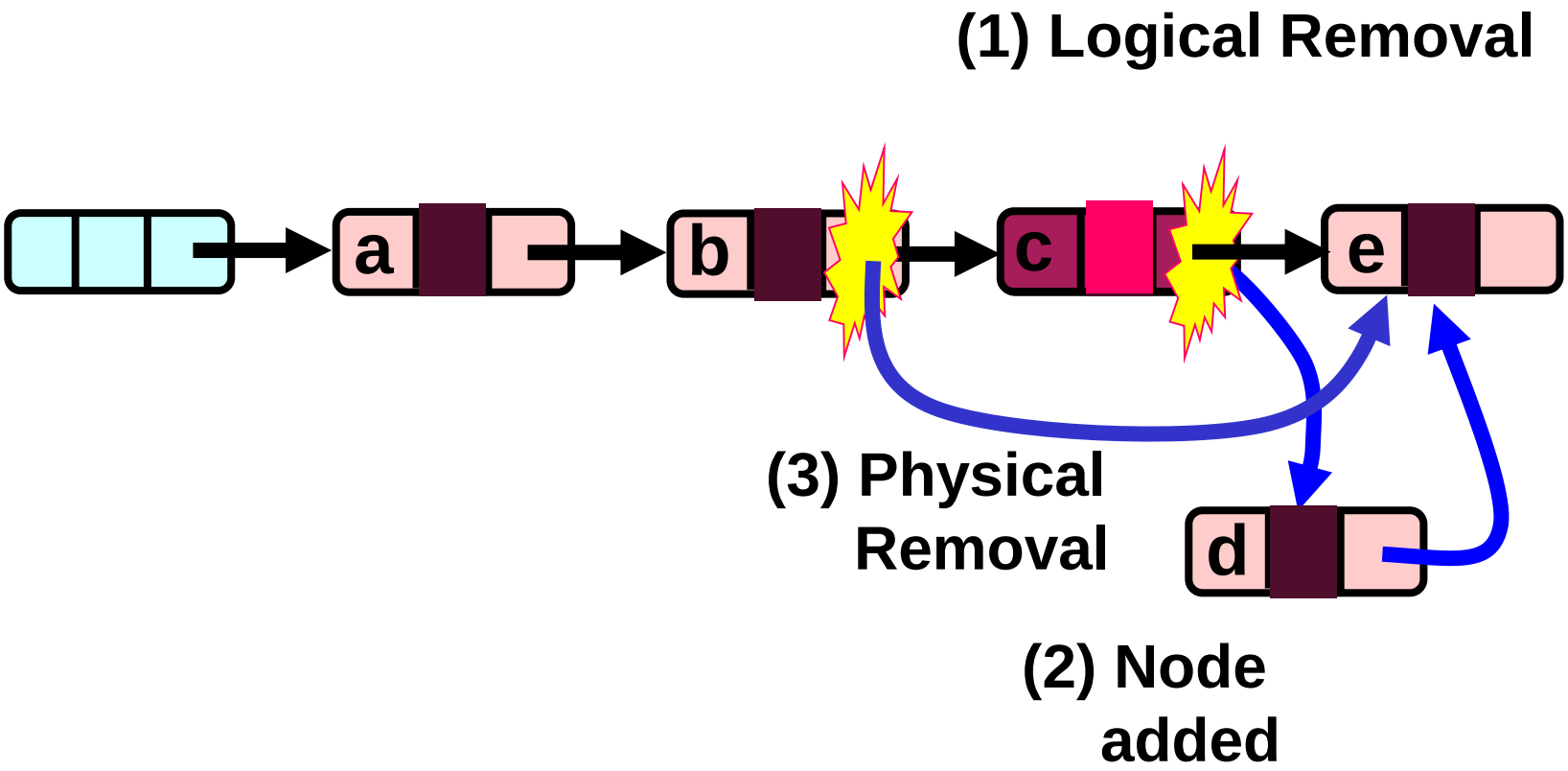


Use CAS to verify pointer is correct

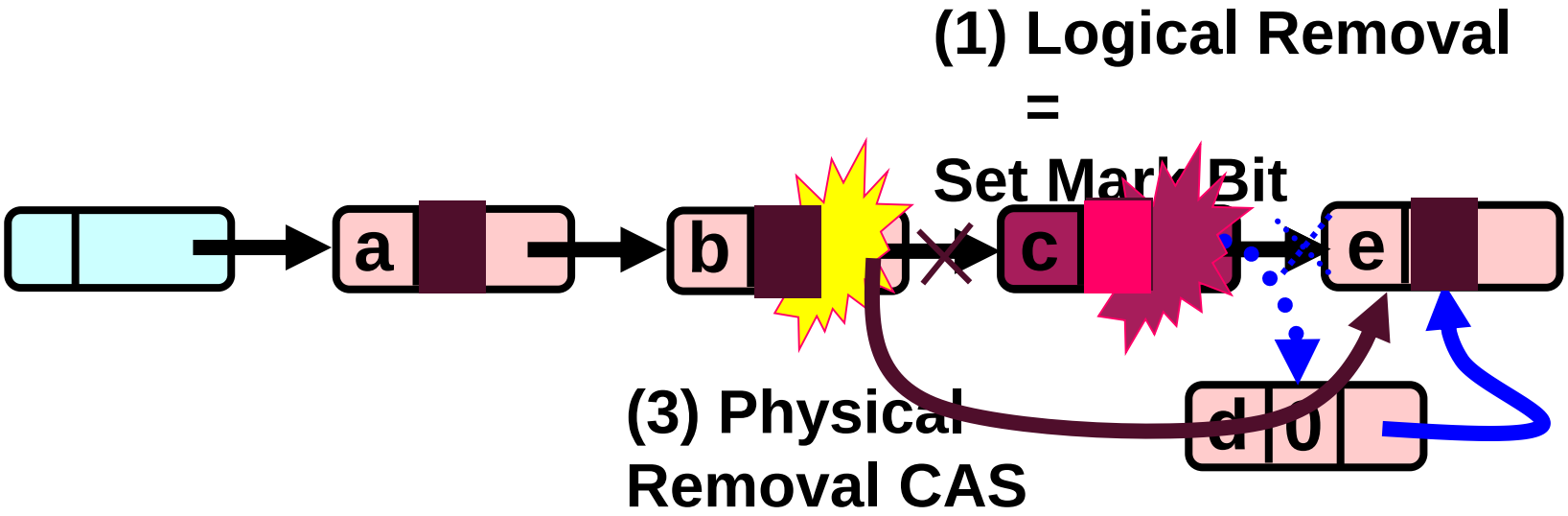
(2) Physical Removal

Not enough! Why?

Problem…



The Solution: Combine Mark and Pointer



Mark-Bit and Pointer are CASed together!

Practical Solution(s)

- **Option 1:**
  - Introduce “atomic markable reference” type
  - “Steal” a bit from a pointer
  - Rather complex and OS specific ☹
- **Option 2:**
  - Use Double CAS (or CAS2) ☺

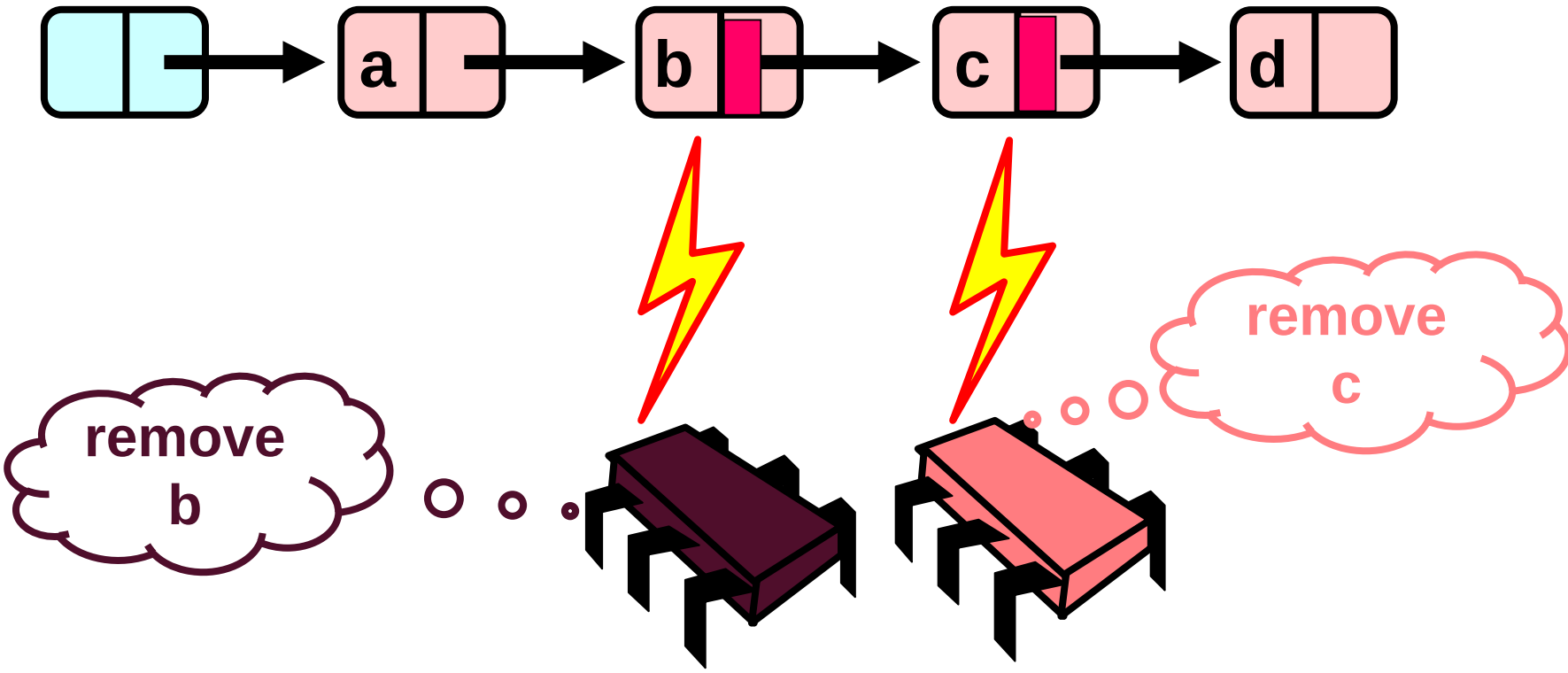
    CAS of two noncontiguous locations
  - Well, not many machines support it ☹

    Any still alive?
- **Option 3:**
  - Our favorite ISA (x86) offers double-width CAS

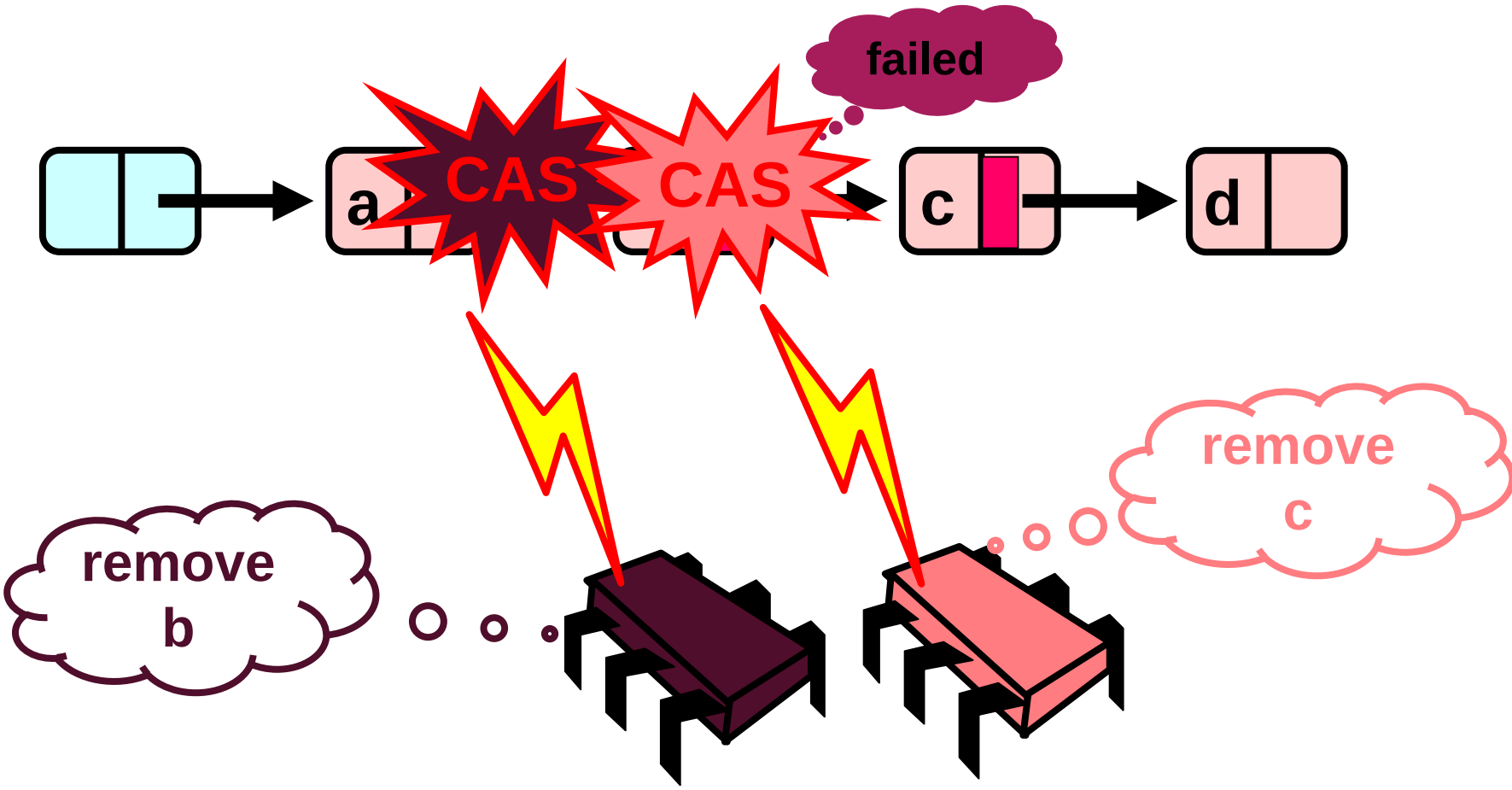
    Contiguous, e.g., lock cmpxchg16b (on 64 bit systems)
- **Option 4:**
  - TM!

    E.g., Intel’s TSX (essentially a cmpxchg64b (operates on a cache line))

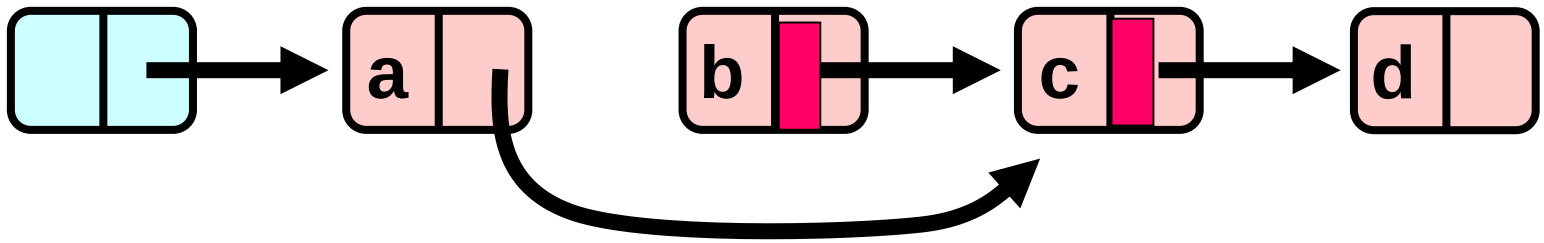
Removing a Node

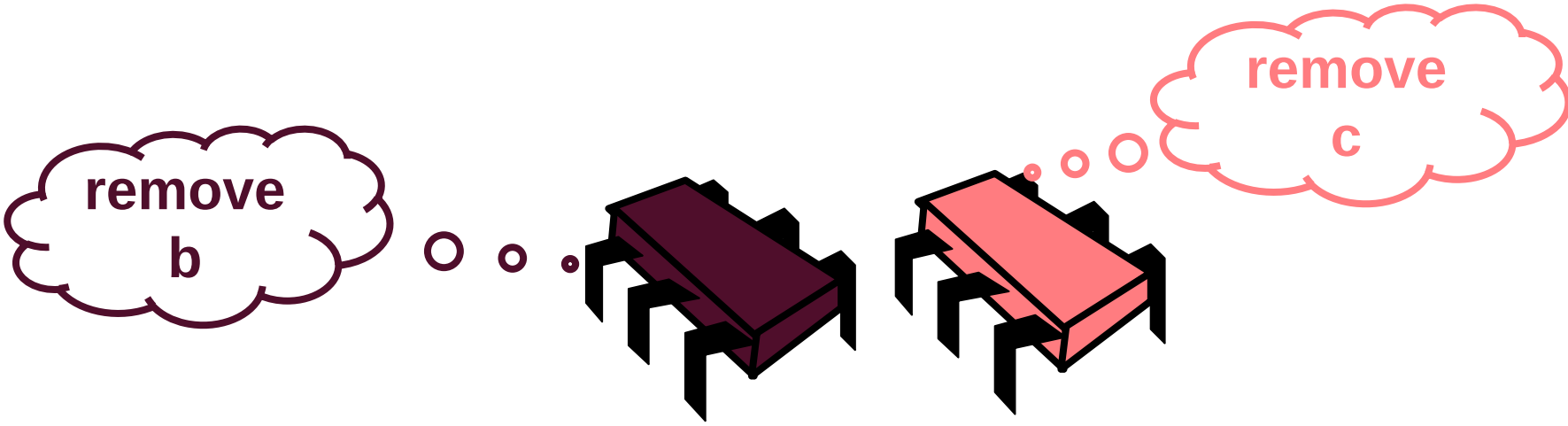


Removing a Node

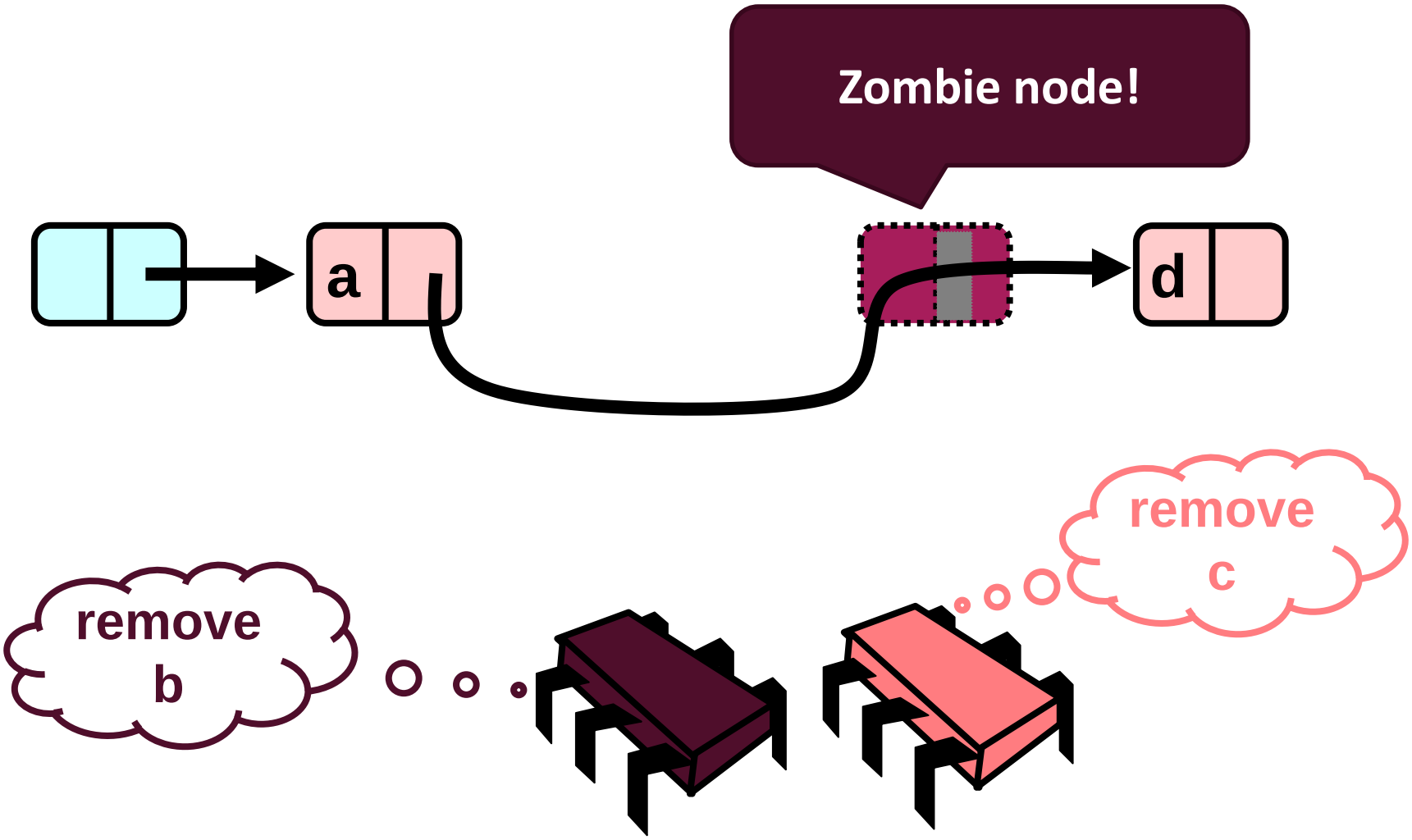


Removing a Node





Uh oh – node marked but not removed!



Dealing With Zombie Nodes

- **Add() and remove() "help to clean up"**
  - Physically remove any marked nodes on their path
  - I.e., if curr is marked: CAS (pred.next, mark) to (curr.next, false) and remove curr

    If CAS fails, restart from beginning!
- **"Helping" is often needed in wait-free algs**
- **This fixes all the issues and makes the algorithm correct!**

Comments

- **Atomically updating two variables (CAS2 etc.) has a non-trivial cost**
- **If CAS fails, routine needs to re-traverse list**
  - Necessary cleanup may lead to unnecessary contention at marked nodes
- **More complex data structures and correctness proofs than for locked versions**
  - But guarantees progress, fault-tolerant and maybe even faster (that really depends)

More Comments

- **Correctness proof techniques**
  - Establish invariants for initial state and transformations

    E.g., head and tail are never removed, every node in the set has to be reachable from head, …
  - Proofs are similar to those we discussed for locks

    Very much the same techniques (just trickier)

    Using sequential consistency (or consistency model of your choice ☺)

    Lock-free gets somewhat tricky
- **Source-codes can be found in Chapter 9 of "The Art of Multiprocessor Programming"**

Lock-free and wait-free

- **A lock-free method**
  - guarantees that infinitely often **some** method call finishes in a finite number of steps
- **A wait-free method**
  - guarantees that **each** method call finishes in a finite number of steps (implies lock-free)

- **Synchronization instructions are not equally powerful!**
  - Indeed, they form an infinite hierarchy; no instruction (primitive) in level x can be used for lock-/wait-free implementations of primitives in level z>x.

Concept: Consensus Number

- **Each level of the hierarchy has a "consensus number" assigned.**
  - Is the maximum number of threads for which primitives in level x can solve the consensus problem
- **The consensus problem:**
  - Has single function: decide(v)
  - Each thread calls it at most once, the function returns a value that meets two conditions:

    consistency: all threads get the same value

    valid: the value is some thread's input
  - Simplification: binary consensus (inputs in {0,1})

Understanding Consensus

- **Can a particular class solve n-thread consensus wait-free?**
  - A class C solves n-thread consensus if there exists a consensus protocol using **any number** of objects of class C and **any number** of atomic registers
  - The protocol has to be wait-free (bounded number of steps per thread)
  - The consensus number of a class C is the largest n for which that class solves n-thread consensus (may be infinite)
  - Assume we have a class D whose objects can be constructed from objects out of class C. If class C has consensus number n, what does class D have?

Starting simple …

- **Binary consensus with two threads (A, B)!**
  - Each thread moves until it decides on a value
  - May update shared objects
  - Protocol state = state of threads + state of shared objects
  - Initial state = state before any thread moved
  - Final state = state after all threads finished
  - States form a tree, wait-free property guarantees a finite tree

    Example with two threads and two moves each!

Atomic Registers

- **Theorem [Herlihy'91]: Atomic registers have consensus number one**
  - Really?
- **Proof outline:**
  - Assume arbitrary consensus protocol, thread A, B
  - Run until it reaches critical state where next action determines outcome (show that it must have a critical state first)
  - Show all options using atomic registers and show that they cannot be used to determine one outcome for all possible executions!
    1) *Any thread reads (other thread runs solo until end)*
    2) *Threads write to different registers (order doesn't matter)*
    3) *Threads write to same register (solo thread can start after each write)*

Atomic Registers

- **Theorem [Herlihy'91]: Atomic registers have consensus number one**
- **Corollary: It is impossible to construct a wait-free implementation of any object with consensus number of >1 using atomic registers**
  - "perhaps one of the most striking impossibility results in Computer Science" (Herlihy, Shavit)
  - → We need hardware atomics or TM!
- **Proof technique borrowed from:**

Impossibility of **distributed consensus** with one faulty process
MJ Fischer, NA Lynch, MS Paterson - Journal of the ACM (JACM), 1985 - dl.acm.org
Abstract The **consensus** problem involves an asynchronous system of processes, some of which may be unreliable. The problem is for the reliable processes to agree on a binary value. In this paper, it is shown that every protocol for this problem has the possibility of **...**
Cited by 3180 Related articles All 164 versions

- **Very influential paper, always worth a read!**
  - Nicely shows proof techniques that are central to parallel and distributed computing!

Other Atomic Operations

- **Simple RMW operations (Test&Set, Fetch&Op, Swap, basically all functions where the op commutes or overwrites) have consensus number 2!**
  - Similar proof technique (bivalence argument)
- **CAS and TM have consensus number ∞**
  - Constructive proof!

Compare and Set/Swap Consensus

```
const int first = -1
volatile int thread = -1;
int proposed[n];

int decide(v) {
  proposed[tid] = v;
  if(CAS(thread, first, tid))
    return  v; // I won!
  else
     return proposed[thread]; // thread won
}
```

- **CAS provides an infinite consensus number**
  - Machines providing CAS are **asynchronous** computation equivalents of the Turing Machine
  - I.e., any concurrent object can be implemented in a wait-free manner (not necessarily fast!)

Now you know everything ☺

- **Not really … ;-)**
  - We'll argue about **performance** now!
- **But you have all the tools for:**
  - Efficient locks
  - Efficient lock-based algorithms
  - Efficient lock-free algorithms (or even wait-free)
  - Reasoning about parallelism!
- **What now?**
  - A different class of problems

    Impact on wait-free/lock-free on actual performance is not well understood
  - Relevant to HPC, applies to shared and distributed memory

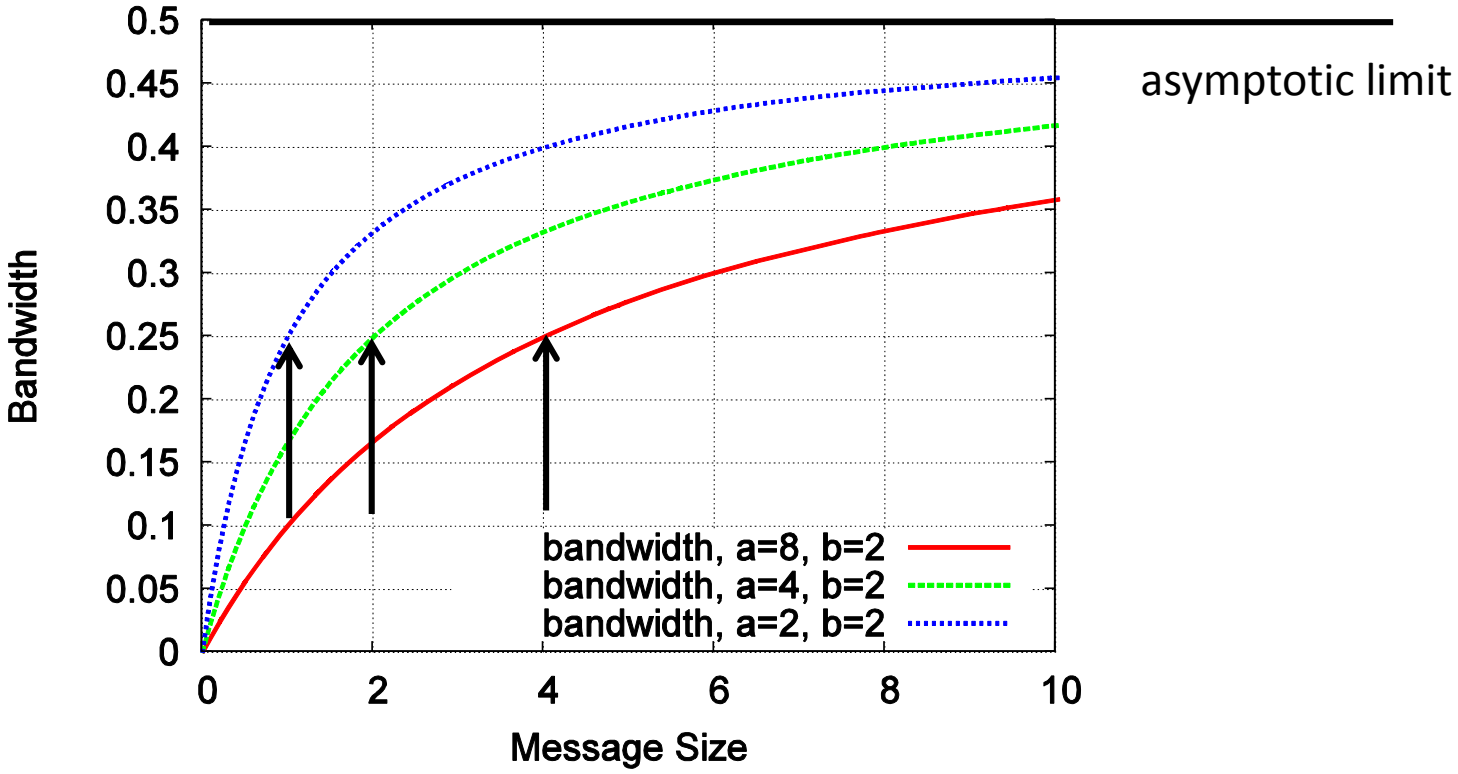
    → *Group communications*

Remember: A Simple Model for Communication

- **Transfer time T(s) = α+βs**
  - α = startup time (latency)
  - β = cost per byte (bandwidth=1/β)
- **As s increases, bandwidth approaches 1/β asymptotically**
  - Convergence rate depends on α
  - $s_{1/2} = \alpha/\beta$
- **Assuming no pipelining (new messages can only be issued from a process after all arrived)**

Bandwidth vs. Latency

- **$s_{1/2} = \alpha/\beta$ often used to distinguish bandwidth- and latency-bound messages**
  - $s_{1/2}$ is in the order of kilobytes on real systems



Quick Example

- **Simplest linear broadcast**
  - One process has a data item to be distributed to all processes
- **Broadcasting s bytes among P processes:**
  - T(s) = (P-1) * (α+βs) = $\mathcal{O}(P)$
- **Class question: Do you know a faster method to accomplish the same?**

k-ary Tree Broadcast

- **Origin process is the root of the tree, passes messages to k neighbors which pass them on**
  - k=2 -> binary tree
- **Class Question: What is the broadcast time in the simple latency/bandwidth model?**
  - $T(s) \approx \lceil log_k(P) \rceil \cdot k \cdot (\alpha + \beta \cdot s) = \mathcal{O}(log(P))$ (for fixed k)
- **Class Question: What is the optimal k?**
  - $0 = \frac{ln(P) \cdot k}{ln(k)} \frac{d}{dk} = \frac{ln(P)ln(k) - ln(P)}{ln^2(k)} \rightarrow k = e = 2.71...$
  - Independent of P, α, βs? Really?

Faster Trees?

- **Class Question: Can we broadcast faster than in a ternary tree?**
  - Yes because each respective root is idle after sending three messages!
  - Those roots could keep sending!
  - Result is a k-nomial tree

    For k=2, it's a binomial tree
- **Class Question: What about the runtime?**
  - $T(s) = \lceil log_k(P) \rceil \cdot (k-1) \cdot (\alpha + \beta \cdot s) = \mathcal{O}(log(P))$
- **Class Question: What is the optimal k here?**
  - T(s) d/dk is monotonically increasing for k>1, thus k_{opt}=2
- **Class Question: Can we broadcast faster than in a k-nomial tree?**
  - $\mathcal{O}(log(P))$ is asymptotically optimal for s=1!
  - But what about large s?

Open Problems

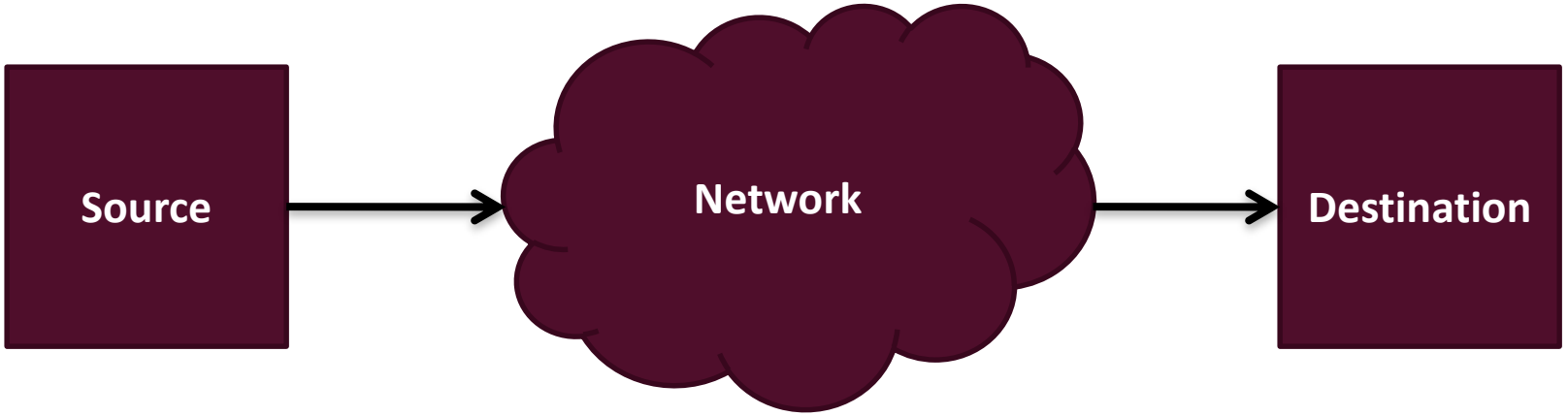
- **Look for optimal parallel algorithms (even in simple models!)**
  - And then check the more realistic models
  - Useful optimization targets are MPI collective operations

    Broadcast/Reduce, Scatter/Gather, Alltoall, Allreduce, Allgather, Scan/Exscan, …
  - Implementations of those (check current MPI libraries ☺)
  - Useful also in scientific computations

    Barnes Hut, linear algebra, FFT, …
- **Lots of work to do!**
  - Contact me for thesis ideas (or check SPCL) if you like this topic
  - Usually involve optimization (ILP/LP) and clever algorithms (algebra) combined with practical experiments on large-scale machines (10,000+ processors)

HPC Networking Basics

- **Familiar (non-HPC) network: Internet TCP/IP**
  - Common model:

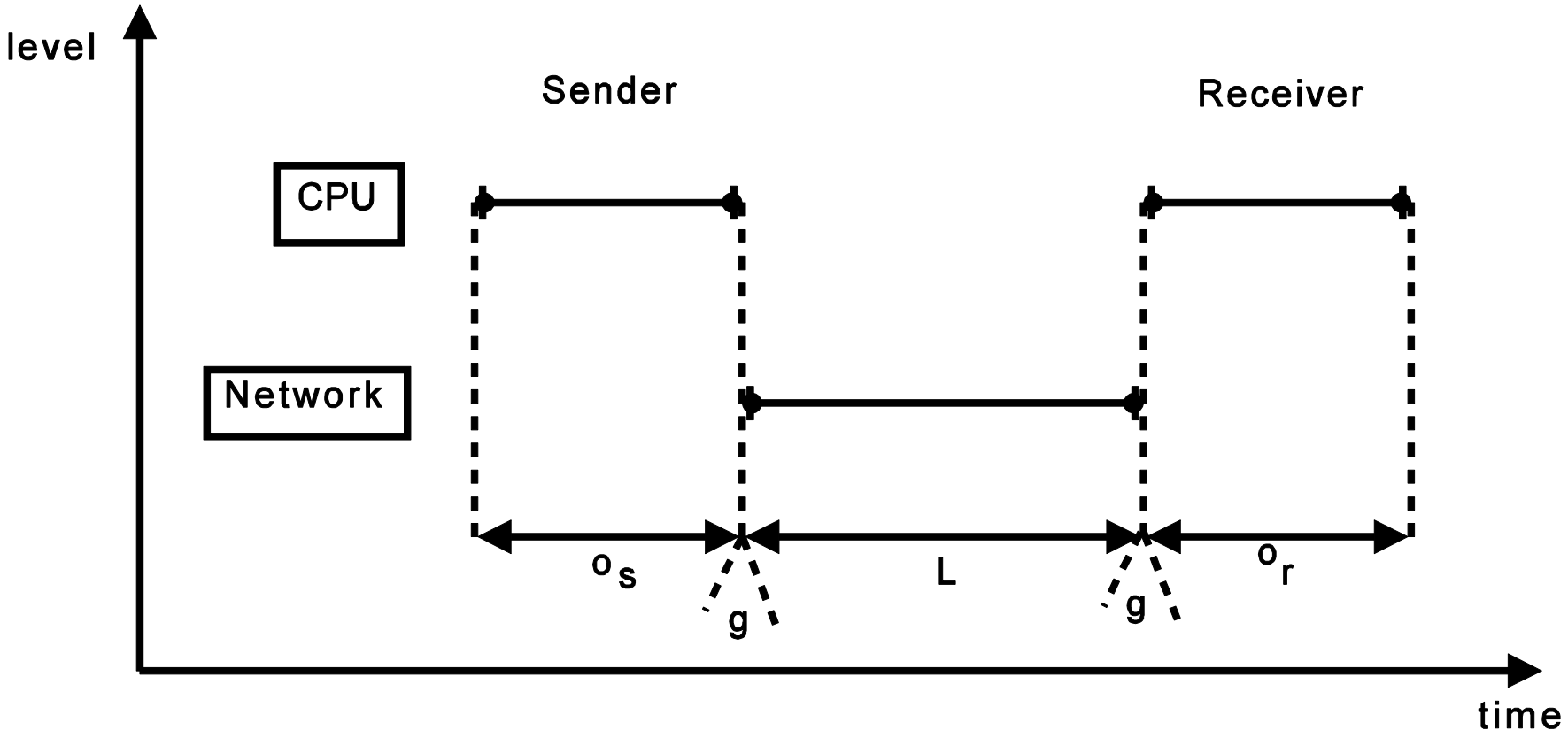


- **Class Question: What parameters are needed to model the performance (including pipelining)?**
  - Latency, Bandwidth, Injection Rate, Host Overhead

The LogP Model

- **Defined by four parameters:**
  - L: an upper bound on the latency, or delay, incurred in communicating a message containing a word (or small number of words) from its source module to its target module.
  - o: the overhead, defined as the length of time that a processor is engaged in the transmission or reception of each message; during this time, the processor cannot perform other operations.
  - g: the gap, defined as the minimum time interval between consecutive message transmissions or consecutive message receptions at a processor. The reciprocal of g corresponds to the available per-processor communication bandwidth.
  - P: the number of processor/memory modules. We assume unit time for local operations and call it a cycle.

The LogP Model



Simple Examples

- **Sending a single message**
  - $T = 2o+L$
- **Ping-Pong Round-Trip**
  - $T_{RTT} = 4o+2L$
- **Transmitting n messages**
  - $T(n) = L+(n-1)*\max(g, o) + 2o$

Simplifications

- **o is bigger than g on some machines**
  - g can be ignored (eliminates max() terms)
  - be careful with multicore!
- **Offloading networks might have very low o**
  - Can be ignored (not yet but hopefully soon)
- **L might be ignored for long message streams**
  - If they are pipelined
- **Account g also for the first message**
  - Eliminates “-1”

Benefits over Latency/Bandwidth Model

- **Models pipelining**
  - L/g messages can be “in flight”
  - Captures state of the art (cf. TCP windows)
- **Models computation/communication overlap**
  - Asynchronous algorithms
- **Models endpoint congestion/overload**
  - Benefits balanced algorithms

Example: Broadcasts

- **Class Question: What is the LogP running time for a linear broadcast of a single packet?**
  - $T_{lin} = L + (P-2) * \max(o,g) + 2o$
- **Class Question: Approximate the LogP runtime for a binary-tree broadcast of a single packet?**
  - $T_{bin} \leq \log_2 P * (L + \max(o,g) + 2o)$
- **Class Question: Approximate the LogP runtime for an k-ary-tree broadcast of a single packet?**
  - $T_{k\text{-}n} \leq \log_k P * (L + (k-1)\max(o,g) + 2o)$

Example: Broadcasts

- **Class Question: Approximate the LogP runtime for a binomial tree broadcast of a single packet (assume L > g!)?**
  - $T_{bin} \leq \log_2 P * (L + 2o)$
- **Class Question: Approximate the LogP runtime for a k-nomial tree broadcast of a single packet?**
  - $T_{k\text{-}n} \leq \log_k P * (L + (k\text{-}2)\max(o,g) + 2o)$
- **Class Question: What is the optimal k (assume o>g)?**
  - Derive by k: $0 = o * \ln(k_{opt}) - L/k_{opt} + o$ (solve numerically)

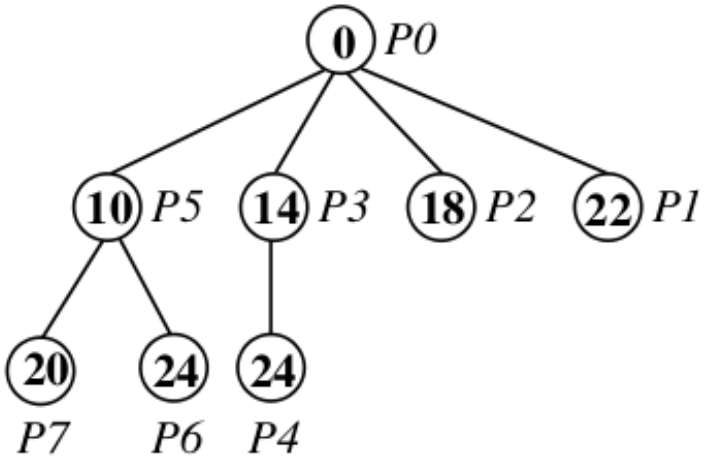
    For larger L, k grows and for larger o, k shrinks
  - Models pipelining capability better than simple model!

Example: Broadcasts

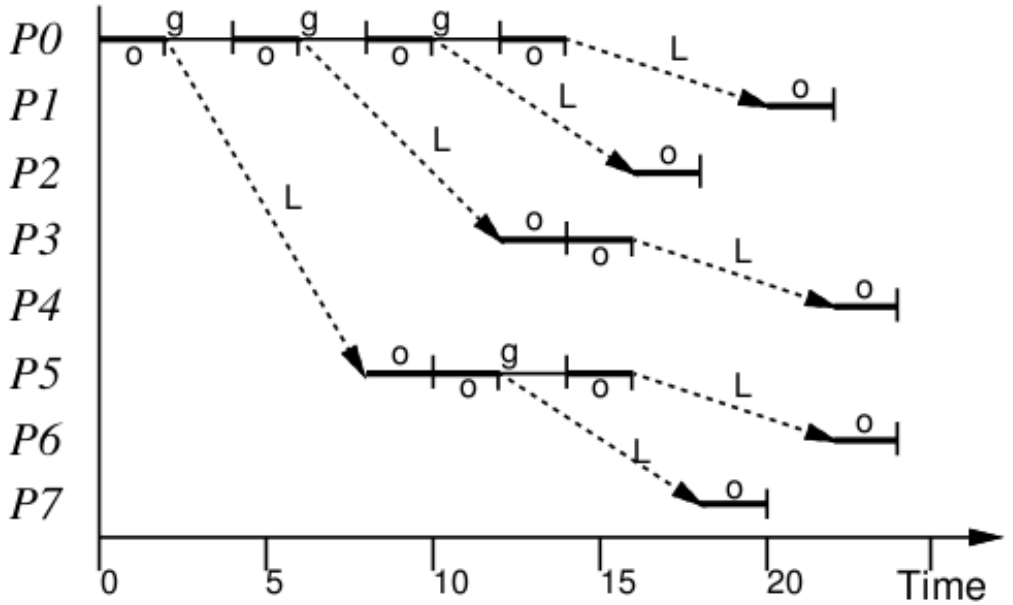
- **Class Question: Can we do better than k_{opt}-ary binomial broadcast?**
  - Problem: fixed k in all stages might not be optimal
  - We can construct a schedule for the optimal broadcast in practical settings
  - First proposed by Karp et al. in “Optimal Broadcast and Summation in the LogP Model”

Example: Optimal Broadcast

- **Broadcast to P-1 processes**
  - Each process who received the value sends it on; each process receives exactly once



P=8, L=6, g=4, o=2



Optimal Broadcast Runtime

- **This determines the maximum number of PEs (P(t)) that can be reached in time t**
- **P(t) can be computed with a generalized Fibonacci recurrence (assuming o>g):**

$$P(t) = \begin{cases} 1: & t < 2o + L \\ P(t-o) + P(t-L-2o): & \text{otherwise.} \end{cases} \quad (1)$$

- **Which can be bounded by (see [1]):** $2^{\left\lfloor \frac{t}{L+2o} \right\rfloor} \leq P(t) \leq 2^{\left\lfloor \frac{t}{o} \right\rfloor}$
  - A closed solution is an interesting open problem!

[1]: Hoefler et al.: "Scalable Communication Protocols for Dynamic Sparse Data Exchange" (Lemma 1)

The Bigger Picture

- **We learned how to program shared memory systems**
  - Coherency & memory models & linearizability
  - Locks as examples for reasoning about correctness and performance
  - List-based sets as examples for lock-free and wait-free algorithms
  - Consensus number
- **We learned about general performance properties and parallelism**
  - Amdahl's and Gustafson's laws
  - Little's law, Work-span, …
  - Balance principles & scheduling
- **We learned how to perform model-based optimizations**
  - Distributed memory broadcast example with two models
- **What next? MPI? OpenMP? UPC?**
  - Next-generation machines "merge" shared and distributed memory concepts → Partitioned Global Address Space (PGAS)