Design of Parallel and High-Performance Computing

Fall 2013

Lecture: Linearizability

Motivational video: https://www.youtube.com/watch?v=qx2dRIQXnbs

Instructor: Torsten Hoefler & Markus Püschel

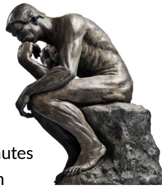
TAs: Timo Schneider

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Review of last lecture

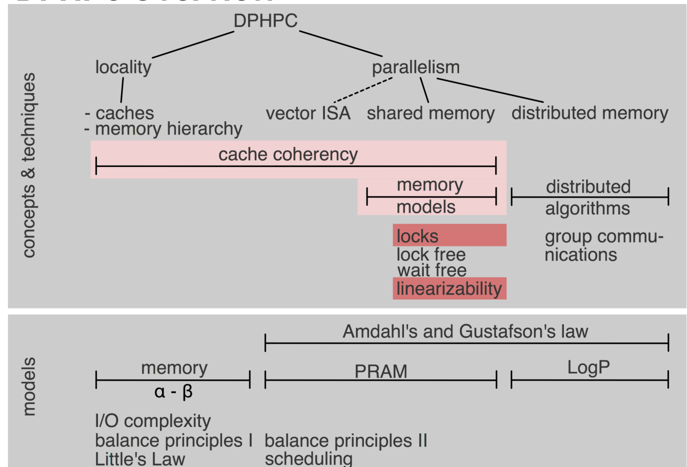
- **Cache-coherence is not enough!**
  - Many more subtle issues for parallel programs!
- **Memory Models**
  - Sequential consistency
  - Why threads cannot be implemented as a library ^^
  - Relaxed consistency models
  - x86 TLO+CC case study
- **Complexity of reasoning about parallel objects**
  - Serial specifications (e.g., pre-/postconditions)
  - Started to lock things …

Peer Quiz



- **Instructions:**
  - Pick some partners (locally) and discuss each question for 2 minutes
  - We then select a random student (team) to answer the question
- **What are the problems with sequential consistency?**
  - Is it practical? Explain!
  - Is it sufficient for simple parallel programming? Explain!
  - How would you improve the situation?
- **How could memory models of practical CPUs be described?**
  - Is Intel's definition useful?
  - Why would one need a better definition?
  - Threads cannot be implemented as a library? Why does Pthreads work?

DPHPC Overview



Goals of this lecture

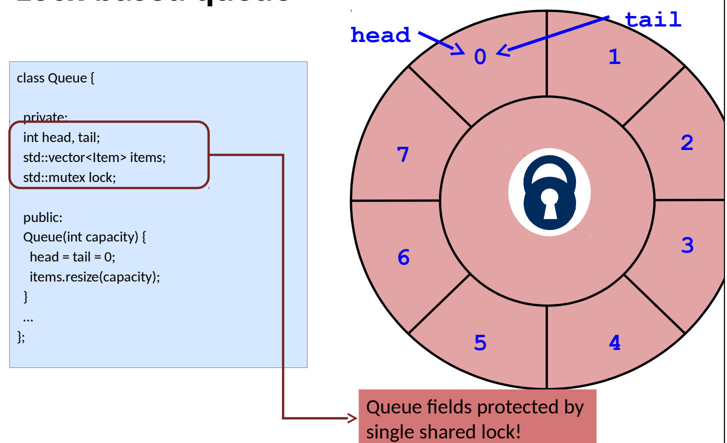
- **Queue:**
  - Problems with the locked queue
  - Wait-free two-thread queue
- **Linearizability**
  - Intuitive understanding (sequential order on objects!)
  - Linearization points
  - Linearizable executions
  - Formal definitions (Histories, Projections, Precedence)
- **Linearizability vs. Sequential Consistency**
  - Modularity
- **Maybe: lock implementations**

Lock-based queue

```
class Queue {

 private:
 int head, tail;
 std::vector<Item> items;
 std::mutex lock;

 public:
 Queue(int capacity) {
   head = tail = 0;
   items.resize(capacity);
 }
 …
};
```



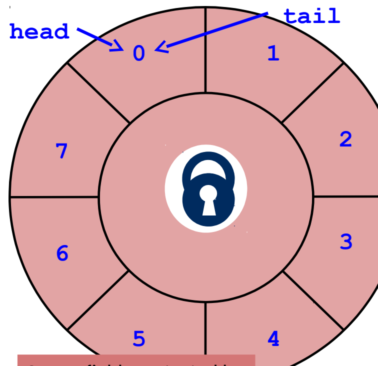
Queue fields protected by single shared lock!

Lock-based queue

```
class Queue {
 ...

 public:
 void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if((tail+1)%items.size()==head) {
   throw FullException;
  }
  items[tail] = x;
  tail = (tail+1)%items.size();
 }

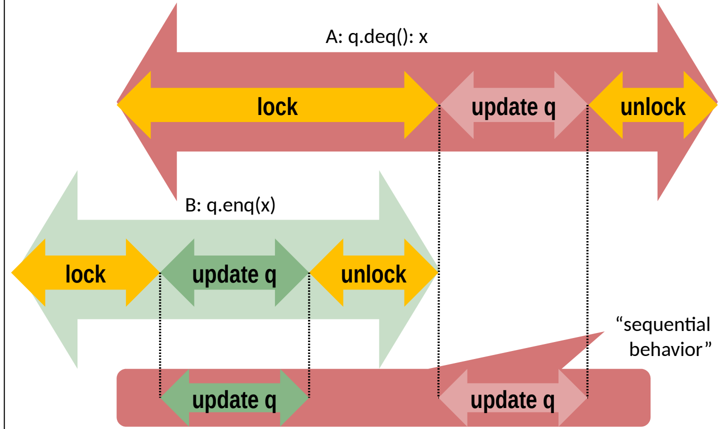
 Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
   throw FullException;
  }
  Item item = items[head];
  head = (head+1)%items.size();
  return item;
 }
};
```



Queue fields protected by single shared lock!

Class question: how is the lock ever unlocked?

Example execution



"sequential behavior"

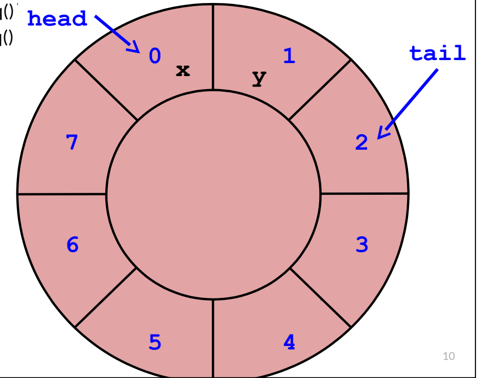
Correctness

- **Is the locked queue correct?**
  - Yes, only one thread has access if locked correctly
  - Allows us again to reason about pre- and postconditions
  - Smells a bit like sequential consistency, no?
- **Class question: What is the problem with this approach?**
  - Same as for SC ^^

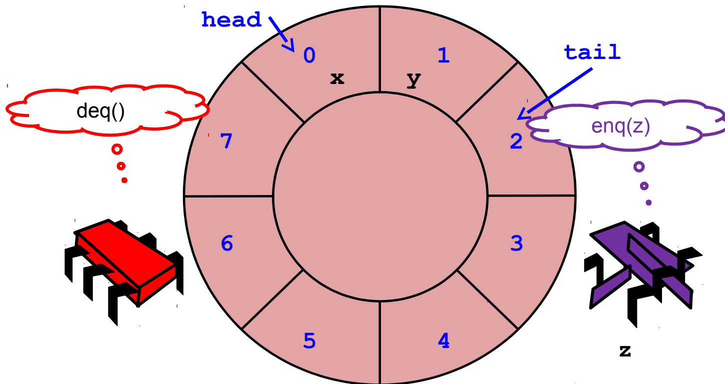
It does not scale!
What is the solution here?

Threads working at the same time?

- **Same thing (concurrent queue)**
- **For simplicity, assume only two threads**
  - Thread A calls only enq()
  - Thread B calls only deq()



Wait-free 2-Thread Queue



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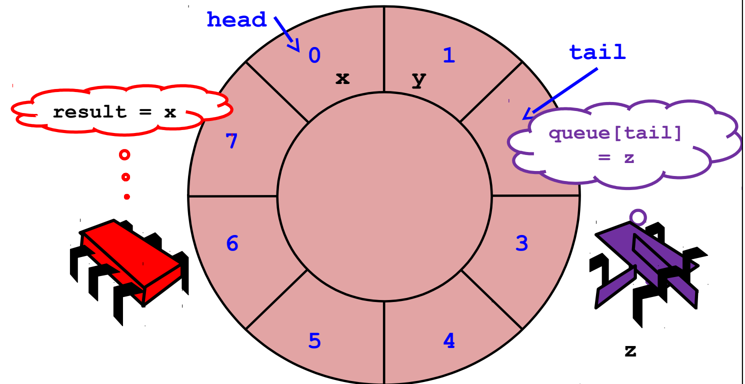

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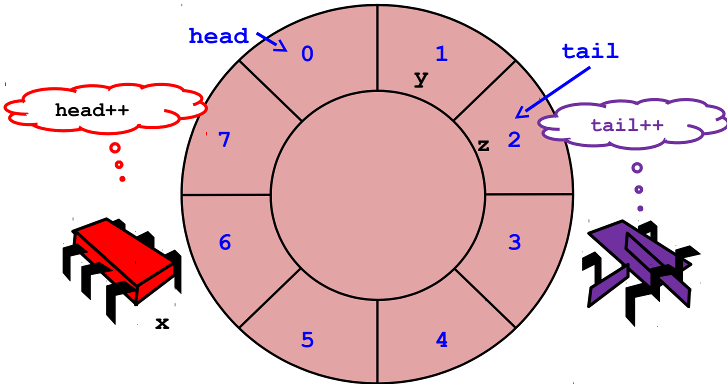


Is this correct?

- Hard to reason about correctness
- What could go wrong?

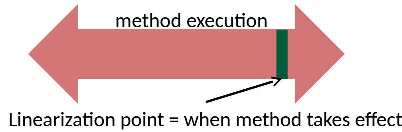
```
void enq(Item x) {
  if((tail+1)%items.size() == head) {
    throw FullException;
  }
  items[tail] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head];
  head = (head+1)%items.size();
  return item;
}
```

- Nothing (at least no crash)
- Yet, the semantics of the queue are funny (define “FIFO” now)!

Serial to Concurrent Specifications

- Serial specifications are complex enough, so lets stick to them
  - Define invocation and response events (start and end of method)
  - Extend the sequential concept to concurrency: linearizability
- Each method should “take effect”
  - Instantaneously
  - Between invocation and response events
- A concurrent object is correct if its “sequential” behavior is correct
  - Called “linearizable”



Linearization point = when method takes effect

Linearizability

- Sounds like a property of an execution …
- An object is called linearizable if all possible executions on the object are linearizable
- Says nothing about the order of executions!

Example

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if((tail+1)%items.size() == head) {
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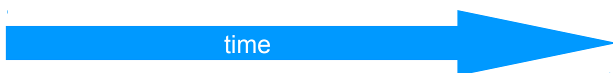
linearization points



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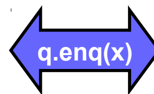

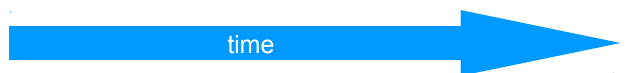

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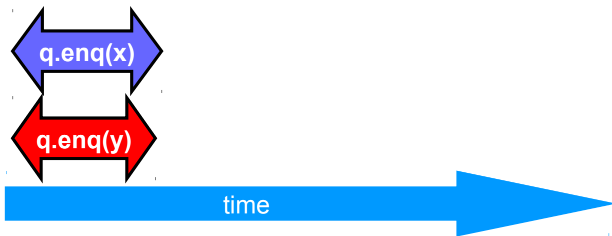

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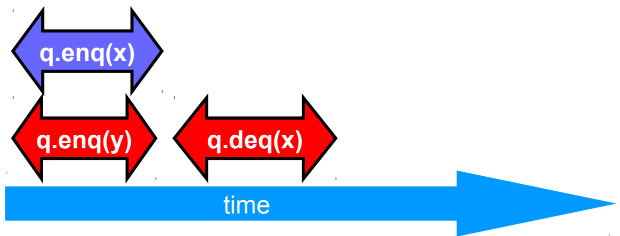

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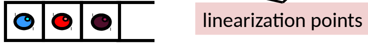

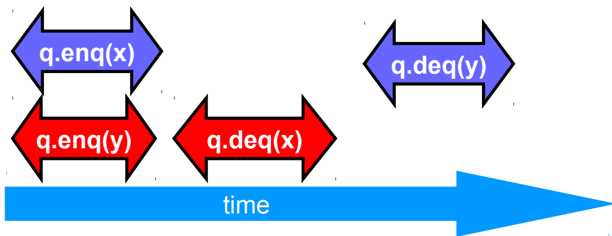

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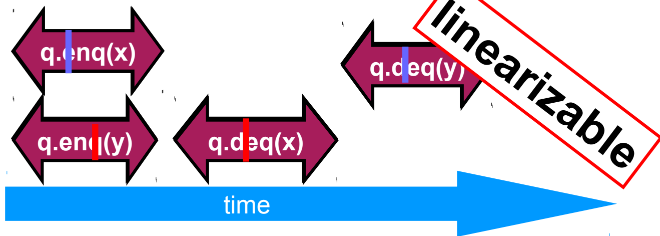

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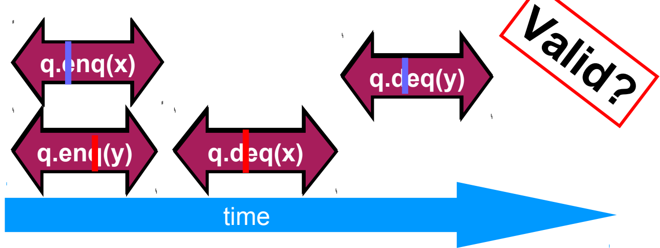

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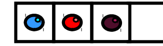

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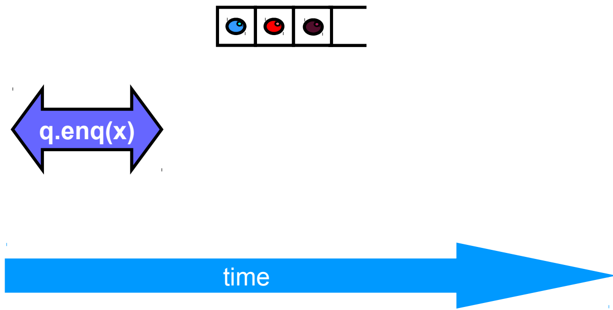

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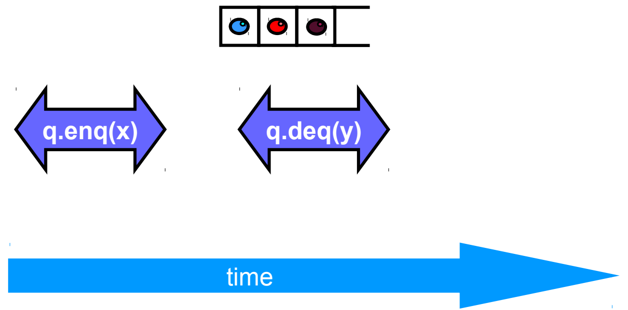

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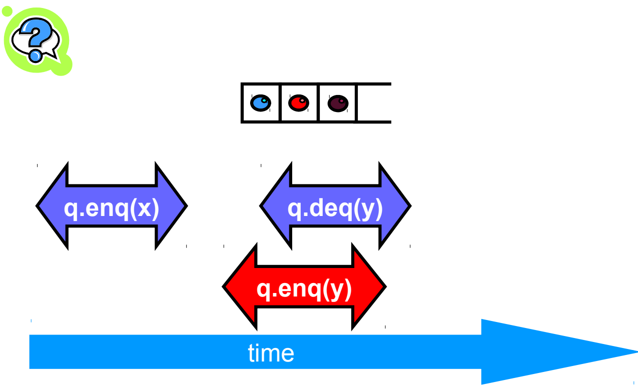

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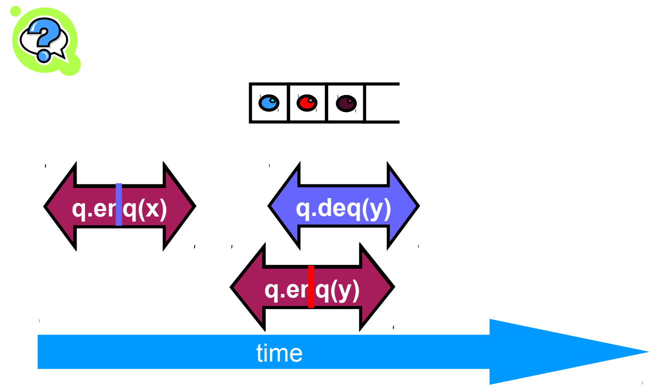

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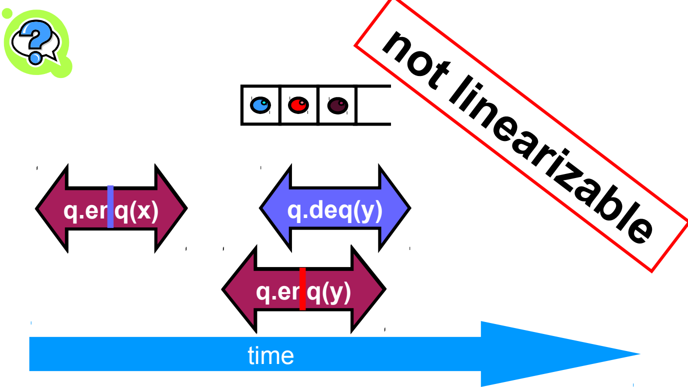
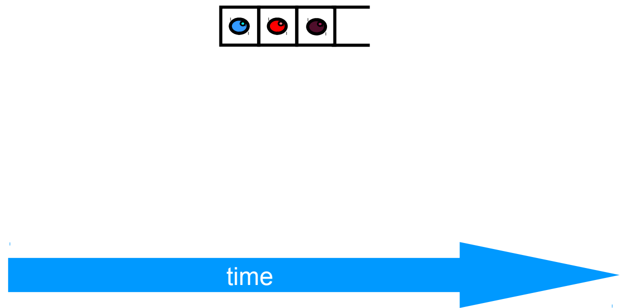


Example 3



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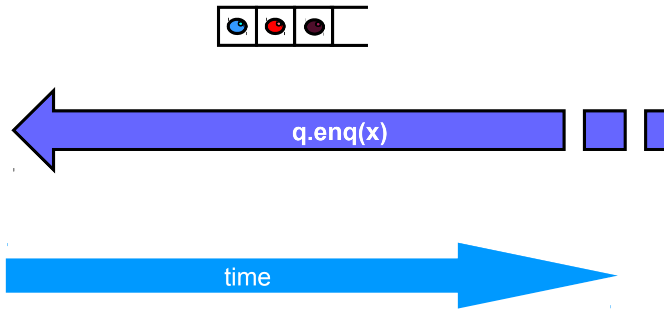

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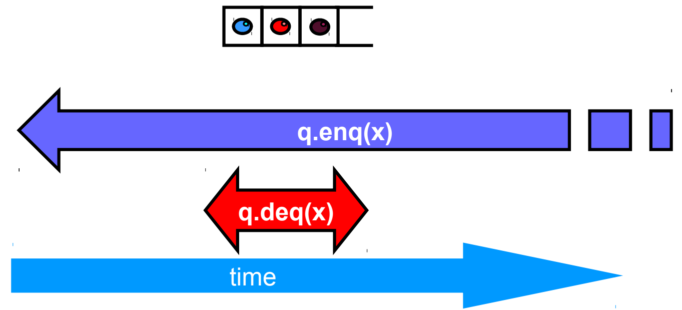

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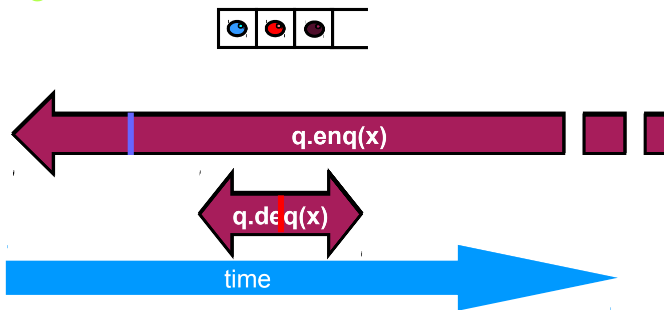

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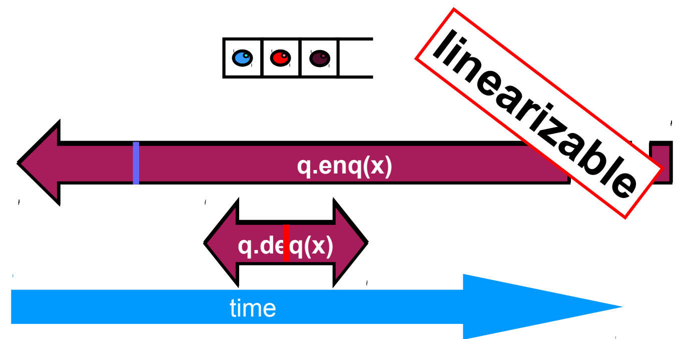

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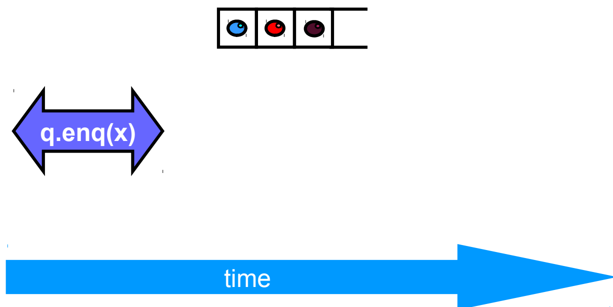

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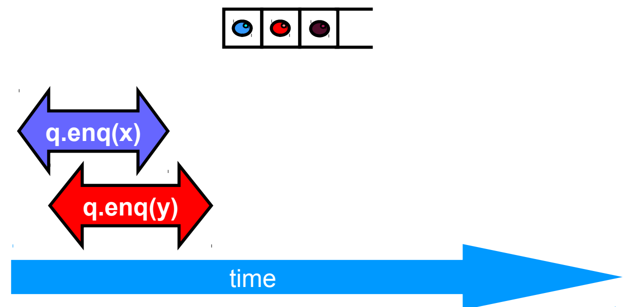

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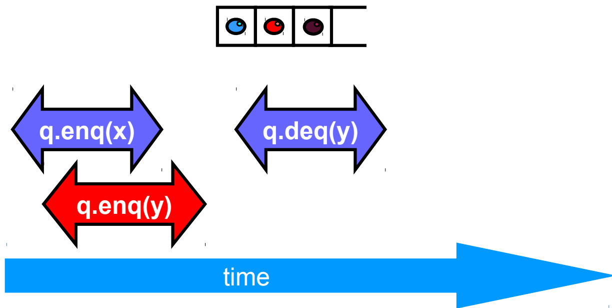

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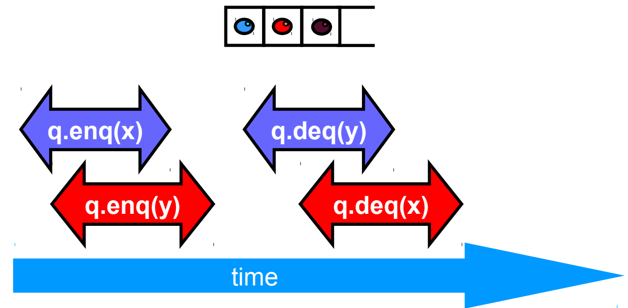

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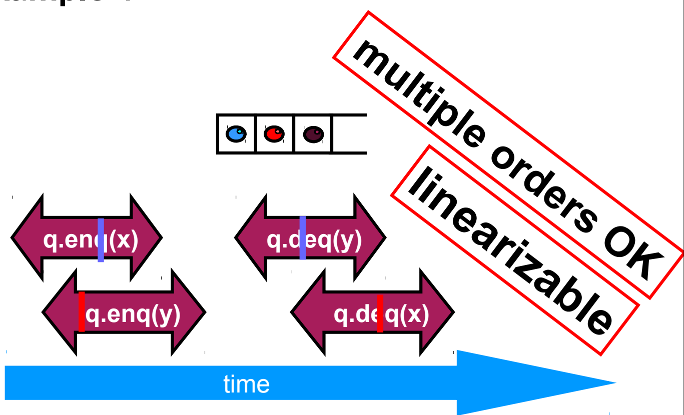


Is the lock-free queue linearizable?

- **A) Only two threads, one calls only deq() and one calls only enq()?**

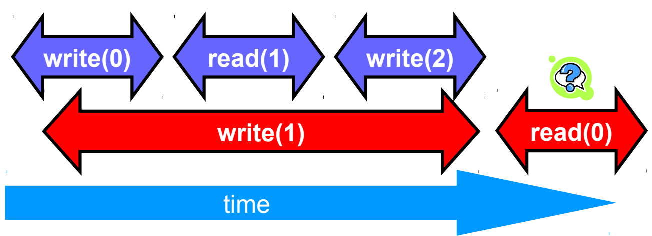
```
void enq(Item x) {
  if((tail+1)%items.size() == head) {
    throw FullException;
  }
  items[tail] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head];
  head = (head+1)%items.size();
  return item;
}
```

- **B) Only two threads but both may call enq() or deq() independently**
- **C) An arbitrary number of threads, but only one calls enq()**
- **D) An arbitrary number of threads can call enq() or deq()**
- **E) If it's linearizable, where are the linearization points?**
  - Remark: typically executions are not constrained, so this is NOT linearizable

Read/Write Register Example

- **Assume atomic update to a single read/write register!**



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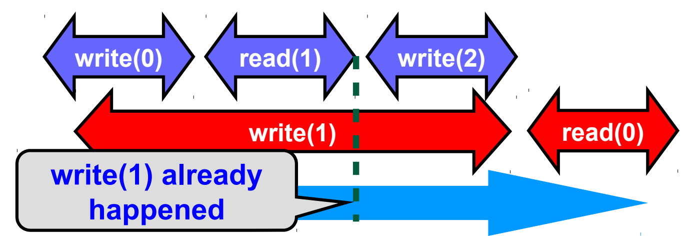

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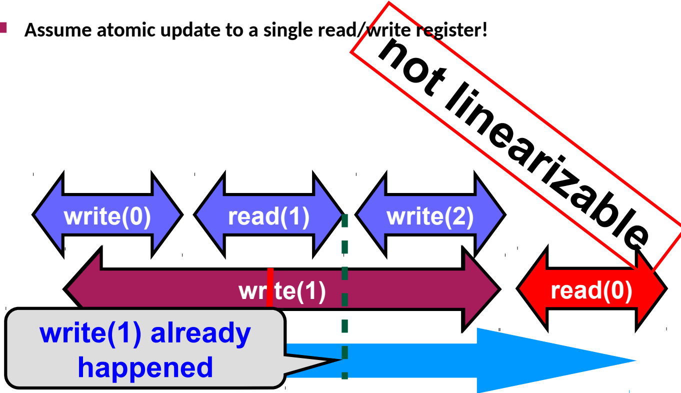

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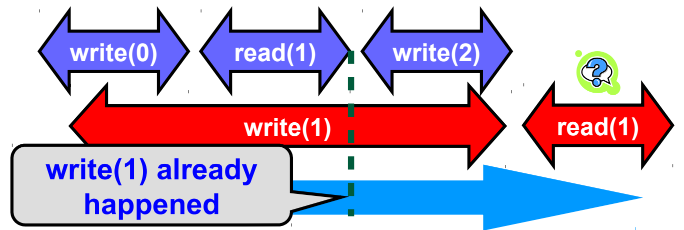

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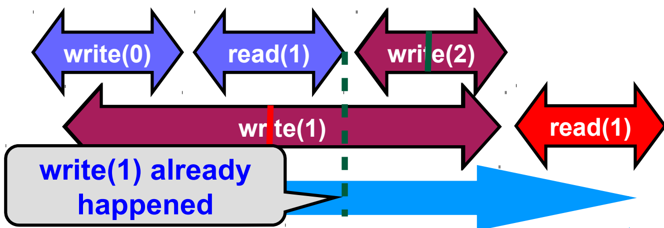

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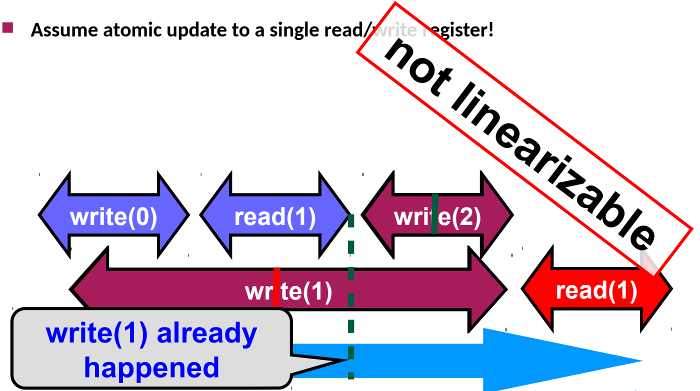

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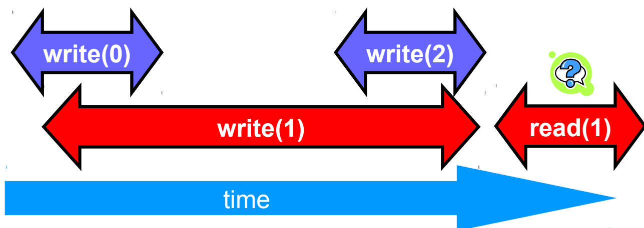

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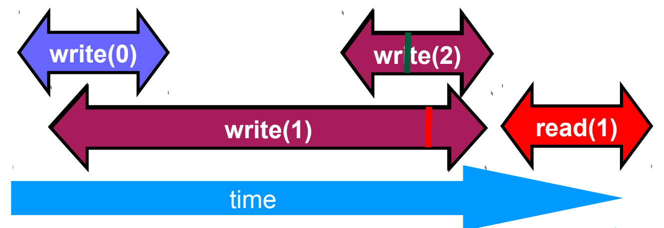

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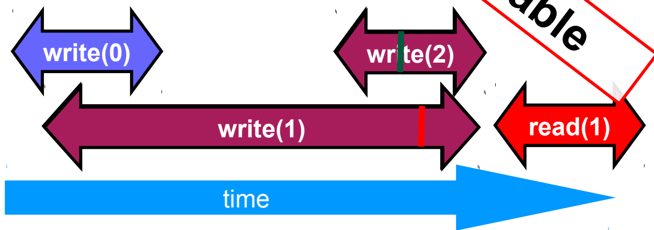

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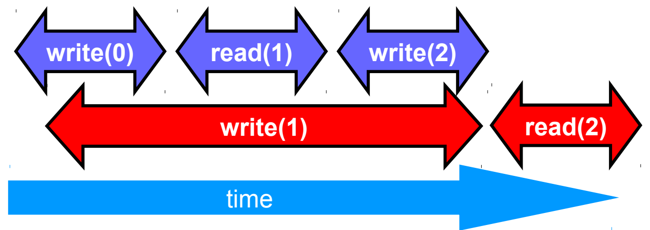

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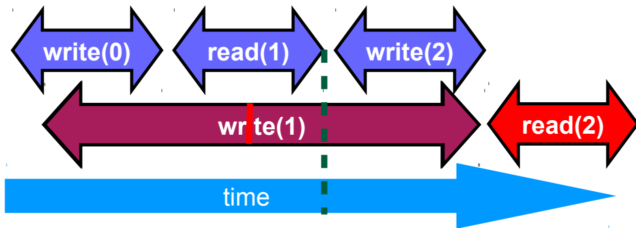

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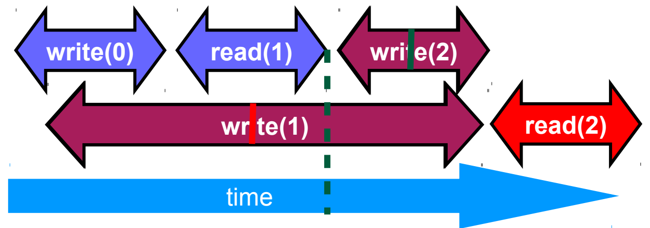
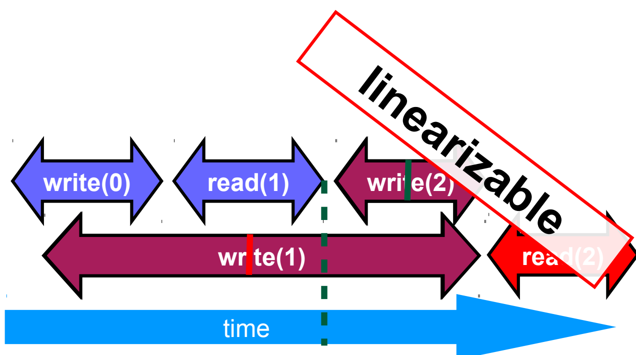


Read/Write Register Example

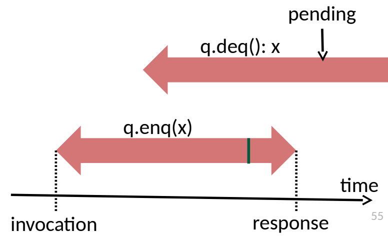


About Executions

- **Why?**
  - Can't we specify the linearization point of each operation statically without describing an execution?
- **Not always**
  - In some cases, the linearization point depends on the execution
    Imagine a "check if one should lock" (not recommended!)
- **Define a formal model for executions!**

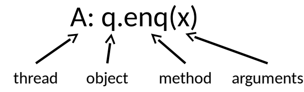
Properties of concurrent method executions

- **Method executions take time**
  - May overlap
- **Method execution = operation**
  - Defined by invocation and response events
- **Duration of method call**
  - Interval between the events

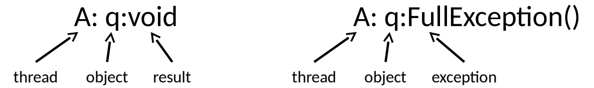


Formalization - Notation

- **Invocation**



- **Response**



  - Question: why is the method name not needed in the response?
    Method is implicit (correctness criterion)!

Concurrency

- **A concurrent system consists of a collection of sequential threads P_i**
- **Threads communicate via shared objects**
  For now!

History

- **Describes an execution**
  - Sequence of invocations and responses
  - H=

A: q.enq(a)
A: q:void
A: q.enq(b)
B: p.enq(c)
B: p:void
B: q.deq()
B: q:a

Invocation and response match if
- thread names are the same
- objects are the same

Remember: Method name is implicit!

Side Question: Is this history linearizable?

Projections on Threads

- **Threads subhistory H|P ("H at P")**
  - Subsequences of all events in H whose thread name is P

H=	H|A=	H|B=
A: q.enq(a)	A: q.enq(a)	
A: q:void	A: q:void	
A: q.enq(b)	A: q.enq(b)	
B: p.enq(c)		B: p.enq(c)
B: p:void		B: p:void
B: q.deq()		B: q.deq()
B: q:a		B: q:a

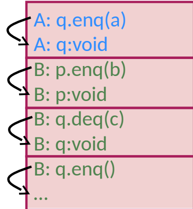
Projections on Objects

- **Objects subhistory H|o ("H at o")**
  - Subsequence of all events in H whose object name is o

H=	H|p=	H|q=
A: q.enq(a)		A: q.enq(a)
A: q:void		A: q:void
A: q.enq(b)		A: q.enq(b)
B: p.enq(c)	B: p.enq(c)	
B: p:void	B: p:void	
B: q.deq()		B: q.deq()
B: q:a		B: q:a

Sequential Histories

- **A history H is sequential if**



- The first event of H is an invocation
- Each invocation (except possibly the last) is immediately followed by a matching response
- Each response is immediately followed by an invocation

Method calls of different threads do not interleave

- **A history H is concurrent if**
  - It is not sequential

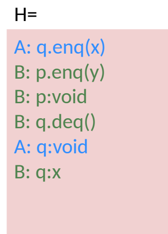
Well-formed histories

a history is sequential if

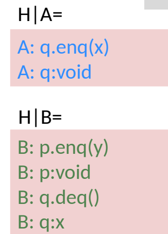
- The first event of H is an invocation
- Each invocation (except possibly the last) is immediately followed by a matching response
- Each response is immediately followed by an invocation

- **Per-thread projections must be sequential**

H=



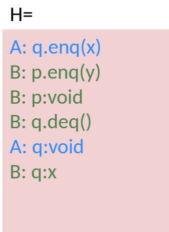
H|A=



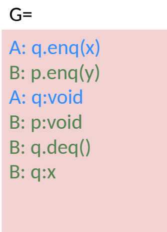
Equivalent histories

- **Per-thread projections must be the same**

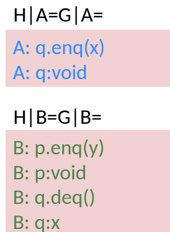
H=



G=



H|A=G|A=

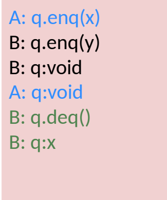


Legal Histories

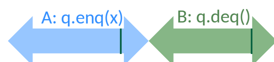
- **Sequential specification allows to describe what behavior we expect and tolerate**
  - When is a single-thread, single-object history legal?
- **Recall: Example**
  - Preconditions and Postconditions
  - Many others exist!
- **A sequential (multi-object) history H is legal if**
  - For every object x
  - H|x adheres to the sequential specification for x
- **Example: FIFO queue**
  - Correct internal state

    Order of removal equals order of addition
  - Full and Empty Exceptions

Precedence

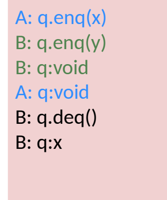


A method execution precedes another if response event precedes invocation event

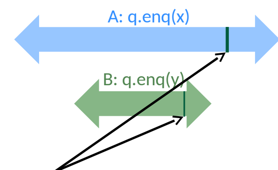


Precedence vs. Overlapping

- **Non-precedence = overlapping**



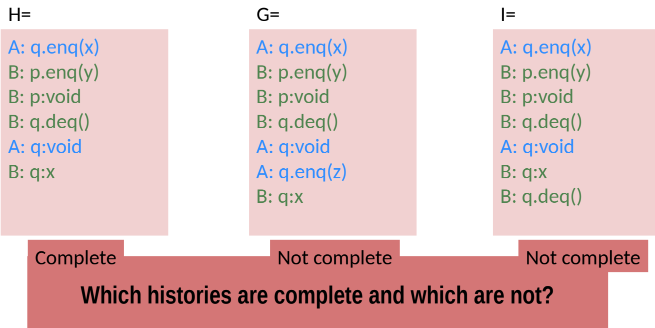
Some method executions overlap with others



Side Question: Is this a correct linearization order?

Complete Histories

- **A history H is complete**
  - If all invocations are matched with a response



Precedence Relations

- **Given history H**
- **Method executions m_0 and m_1 in H**
  - $m_0 \rightarrow_H m_1$ (m_0 precedes m_1 in H) if
  - Response event of m_0 precedes invocation event of m_1
- **Precedence relation $m_0 \rightarrow_H m_1$ is a**
  - Strict partial order on method executions
    Irreflexive, antisymmetric, transitive
- **Considerations**
  - Precedence forms a total order if H is sequential
  - Unrelated method calls ⇒ may overlap ⇒ concurrent

Definition Linearizability

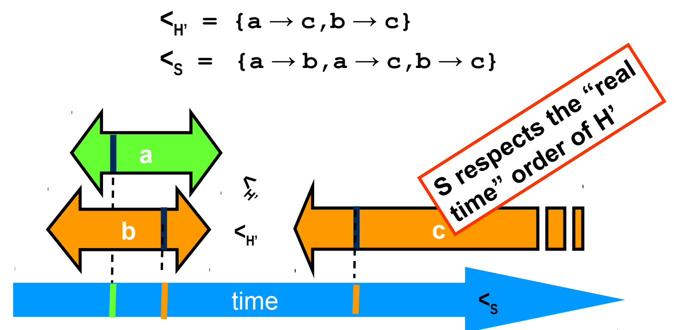
- **A history H induces a strict partial order $<_H$ on operations**
  - $m_0 <_H m_1$ if $m_0 \rightarrow_H m_1$
- **A history H is linearizable if**
  - H can be extended to a complete history H'
    by appending responses to pending operations or dropping pending operations
  - H' is equivalent to some legal sequential history S and
  - $<_{H'} \subseteq <_S$
- **S is a linearization of H**
- **Remarks:**
  - For each H, there may be many valid extensions to H'
  - For each extension H', there may be many S
  - Interleaving at the granularity of methods

Ensuring $<_{H'} \subseteq <_S$

- **Find an S that contains H'**

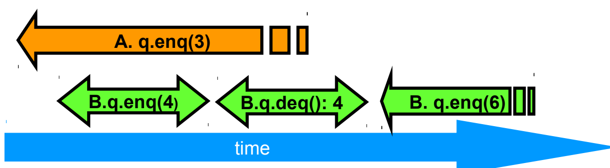
$<_{H'}$ = {a → c, b → c}

$<_S$ = {a → b, a → c, b → c}



Example

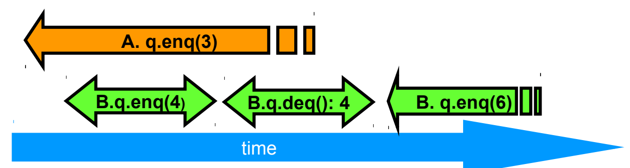
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



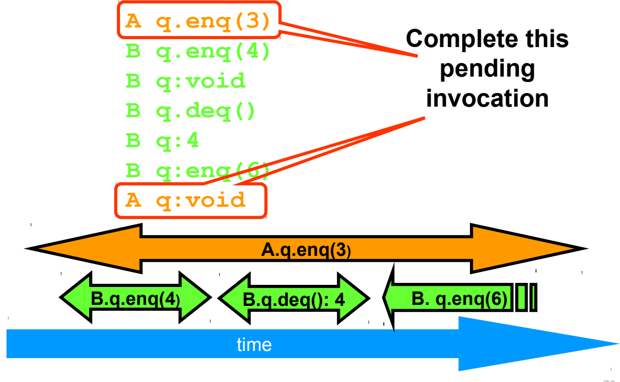
Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```

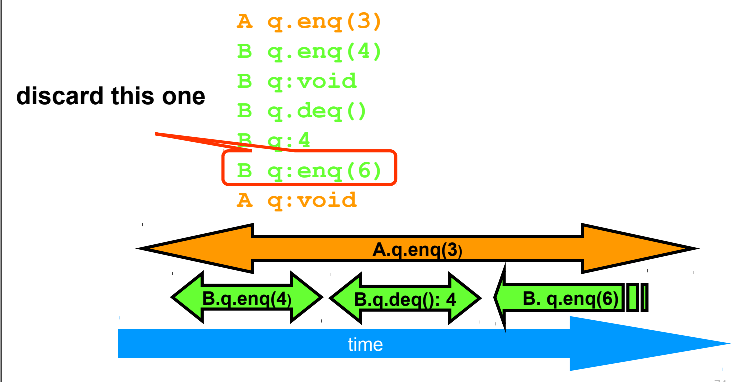
Complete this pending invocation



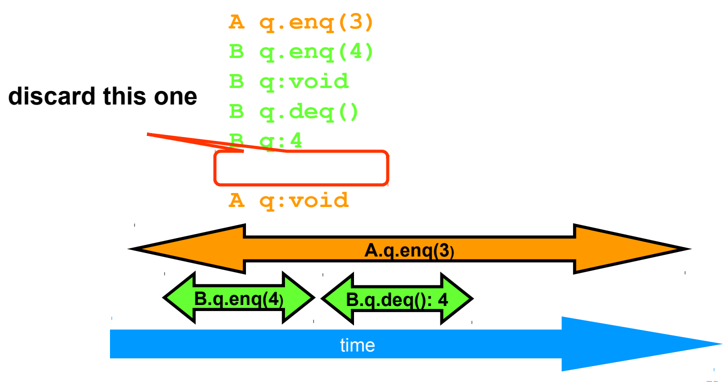
Example



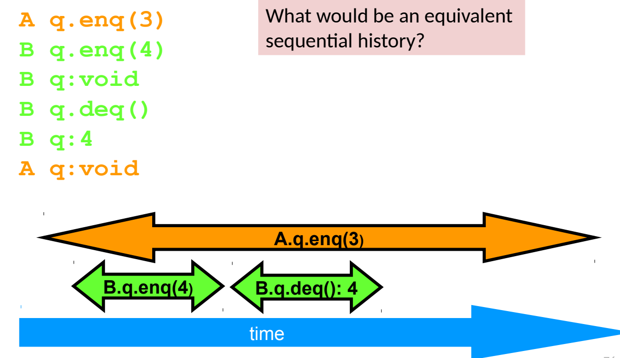
Example



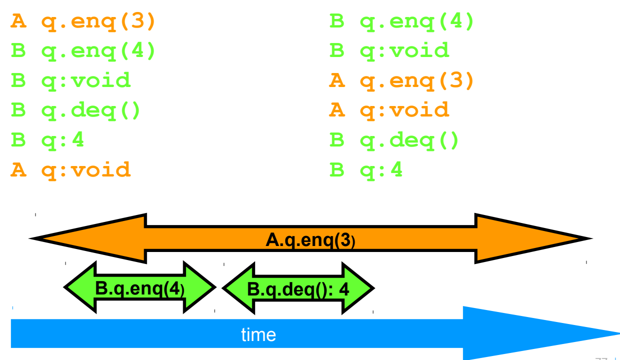
Example



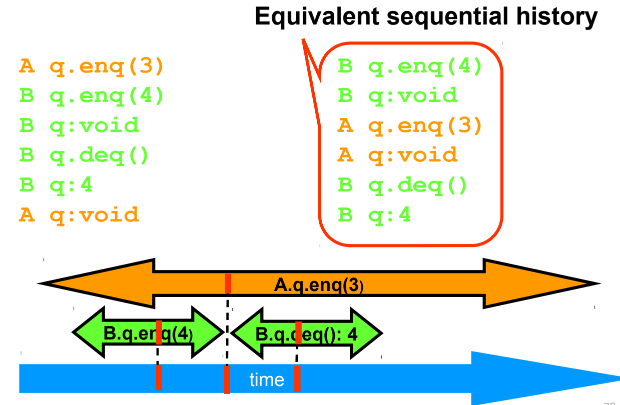
Example



Example



Example



Remember: Linearization Points

- **Identify one atomic step where a method "happens" (effects become visible to others)**
  - Critical section
  - Machine instruction (atomics, transactional memory ...)
- **Does not always succeed**
  - One may need to define several different steps for a given method
  - If so, extreme care must be taken to ensure pre-/postconditions
- **All possible executi[...] linearizable**

Now assuming wait-free two-thread queue?

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock);
  if((tail+1)%items.size() == head) {
    throw FullException;
  }
  items[tail] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock);
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head];
  head = (head+1)%items.size();
  return item;
}
```

Linearization points?

Composition

- **H is linearizable iff for every object x, H|x is linearizable!**
  - Corrollary: Composing linearizable objects results in a linearizable system
- **Reasoning**
  - Consider linearizability of objects in isolation
- **Modularity**
  - Allows concurrent systems to be constructed in a modular fashion
  - Compose independently-implemented objects

Linearizability vs. Sequential Consistency

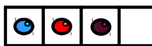
- **Sequential consistency**
  - Correctness condition
  - For describing hardware memory interfaces
  - Remember: not *actual* ones!
- **Linearizability**
  - Stronger correctness condition
  - For describing higher-level systems composed from linearizable components

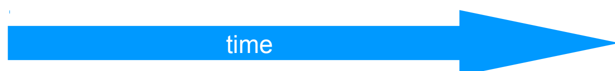
    Requires understanding of object semantics

Map linearizability to sequential consistency

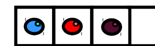
- **Variables with read and write operations**
  - Sequential consistency
- **Objects with a type and methods**
  - Linearizability
- **Map sequential consistency ↔ linearizability**
  - ← Reduce data types to variables with read and write operations
  - → Model variables as data types with read() and write() methods
- **Remember: Sequential consistency**
  - A history H is sequential if it can be extended to H' and H' is equivalent to some sequential history S
  - Note: Precedence order ($<_H \subseteq <_S$) does not need to be maintained

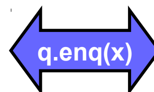
Example

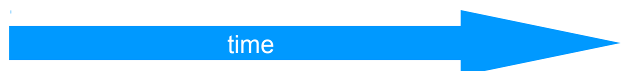




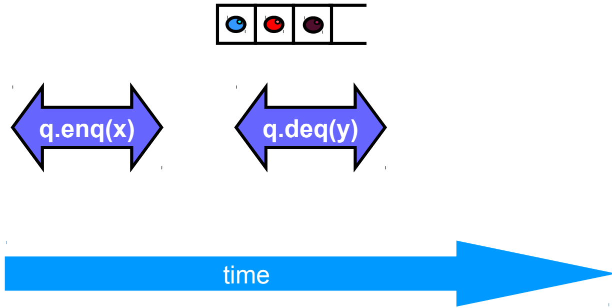
Example





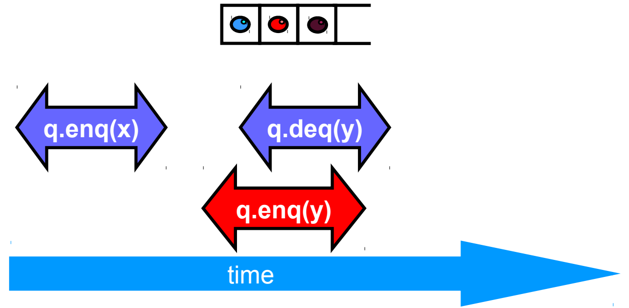


Example



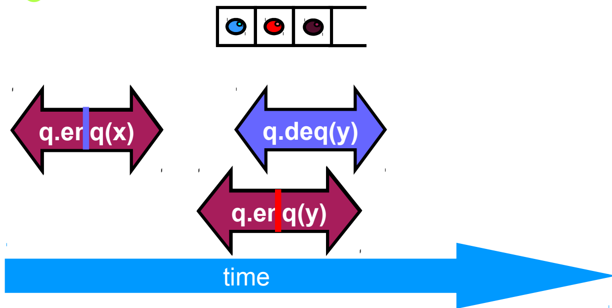
Example

Linearizable?



Example

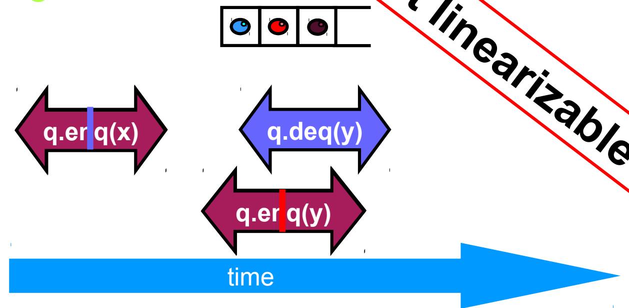
Linearizable?



Example

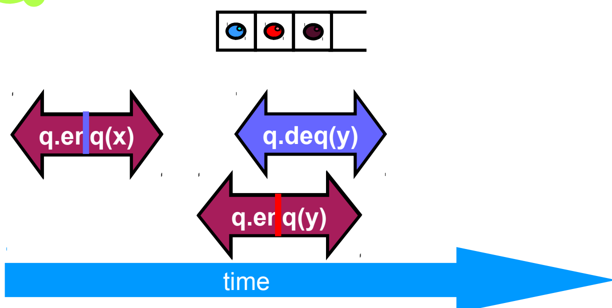
Linearizable?

not linearizable



Example

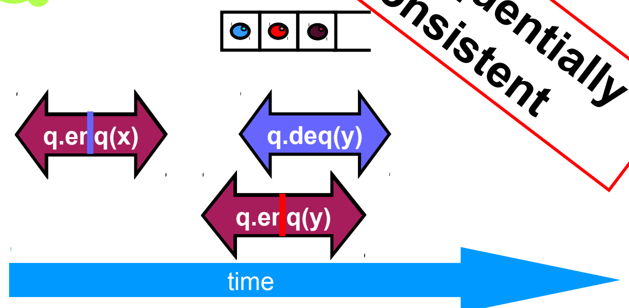
Sequentially consistent?



Example

Sequentially consistent?

yet sequentially consistent



Properties of sequential consistency

- **Theorem: Sequential consistency is not compositional**

H=

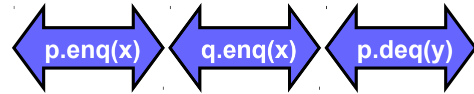
A: p.enq(x)
A: p:void
B: q.enq(y)
B: q:void
A: q.enq(x)
A: q:void
B: p.enq(y)
B: p:void
A: p.deq()
A: p:y
B: q.deq()
B: q:x

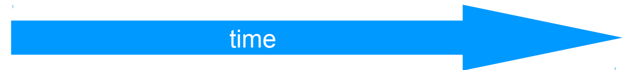
Compositional would mean:
"If H|p and H|q are sequentially consistent, then H is sequentially consistent!"

This is not guaranteed for SC schedules!

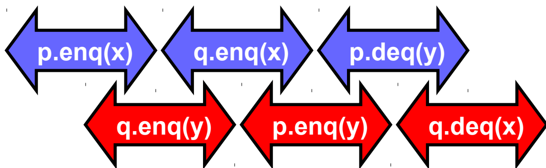
See following example!

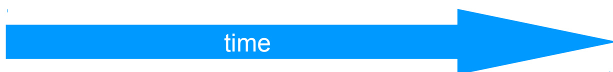
FIFO Queue Example



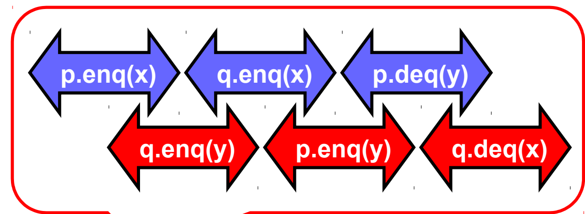


FIFO Queue Example

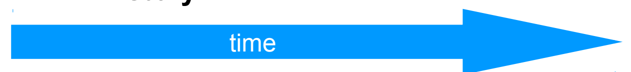




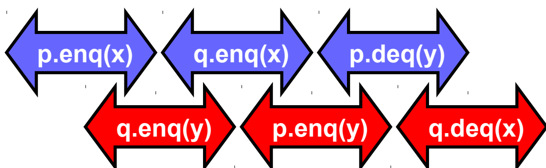
FIFO Queue Example

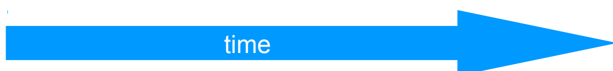


History H

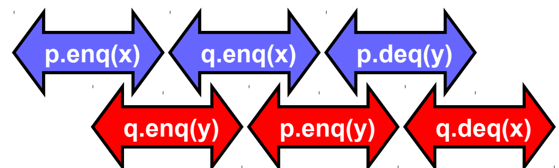


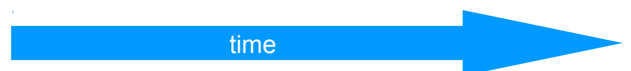
H|p Sequentially Consistent



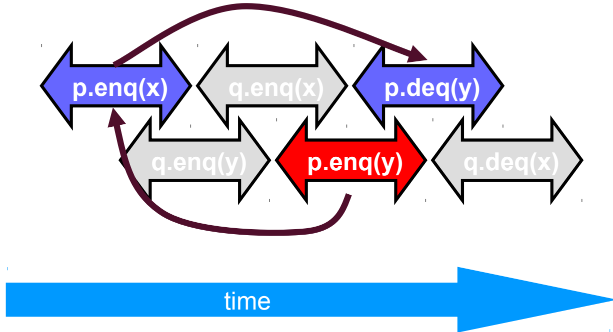


H|q Sequentially Consistent

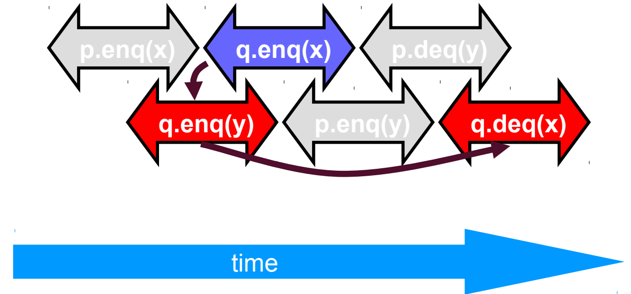




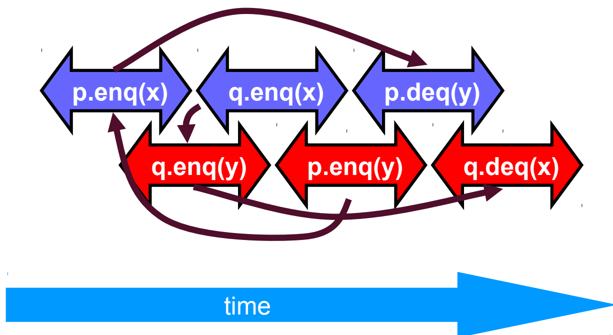
Ordering imposed by p



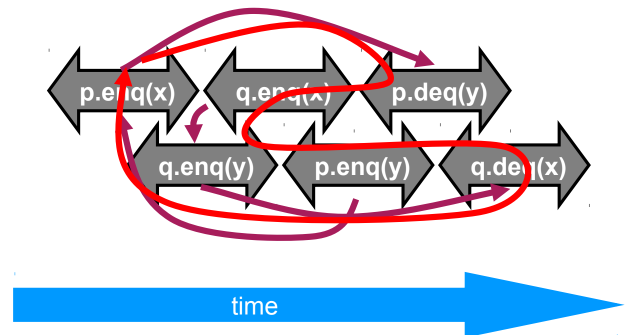
Ordering imposed by q



Ordering imposed by both



Combining orders



Example in our notation

- **Sequential consistency is not compositional – H|p**

H=	H|p=	(H|p)|A=	(H|p)|B=
A: p.enq(x)	A: p.enq(x)	A: p.enq(x)	B: p.enq(y)
A: p:void	A: p:void	A: p:void	B: p:void
B: q.enq(y)	B: p.enq(y)	A: p.deq()	
B: q:void	B: p:void	A: p:y	
A: q.enq(x)	A: p.deq()		
A: q:void	A: p:y		
B: p.enq(y)			
B: p:void			
A: p.deq()			
A: p:y			
B: q.deq()			
B: q:x			

H|p is sequentially consistent!

Example in our notation

- **Sequential consistency is not compositional – H|q**

H=	H|q=	(H|q)|A=	(H|q)|B=
A: p.enq(x)	B: q.enq(y)	A: q.enq(x)	B: q.enq(y)
A: p:void	B: q:void	A: q:void	B: q:void
B: q.enq(y)	A: q.enq(x)		B: q.deq()
B: q:void	A: q:void		B: q:x
A: q.enq(x)	B: q.deq()		
A: q:void	B: q:x		
B: p.enq(y)			
B: p:void			
A: p.deq()			
A: p:y			
B: q.deq()			
B: q:x			

H|q is sequentially consistent!

Example in our notation

- **Sequential consistency is not compositional**

H=	H|A=	H|B=
A: p.enq(x)	A: p.enq(x)	B: q.enq(y)
A: p:void	A: p:void	B: q:void
B: q.enq(y)	A: q.enq(x)	B: p.enq(y)
B: q:void	A: q:void	B: p:void
A: q.enq(x)	A: p.deq()	B: q.deq()
A: q:void	A: p:y	B: q:x
B: p.enq(y)		
B: p:void		
A: p.deq()		
A: p:y		
B: q.deq()		
B: q:x		

H is not sequentially consistent!

Correctness: Linearizability

- **Sequential Consistency**
  - Not composable
  - Harder to work with
  - Good (simple) way to think about hardware models
    Few assumptions (no semantics or time)
- **We will use *linearizability* in the remainder of this course unless stated otherwise**
  Consider routine entry and exit

Study Goals (Homework)

- **Define linearizability with your own words!**
- **Describe the properties of linearizability!**
- **Explain the differences between sequential consistency and linearizability!**
- **Given a history H**
  - Identify linearization points
  - Find equivalent sequential history S
  - Decide and explain whether H is linearizable
  - Decide and explain whether H is sequentially consistent
  - Give values for the response events such that the execution is linearizable

Language Memory Models

- **Which transformations/reorderings can be applied to a program**
- **Affects platform/system**
  - Compiler, (VM), hardware
- **Affects programmer**
  - What are possible semantics/output
  - Which communication between threads is legal?
- **Without memory model**
  - Impossible to even define “legal” or “semantics” when data is accessed concurrently
- **A memory model is a contract**
  - Between platform and programmer

History of Memory Models

- **Java’s original memory model was broken [1]**
  - Difficult to understand => widely violated
  - Did not allow reorderings as implemented in standard VMs
  - Final fields could appear to change value without synchronization
  - Volatile writes could be reordered with normal reads and writes
    => *counter-intuitive for most developers*
- **Java memory model was revised [2]**
  - Java 1.5 (JSR-133)
  - Still some issues (operational semantics definition [3])
- **C/C++ didn’t even have a memory model until recently**
  - Not able to make any statement about threaded semantics!
  - Introduced in C++11 and C11
  - Based on experience from Java, more conservative

[1] Pugh: “The Java Memory Model is Fatally Flawed”, CCPE 2000
[2] Manson, Pugh, Adve: “The Java memory model”, POPL’05
[3] Aspinall, Sevcik: “Java memory model examples: Good, bad and ugly”, VAMP’07

Everybody wants to optimize

- **Language constructs for synchronization**
  - Java: volatile, synchronized, …
  - C++: atomic, (**NOT volatile**!), mutex, …
- **Without synchronization (defined language-specific)**
  - Compiler, (VM), architecture
  - Reorder and appear to reorder memory operations
  - Maintain sequential semantics per thread
  - Other threads may observe any order (have seen examples before)

Java and C++ High-level overview

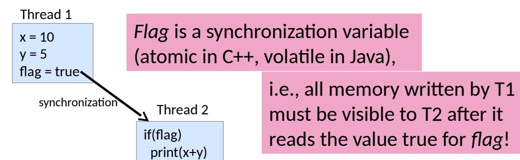
- **Relaxed memory model**
  - No global visibility ordering of operations
  - Allows for standard compiler optimizations
- **But**
  - Program order for each thread (sequential semantics)
  - Partial order on memory operations (with respect to synchronizations)
  - Visibility function defined
- **Correctly synchronized programs**
  - Guarantee sequential consistency
- **Incorrectly synchronized programs**
  - Java: maintain safety and security guarantees

    Type safety etc. (require behavior bounded by causality)
  - C++: undefined behavior

    No safety (anything can happen/change)

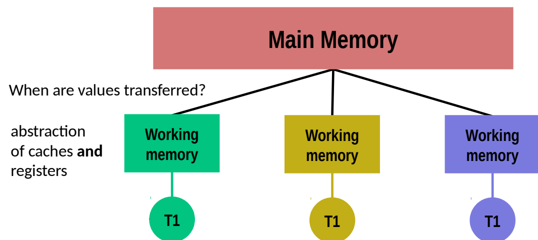
Communication between Threads: Intuition

- **Not guaranteed unless by:**
  - Synchronization
  - Volatile/atomic variables
  - Specialized functions/classes (e.g., java.util.concurrent, ...)



Memory Model: Intuition

- **Abstract relation between threads and memory**
  - Local thread view!



- **Does not talk about classes, objects, methods, ...**
  - Linearizability is a higher-level concept!

Lock Synchronization

- **Java**

```
synchronized (lock) {
  // critical region
}
```

  - Synchronized methods as syntactic sugar

- **C++**

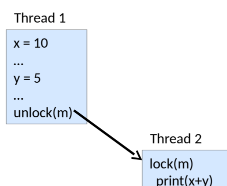
```
{
  unique_lock<mutex> l(lock);
  // critical region
}
```

  - Many flexible variants

- **Semantics:**
  - mutual exclusion
  - at most one thread may own a lock
  - a thread B trying to acquire a lock held by thread A blocks until thread A releases lock
  - note: threads may wait forever (no progress guarantee!)

Memory semantics

- **Similar to synchronization variables**



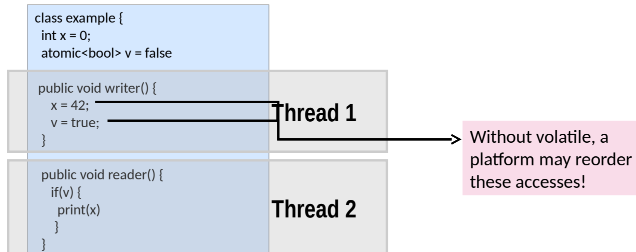
  - All memory accesses before an unlock ...
  - are ordered before and are visible to ...
  - any memory access after a matching lock!

Synchronization Variables

- **Variables can be declared volatile (Java) or atomic (C++)**
- **Reads and writes to synchronization variables**
  - Are totally ordered with respect to all threads
  - Must not be reordered with normal reads and writes
- **Compiler**
  - Must not allocate synchronization variables in registers
  - Must not swap variables with synchronization variables
  - May need to issue memory fences/barriers
  - ...

Synchronization Variables

- **Write to a synchronization variable**
  - Similar memory semantics as unlock (no process synchronization!)
- **Read from a synchronization variable**
  - Similar memory semantics as lock (no process synchronization!)



Memory Model Rules

- **Java/C++: Correctly synchronized programs will execute sequentially consistent**
- **Correctly synchronized = data-race free**
  - iff all sequentially consistent executions are free of data races
- **Two accesses to a shared memory location form a data race in the execution of a program if**
  - The two accesses are from different threads
  - At least one access is a write and
  - The accesses are not synchronized