Design of Parallel and High-Performance Computing

Fall 2013

Lecture: Linearizability

Instructor: Torsten Hoefler & Markus Püschel

TA: Timo Schneider

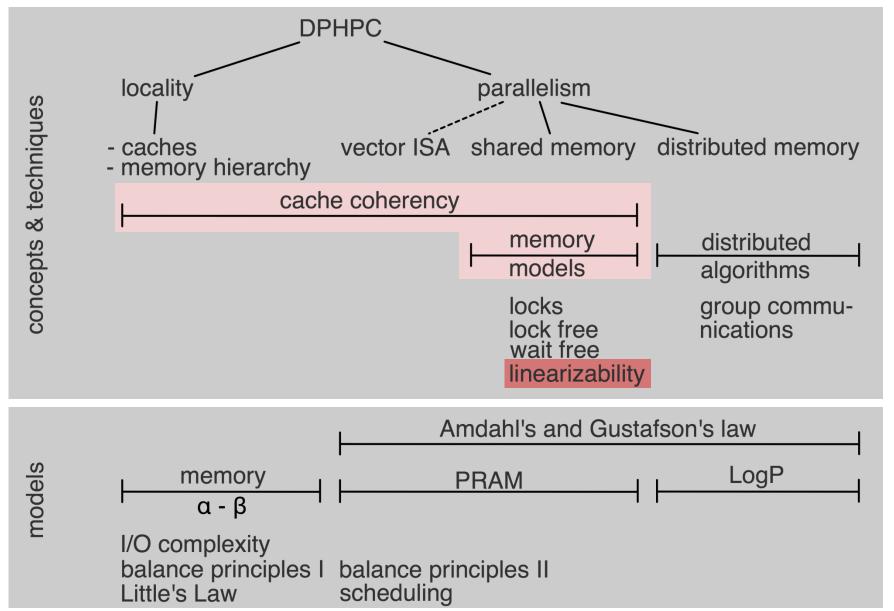


Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

Review of last lecture

- Cache-coherence is not enough!
 - Many more subtle issues for parallel programs!
- Memory Models
 - Sequential consistency
 - Why threads cannot be implemented as a library ©
 - Relaxed consistency models
 - x86 TLO+CC case study
- Complexity of reasoning about parallel objects

DPHPC Overview



Goals of this lecture

Queue:

- Locked
 - C++ locking (small detour)
- Wait-free two-thread queue

Linearizability

- Intuitive understanding (sequential order on objects!)
- Linearization points
- Linearizable executions
- Formal definitions (Histories, Projections, Precedence)
- Linearizability vs. Sequential Consistency
 - Modularity

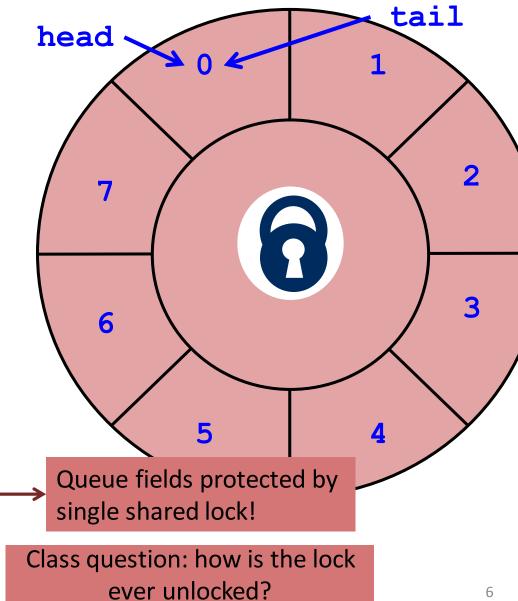
Lock-based queue

```
head
class Queue {
private:
int head, tail;
std::vector<Item> items;
 std::mutex lock;
public:
 Queue(int capacity) {
 head = tail = 0;
 items.resize(capacity);
                                                     Queue fields protected by
                                                     single shared lock!
```

tail

Lock-based queue

```
class Queue {
 public:
void eng(Item x) {
  std::lock guard<std::mutex> I(lock)
  if(tail-head == items.size()) {
   throw FullException;
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
 Item deg() {
  std::lock_guard<std::mutex> I(lock)
  if(tail == head) {
   throw FullException;
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
```



C++ Resource Acquisition is Initialization

- RAII suboptimal name
- Can be used for locks (or any other resource acquisitions)
 - Constructor grabs resource
 - Destructor frees resource

Behaves as if

Implicit unlock at end of block!

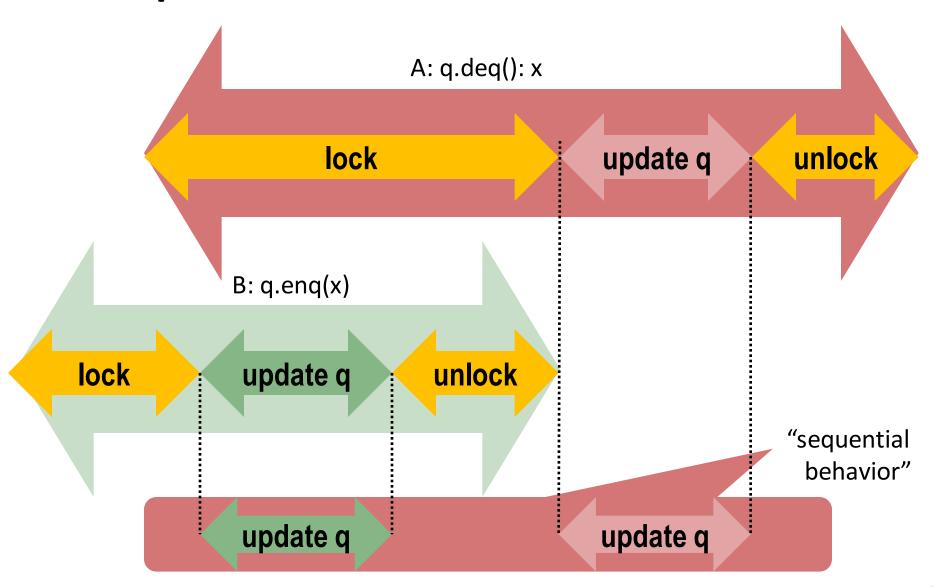
Main advantages

- Always free lock at exit
- No "lost" locks due to exceptions or strange control flow (goto ②)
- Very easy to use

```
class lock_guard<typename mutex_impl> {
    mutex_impl &_mtx; // ref to the mutex

public:
    scoped_lock(mutex_impl & mtx ) : _mtx(mtx) {
        _mtx.lock(); //lock mutex in constructor
    }
    ~scoped_lock() {
        _mtx.unlock(); //unlock mutex in destructor
    }
};
```

Example execution



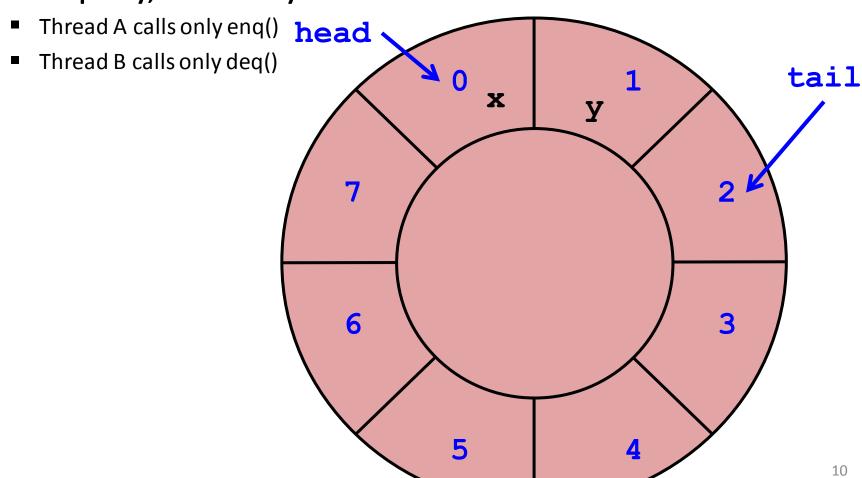
Correctness

- Is the locked queue correct?
 - Yes, only one thread has access if locked correctly
 - Allows us again to reason about pre- and postconditions
 - Smells a bit like sequential consistency, no?
- Class question: What is the problem with this approach?
 - Same as for SC ©

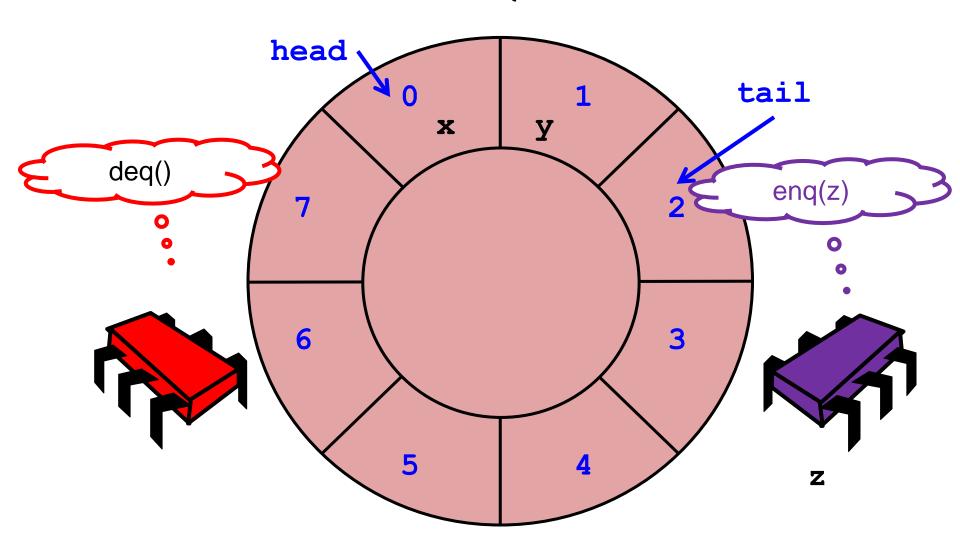
It does not scale!
What is the solution here?

Threads working at the same time?

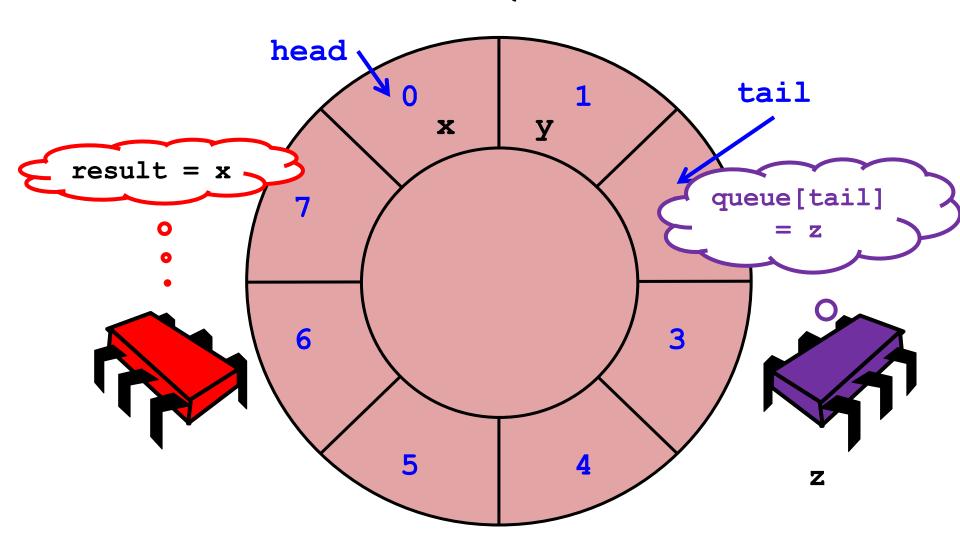
- Same thing (concurrent queue)
- For simplicity, assume only two threads



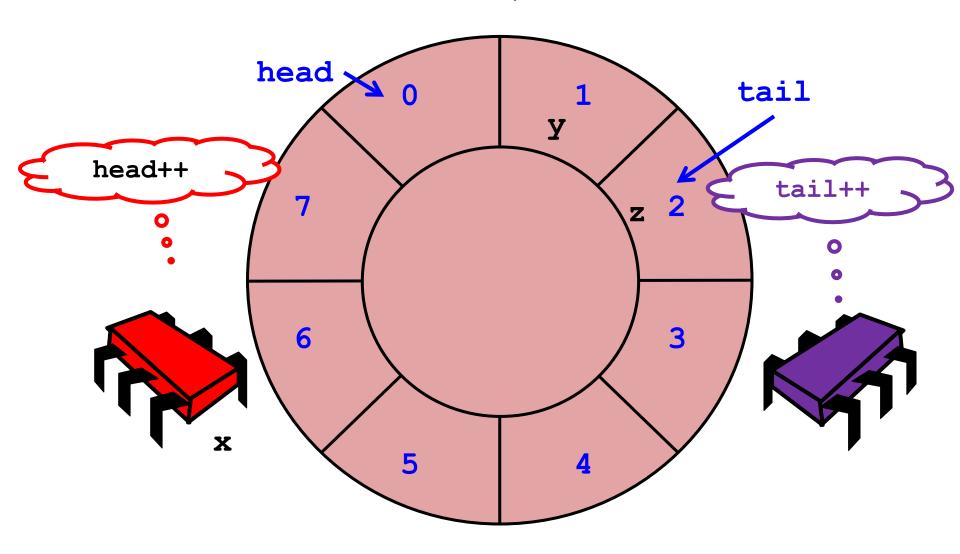
Wait-free 2-Thread Queue



Wait-free 2-Thread Queue



Wait-free 2-Thread Queue



Is this correct?

- Hard to reason about correctness
- What could go wrong?

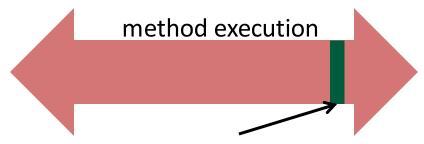
```
void enq(Item x) {
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```

- Nothing (at least no crash)
- Yet, the semantics of the queue are funny (define "FIFO" now)!

Serial to Concurrent Specifications

- Serial specifications are complex enough, so lets stick to them
 - Define invocation and response events (start and end of method)
- Extend the concept to concurrency: linearizability
- Each method should "take effect"
 - Instantaneously
 - Between invocation and response events
- Concurrent object is correct if this "sequential" behavior is correct
 - Called "linearizable"



Linearization point = when method takes effect

Linearizability

- Sounds like a property of an execution ...
- An object is called linearizable if all possible executions on the object are linearizable
- Says nothing about the order of executions!

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
O O O
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```

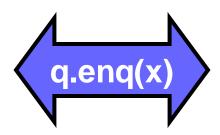
linearization points

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```



linearization points

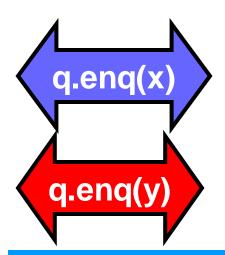


```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```



linearization points

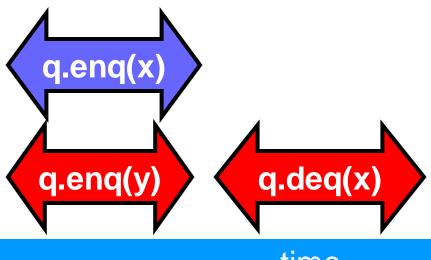


time

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```



linearization points



time

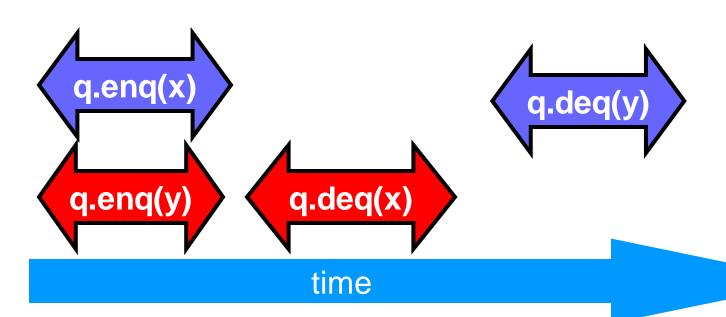


```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```



linearization points



q.enq(x)

q.en((y)

```
void eng(Item x) {
                                     Item deq() {
 std::lock guard<std::mutex> I(lock)
                                      std::lock guard<std::mutex> l(lock)
 if(tail-head == items.size()) {
                                      if(tail == head) {
  throw FullException;
                                       throw EmptyException;
 items[tail % items.size()] = x;
                                       Item item = items[head % items.size()];
 tail = (tail+1)%items.size();
                                       head = (head+1)%items.size();
                                     linearization points
                            q.c eq(y)
      q.d \ni q(x)
        time
```

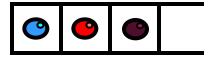
q.enq(x)

q.en((y)

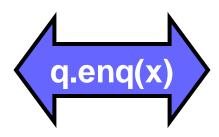


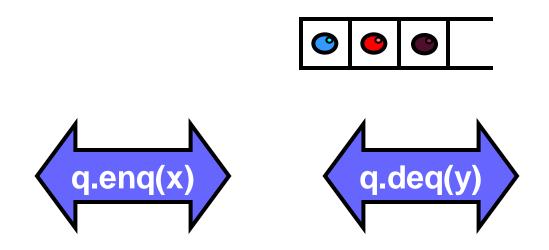
```
Item deq() {
void eng(Item x) {
 std::lock guard<std::mutex> I(lock)
                                          std::lock guard<std::mutex> l(lock)
 if(tail-head == items.size()) {
                                           if(tail == head) {
  throw FullException;
                                            throw EmptyException;
 items[tail % items.size()] = x;
                                           Item item = items[head % items.size()];
 tail = (tail+1)%items.size();
                                           head = (head+1)%items.size();
                                         linearization points
                                 q.ceq(y)
      q.d \ni q(x)
```

time



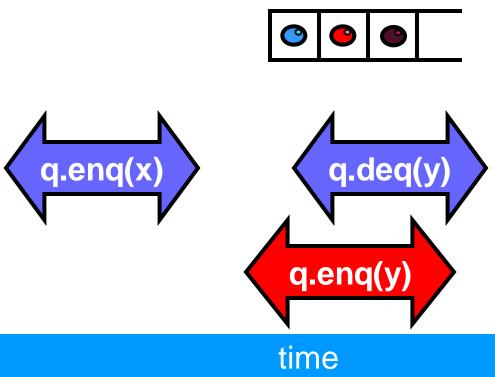




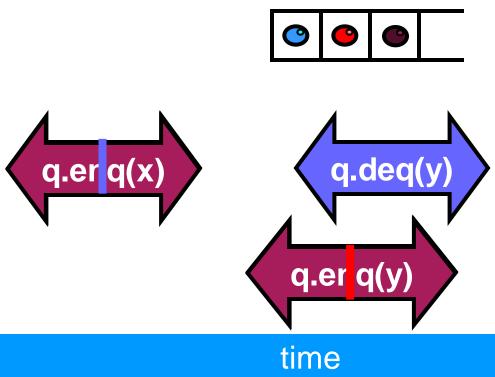


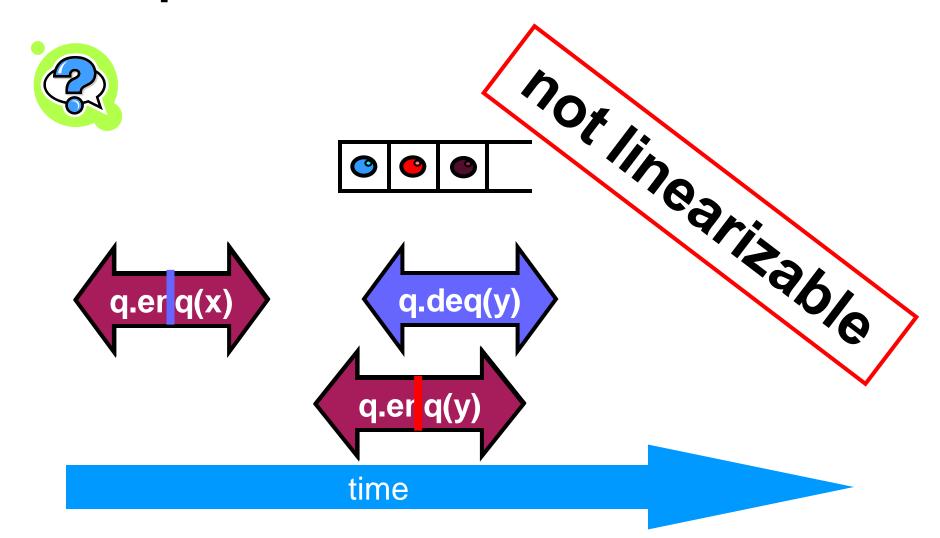
time



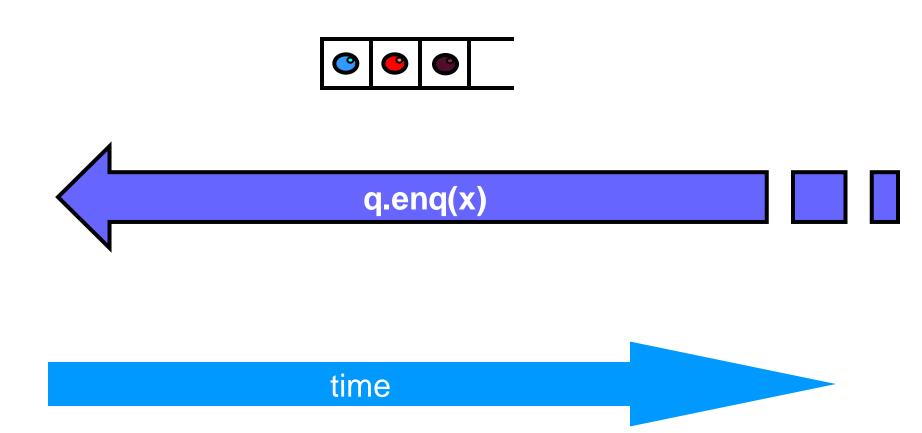




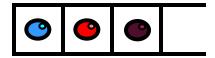


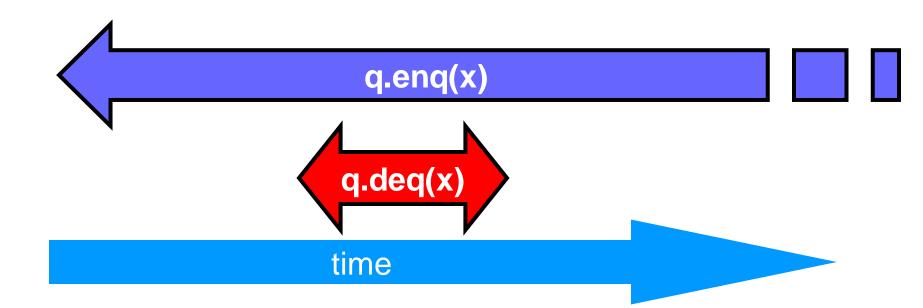




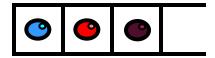


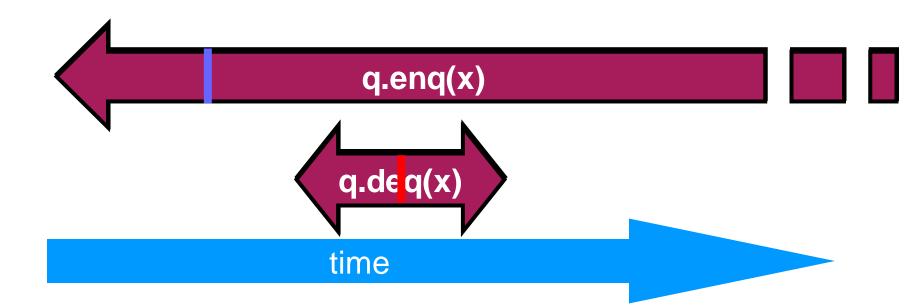


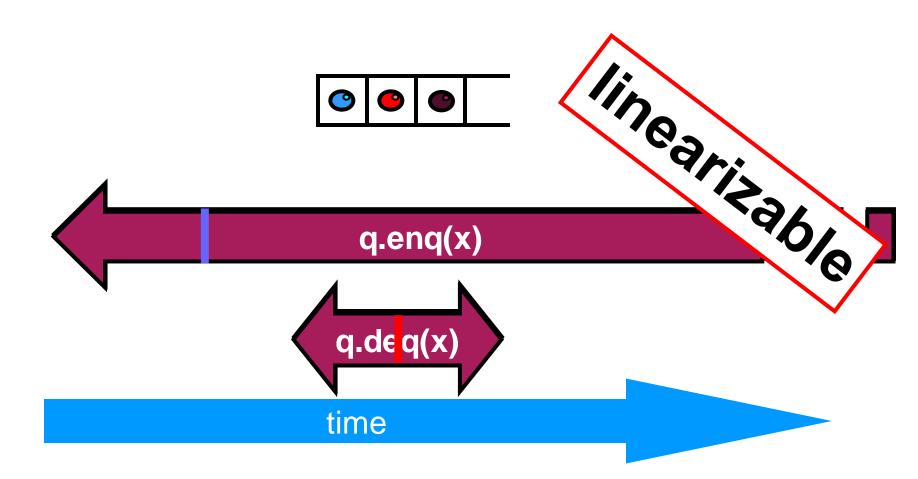


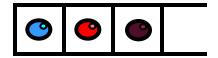


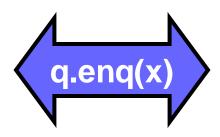


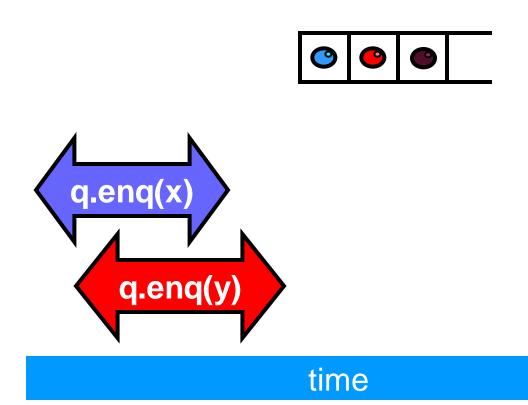


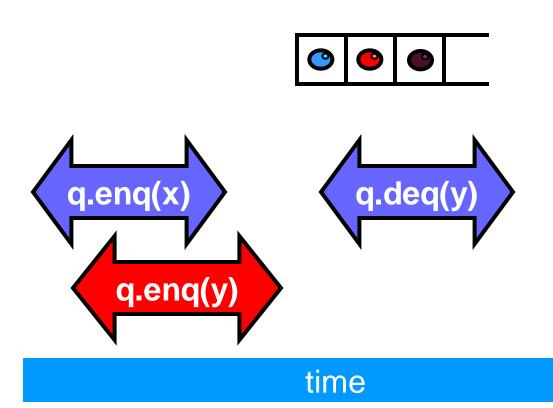




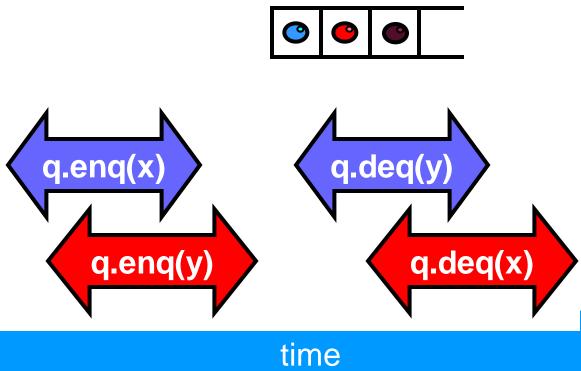


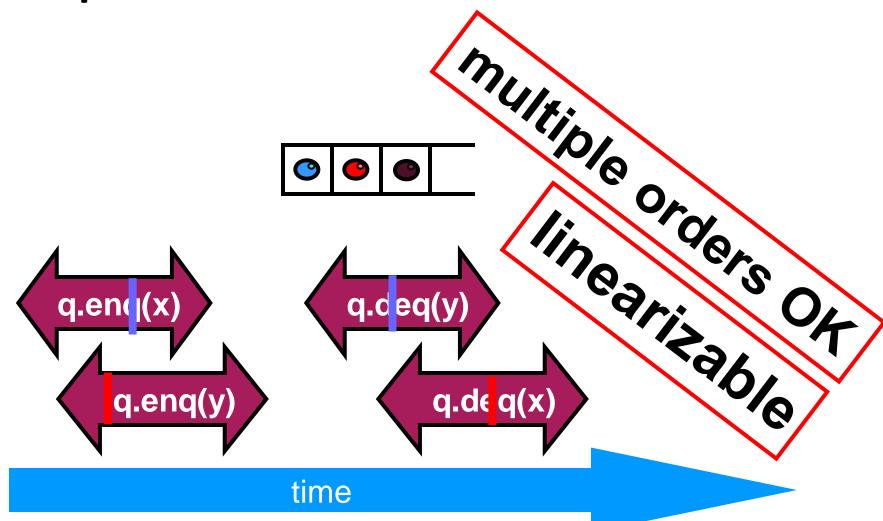


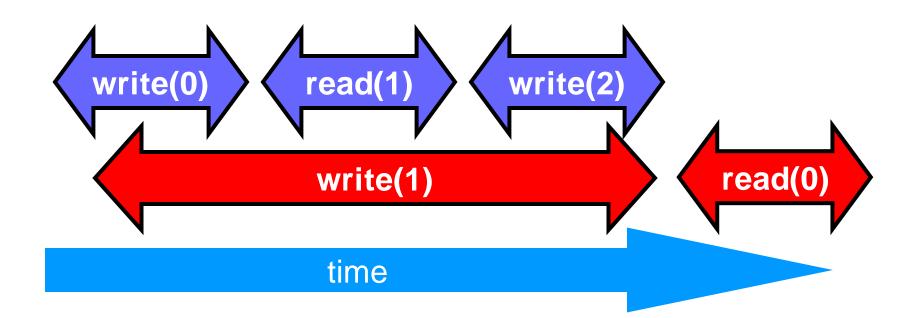


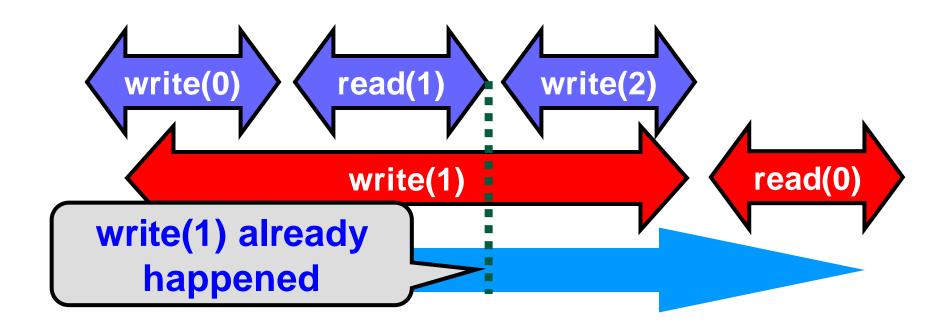


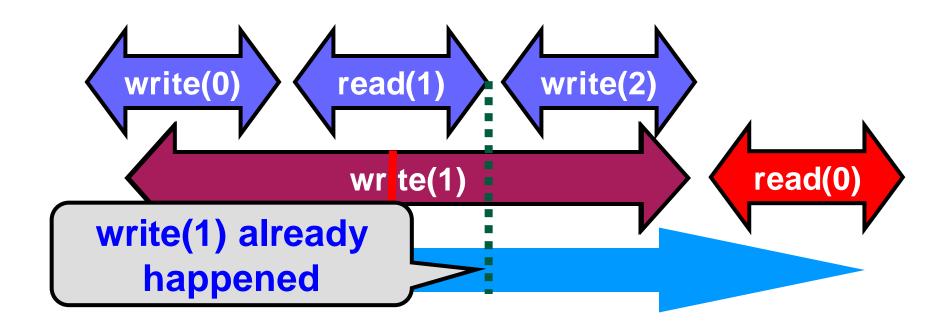


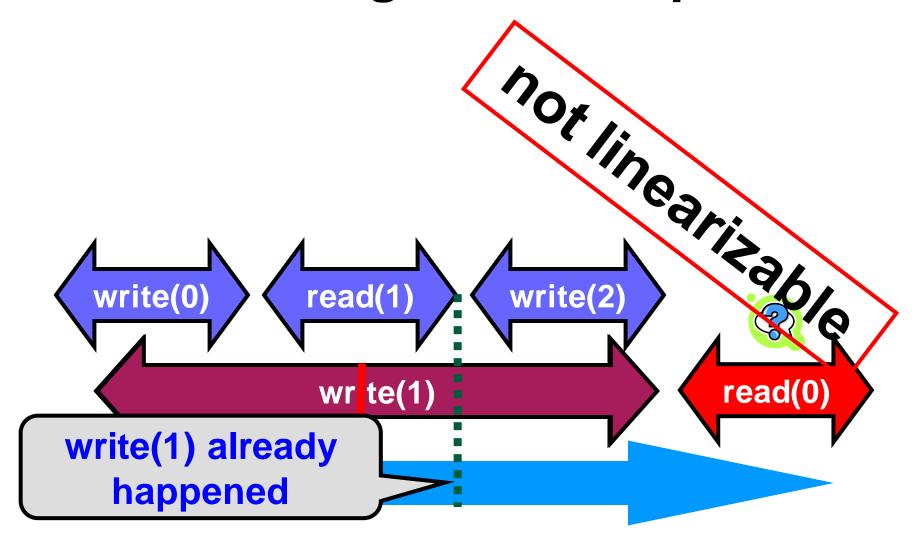


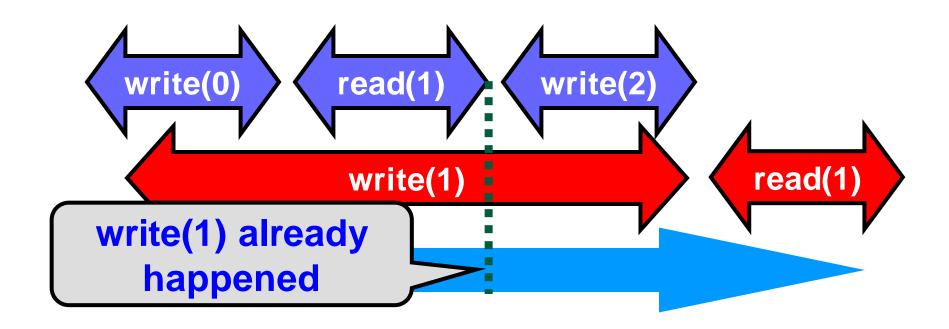


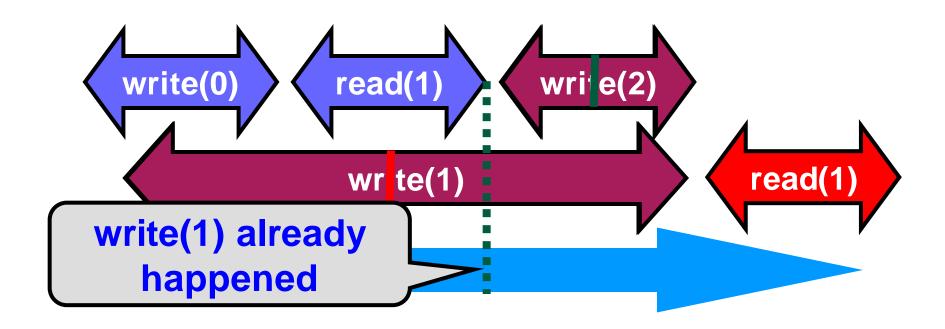


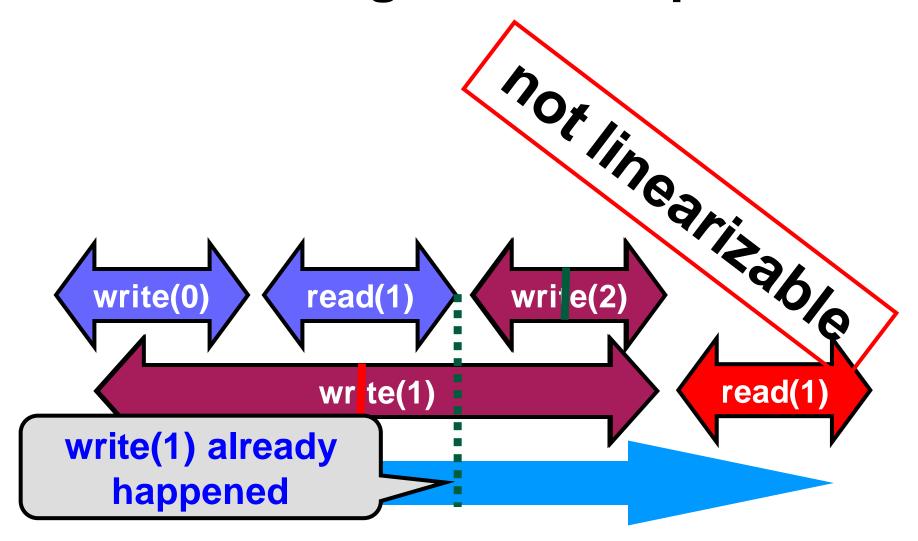


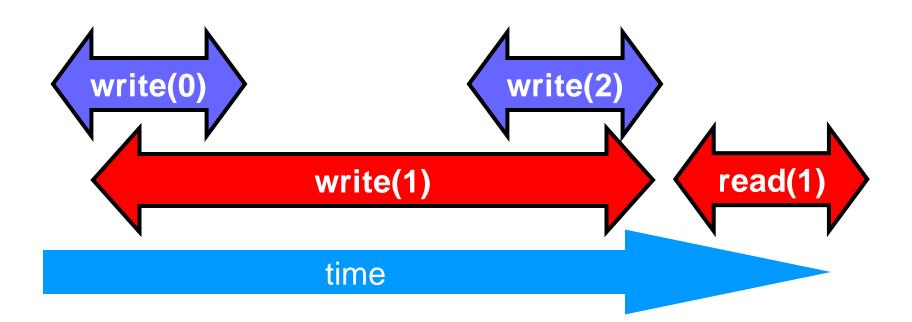


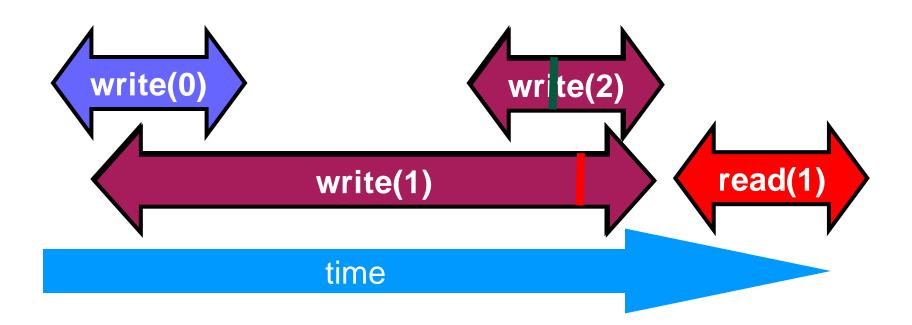


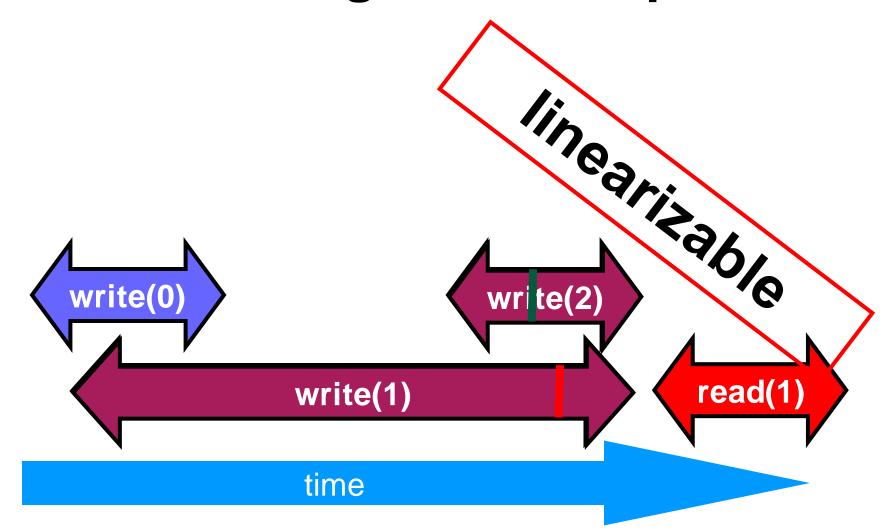


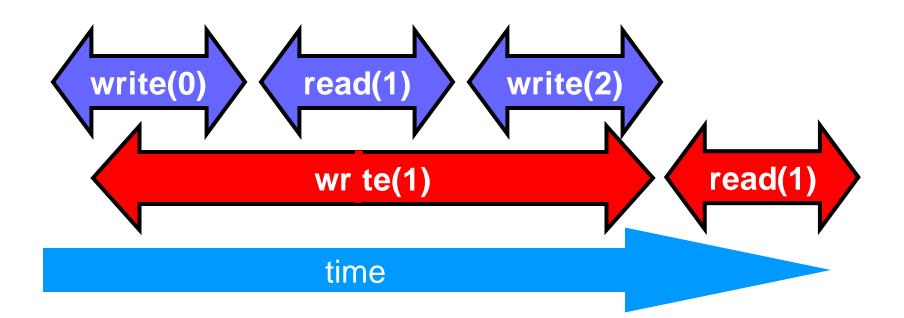


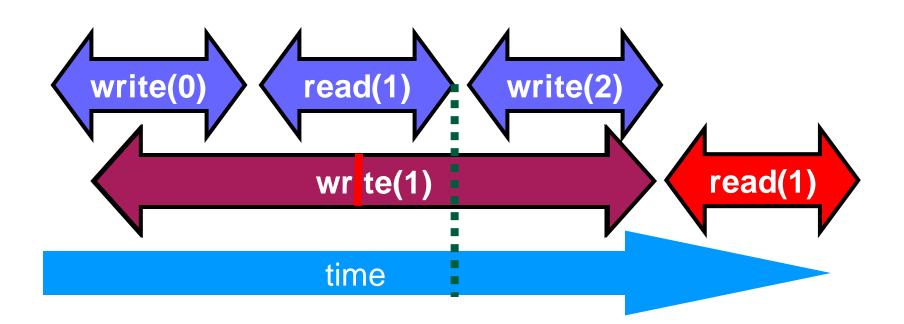


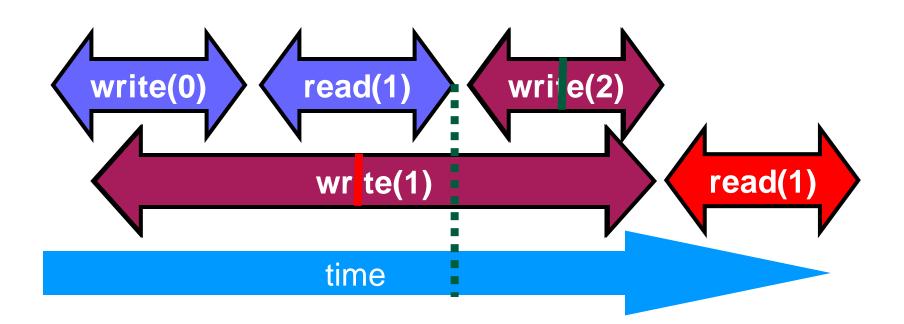


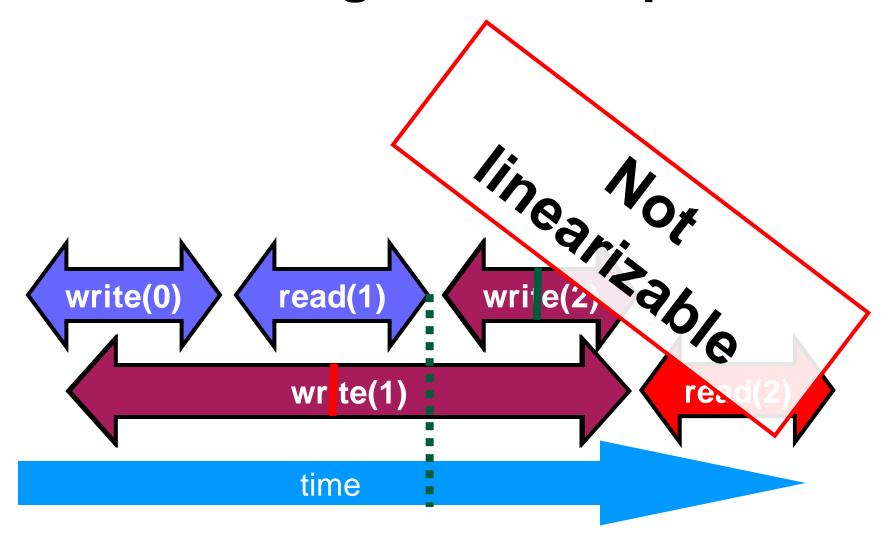












About Executions

Why?

Can't we specify the linearization point of each operation without describing an execution?

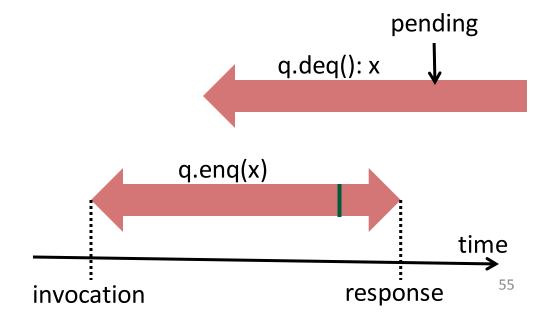
Not always

In some cases, linearization point depends on the execution

Define a formal model for executions!

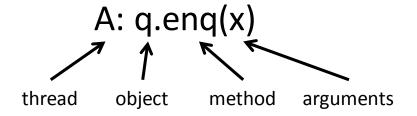
Properties of concurrent method executions

- Method executions take time
 - May overlap
- Method execution = operation
 - Defined by invocation and response events
- Duration of method call
 - Interval between the events

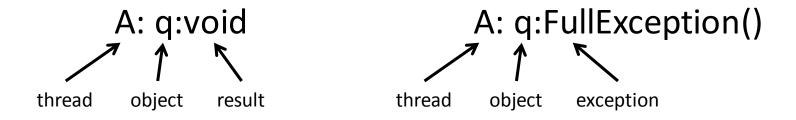


Formalization - Notation

Invocation



Response



Method is implicit (correctness criterion)!

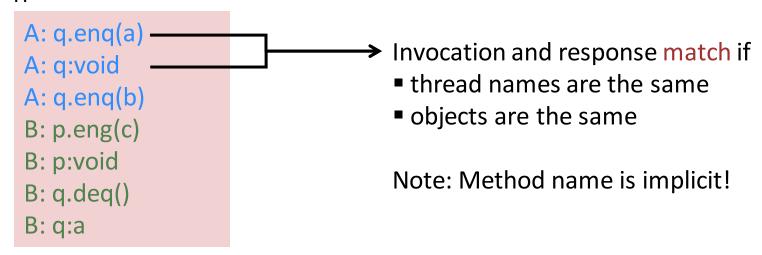
Concurrency

- A concurrent system consists of a collection of sequential threads P_i
- Threads communicate via shared objects

History

Describes an execution

- Sequence of invocations and responses
- H=



Projections on Threads

- Threads subhistory H|P ("H at P")
 - Subsequences of all events in H whose thread name is P

```
H|A=
                                                   H|B=
H=
A: q.enq(a)
                         A: q.enq(a)
A: q:void
                         A: q:void
A: q.enq(b)
                         A: q.enq(b)
B: p.enq(c)
                                                   B: p.enq(c)
B: p:void
                                                   B: p:void
B: q.deq()
                                                   B: q.deq()
B: q:a
                                                   B: q:a
```

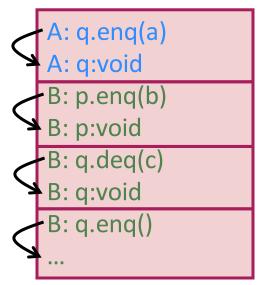
Projections on Objects

- Objects subhistory H | o ("H at o")
 - Subsequence of all events in H whose object name is o

| H= | H p= | H q= |
|-----------------------------------|-----------------------|-----------------------------------|
| A: q.enq(a) A: q:void A: q.enq(b) | | A: q.enq(a) A: q:void A: q.enq(b) |
| B: p.eng(c) B: p:void | B: p.eng(c) B: p:void | |
| B: q.deq() B: q:a | 2. p. v 310 | B: q.deq() B: q:a |

Sequential Histories

A history H is sequential if



- First event of H is an invocation
- Each invocation (except possibly the last is immediately followed by a matching response
- Each response is immediately followed by an invocation

Method calls of different threads do not interleave

- A history H is concurrent if
 - It is not sequential

Well-formed histories

Per-thread projections must be sequential

```
H= H|A=

A: q.enq(x)

B: p.enq(y)

A: q:void

B: p:void

B: q.deq()

A: q:void

B: p.enq(y)

B: p:void

B: p:void

B: p:void

B: p:void

B: q.deq()

B: q:x
```

Equivalent histories

Per-thread projections must be the same

A: q.enq(x)
B: p.enq(y)
B: p:void
B: q.deq()
A: q:void
B: q:x

A: q.enq(x)
B: p.enq(y)
A: q:void
B: p:void
B: q.deq()
B: q:x

H|A=G|A=

A: q.enq(x)
A: q:void

H|B=G|B=

B: p.enq(y)
B: p:void
B: q.deq()
B: q:x

Legal Histories

- Sequential specification allows to describe what behavior we expect and tolerate
 - When is a single-thread, single-object history legal?

- Recall: Example
 - Preconditions and Postconditions
 - Many others exist!
- A sequential (multi-object) history H is legal if
 - For every object x
 - H|x adheres to the sequential specification for x

Precedence

A: q.enq(x)

B: q.enq(y)

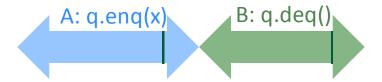
B: q:void

A: q:void

B: q.deq()

B: q:x

A method execution precedes another if response event precedes invocation event



Precedence vs. Overlapping

Non-precedence = overlapping

A: q.enq(x)

B: q.enq(y)

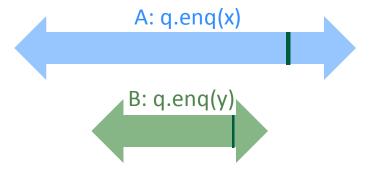
B: q:void

A: q:void

B: q.deq()

B: q:x

Some method executions overlap with others



Precedence relations

- Given history H
- Method executions m₀ and m₁ in H
 - $m_0 \rightarrow_H m_1$ (m_0 precedes m_1 in H) if
 - Response event of m₀ precedes invocation event of m₁
- Precedence relation $m_0 \rightarrow_H m_1$ is a
 - Strict partial order on method executions
 Irreflexive, antisymmetric, transitive
- Considerations
 - Precedence forms a total order if H is sequential
 - Unrelated method calls → overlap → concurrent

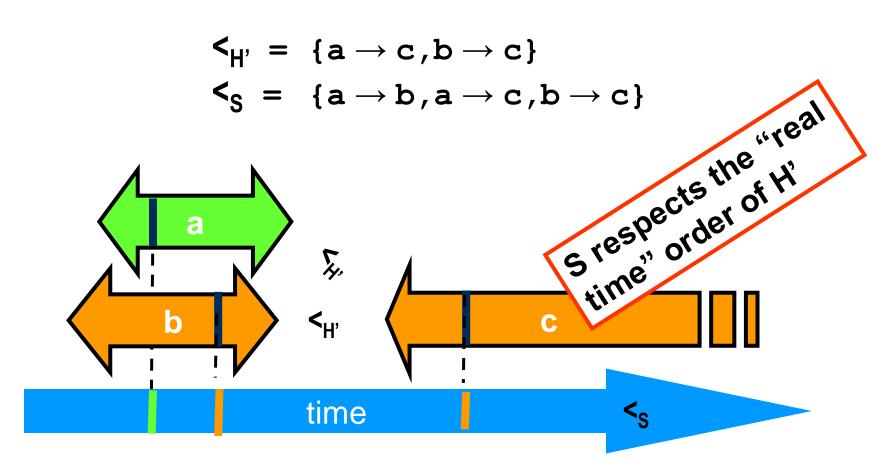
Definition Linearizability

- A history H induces a strict partial order <_H on operations
 - $m_0 <_H m_1 \text{ if } m_0 \rightarrow_H m_1$
- A history H is linearizable if
 - H can be extended to a history H'
 by appending responses to pending operations or dropping pending operations
 - H' is equivalent to some legal sequential history S and
 - <p
- S is a linearization of H

- Remarks:
 - For each H, there may be many valid extensions to H'
 - For each extension H', there may be many S
 - Interleaving at the granularity of methods

Ensuring <_{H'} ⊆ <_s

Find an S that contains H'

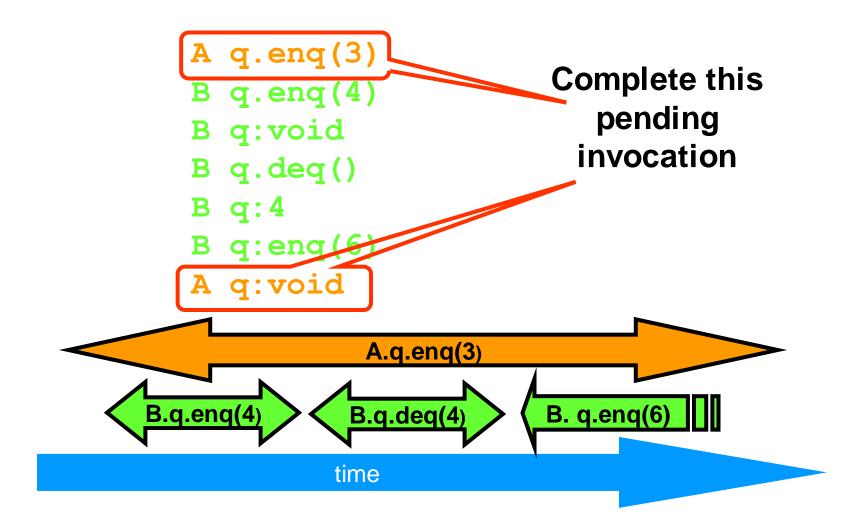


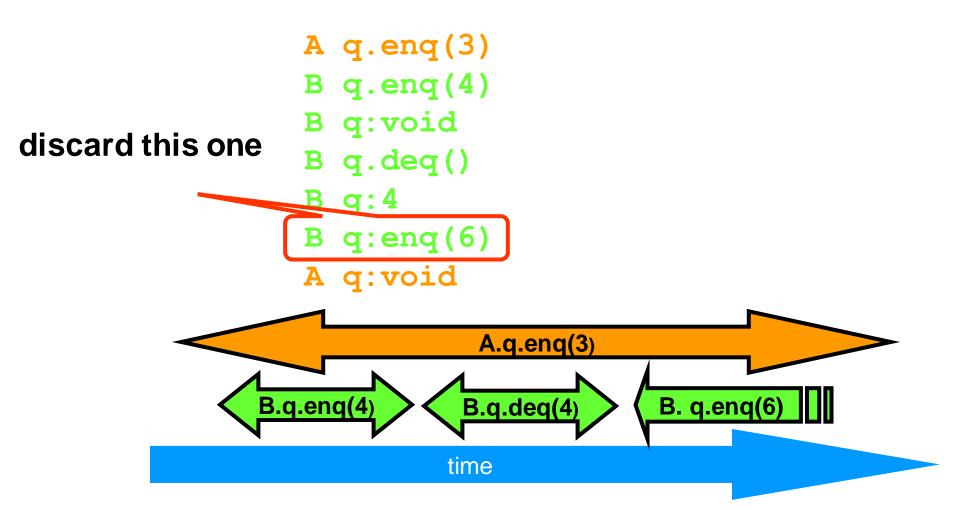
```
B q:void
    B q.deq()
    B q:4
    B q:enq(6)
 A. q.enq(3)
              B.q.deq(4)
                             B. q.enq(6)
B.q.enq(4)
             time
```

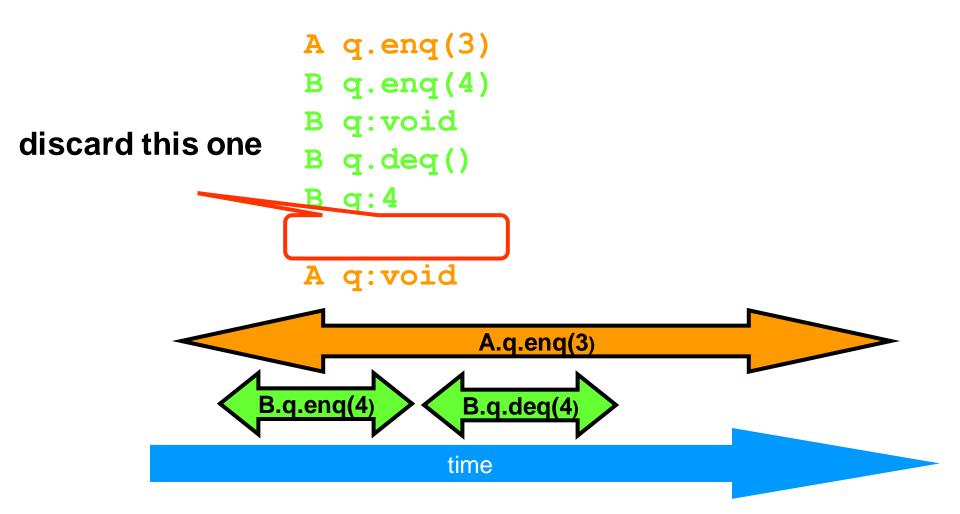
A q.enq(3)

Bq.enq(4)

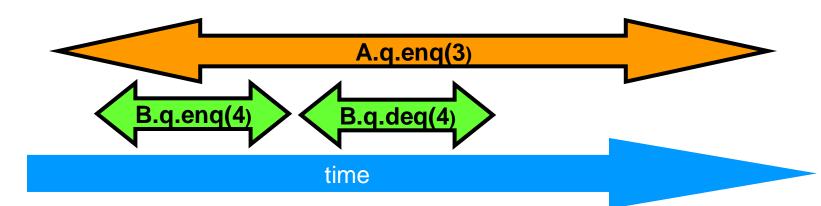
```
q.enq(3)
                            Complete this
     q.enq(4)
                               pending
     q:void
                              invocation
   B q.deq()
   B q:4
   B q:enq(6)
A. q.enq(3)
B.q.enq(4)
              B.q.deq(3)
                           B. q.enq(6)
             time
```



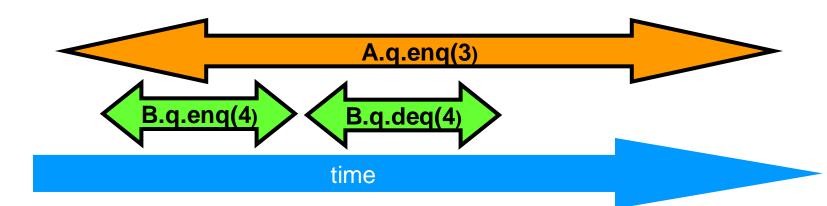




```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



```
A q.enq(3)
B q.enq(4)
B q.enq(4)
B q:void
A q.enq(3)
B q.deq()
A q:void
B q:4
B q.deq()
A q:void
B q:4
B q:4
```



```
Equivalent sequential history
                          B q.enq(4)
A q.enq(3)
                          B q:void
Bq.enq(4)
                          A q.enq(3)
B q:void
B q.deq()
                          A q:void
                          B q.deq()
B q:4
A q:void
                          B q:4
                     A.q.enq(3)
                    B.q. deq(4)
       B.q.er q(4)
                   time
```

Linearization Points

- Identify one atomic step where a method "happens" (effects become visible to others)
 - Critical section
 - Machine instruction (atomics, transactional memory ...)
- Does not always succeed
 - One may need to define several different steps for a given method
 - If so, extreme care must be taken to ensure pre-/postconditions
- All possible executions on the object must be linearizable

```
void enq(Item x) {
  std::lock_guard<std::mutex> l(lock)
  if(tail-head == items.size()) {
    throw FullException;
  }
  items[tail % items.size()] = x;
  tail = (tail+1)%items.size();
}
```

```
Item deq() {
  std::lock_guard<std::mutex> l(lock)
  if(tail == head) {
    throw EmptyException;
  }
  Item item = items[head % items.size()];
  head = (head+1)%items.size();
}
```

Composition

- H is linearizable iff for every object x, H|x is linearizable!
 - Composing linearizable objects results in a linearizable system

Reasoning

Consider linearizability of objects in isolation

Modularity

- Allows concurrent systems to be constructed in a modular fashion
- Compose independently-implemented objects

Linearizability vs. Sequential Consistency

Sequential consistency

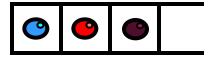
- Correctness condition
- For describing hardware memory interfaces
- Remember: not existing ones!

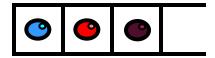
Linearizability

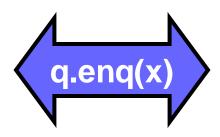
- Stronger correctness condition
- For describing higher-level systems composed from linearizable components

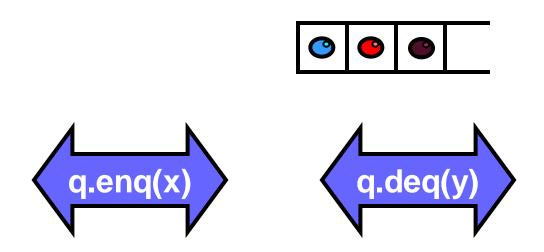
Map linearizability to sequential consistency

- Variables with read and write operations
 - Sequential consistency
- Objects with a type and methods
 - Linearizability
- Map sequential consistency → linearizability
 - Reduce data types to variables with read and write operations
 - Model variables as data types with read() and write() methods
- Sequential consistency
 - A history H is sequential if it can be extended to H' and H' is equivalent to some sequential history S
 - Note: Precedence order (<H ⊆ <S) does not need to be maintained</p>

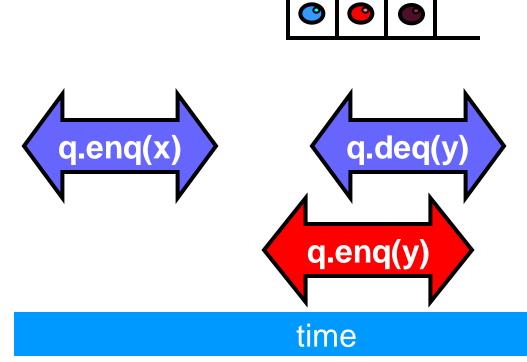




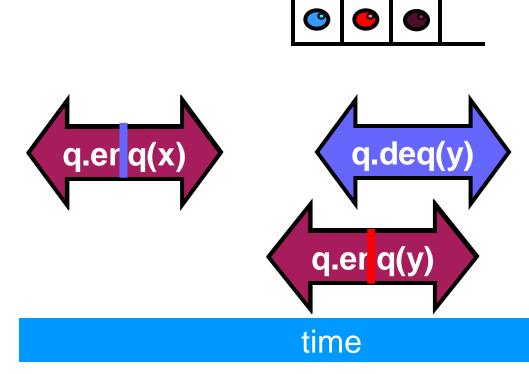


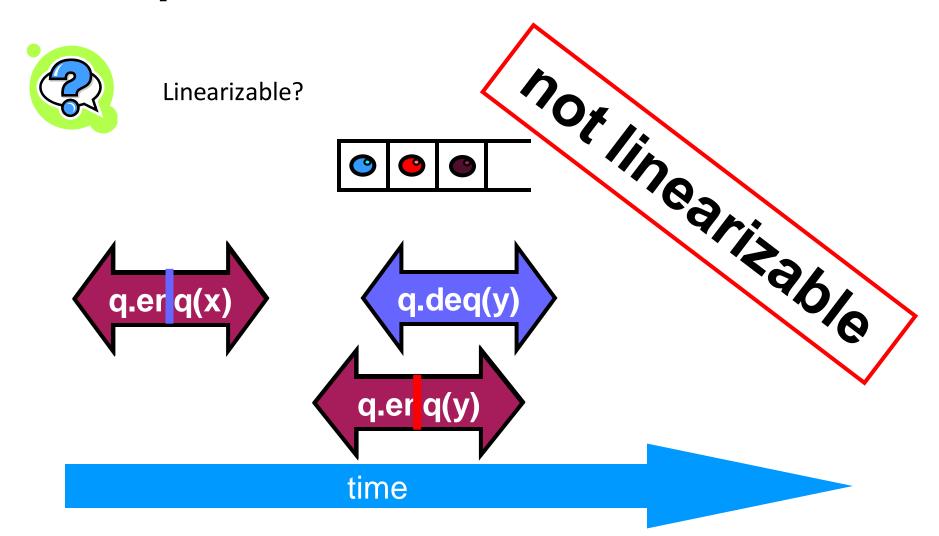






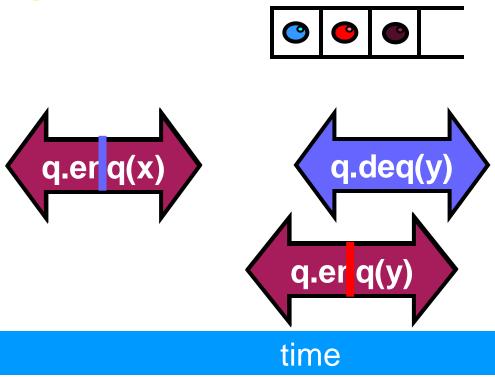


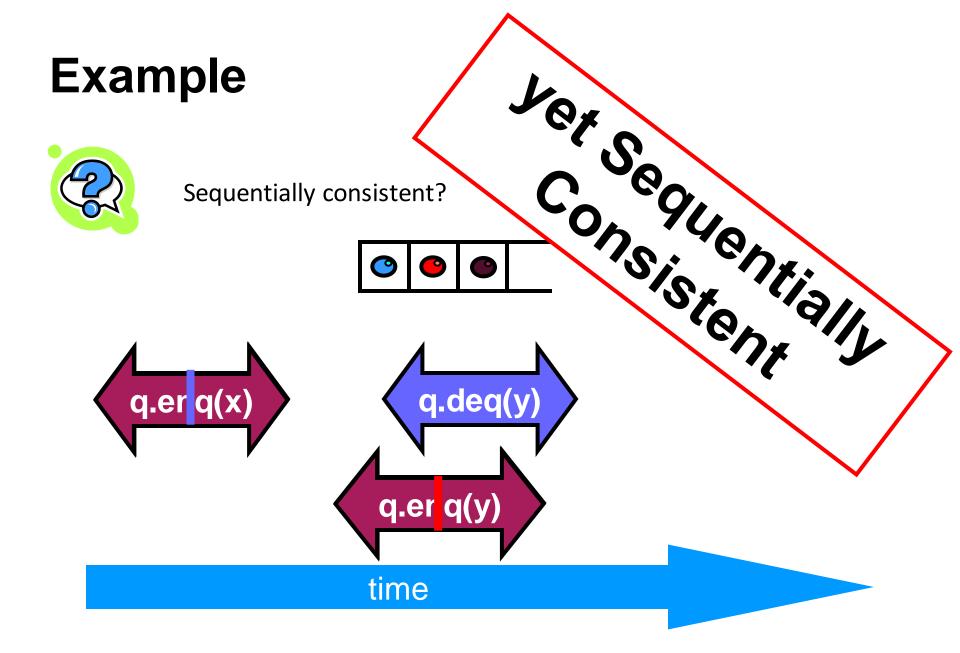






Sequentially consistent?





Properties of sequential consistency

Theorem: Sequential consistency is not compositional

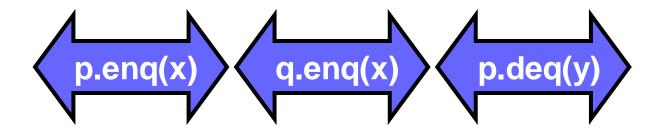
```
H=
A: p.enq(x)
A: p:void
B: q.enq(y)
B: q:void
A: q.enq(x)
A: q:void
B: p.enq(y)
B: p:void
A: p.deq()
A: p:y
B: q.deq()
B: q:x
```

```
Compositional would mean:
"If H|p and H|q are sequentially consistent,
then H is sequentially consistent!"
```

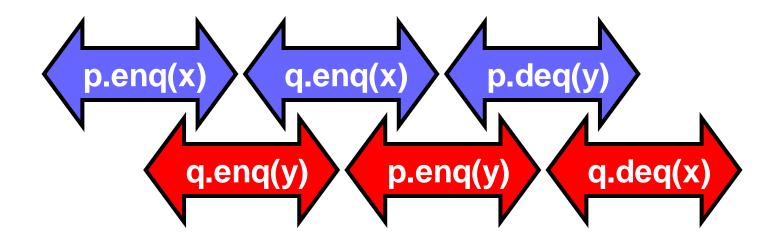
This is not guaranteed for SC schedules!

See following example!

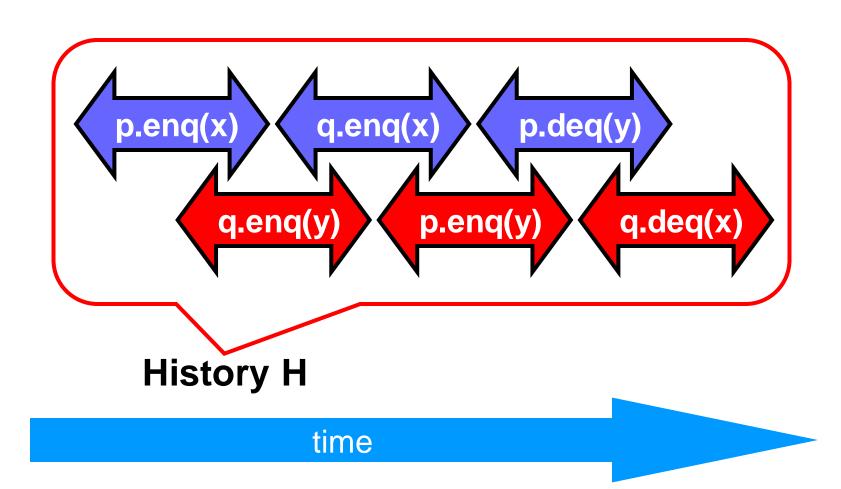
FIFO Queue Example



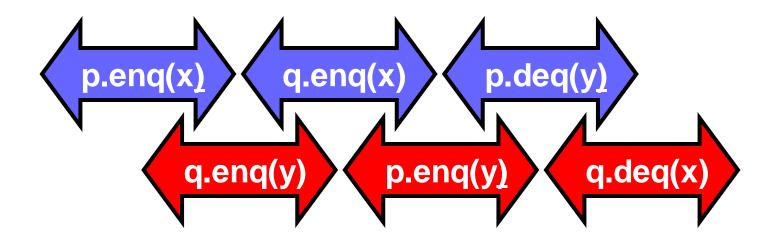
FIFO Queue Example



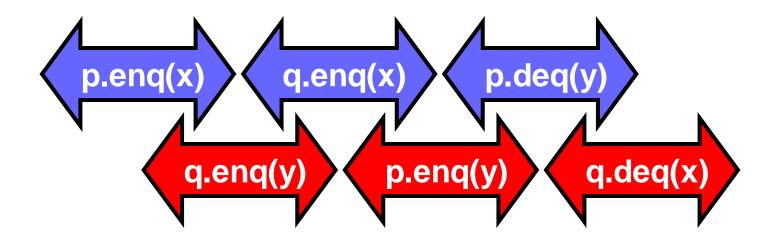
FIFO Queue Example



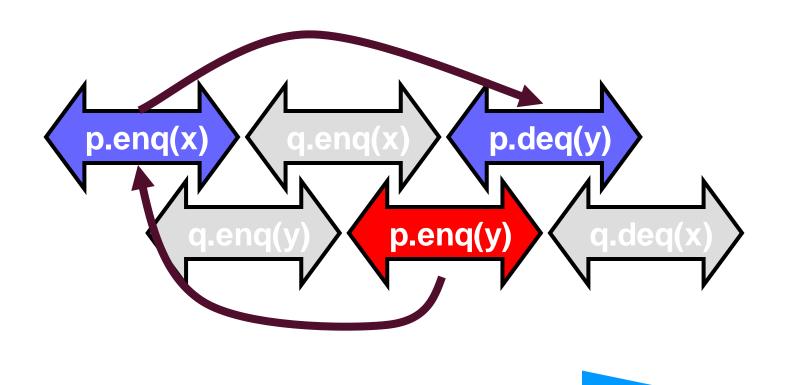
H|p Sequentially Consistent



H|q Sequentially Consistent

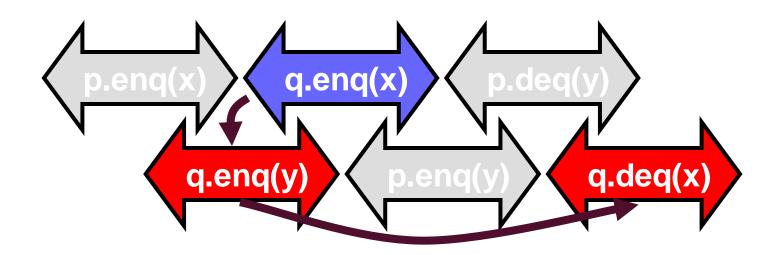


Ordering imposed by p



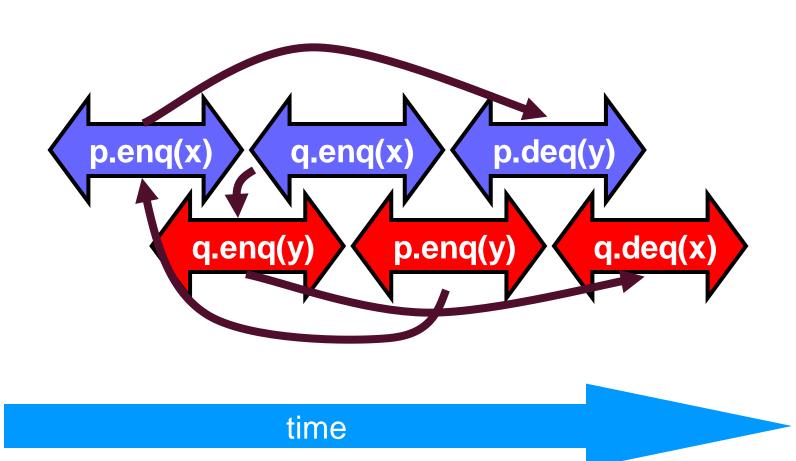
time

Ordering imposed by q

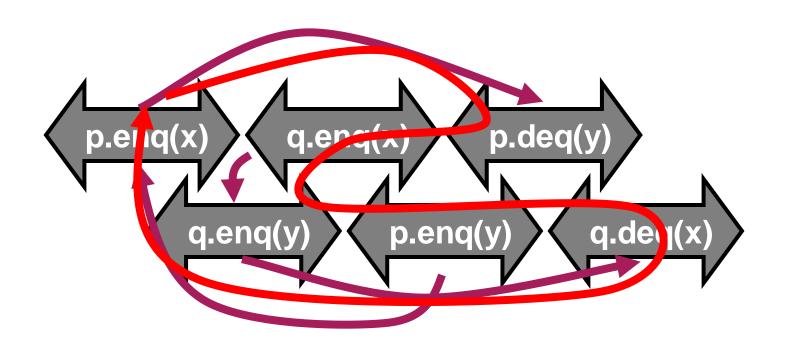


time

Ordering imposed by both



Combining orders



time

Example in our notation

Sequential consistency is not compositional

H=

A: p.enq(x)

A: p:void

B: q.enq(y)

B: q:void

A: q.enq(x)

A: q:void

B: p.enq(y)

B: p:void

A: p.deq()

A: p:y

B: q.deq()

B: q:x

H|p=

A: p.enq(x)

A: p:void

B: p.enq(y)

B: p:void

A: p.deq()

A: p:y

(H|p)|A=

A: p.enq(x)

A: p:void

A: p.deq()

A: p:y

(H|p)|B=

B: p.enq(y)

B: p:void

H|p is sequentially consistent!

Example in our notation

Sequential consistency is not compositional

H= A: p.enq(x)A: p:void B: q.enq(y) B: q:void A: q.enq(x)A: q:void B: p.enq(y) B: p:void A: p.deq() A: p:y B: q.deq() B: q:x

H|q= (H|q)|A= (H|q)|B=

B: q.enq(y)
B: q:void
A: q:void
A: q:void
B: q:void
B: q:void
B: q:void
B: q:x

H|q is sequentially consistent!

Example in our notation

Sequential consistency is not compositional

```
H=
A: p.enq(x)
A: p:void
B: q.enq(y)
B: q:void
A: q.enq(x)
A: q:void
B: p.enq(y)
B: p:void
A: p.deq()
A: p:y
B: q.deq()
B: q:x
```

```
H|A= H|B=

A: p.enq(x)
A: p:void
B: q:void
A: q.enq(x)
A: q:void
B: p.enq(y)
A: q:void
A: p:void
B: p:void
B: p:void
B: q.deq()
A: p:y
B: q:x
```

H is not sequentially consistent!

Correctness: Linearizability

- Sequential Consistency
 - Not composable
 - Harder to work with
 - Good way to think about hardware models
- We will use *linearizability* as in the remainder of this course unless stated otherwise

Study Goals

- Define linearizability with your own words!
- Describe the properties of linearizability!
- Explain the differences between sequential consistency and linearizability!

Given a history H

- Identify linearization points
- Find equivalent sequential history S
- Decide and explain whether H is linearizable
- Decide and explain whether H is sequentially consistent
- Give values for the response events such that the execution is linearizable

Language Memory Models

- Which transformations/reorderings can be applied to a program
- Affects platform/system
 - Compiler, (VM), hardware
- Affects programmer
 - What are possible semantics/output
 - Which communication between threads is legal?
- Without memory model
 - Impossible to even define "legal" or "semantics" when data is accessed concurrently
- A memory model is a contract
 - Between platform and programmer

History of Memory Models

- Java's original memory model was broken
 - Difficult to understand => widely violated
 - Did not allow reorderings as implemented in standard VMs
 - Final fields could appear to change value without synchronization
 - Volatile writes could be reordered with normal reads and writes
 => counter-intuitive for most developers
- Java memory model was revised
 - Java 1.5 (JSR-133)
 - Still some issues (operational semantics definition)
- C/C++ didn't even have a memory model until recently
 - Not able to make any statement about threaded semantics!
 - Introduced in C++11 and C11
 - Based on experience from Java, more conservative

Everybody wants to optimize

- Language constructs for synchronization
 - Java: volatile, final, synchronized, ...
 - C++: atomic, (NOT volatile!) ...

- Without synchronization (defined language-specific)
 - Compiler, (VM), architecture
 - Reorder and appear to reorder memory operations
 - Maintain sequential semantics per thread
 - Other threads may observe any order (have seen examples before)

Java and C++ High-level overview

Relaxed memory model

- No global visibility ordering of operations
- Allows for standard compiler optimizations

But

- Program order for each thread (sequential semantics)
- Partial order on memory operations (with respect to synchronizations)
- Visibility function defined

Correctly synchronized programs

Guarantee sequential consistency

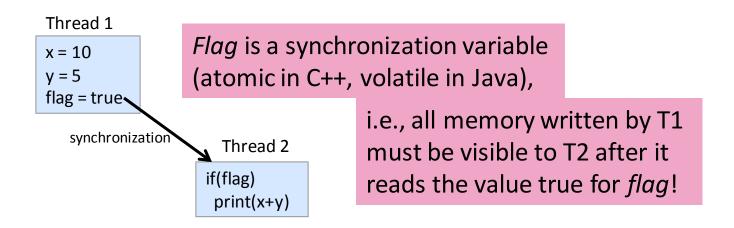
Incorrectly synchronized programs

- Java: maintain safety and security guarantees
 Type safety etc. (require behavior bounded by casuality)
- C++: undefined behavior
 No safety (anything can happen/change)

Communication between Threads Intuition

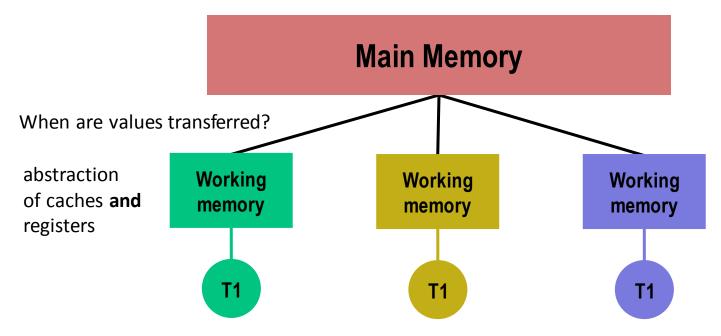
Not guaranteed unless by:

- Synchronization
- Volatile/atomic variables
- Specialized functions/classes (e.g., java.util.concurrent,)



Memory Model Intuition

- Abstract relation between threads and memory
 - Local thread view!



- Does not talk about classes, objects, methods, ...
 - Linearizability is a higher-level concept!

Lock Synchronization

synchronized (lock) { // critical region } Synchronized methods as syntactic sugar

```
    C++

{
    unique_lock<mutex>l(lock);
    // critical region
}

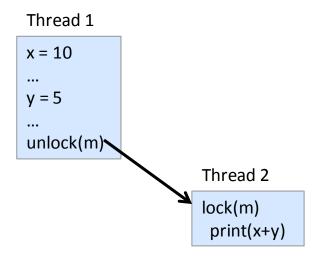
    Many flexible variants
```

Semantics:

- mutual exclusion
- at most one thread may own a lock
- a thread B trying to acquire a lock held by thread A blocks until thread A releases lock
- note: threads may wait forever (no progress guarantee!)

Memory semantics

Similar to synchronization variables



- All memory accesses before an unlock ...
- are ordered before and are visible to ...
- any memory access after a matching lock!

Synchronization Variables

Variables can be declared volatile (Java) or atomic (C++)

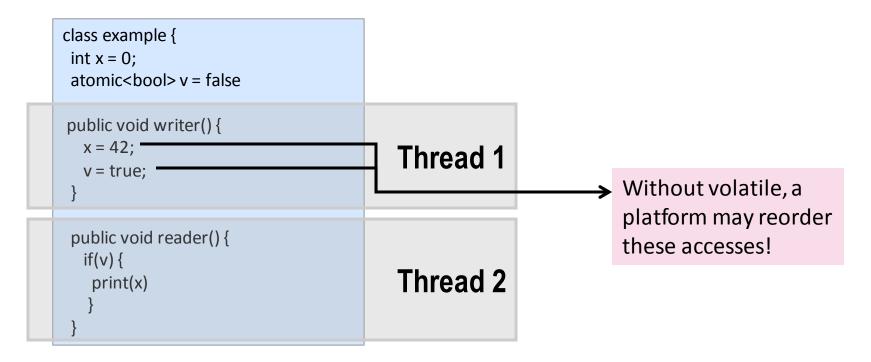
- Reads and writes to synchronization variables
 - Are totally ordered with respect to all threads
 - Must not be reordered with normal reads and writes

Compiler

- Must not allocate synchronization variables in registers
- Must not swap variables with synchronization variables
- May need to issue memory fences/barriers
- **-** ...

Synchronization Variables

- Write to a synchronization variable
 - Similar memory semantics as unlock (no process synchronization!)
- Read from a synchronization variable
 - Similar memory semantics as lock (no process synchronization!)



Memory Model Rules

- Java/C++: Correctly synchronized programs will execute sequentially consistent
- Correctly synchronized = data-race free
 - iff all sequentially consistent executions are free of data races
- Two accesses to a shared memory location form a data race in the execution of a program if
 - The two accesses are from different threads
 - At least one access is a write and
 - The accesses are not synchronized

