# Consensus

Give a constructive proof that the consensus number (for wait-free consensus) of fetch-and-add is (at least) two. Give a constructive proof that the consensus number (for wait-free consensus) of compare-and-swap is unbounded.

## Wait-free consensus among n Processes using compare and swap

```
// reg is initialized to X which is different than all possible inputs
int decide(val) {
  first = compare_and_swap(reg, X, val);
  if (first = X) return(val);
  else return(first);
}
```

## Wait-free consensus among 2 Processes using fetch-and-add

```
// reg is initialized to 0
int decide(val) {
    prefer[thread-id] = val;
    if (fetch-and-add(reg, 1) == 0)
        return(prefer[thread-id]);
    else
        return(prefer[thread-id+1%2]);
}
```

# Broadcast in the $\alpha$-$\beta$-Model

The time taken to send a message of size $s$ from one process to another is $T(s) = \alpha + s\beta$. If a process sends a message of size s at the time $t_0$ it can not send another message before $t_0 + T(s)$.

For $\alpha = 10$ and $\beta = 1$ evaluate three different algorithms which replicate a data item of sizes $s_1 = 1$, $s_2 = 500$, and $s_3 = 40000000$, available on process 0, to all $P = 1000$ processes.

If your algorithm depends on more parameters than $\alpha$, $\beta$ and $s$, also explain how suitable values for those parameters can be found.

## Solution: Broadcast in the $\alpha$-$\beta$-Model

Linear: $T(s) = (P-1) * (\alpha + s\beta)$

k-ary Tree: $T(s) = log_k(P) * k * (\alpha + s\beta)$ (optimal $k$: 3)

k-nomial Tree: $T(s) = log_k(P) * (k-1) * (\alpha + s\beta)$ (optimal $k$: 2)

Pipelined with segmentsize $z$: $T(s) = (s/z + P - 1) * (\alpha + z\beta)$ (optimal $z$: $\sqrt{\frac{s\alpha}{(P-2)\beta}}$)

Pipelined Tree: $T(s) = (s/z + log_2(P) - 1) * 2 * (\alpha + s\beta)$ (optimal z: $\sqrt{\frac{s\alpha}{\beta(log_2(P)-1)}}$)

| Parameters | $P = 1000, \alpha = 10, \beta = 2, s = 1$ |
|---|---|
| Linear | 11988 |
| k-ary Tree (k=3) | 252 |
| **k-nomial Tree (k=2)** | 120 |

| Parameters: | $P = 1000, \alpha = 10, \beta = 2, s = 500$ |
|---|---|
| Linear: | 1008990 |
| k-ary Tree (k=3) | 21210 |
| k-nomial Tree (k=2) | 10100 |
| Pipeline (z=2) | 17486 |
| **Pipelined Tree (z=17, k=2)** | 3432 |

| Parameters: | $P = 1000, \alpha = 10, \beta = 2, s = 40000000$ |
|---|---|
| Linear: | 79920009990 |
| k-ary Tree (k=3): | 1680000210 |
| k-nomial Tree (k=2): | 800000100 |
| **Pipeline (z=448):** | 81798210 |
| Pipelined Tree (z=4715, k=2) | 160347840 |