M. Besta, T. Hoefler

Towards High-Performance Processing, Storage, and Analytics of Extreme-Scale Graphs

With contributions from Dimitri Stanojevic, Simon Weber, Lukas Gianinazzi, Andrey Ivanov, Marc Fischer, Robert Gerstenberger, Heng Lin, Xiaowei Zhu, Bowen Yu, Xiongchao Tang, Wei Xue, Wenguang Chen, Lufei Zhang, Xiaosong Ma, Xin Liu, Weimin Zheng, and Jingfang Xu and others.

[Extreme-Scale] Graphs

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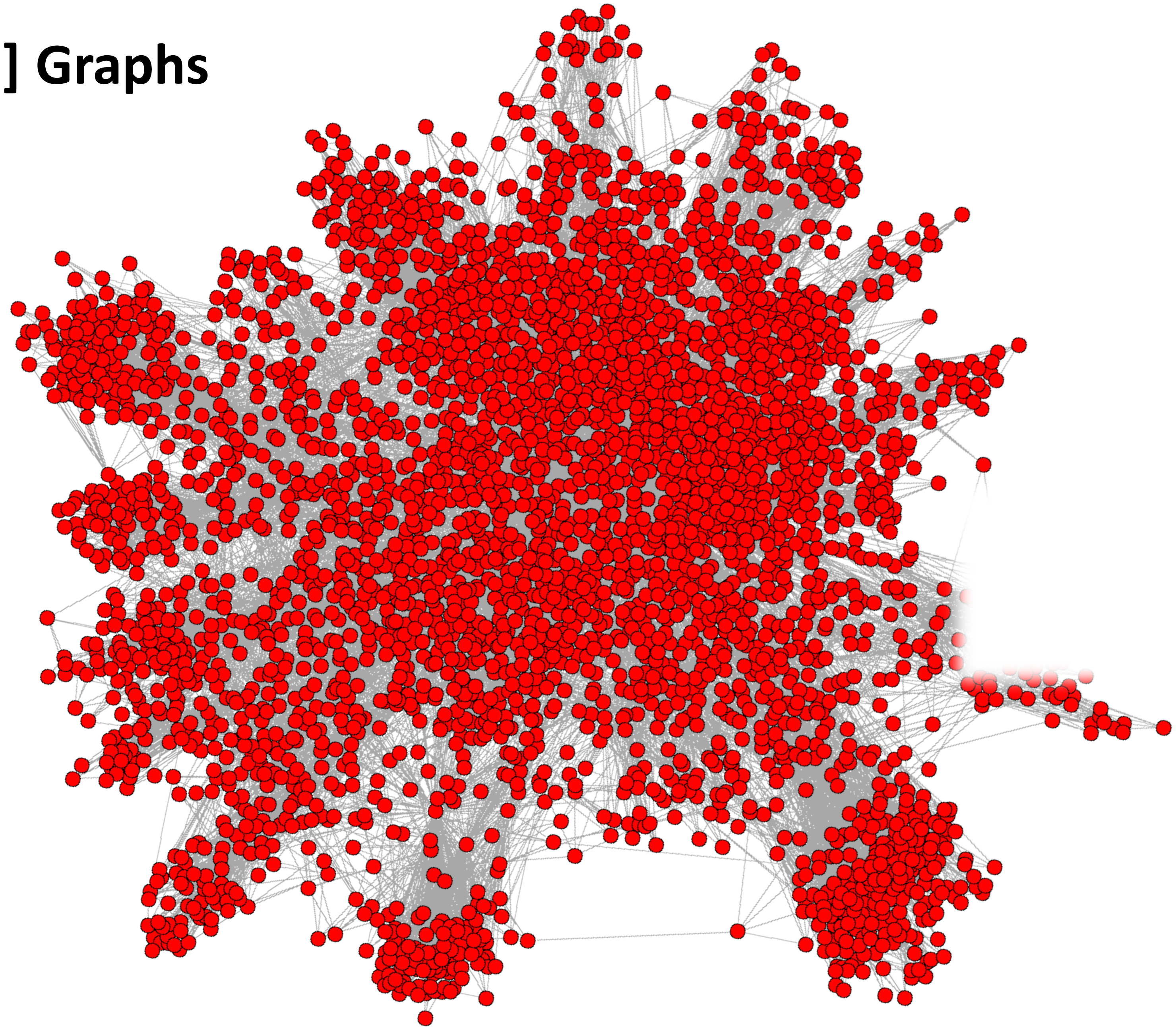
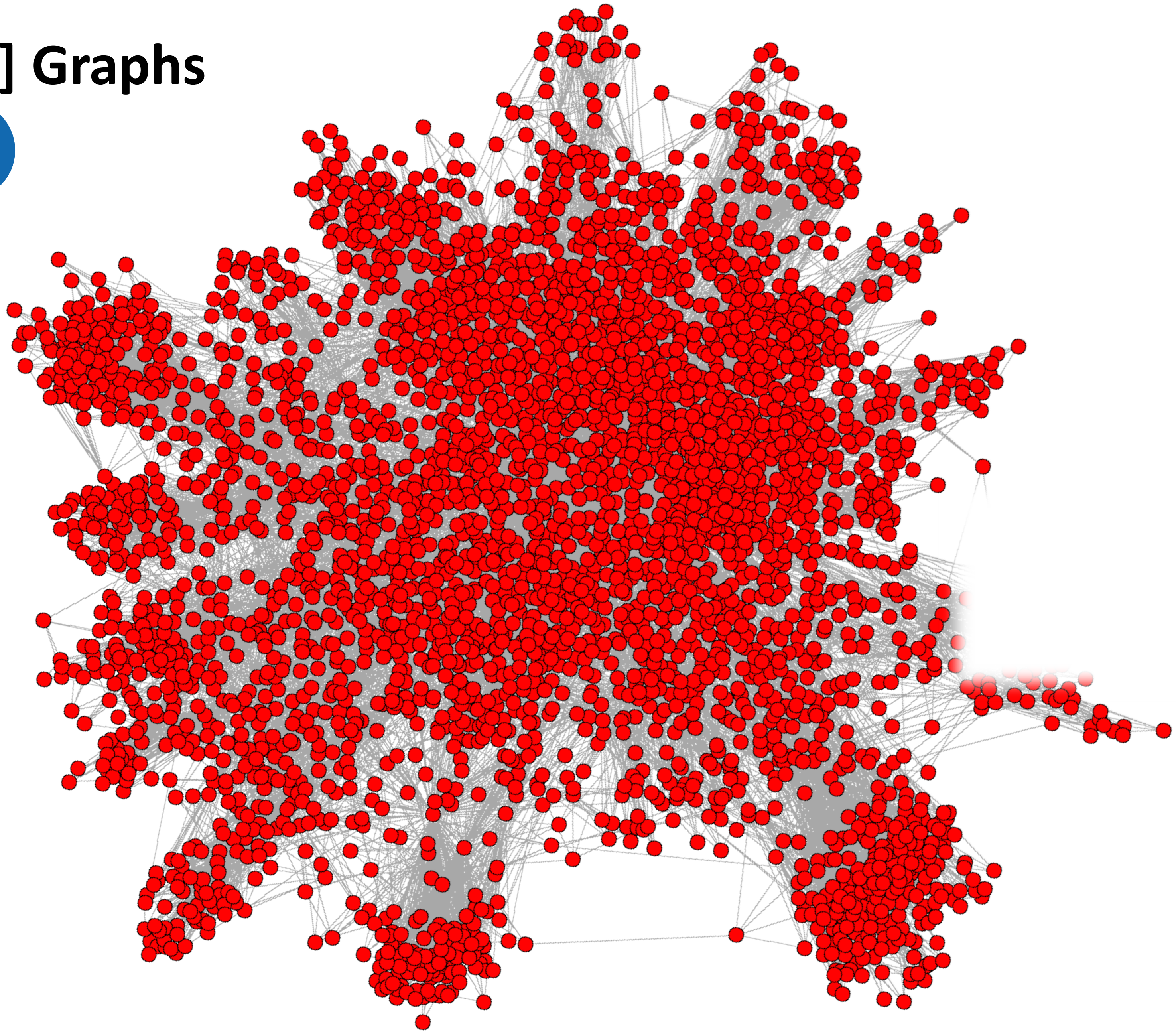


[Extreme-Scale] Graphs

Why do we care?



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Useful model

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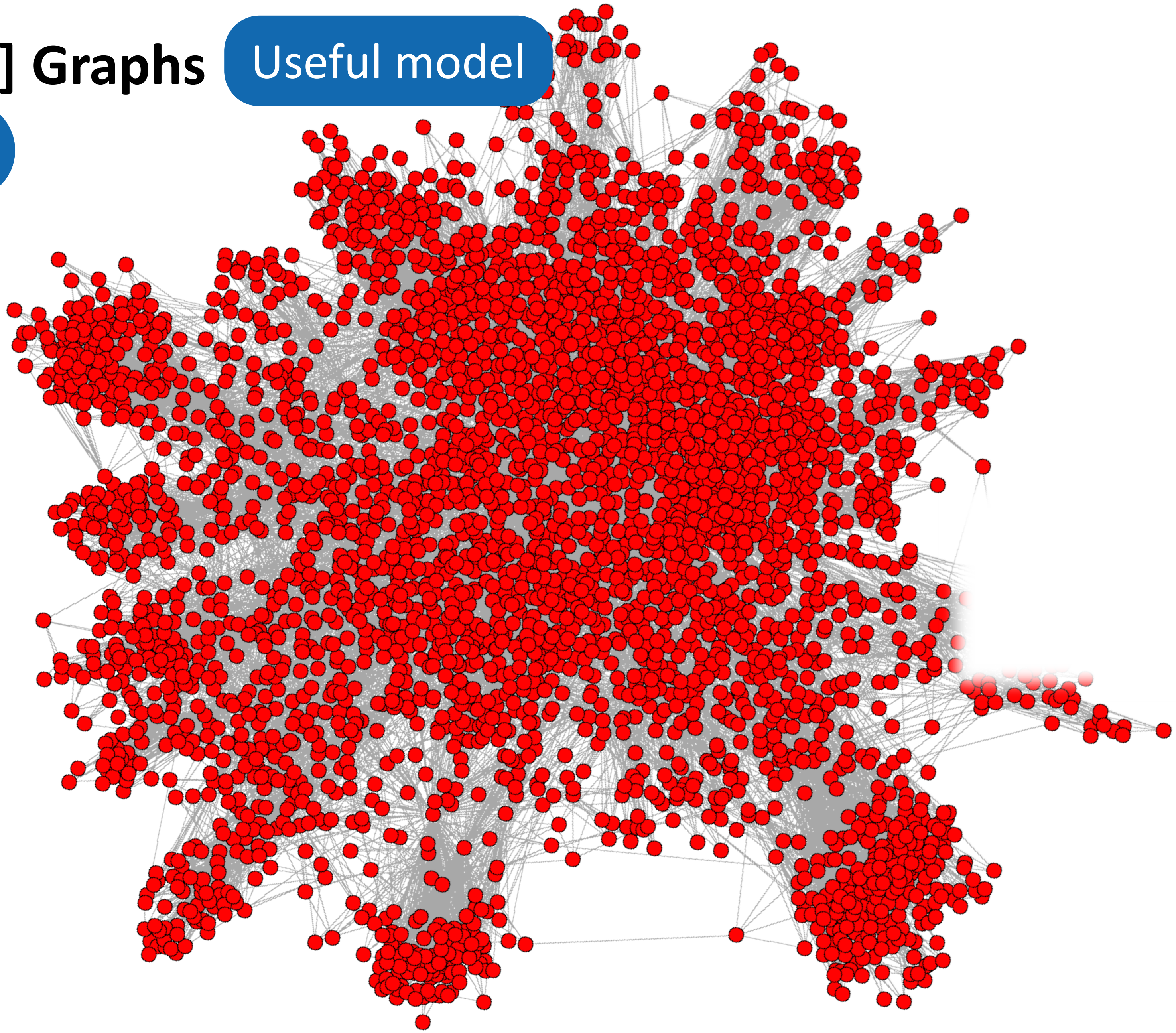

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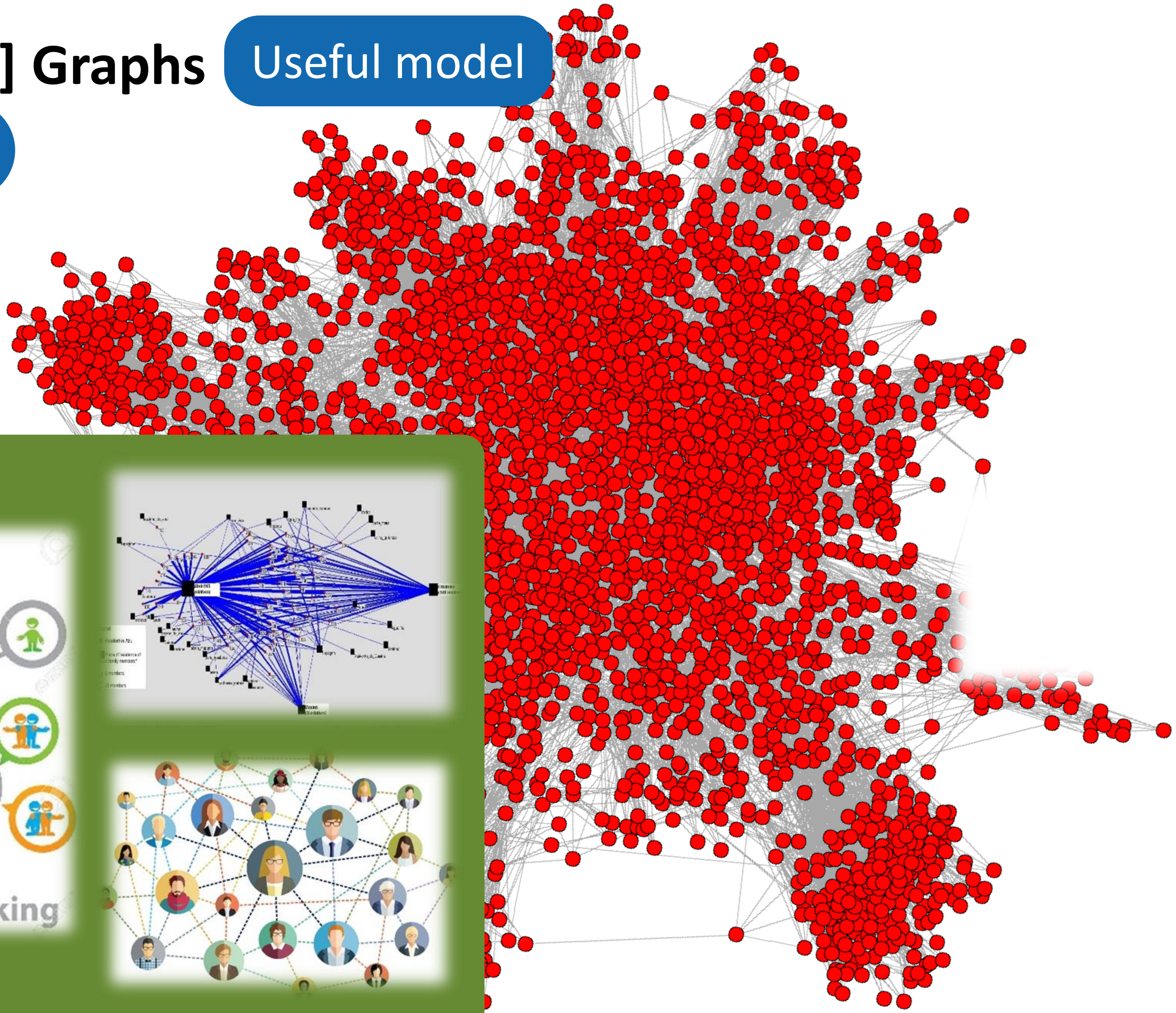

Social networks

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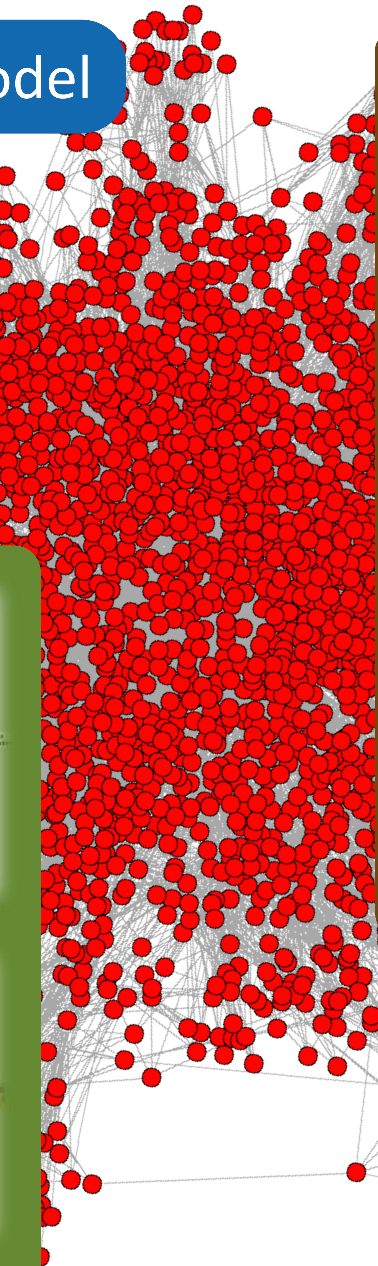

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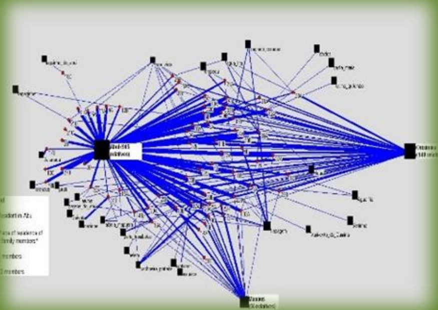

Engineering networks

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Why do we care?

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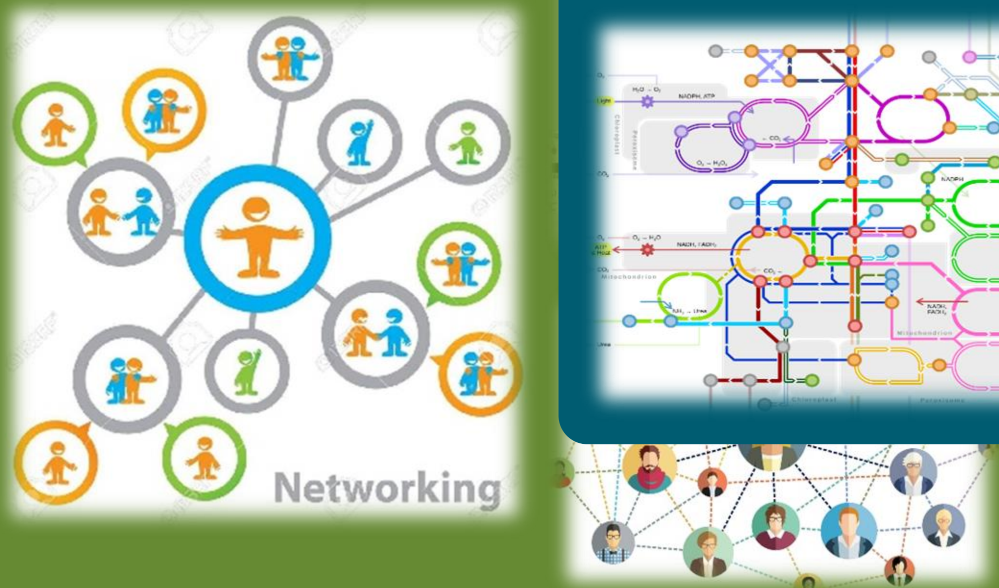
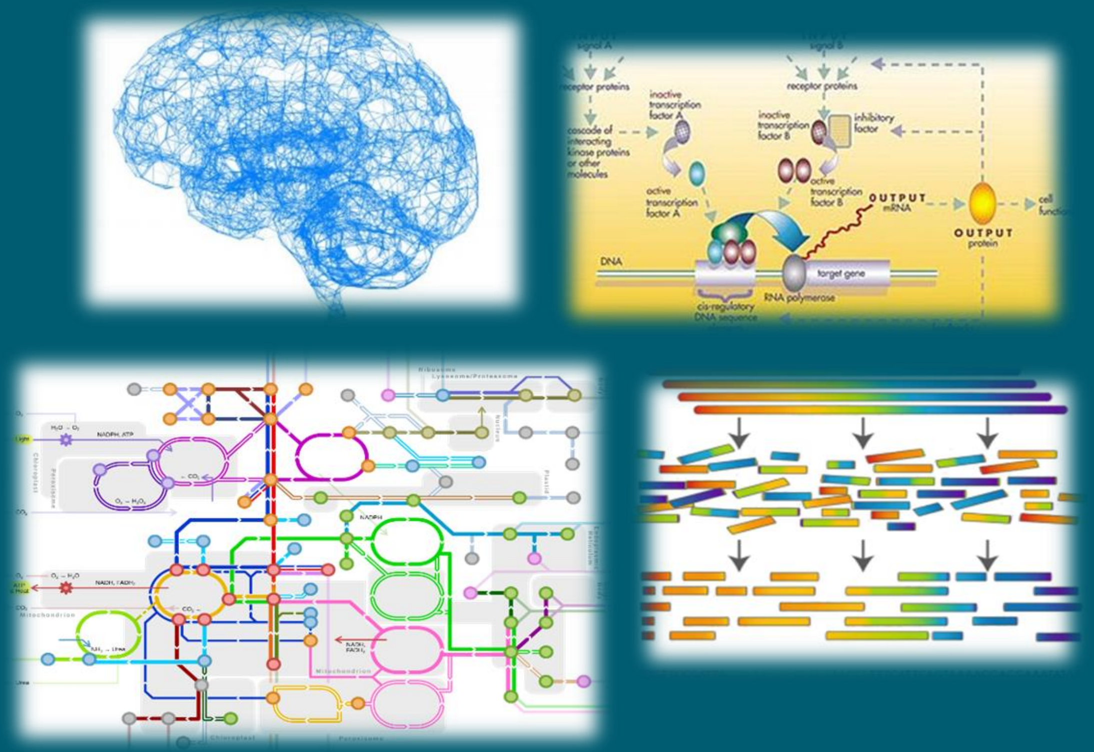


Biological networks



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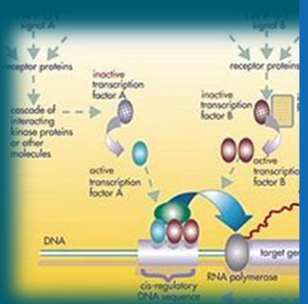
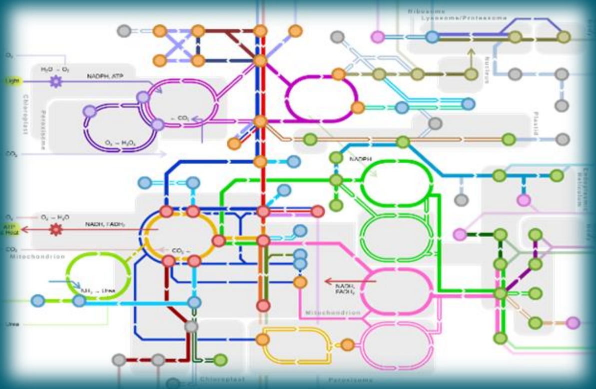
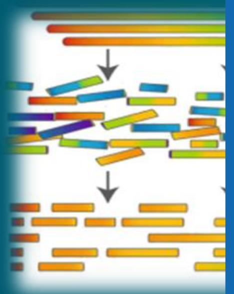
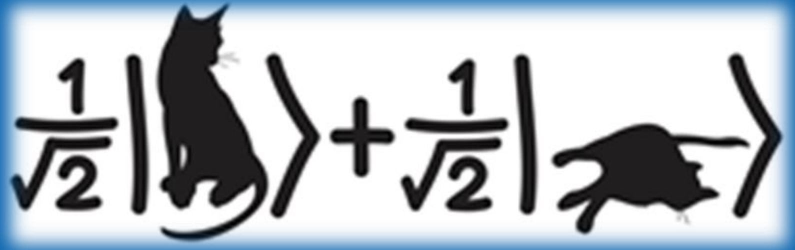


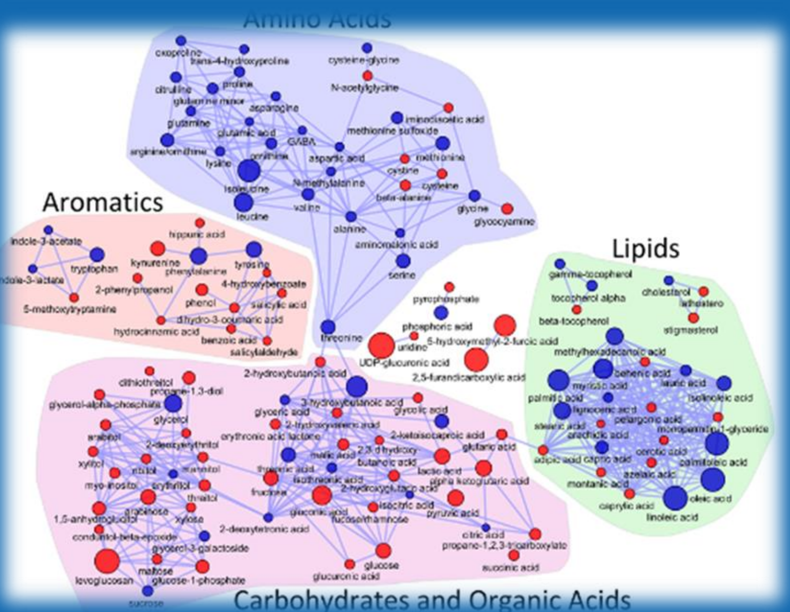




Engineering networks

Physics, chemistry





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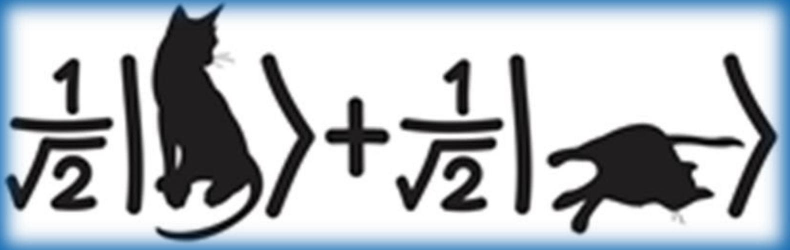

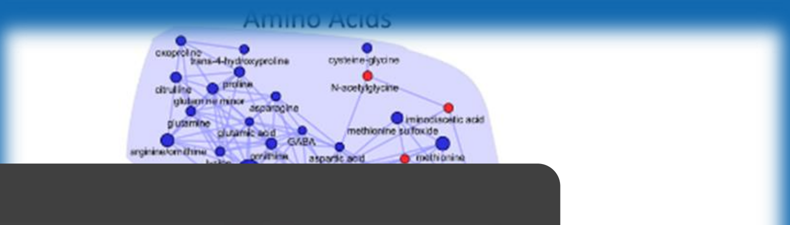

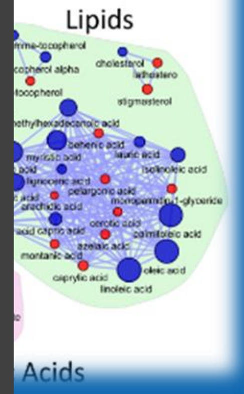

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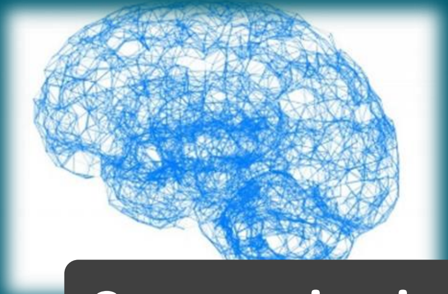
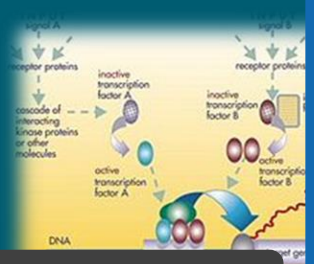
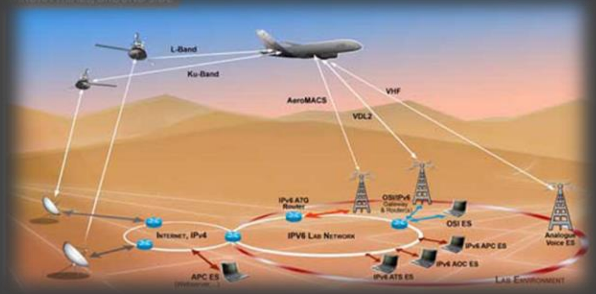
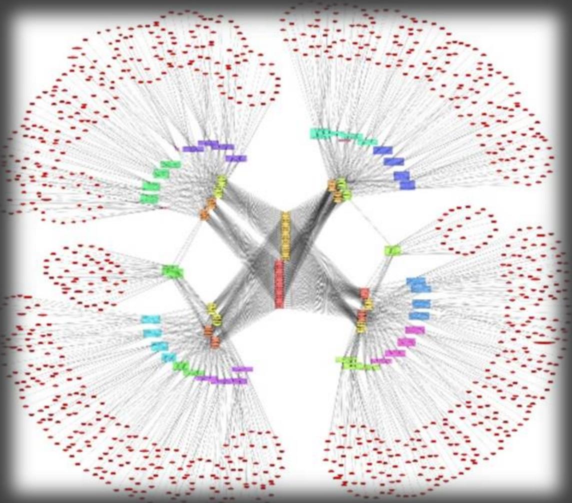




Social networks

Communication networks





[Extreme-Scale] Graphs

Useful model

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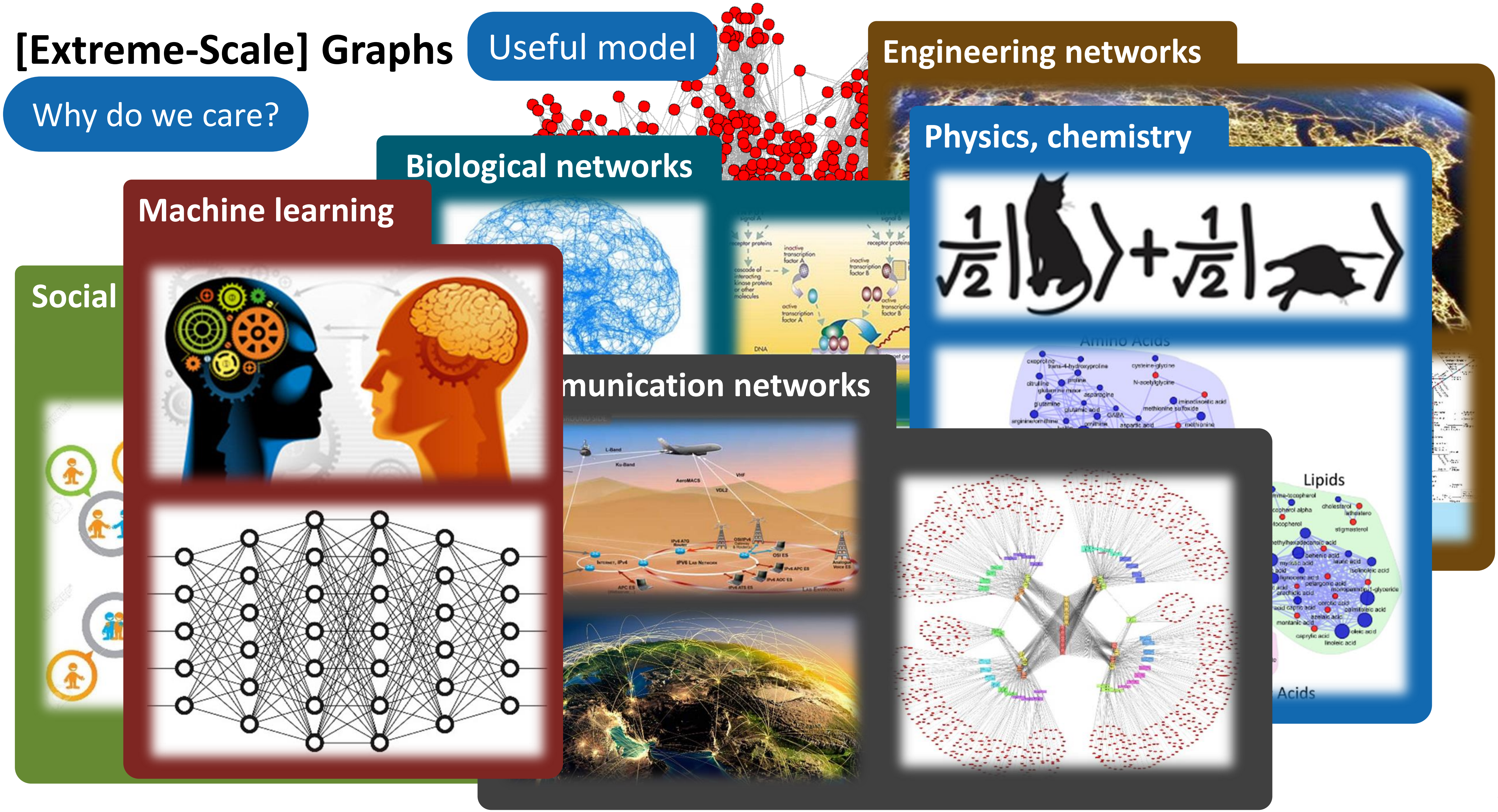


[Extreme-Scale] Graphs

Useful model

Why do we care?

Engineering networks

...even philosophy ☺

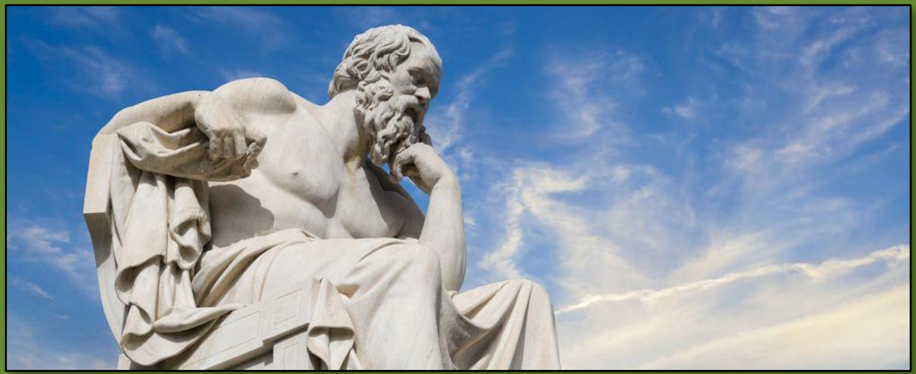
hysics, chemistry

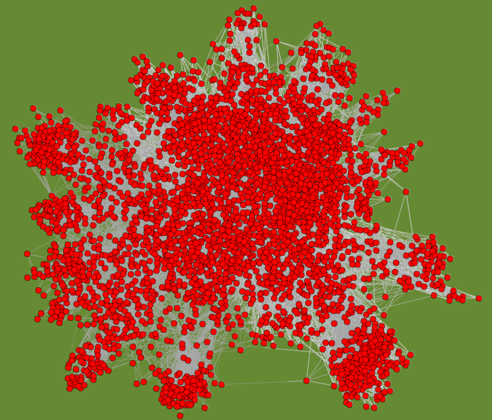
Biological ne

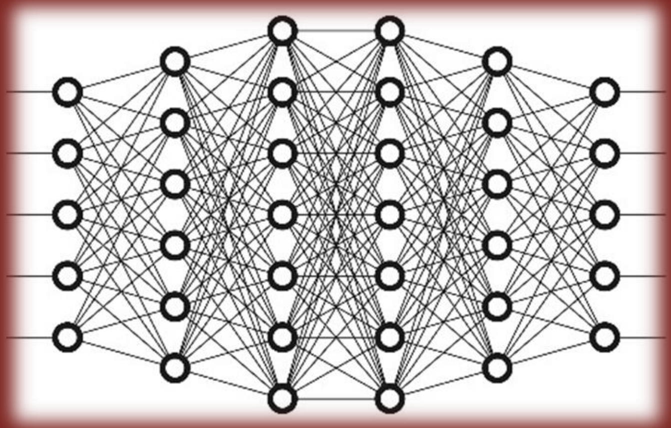
Machine learning

Social

m







FOSDEM 2016 / Schedule / Events / Developer rooms / Graph Processing / Modeling a Philosophical Inquiry: from MySQL to a graph database

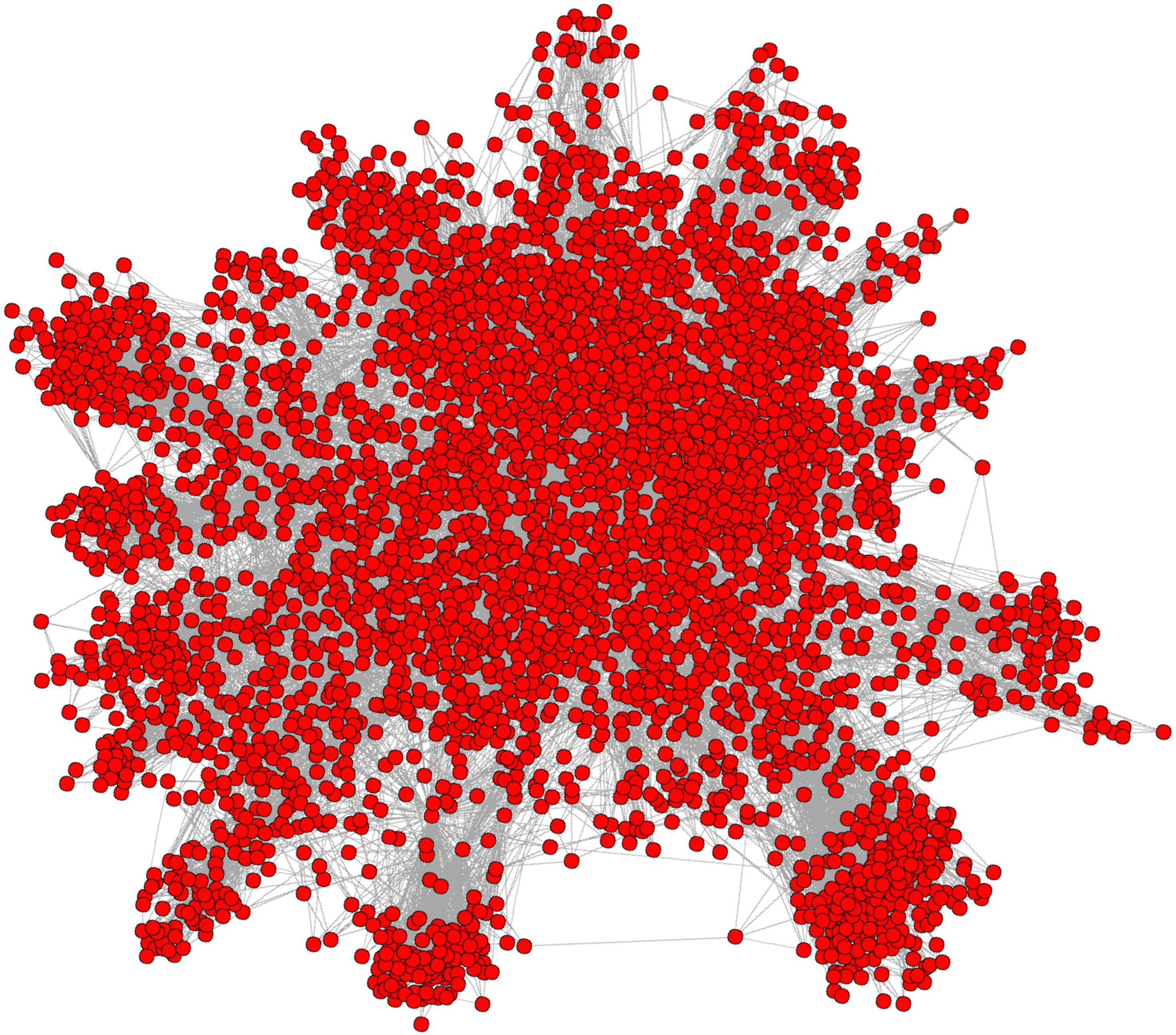
Modeling a Philosophical Inquiry: from MySQL to a graph database

The short story of a long refactoring process

- **Track**: Graph Processing devroom
- **Room**: AW1.126
- **Day**: Saturday
- **Start**: 12:45
- **End**: 13:35

Bruno Latour wrote a book about philosophy (an inquiry into modes of existence). He decided that the paper book was no place for the numerous footnotes, documentation or glossary, instead giving access to all this information surrounding the book through a web application which would present itself as a reading companion. He also offered to the community of readers to submit their contributions to his inquiry by writing new documents to be added to the platform. The first version

Problems!



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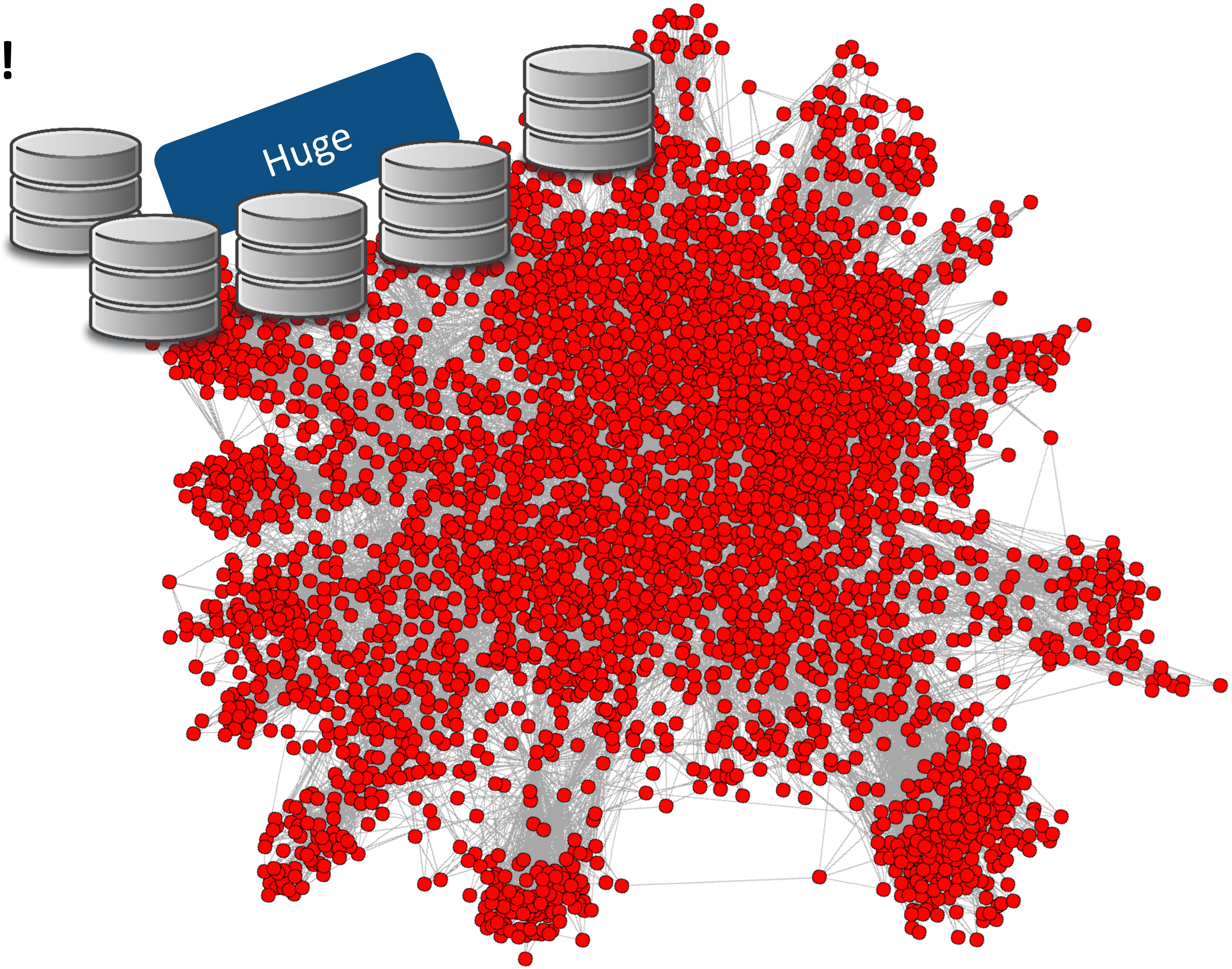

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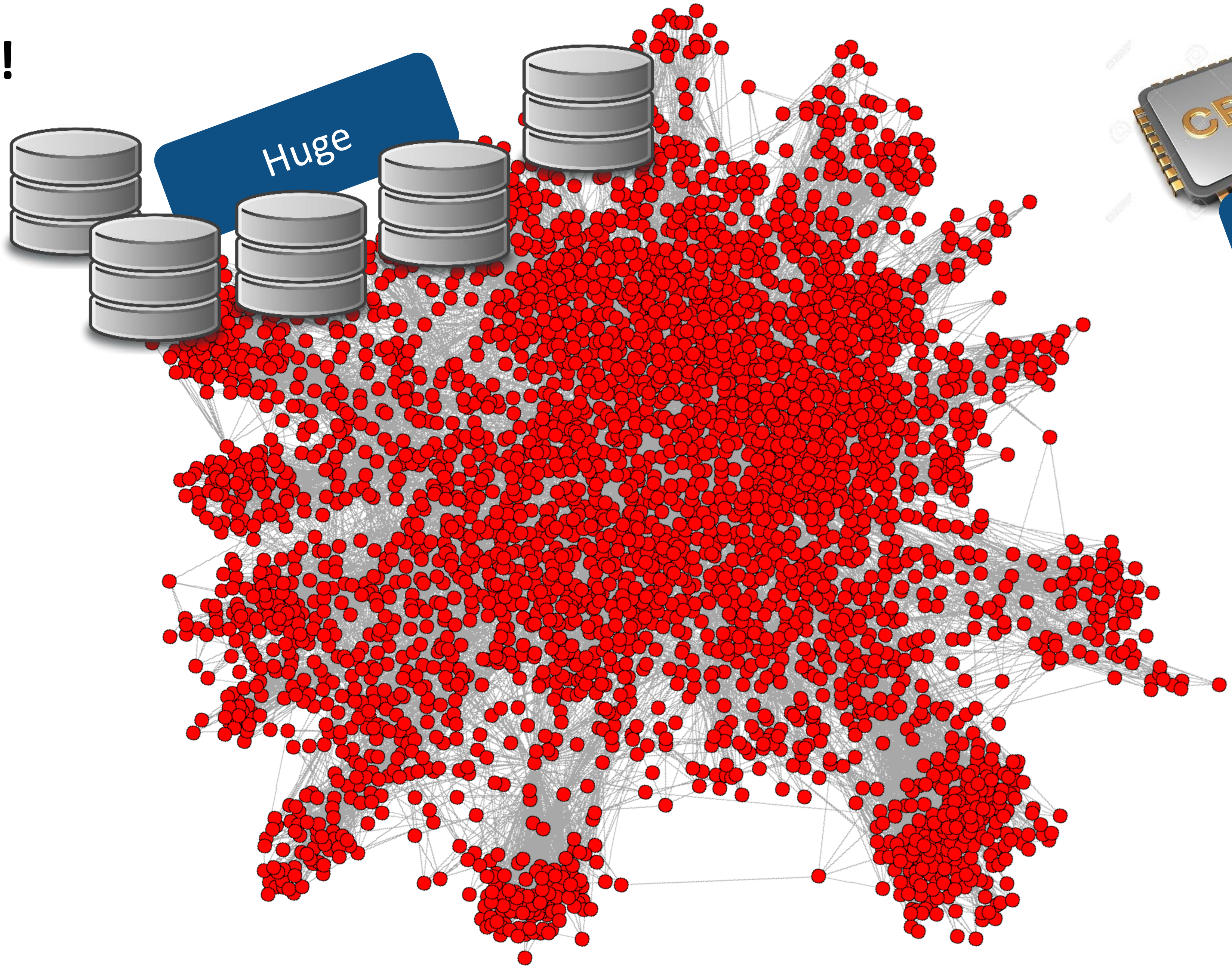
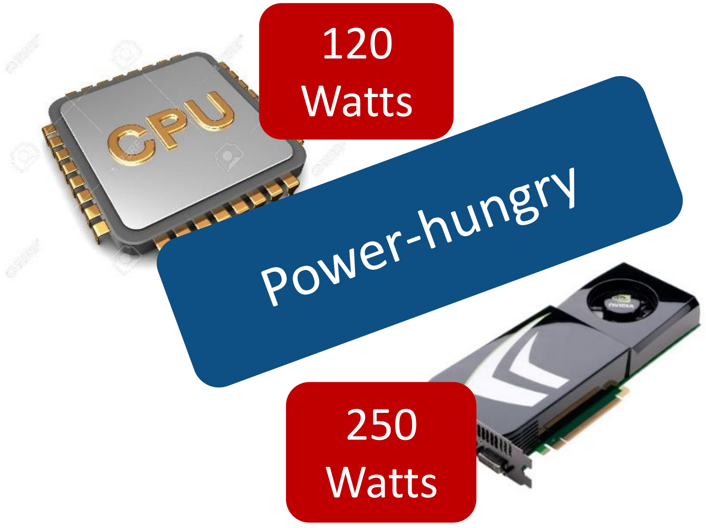




Problems!


Huge
Communication-heavy
CPU
120 Watts
Power-hungry
250 Watts

Problems!


Huge
Communication-heavy
Synchronization-heavy
Power-hungry
CPU
120
Watts
250
Watts

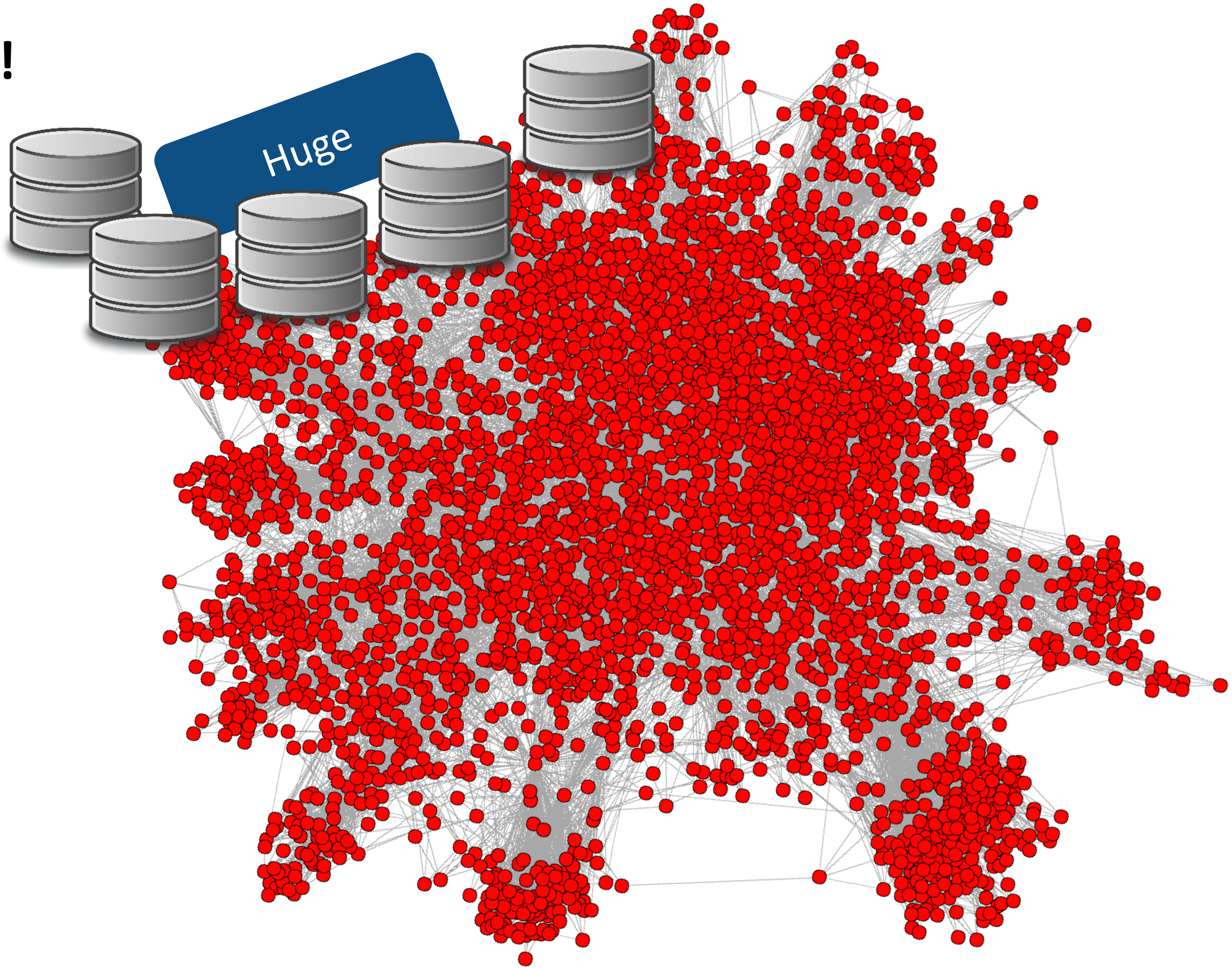
Problems!


Huge
Power-hungry
120 Watts
250 Watts
Communication-heavy
Synchronization-heavy
Irregular

Problems!


Huge
120
Watts
CPU
Power-hungry
250
Watts
It's Complicated...
Communication-heavy
Synchronization-heavy
Irregular

Problems!



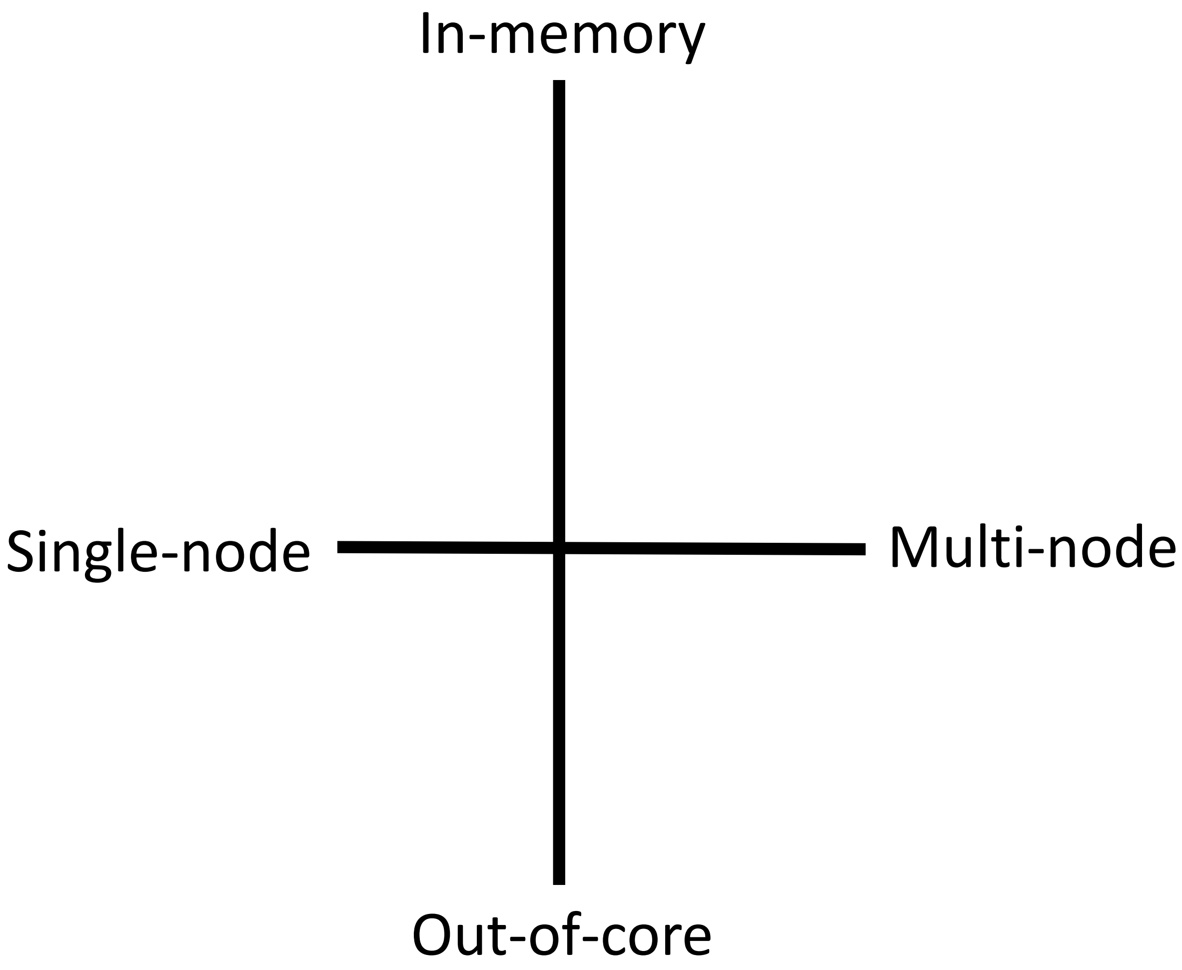
Problems!

What does "huge" mean?

Practice of Extreme-Scale Graph Processing

[1] Heng Lin et al.: ShenTu: Processing Multi-Trillion Edge Graphs on Millions of Cores in Seconds, SC18, Gordon Bell Finalist

Practice of Extreme-Scale Graph Processing



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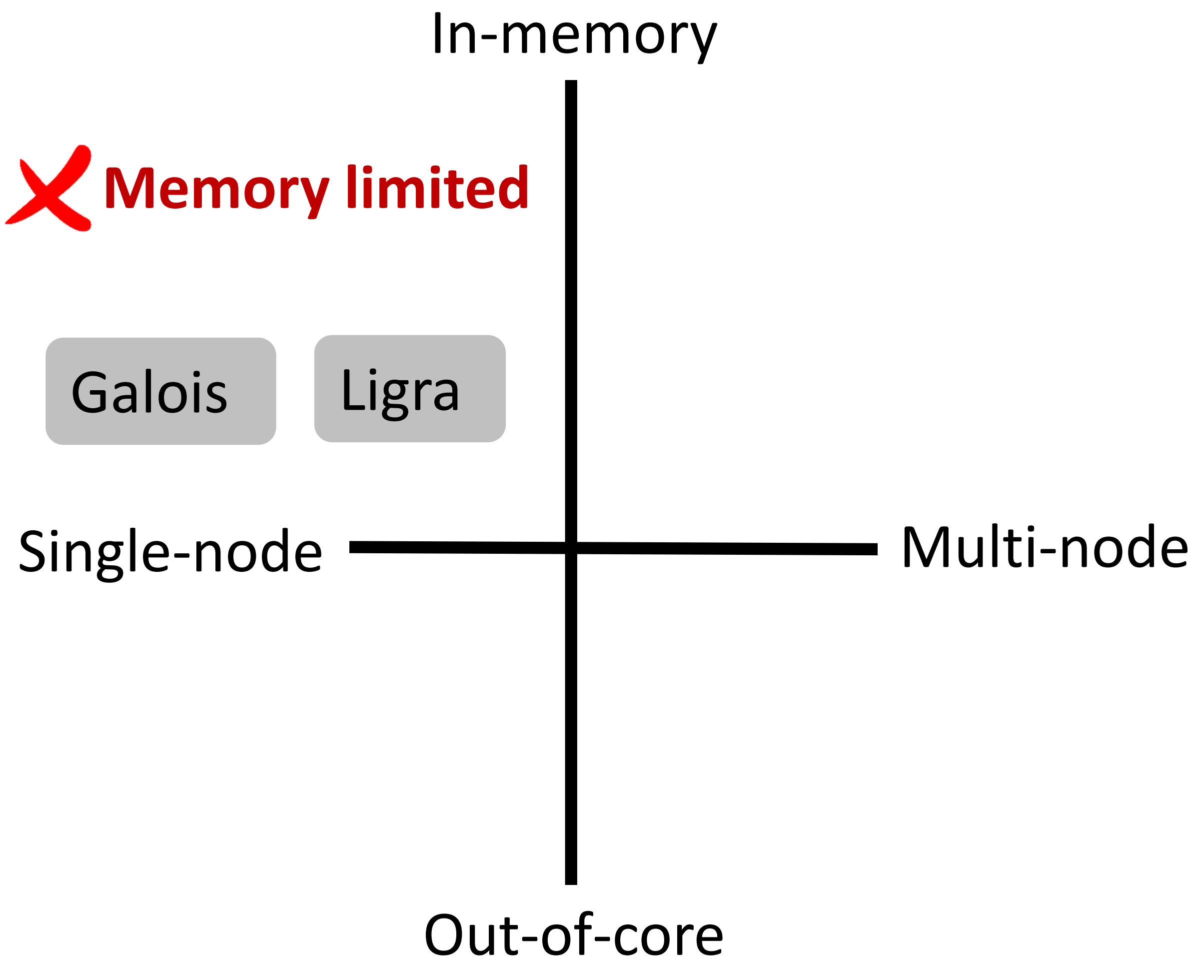

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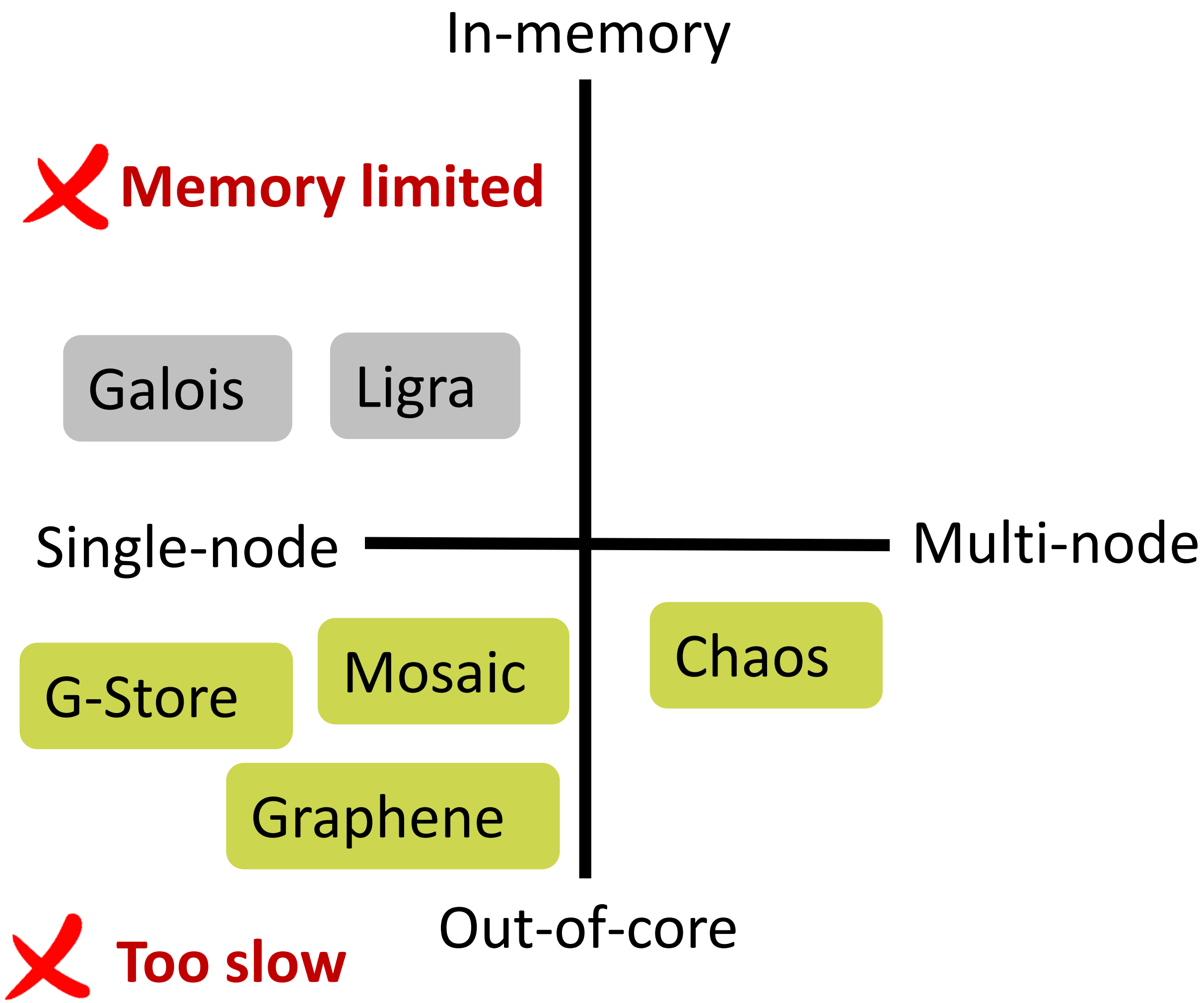

(> 20 mins per PageRank iteration on 1-trillion-edge graph)

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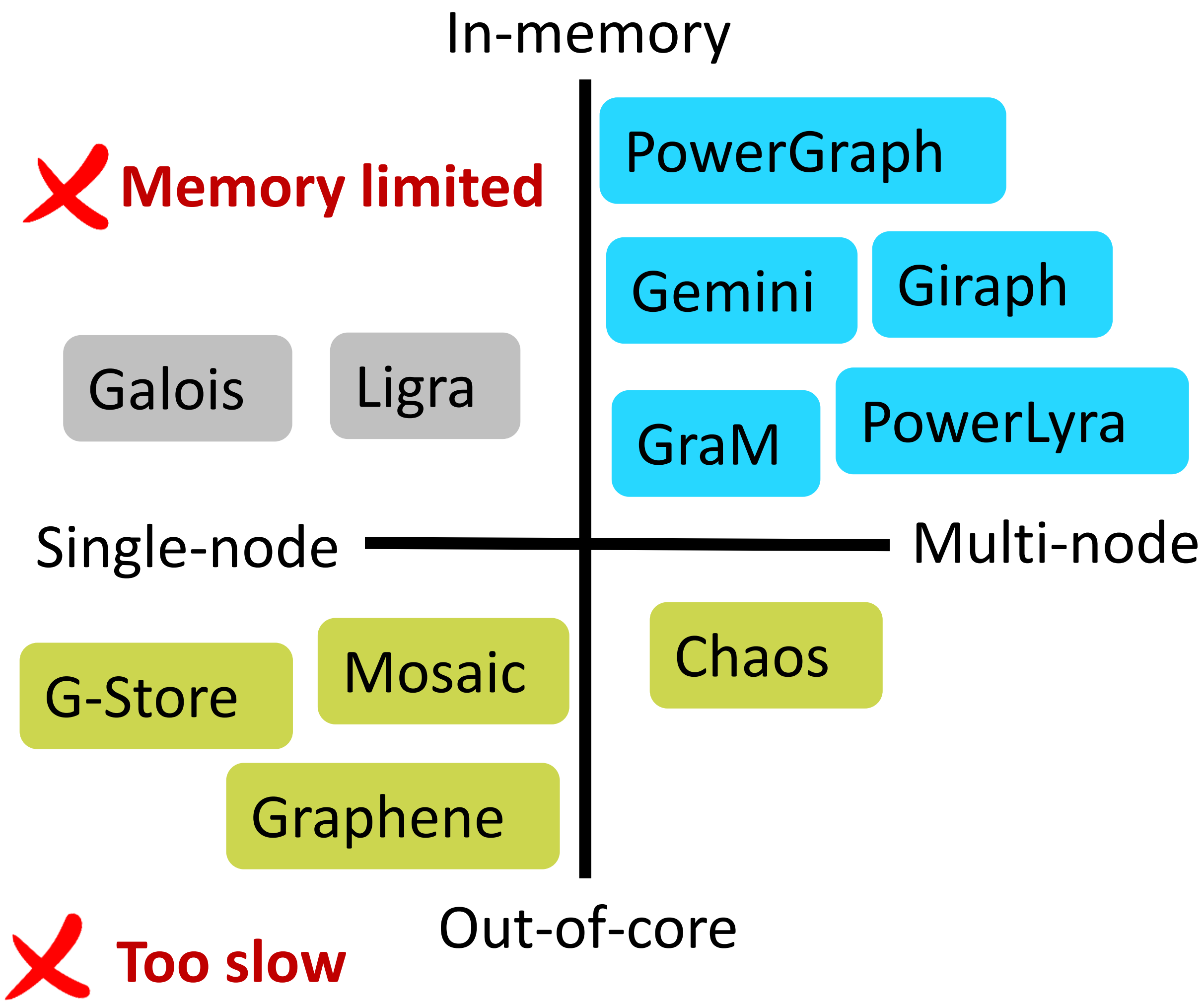

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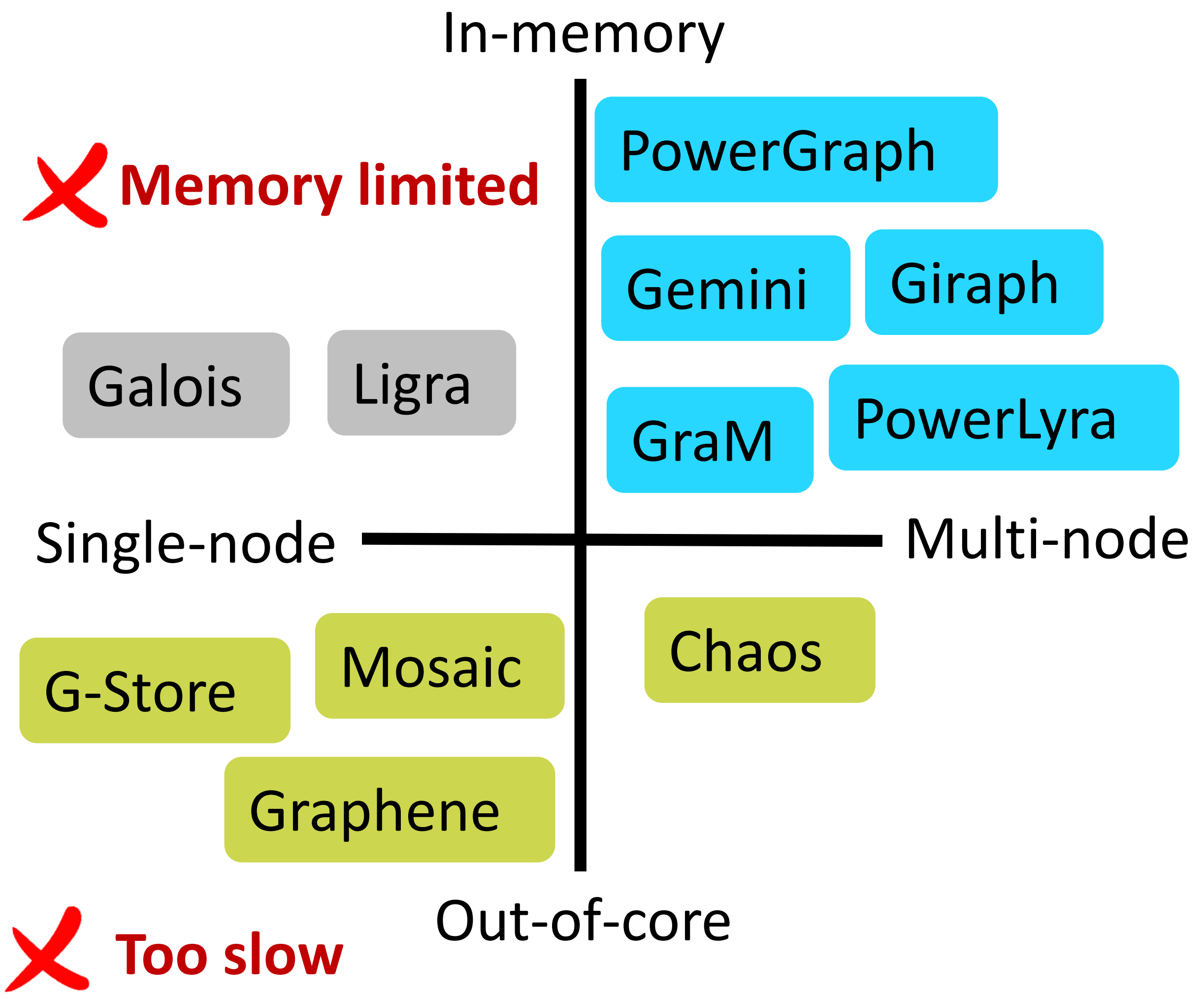

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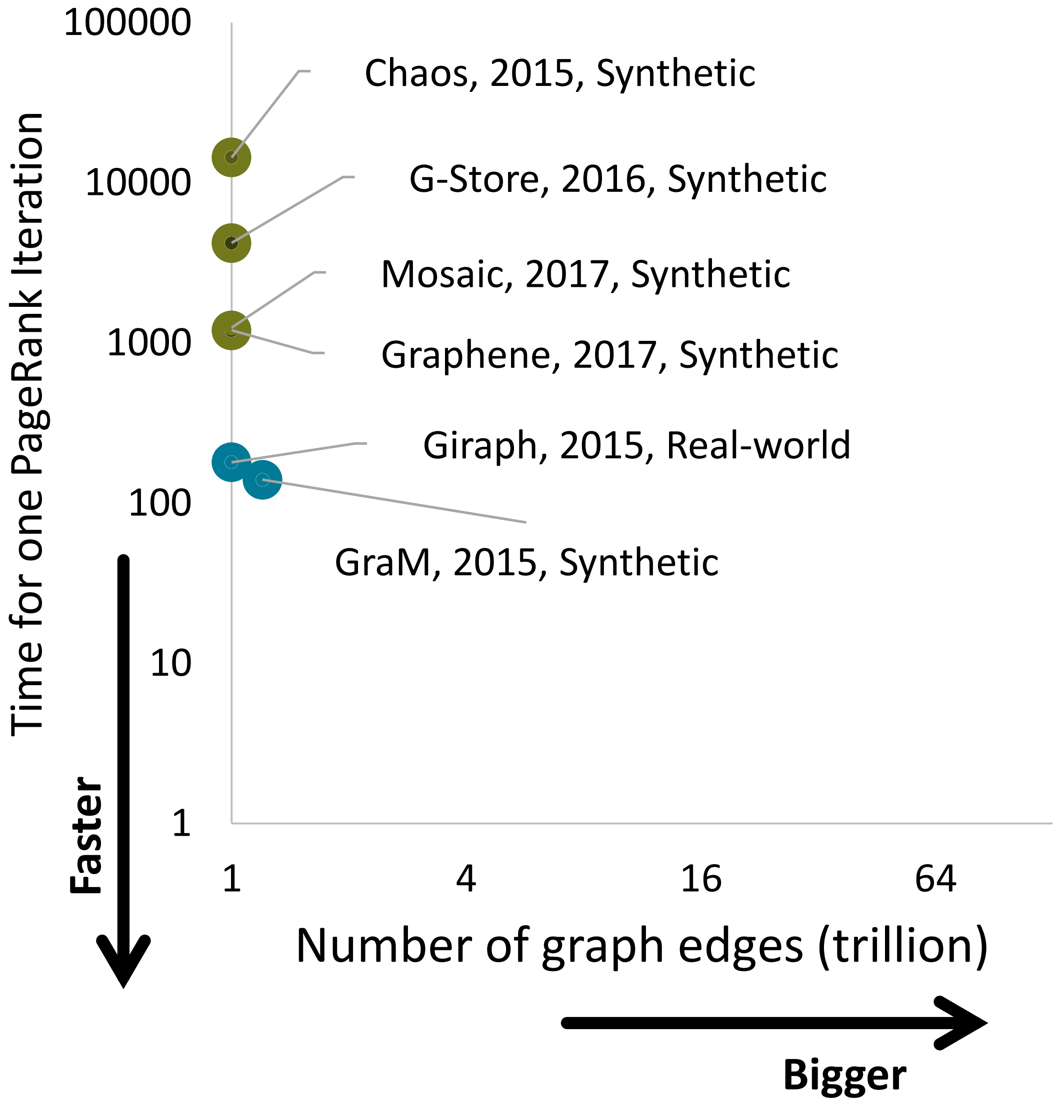

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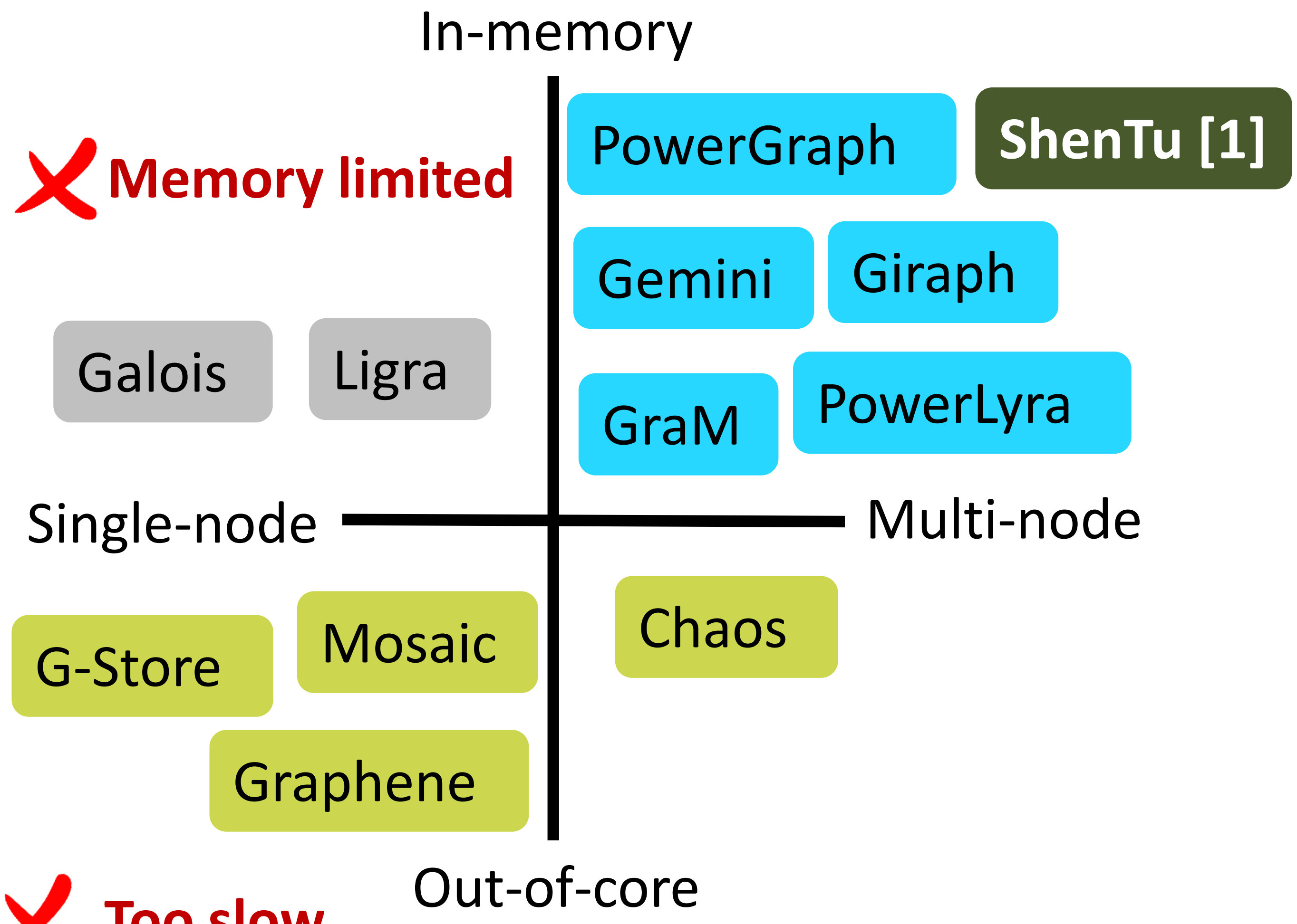
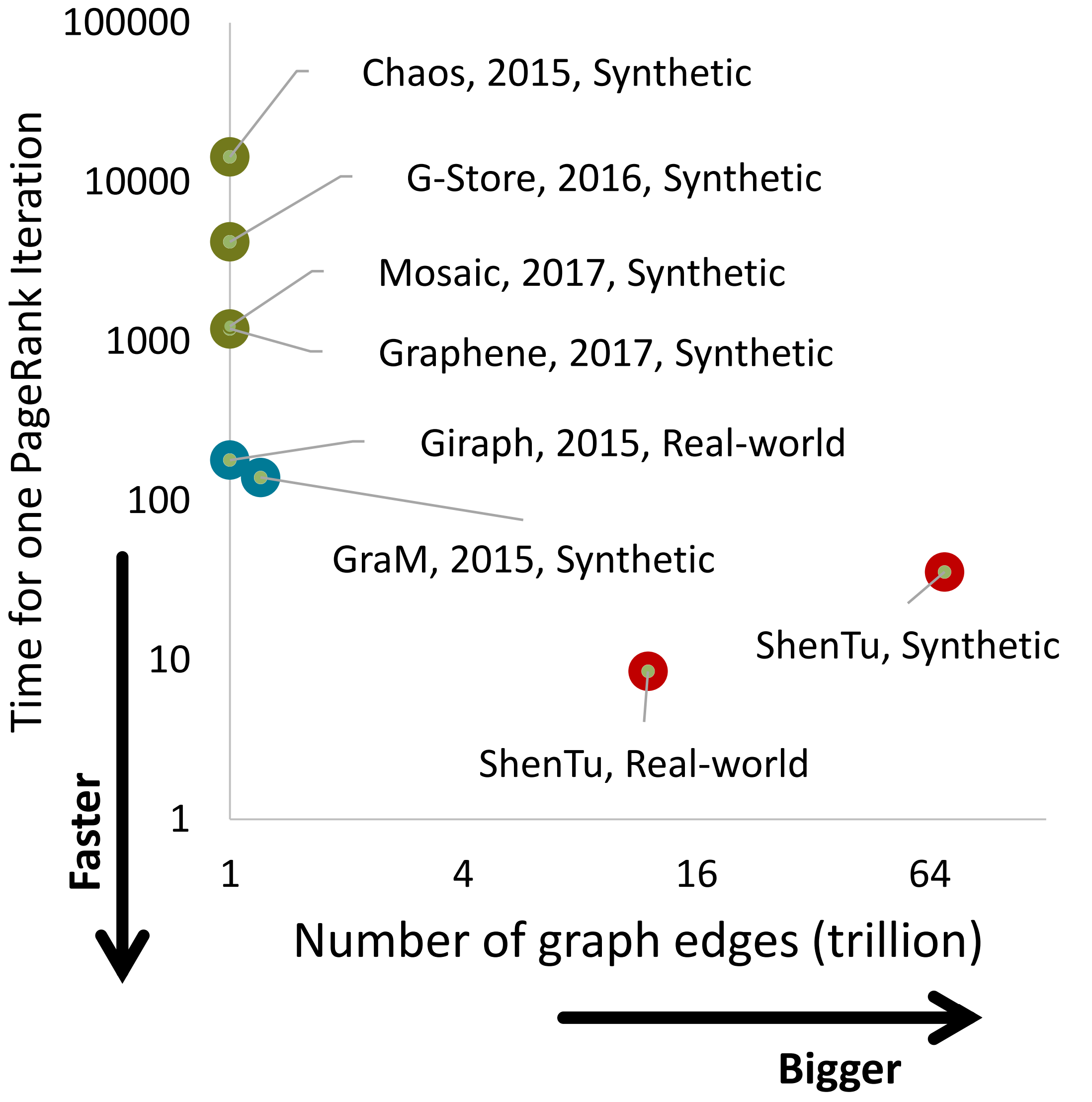


Memory limited

Too slow

(> 20 mins per PageRank iteration on 1-trillion-edge graph)



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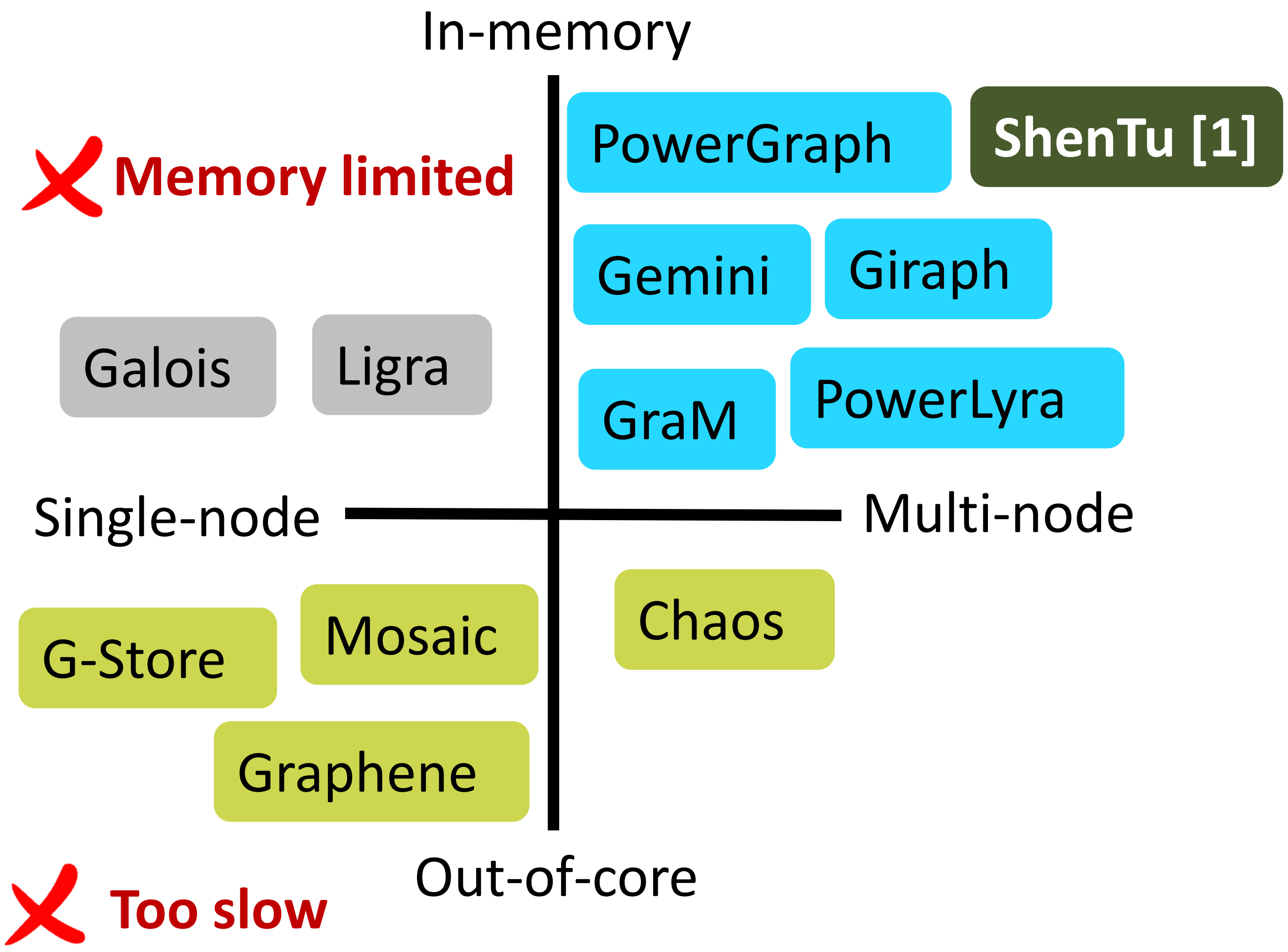

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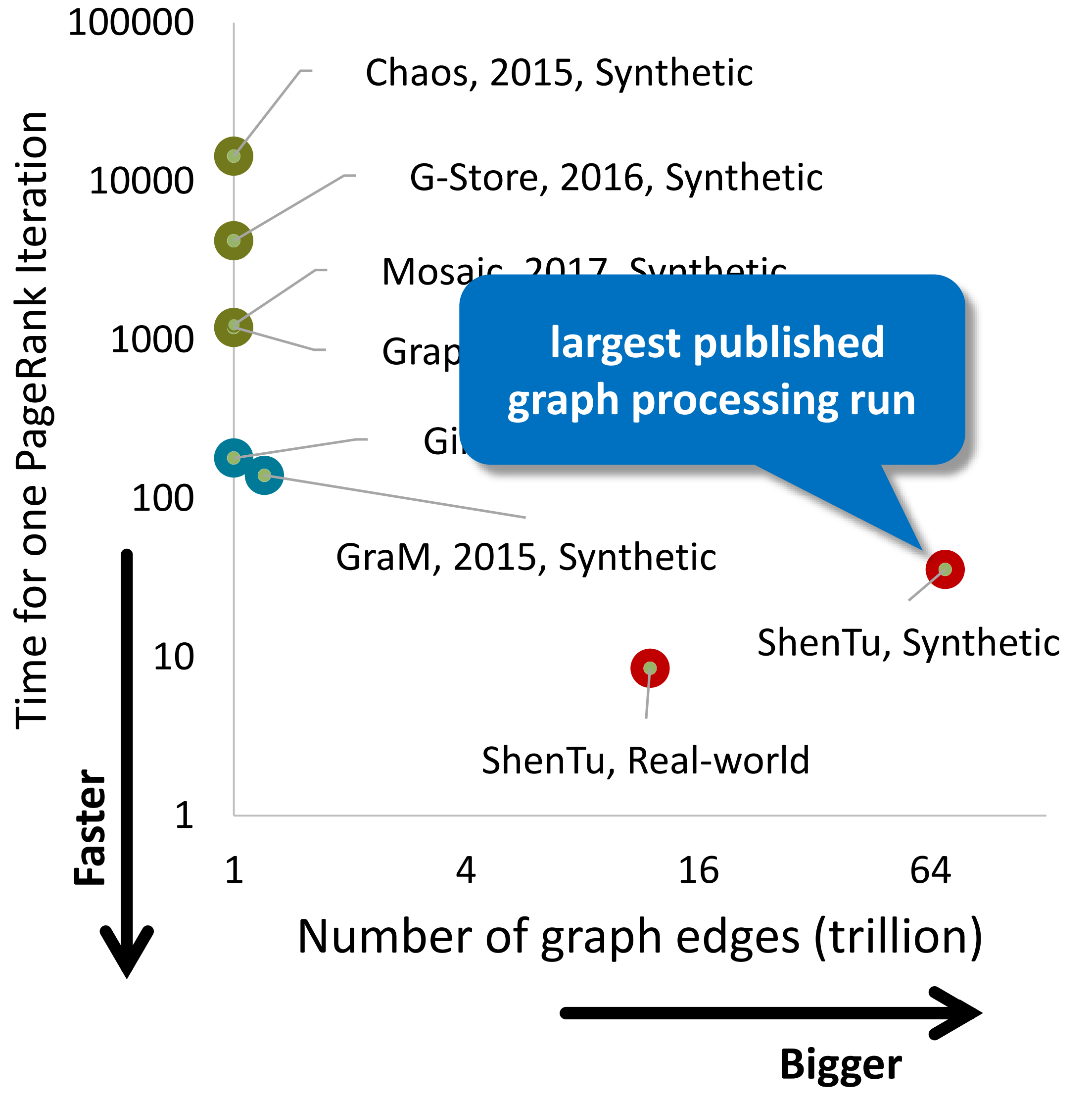

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How large are extreme-scale graphs today?

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Webgraph datasets

Graph	Crawl date	Nodes	Arcs
uk-2014	2014	787 801 471	47 614 527 250
eu-2015	2015	1 070 557 254	91 792 261 600
gsh-2015	2015	988 490 691	33 877 399 152

KONECT graph datasets

en	Wikipedia edits (en)	50,757,442	572,591,272
TW	Twitter (WWW)	41,652,230	1,468,365,182
TF	Twitter (MPI)	52,579,682	1,963,263,821
FR	Friendster	68,349,466	2,586,147,869
UL	UK domain (2007)	105,153,952	3,301,876,564

Web data commons datasets

Granularity	#Nodes	#Arcs
Page	3,563 million	128,736 million
Host	101 million	2,043 million
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Largest Published Graph Computation [1]
Gordon Bell Finalist 2018
ShenTu on Sunway TaihuLight

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Sogou 搜狗 271 billion vertices, 12 trillion edges

Kronecker graph: 4.4 trillion vertices, 70 trillion edges

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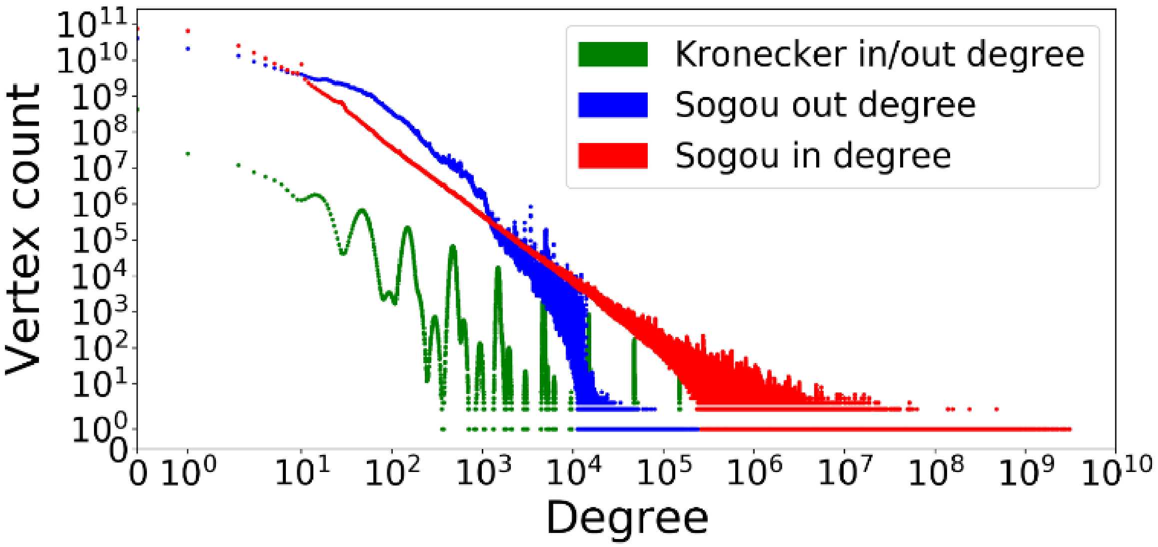

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Sunway TaihuLight

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Sunway TaihuLight

TaihuLight Top500 ranking: #3 (2018 Nov), #1 (2016, 2017)

Heng Lin et al.: ShenTu: Processing Multi-Trillion Edge Graphs on Millions of Cores in Seconds, SC18, Gordon Bell Finalist

Sunway TaihuLight

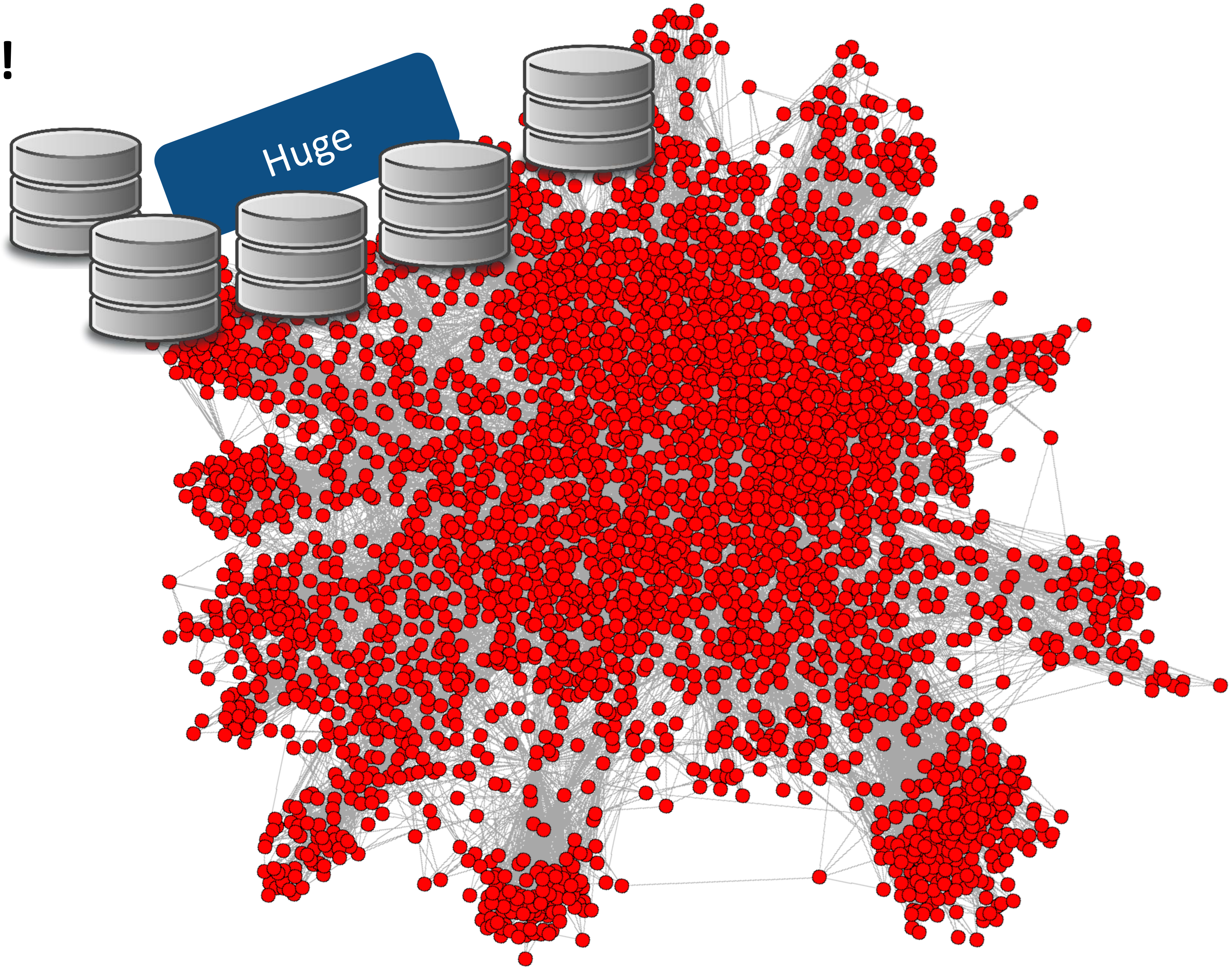
Nearly full system run (~41,000 nodes, ~1 million cores)

- PageRank iteration on 12 trillion edges in 8.5s (1.4 TPEPS)
- On 70 trillion edges, nearly 2 TPEPS for PageRank and WCC
  - 774 GPEPS for BFS

  (PEPS = processes edges per second as opposed to TEPS)

Heng Lin et al.: ShenTu: Processing Multi-Trillion Edge Graphs on Millions of Cores in Seconds, SC18, Gordon Bell Finalist

Problems!



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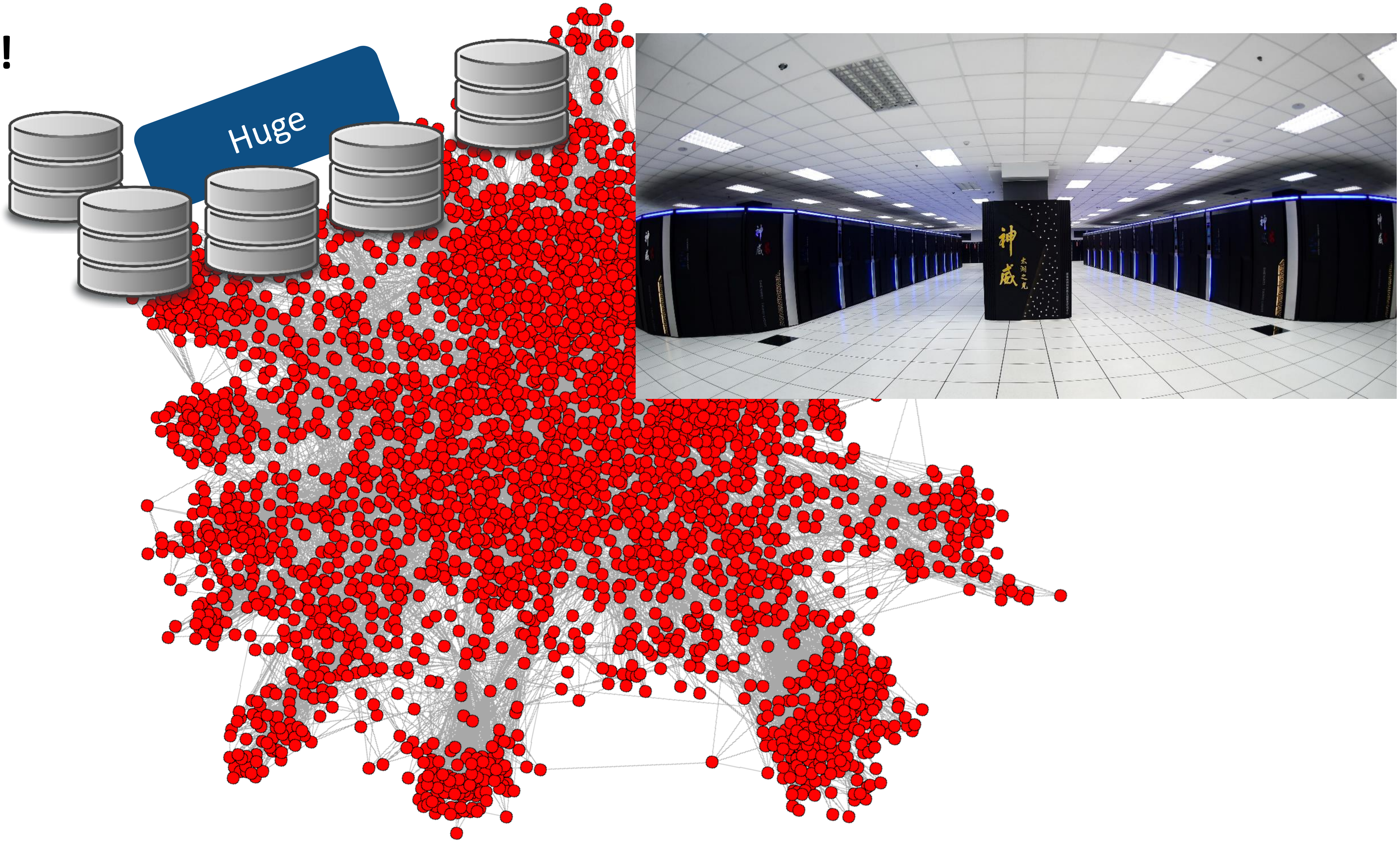

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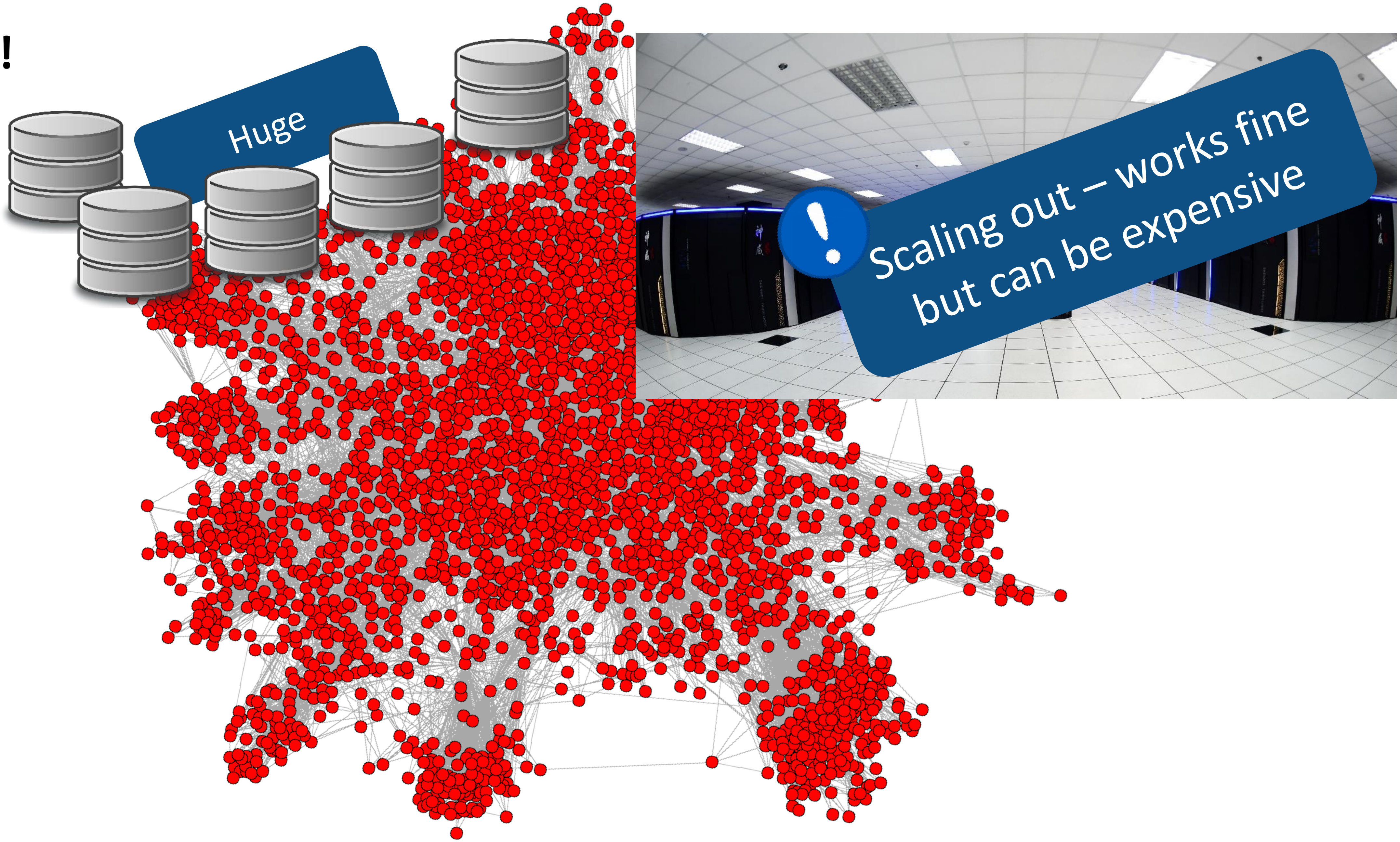

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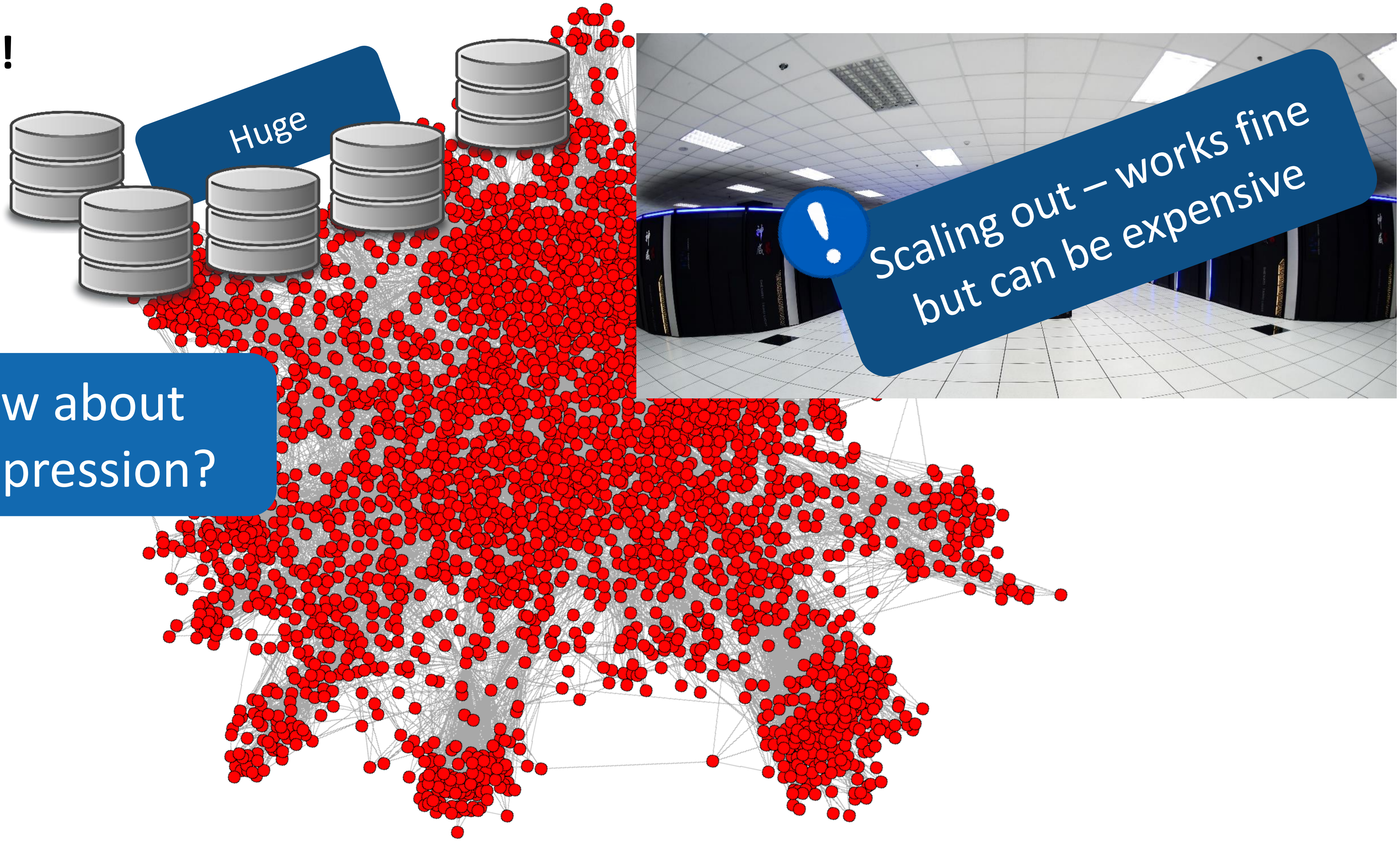


Scaling out – works fine but can be expensive

How about compression?

Problems!

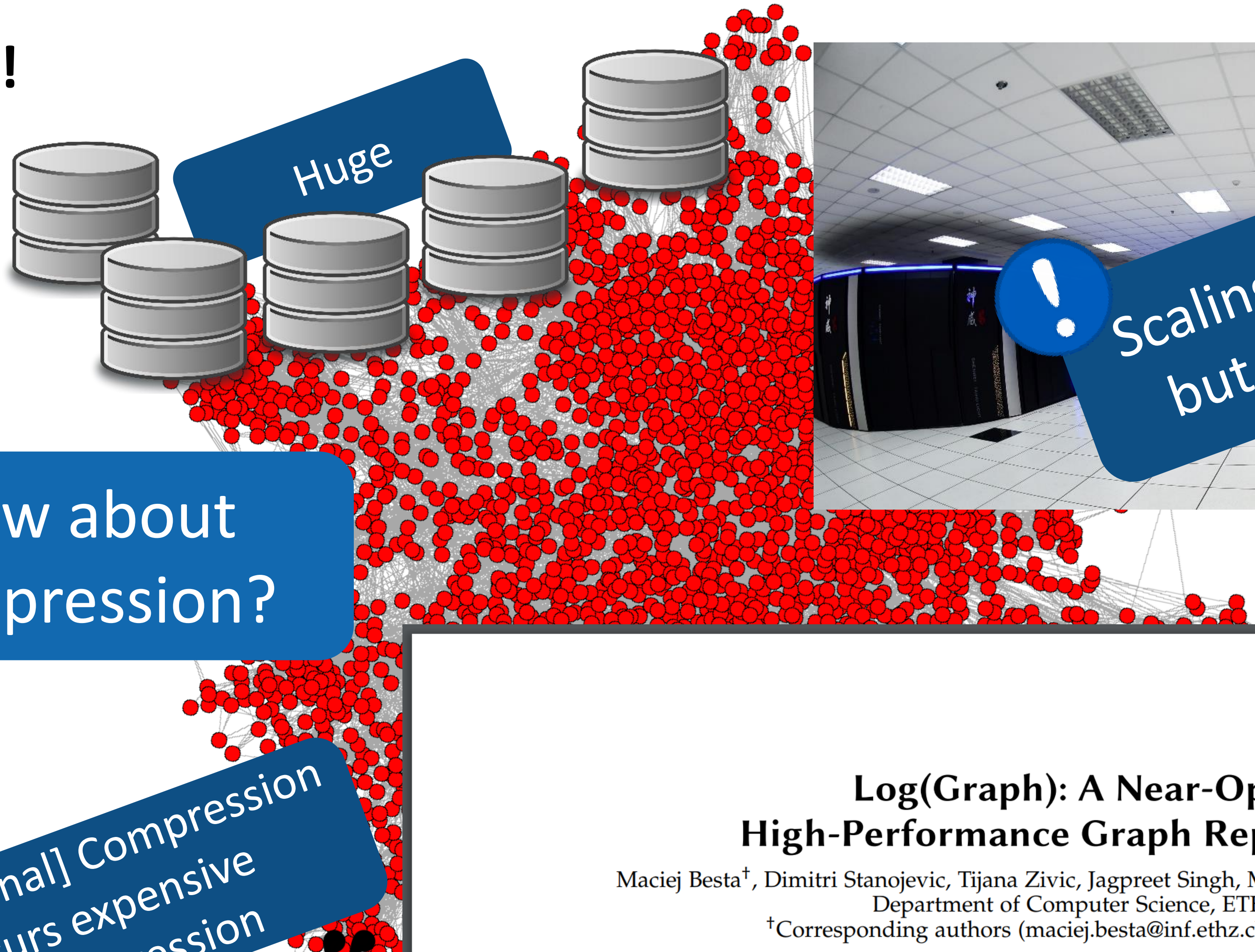
Huge

Scaling out – works fine but can be expensive

How about compression?

[Traditional] Compression incurs expensive decompression

Problems!



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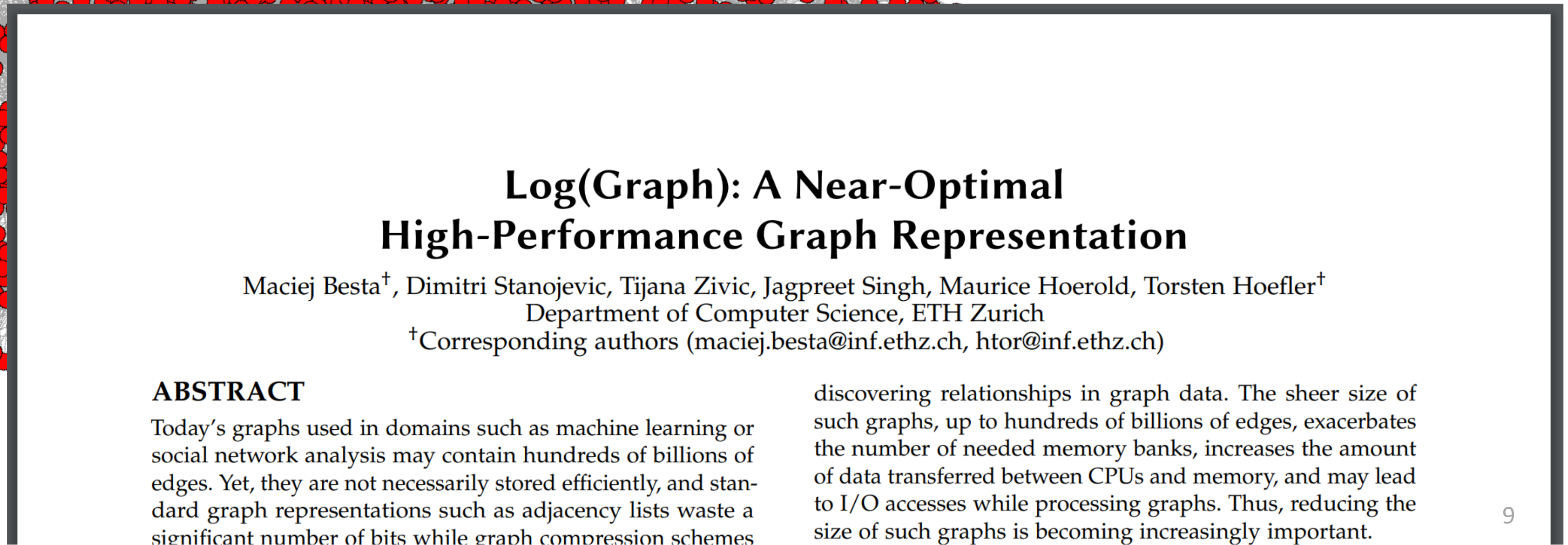

Log(Graph): A Near-Optimal High-Performance Graph Representation

Maciej Besta[†], Dimitri Stanojevic, Tijana Zivic, Jagpreet Singh, Maurice Hoerold, Torsten Hoefler[†]
Department of Computer Science, ETH Zurich
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ABSTRACT

Today's graphs used in domains such as machine learning or social network analysis may contain hundreds of billions of edges. Yet, they are not necessarily stored efficiently, and standard graph representations such as adjacency lists waste a significant number of bits while graph compression schemes discovering relationships in graph data. The sheer size of such graphs, up to hundreds of billions of edges, exacerbates the number of needed memory banks, increases the amount of data transferred between CPUs and memory, and may lead to I/O accesses while processing graphs. Thus, reducing the size of such graphs is becoming increasingly important.

Problems!

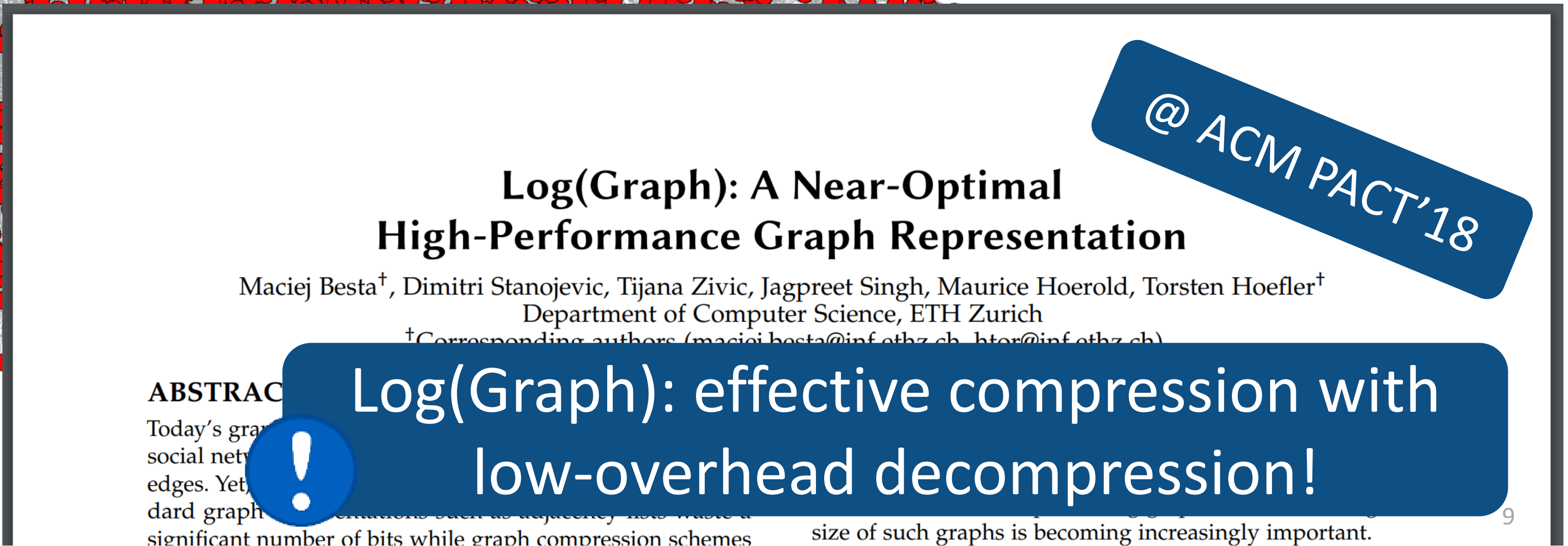
Huge

Scaling out – works fine but can be expensive

How about compression?

[Traditional] Compression incurs expensive decompression

@ ACM PACT'18



Log(Graph): A Near-Optimal High-Performance Graph Representation

Maciej Besta†, Dimitri Stanojevic, Tijana Zivic, Jagpreet Singh, Maurice Hoerold, Torsten Hoefler†
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ABSTRAC
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size of such graphs is becoming increasingly important.

Log(Graph): effective compression with low-overhead decompression!

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What is **the lowest storage** we can (hope to) use to store a graph?

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The storage **lower bound** Ω

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Shannon's approach
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(one needs at least log|S| bits to store an object from an arbitrary set S)

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$x_1 \rightarrow 0 \dots 01$
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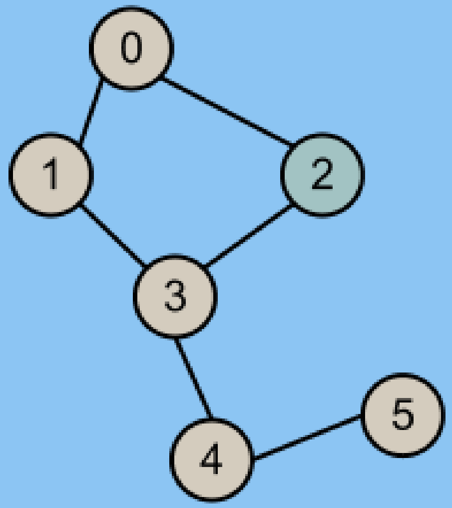
Encode different parts of a graph representation using (logarithmic) **storage lower bounds**



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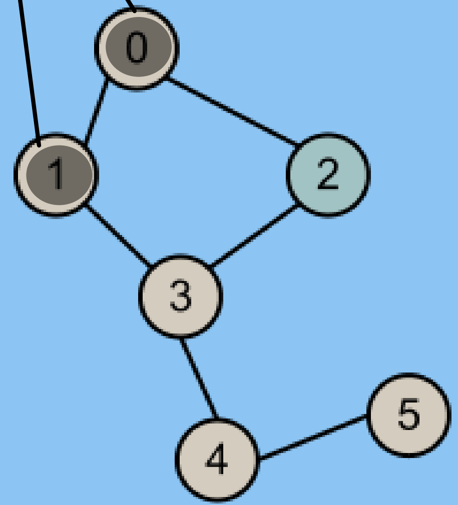

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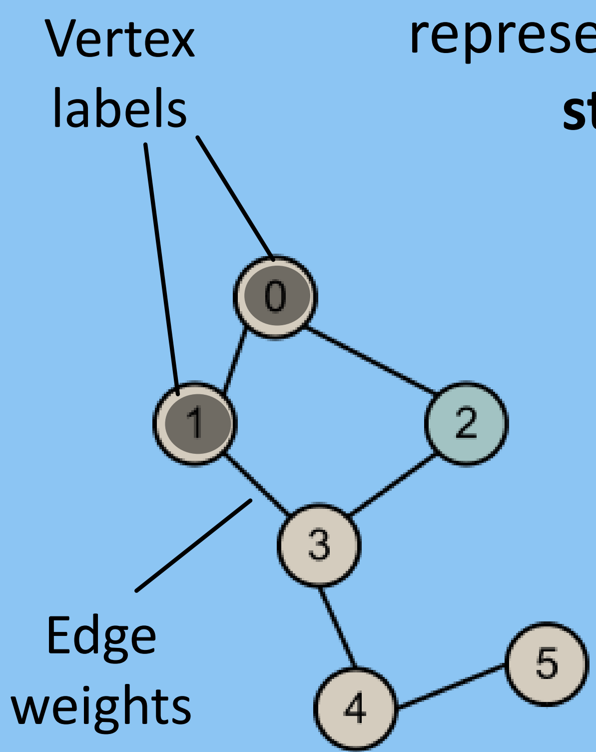

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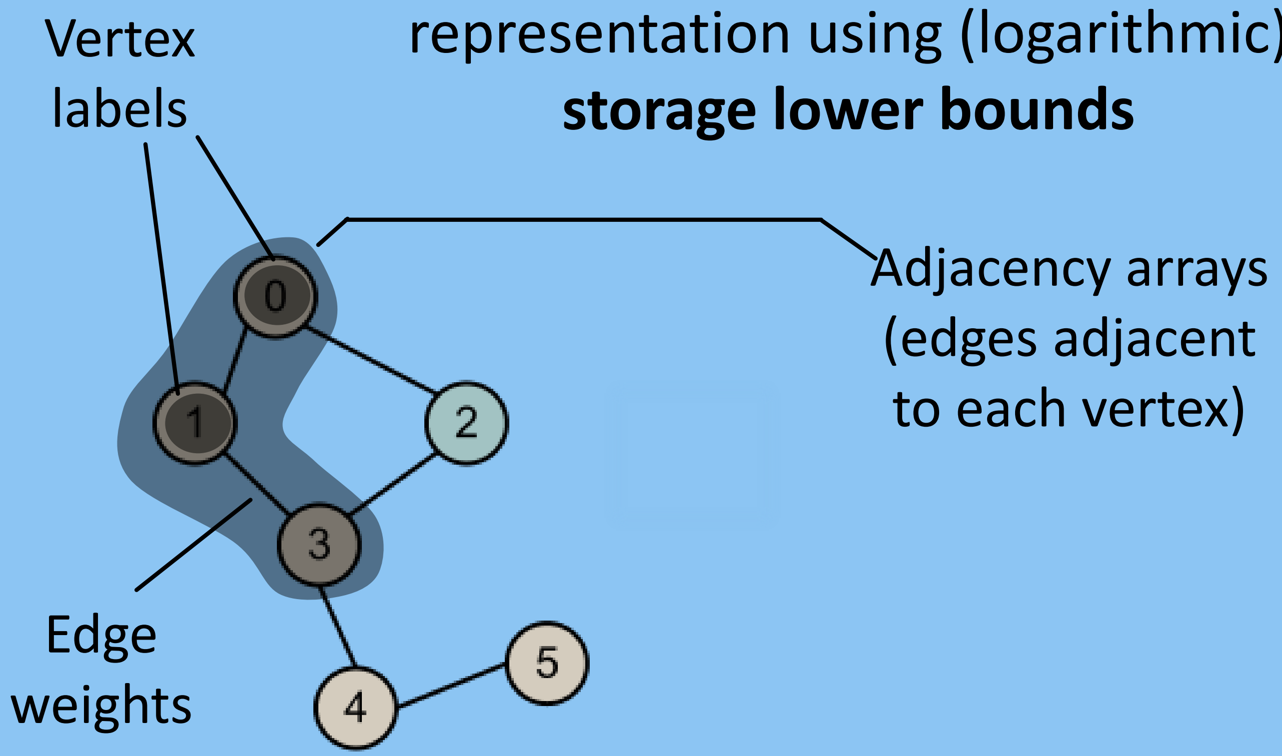

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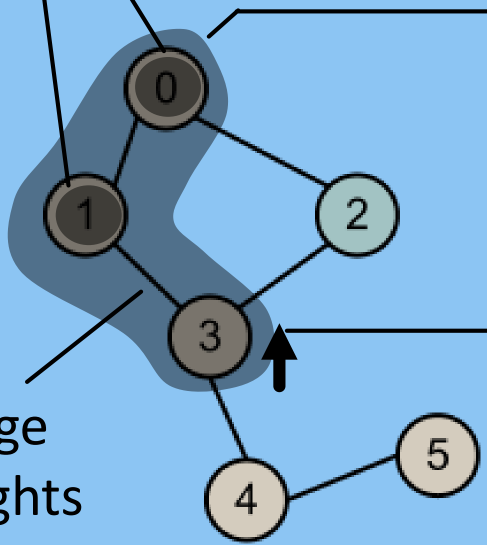

Adjacency arrays (edges adjacent to each vertex)

Edge weights

Offsets (locations) of adj. arrays

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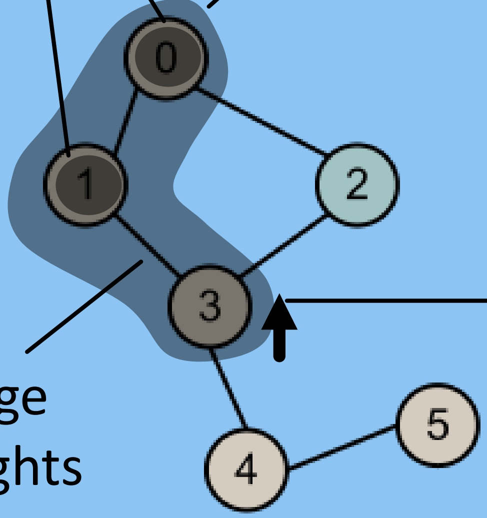

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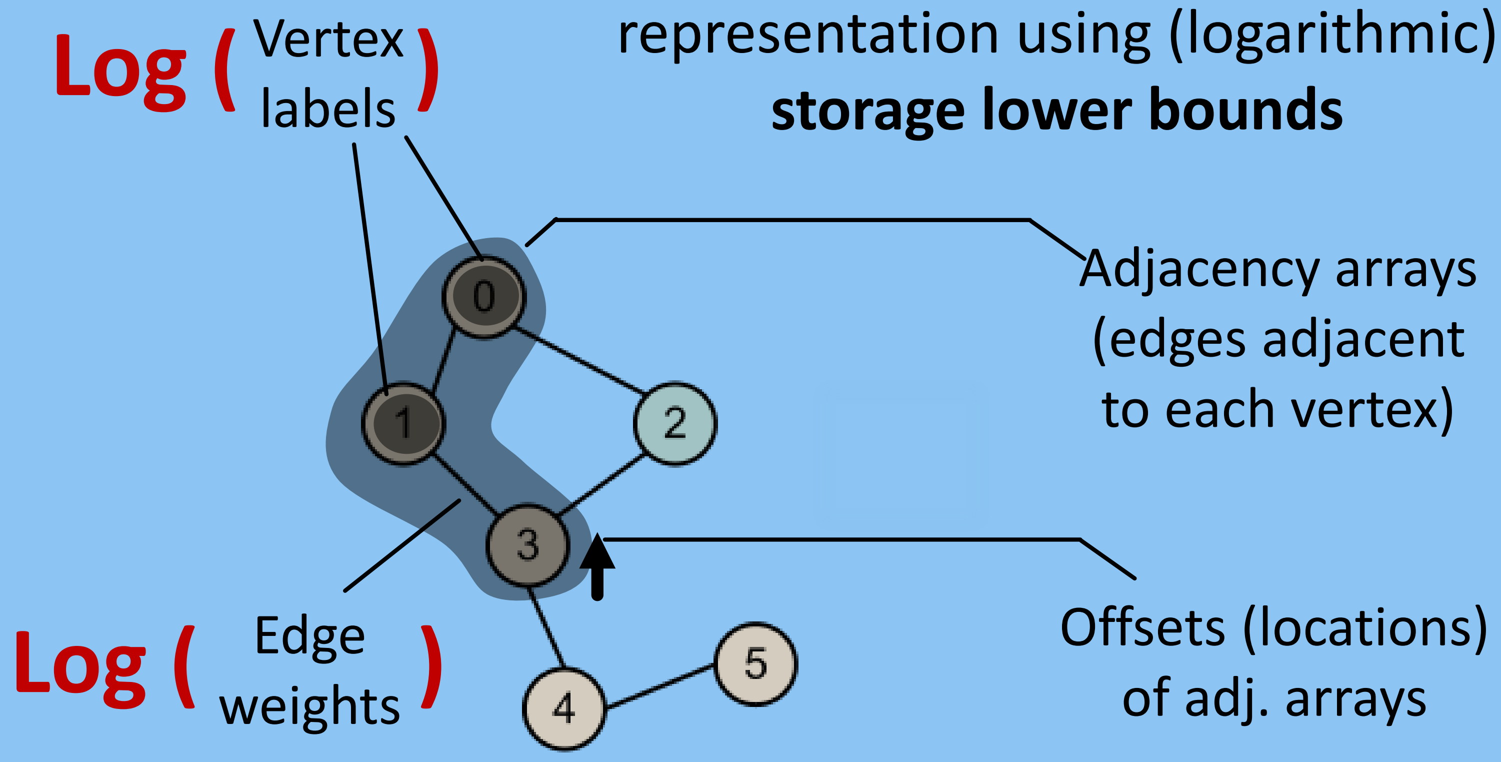

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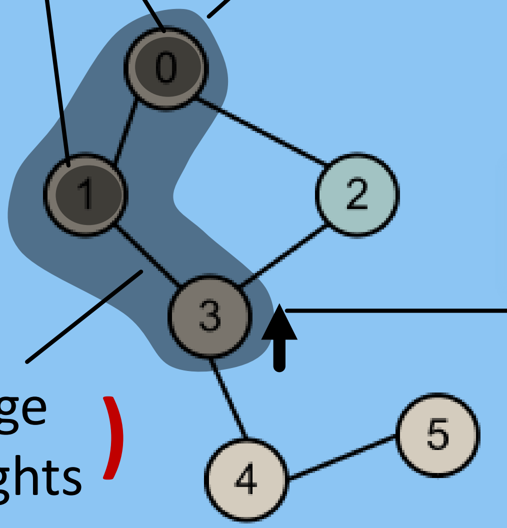

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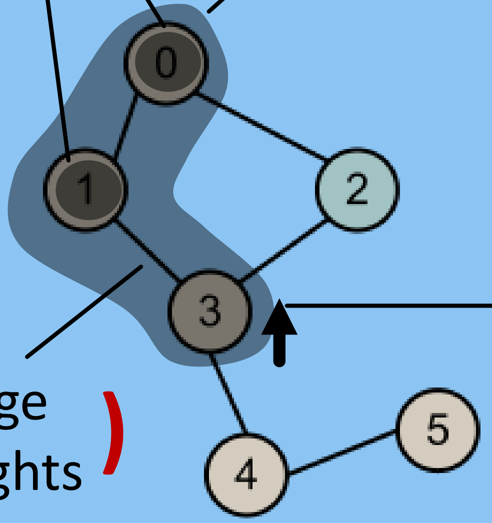

ADJACENCY ARRAY
GRAPH REPRESENTATION

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ADJACENCY ARRAY GRAPH REPRESENTATION

Physical realization

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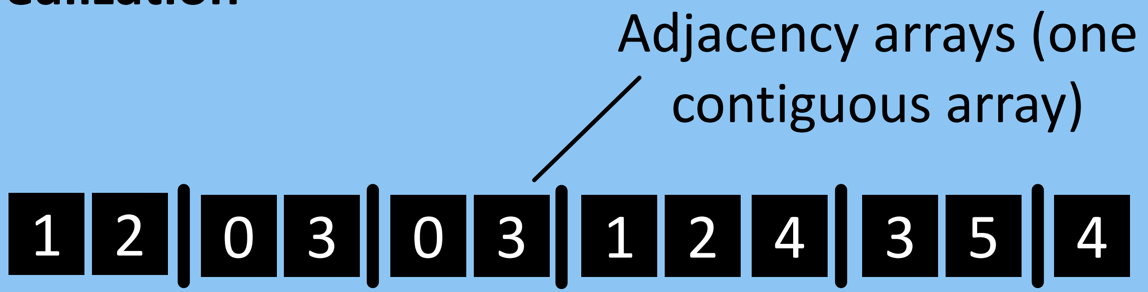
Adjacency Array
Graph Representation

Physical realization



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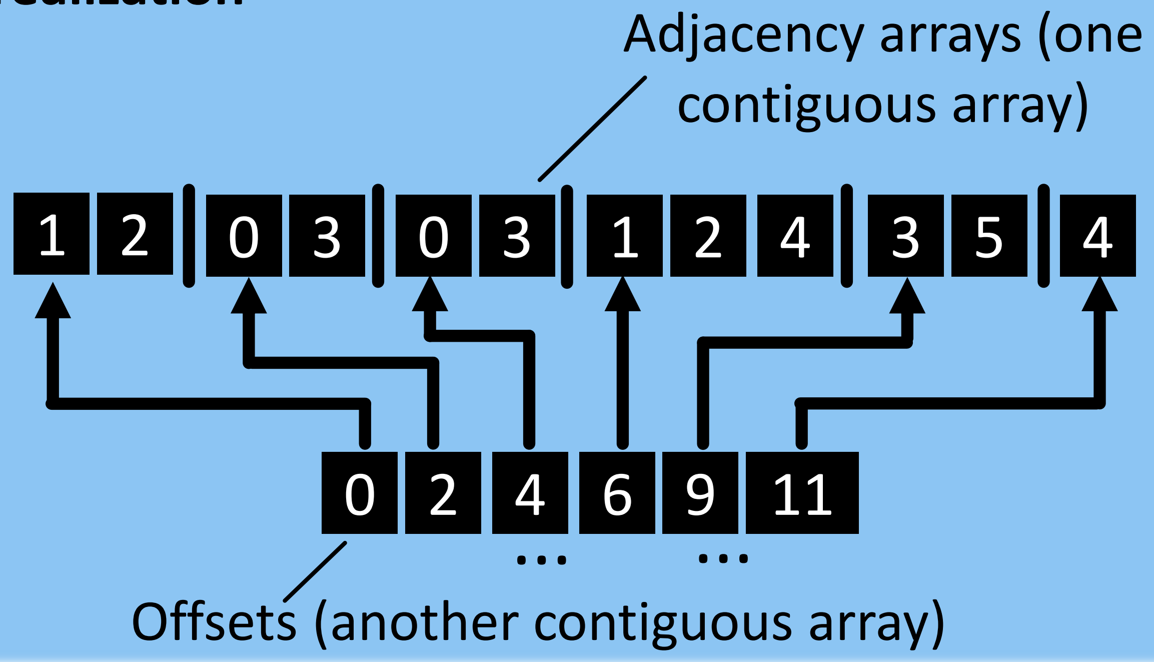

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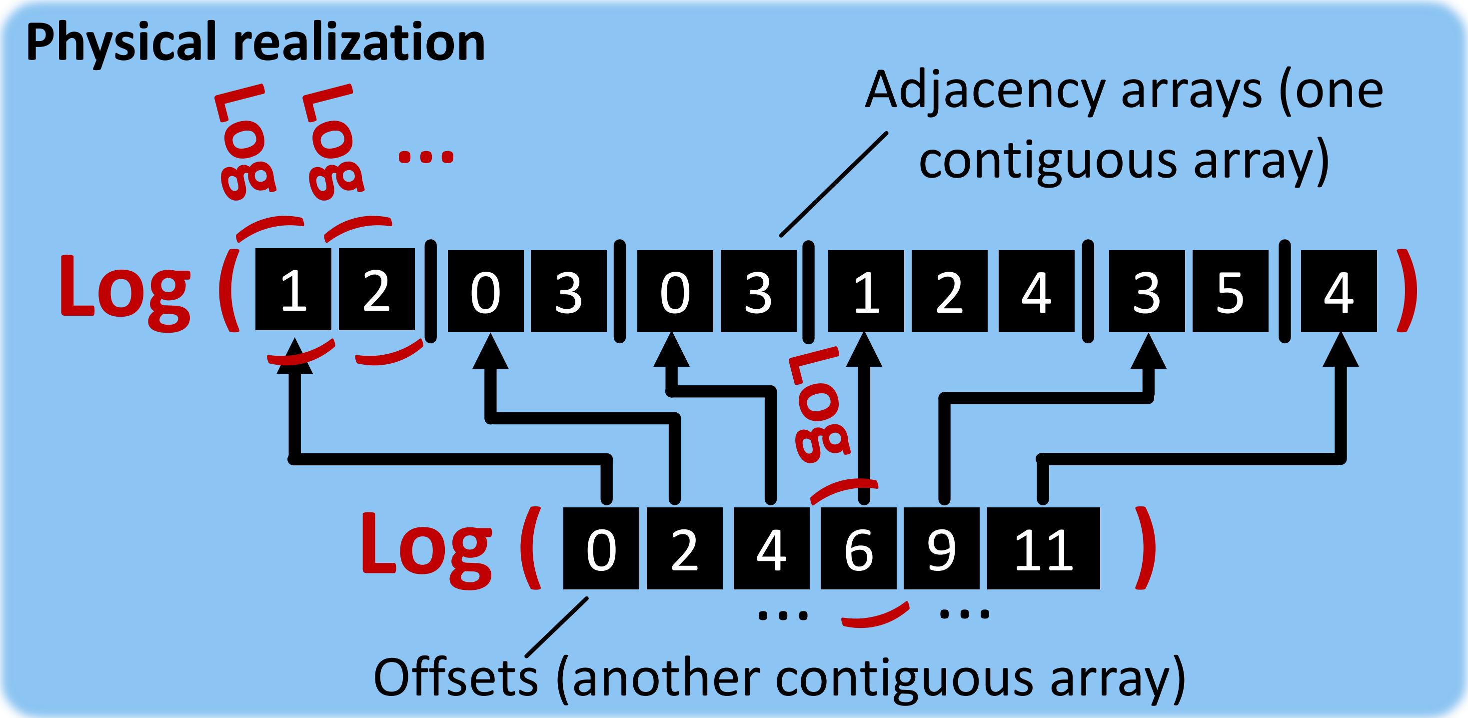

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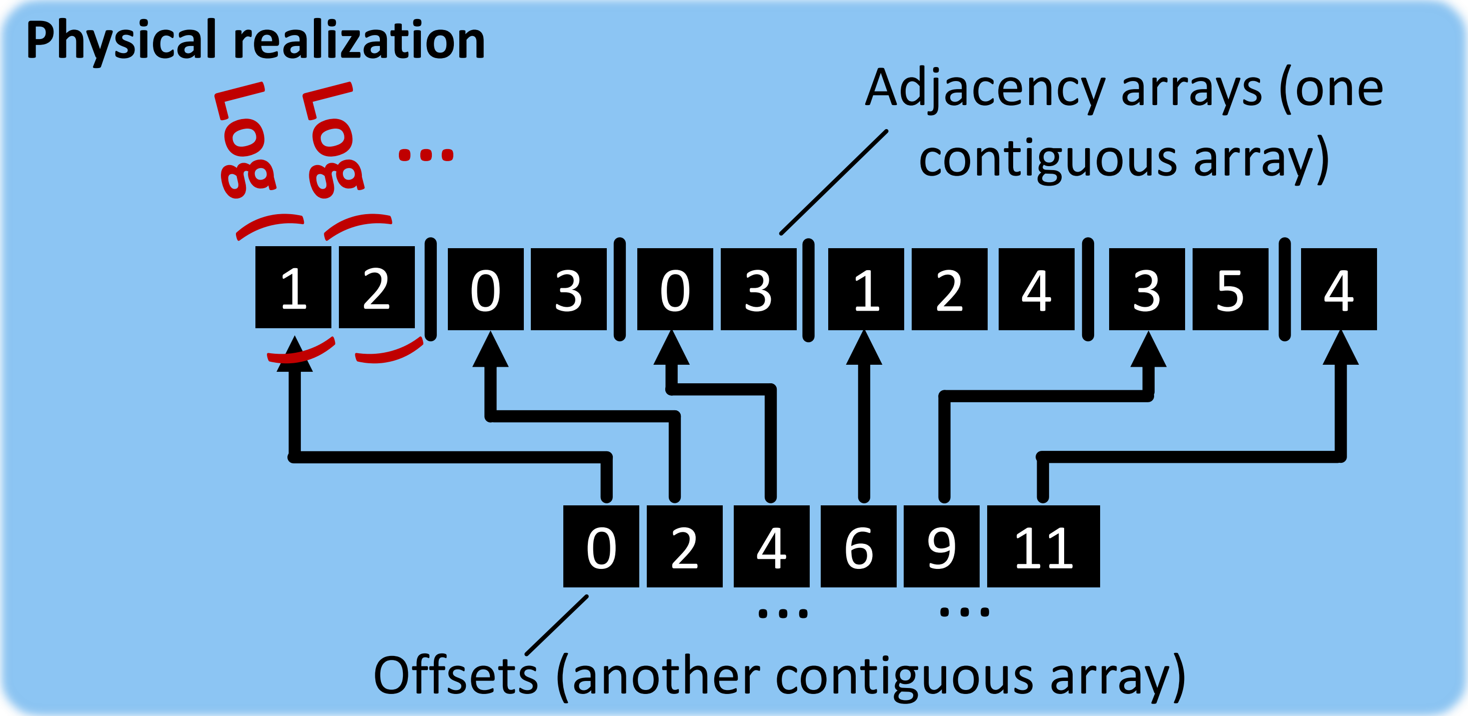

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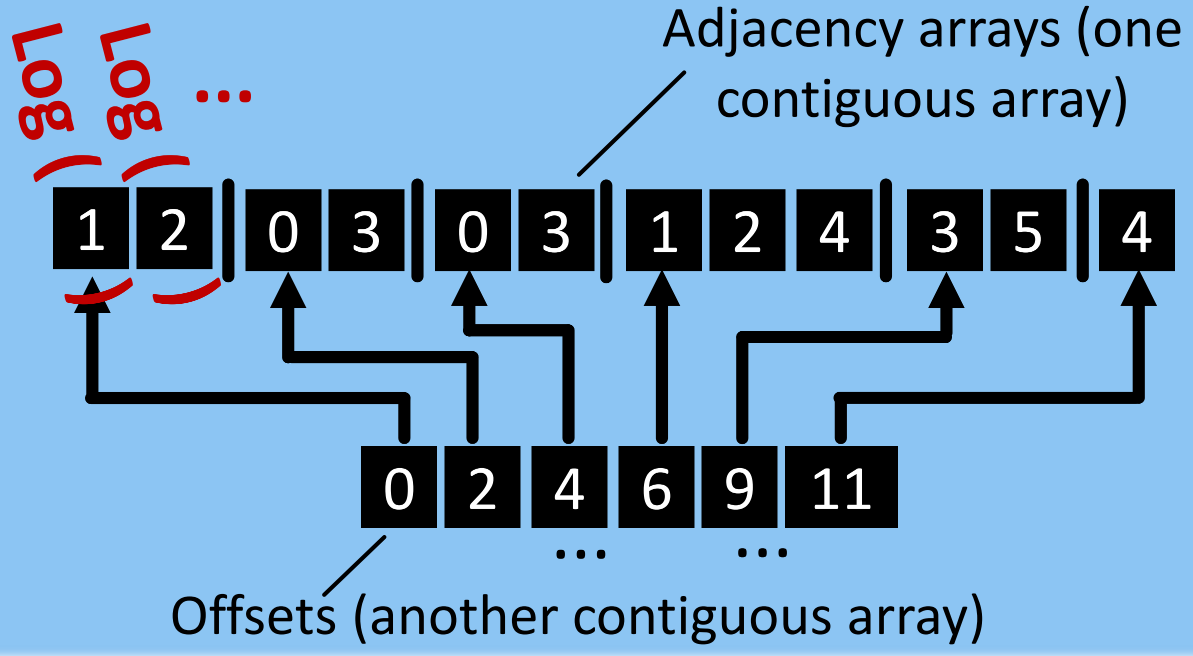
Adjacency Array Graph Representation

1 **Log (Vertex labels), Log (Edge weights)**

Physical realization



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Adjacency Array Graph Representation

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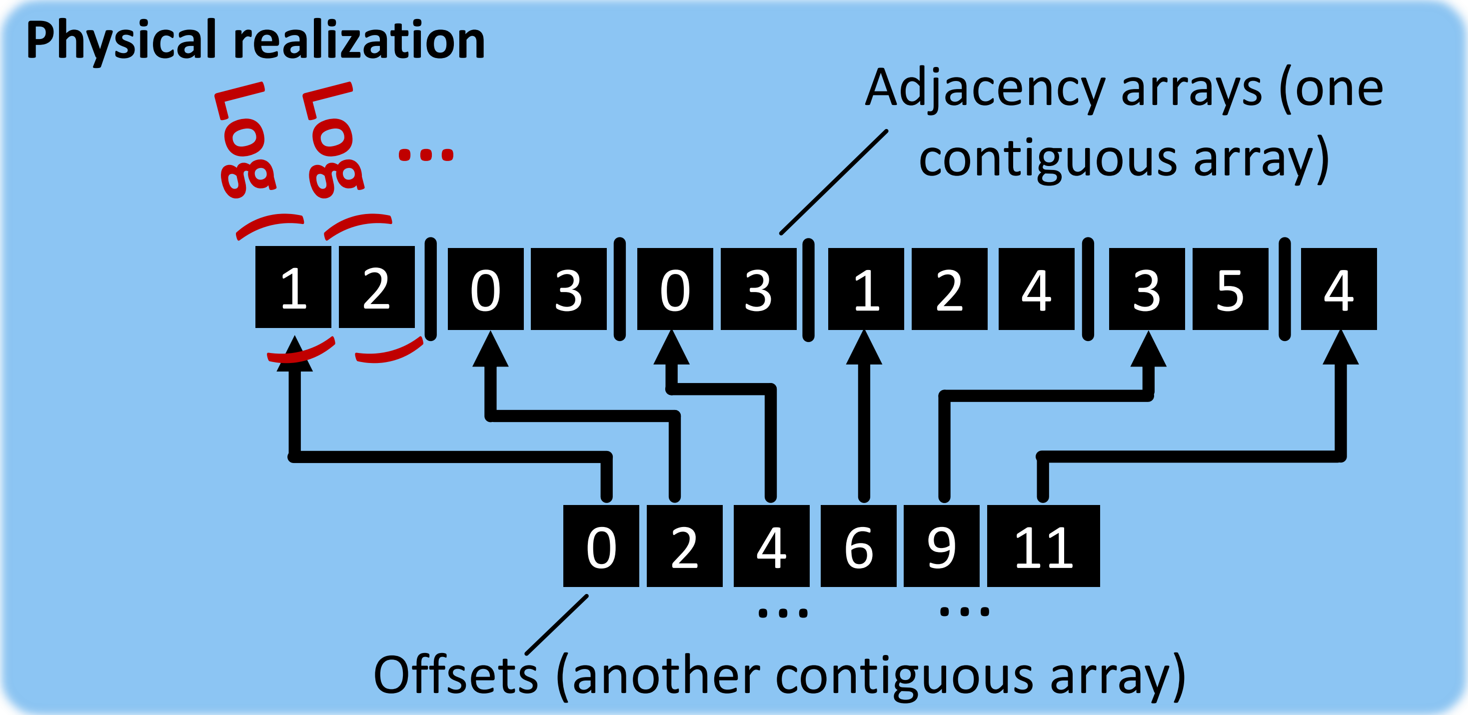
Global bound $\lceil \log n \rceil$



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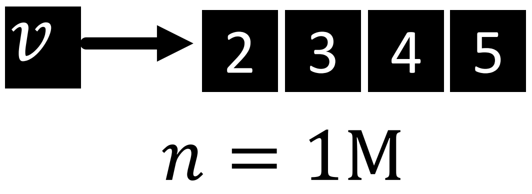

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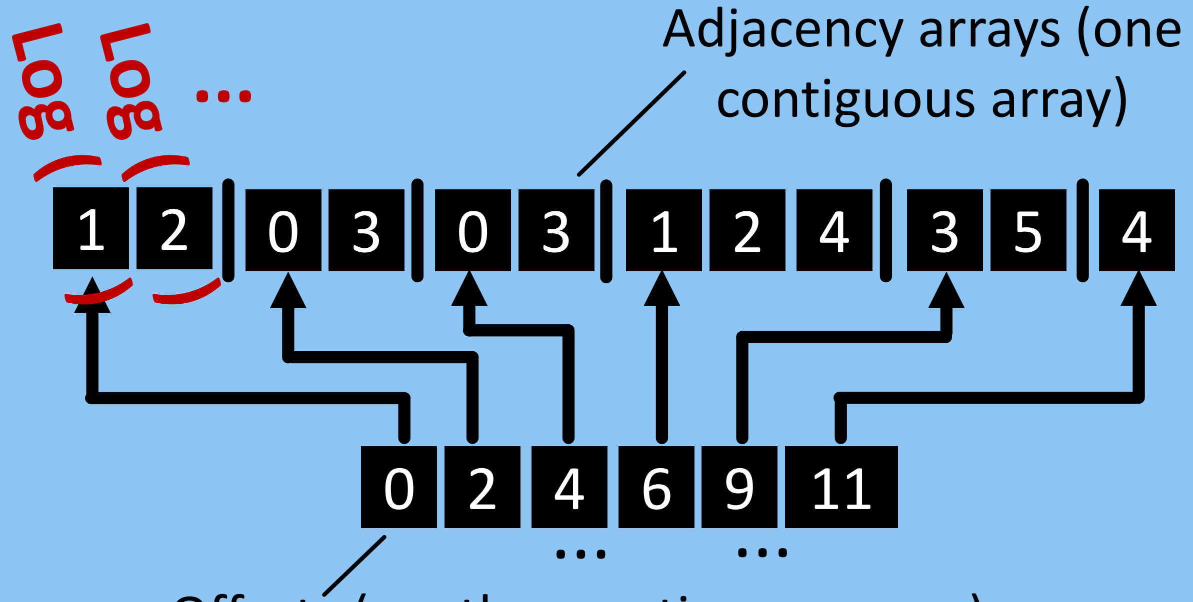

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Adjacency Array Graph Representation

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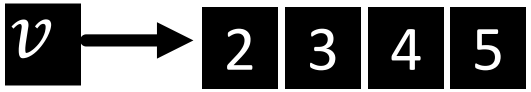
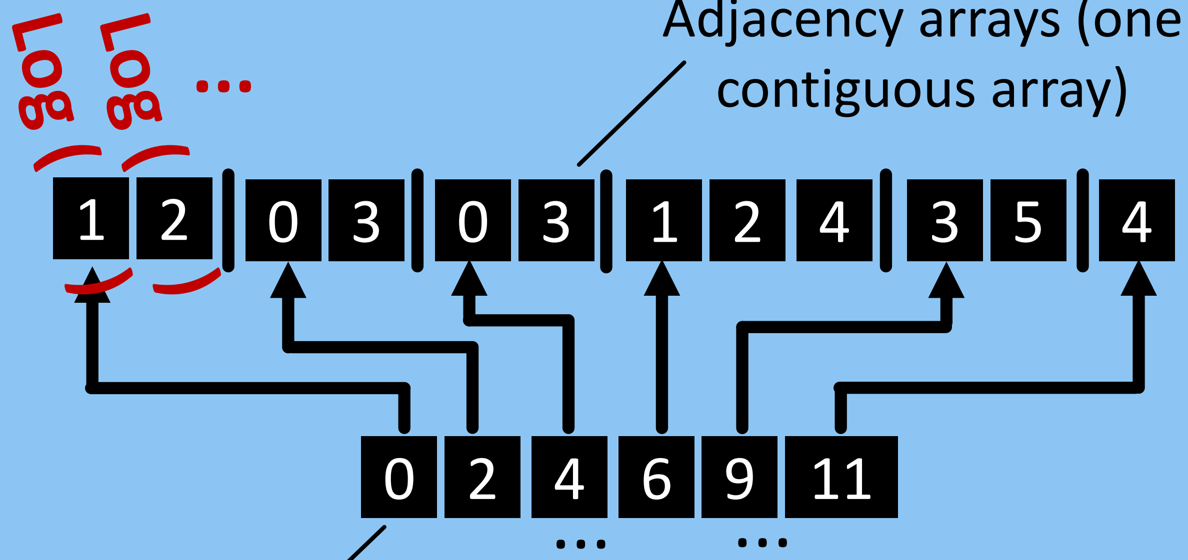
Global bound $\lceil \log n \rceil$

Local bounds $\lceil \log(\max(v_i)) \rceil$



$n = 1\text{M}$

Physical realization



Offsets (another contiguous array)

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Adjacency Array Graph Representation

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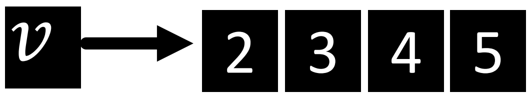

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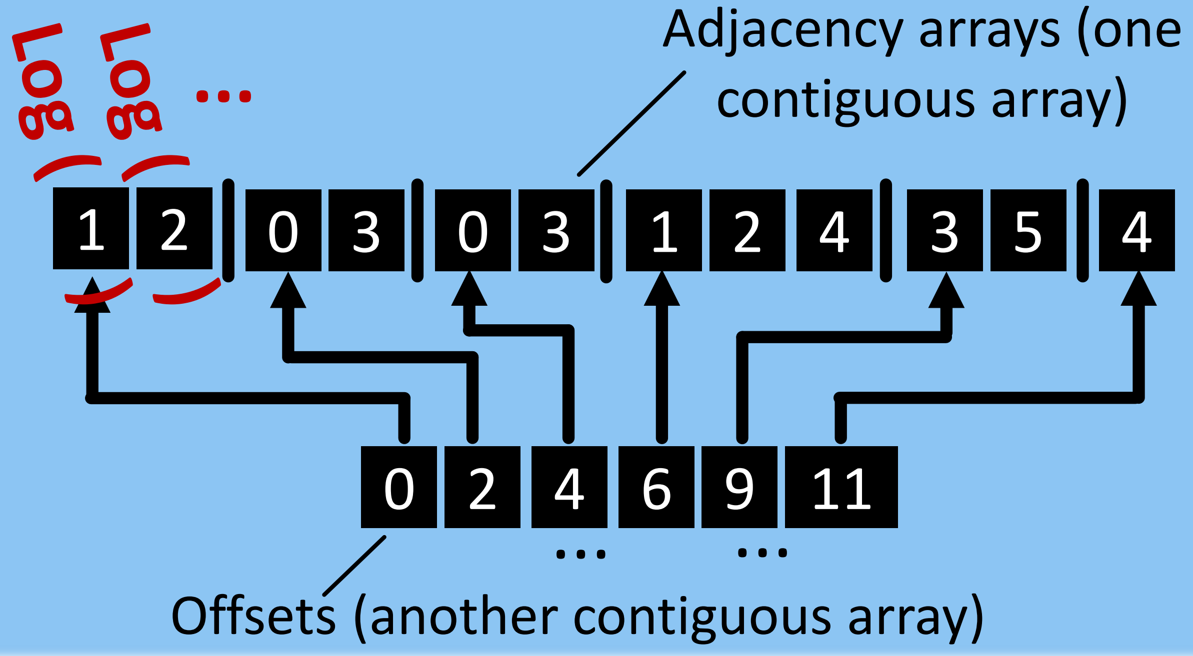

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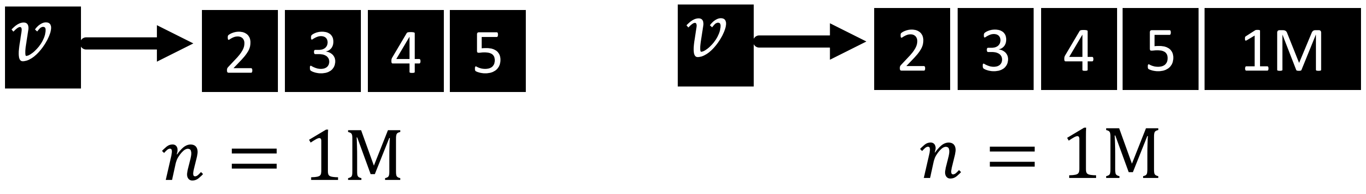
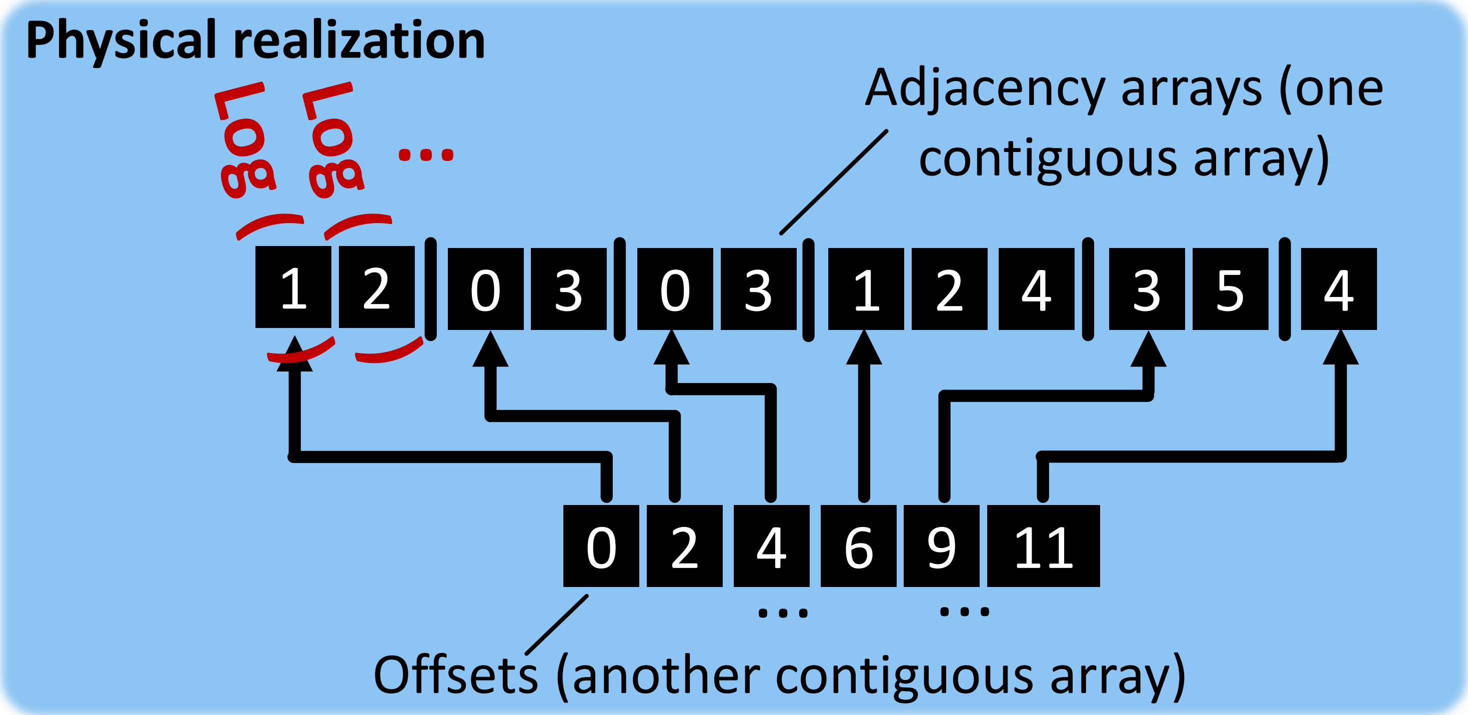
Adjacency Array Graph Representation

1 **Log (** Vertex labels **), Log (** Edge weights **)**





Global bound $\lceil \log n \rceil$

Local bounds $\lceil \log(\max(v_i)) \rceil$

Permute vertex labels **to reduce such maximum labels** in as many neighborhoods as possible

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2 **Log (** Offset structure **)**

3 **Log (** Adjacency structure **)**

[1] G. J. Jacobson. Succinct Static Data Structures. 1988

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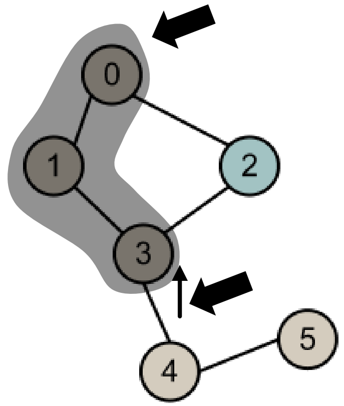

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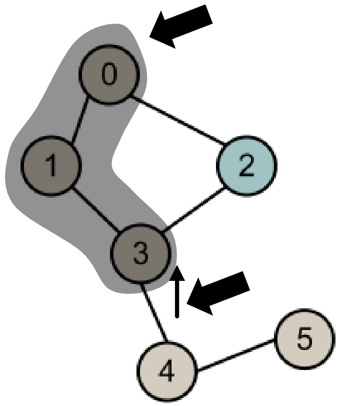

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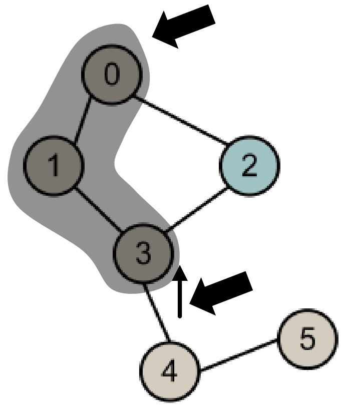

Succinct data structures [1]

They use **[Q] + *o*(Q) bits** ([Q] - lower bound), they answer various queries in ***o*(Q) time**.

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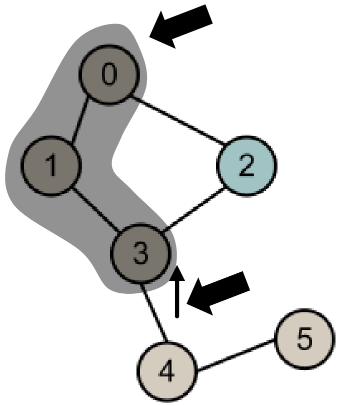

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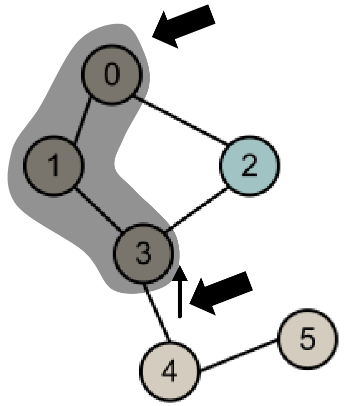

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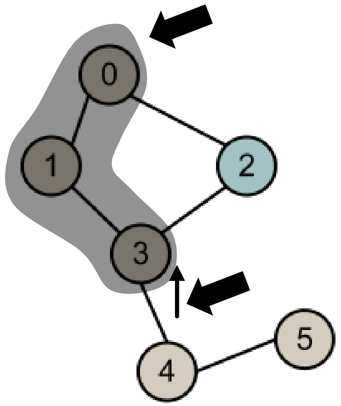

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We show that they are in practice both small and fast!

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Overview of Full Log(Graph) Design

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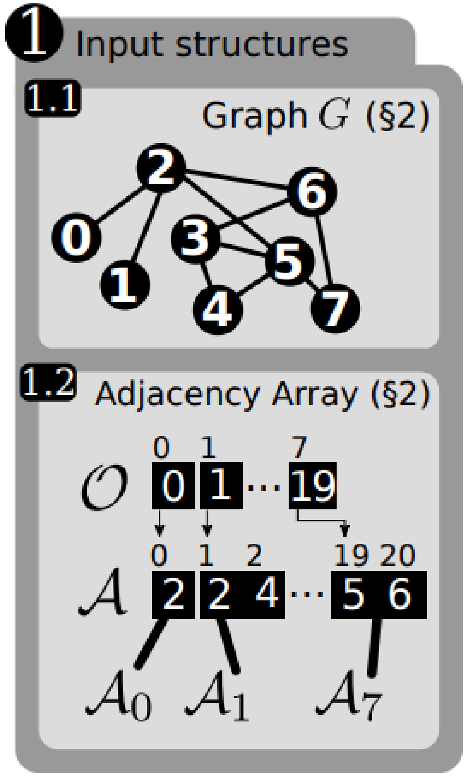

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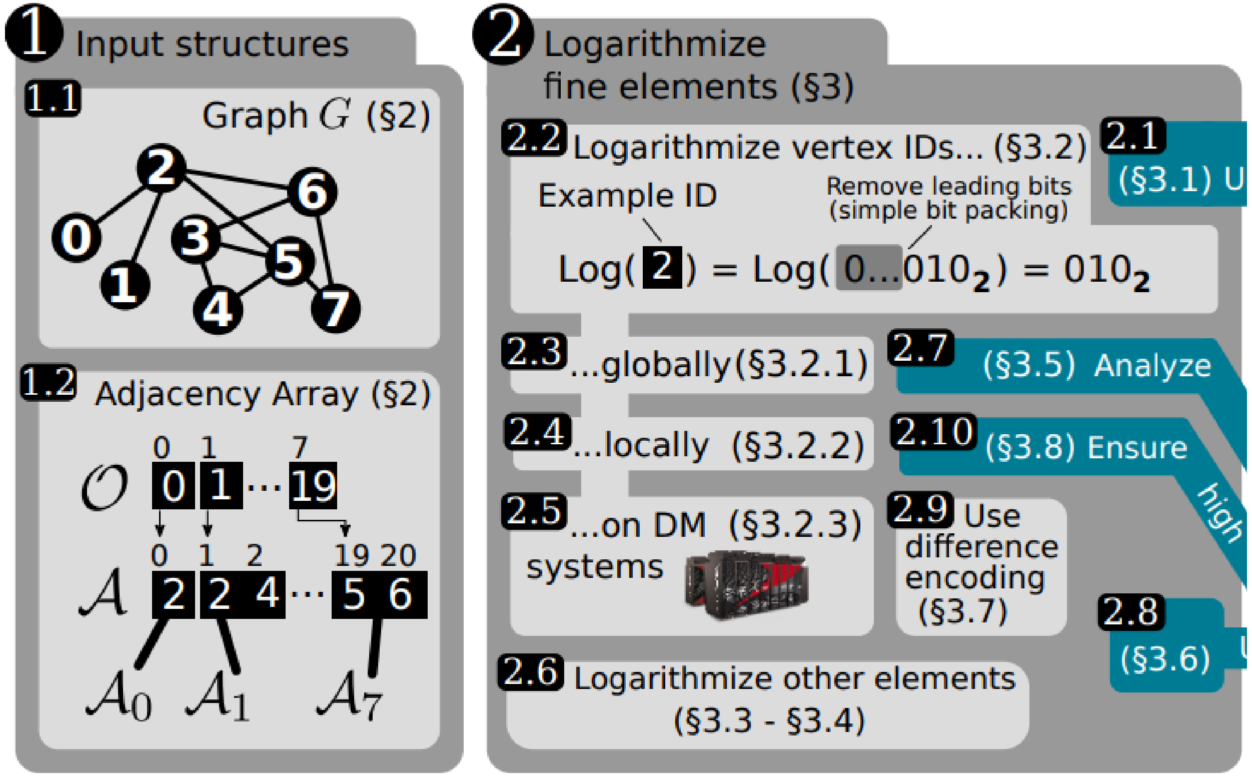

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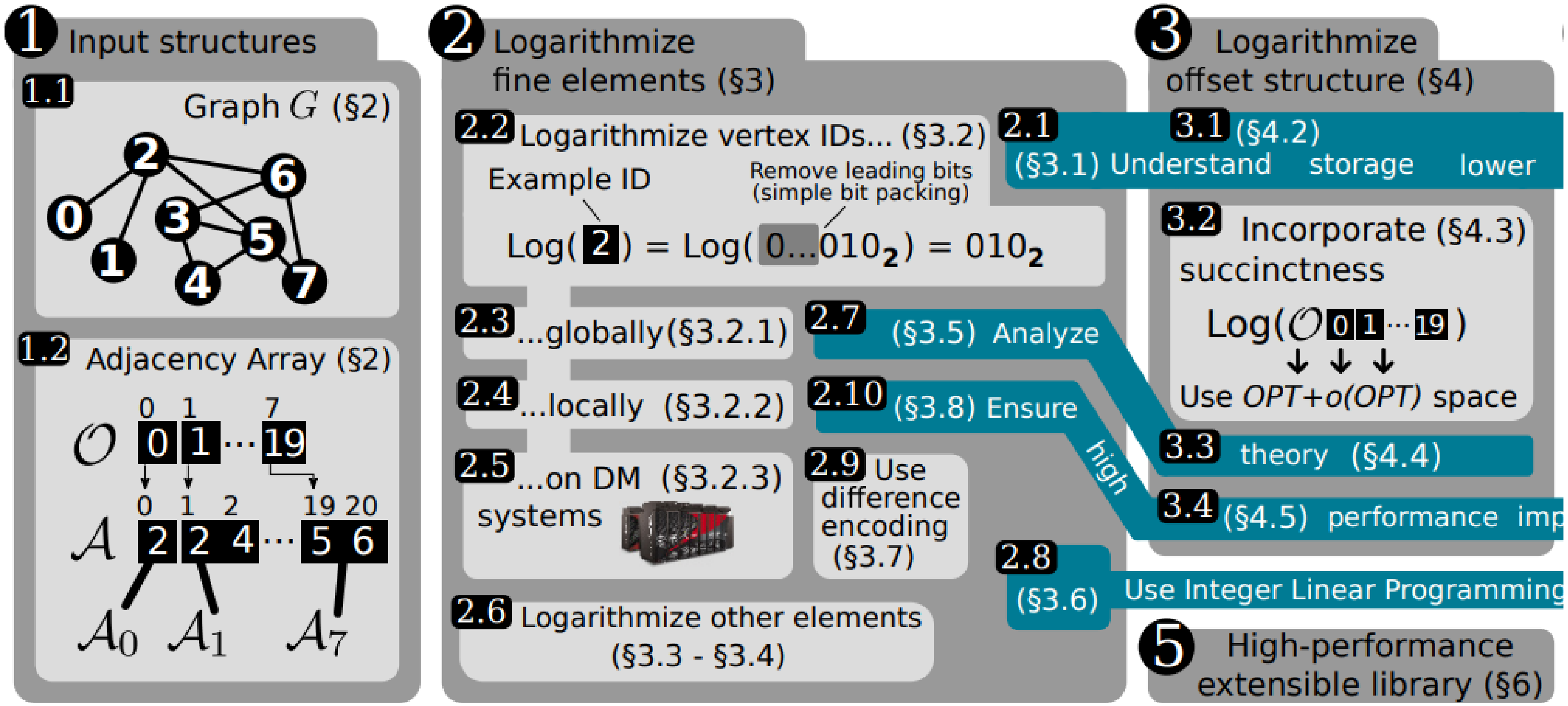

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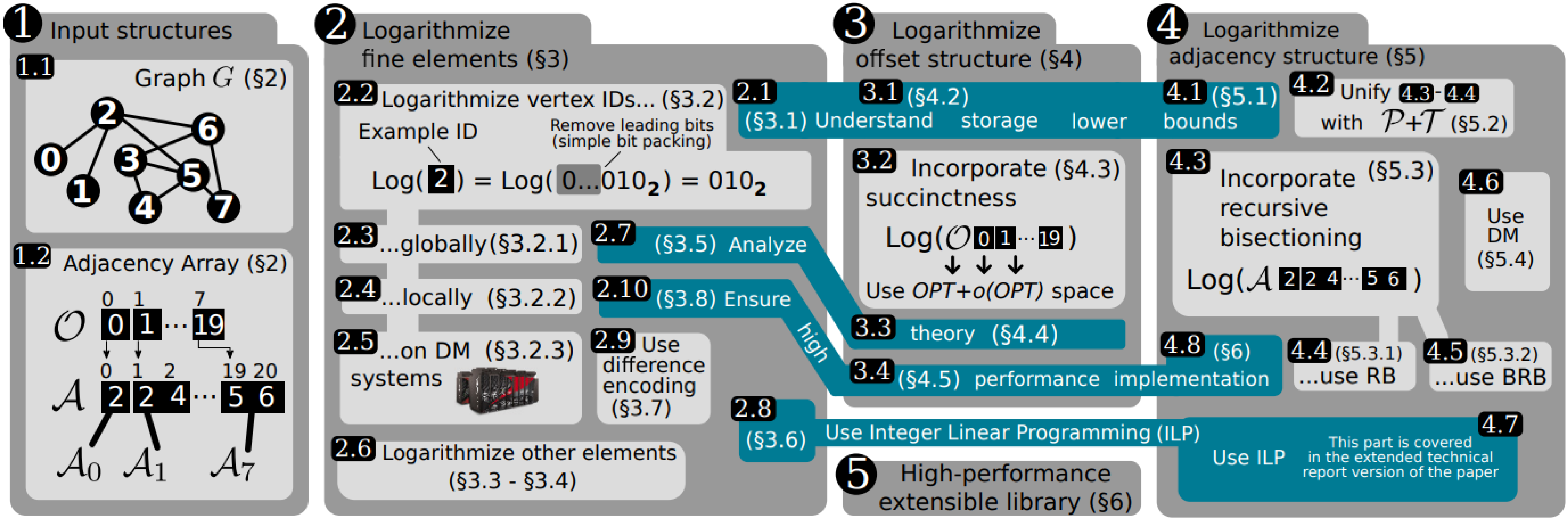

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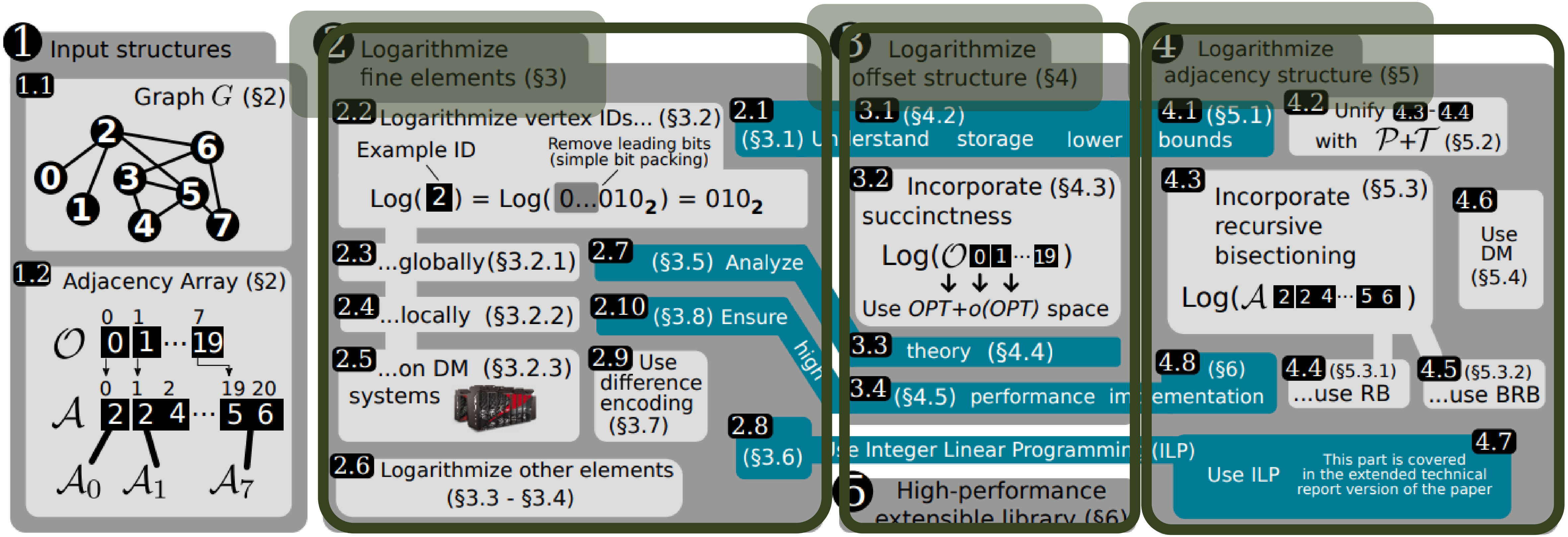

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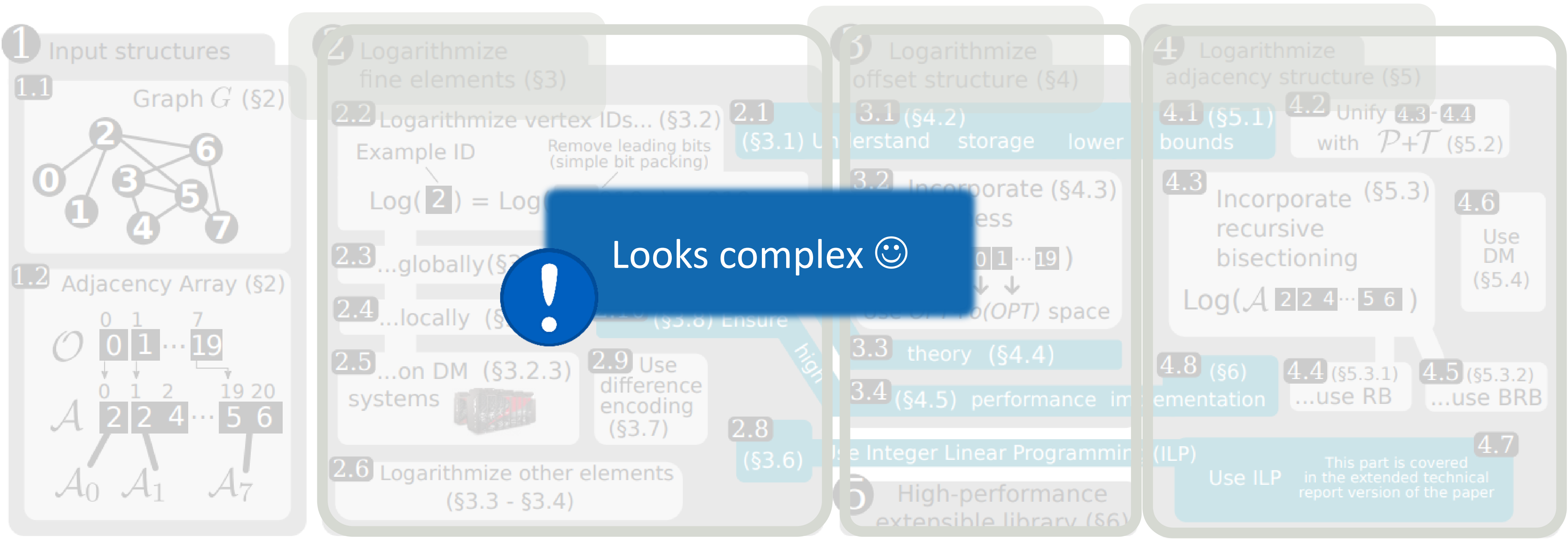

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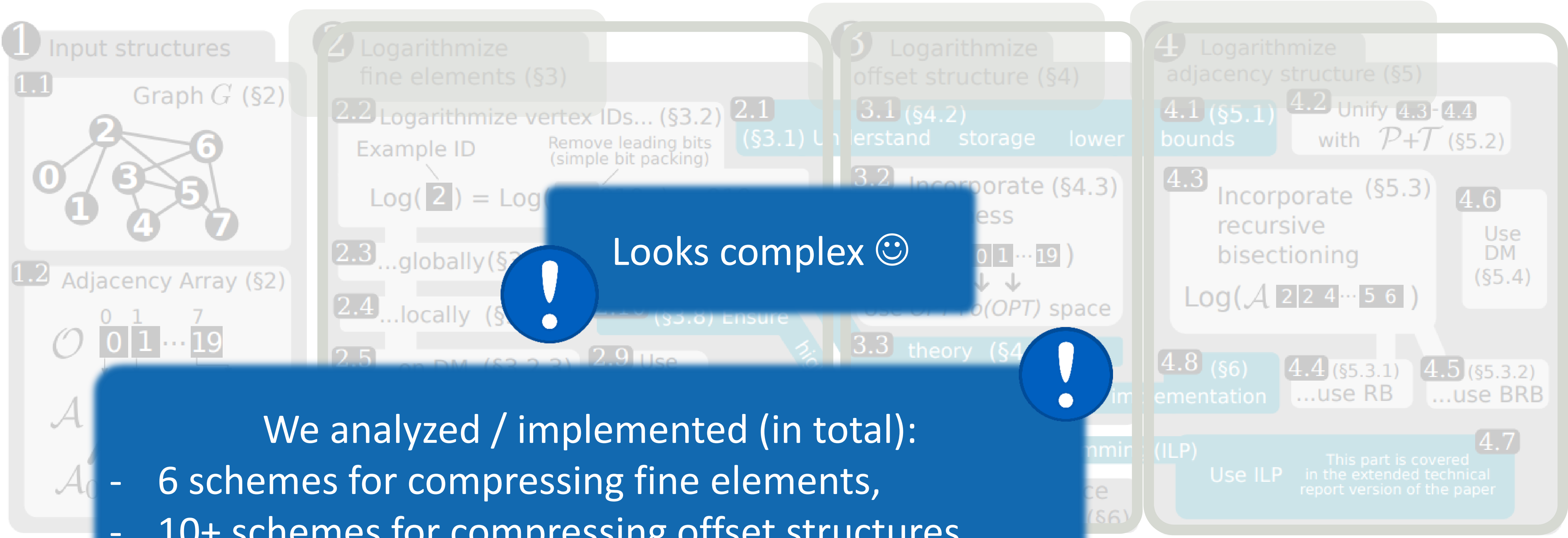

Looks complex ☺

We analyzed / implemented (in total):

- 6 schemes for compressing fine elements,
- 10+ schemes for compressing offset structures,
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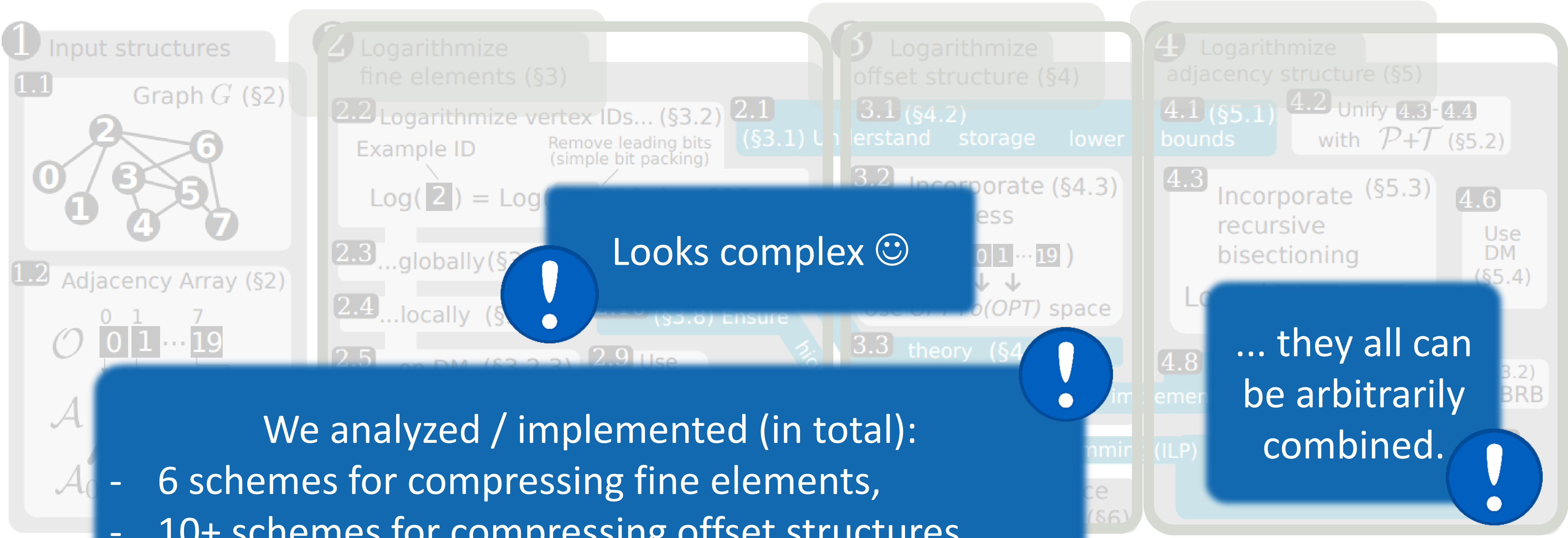

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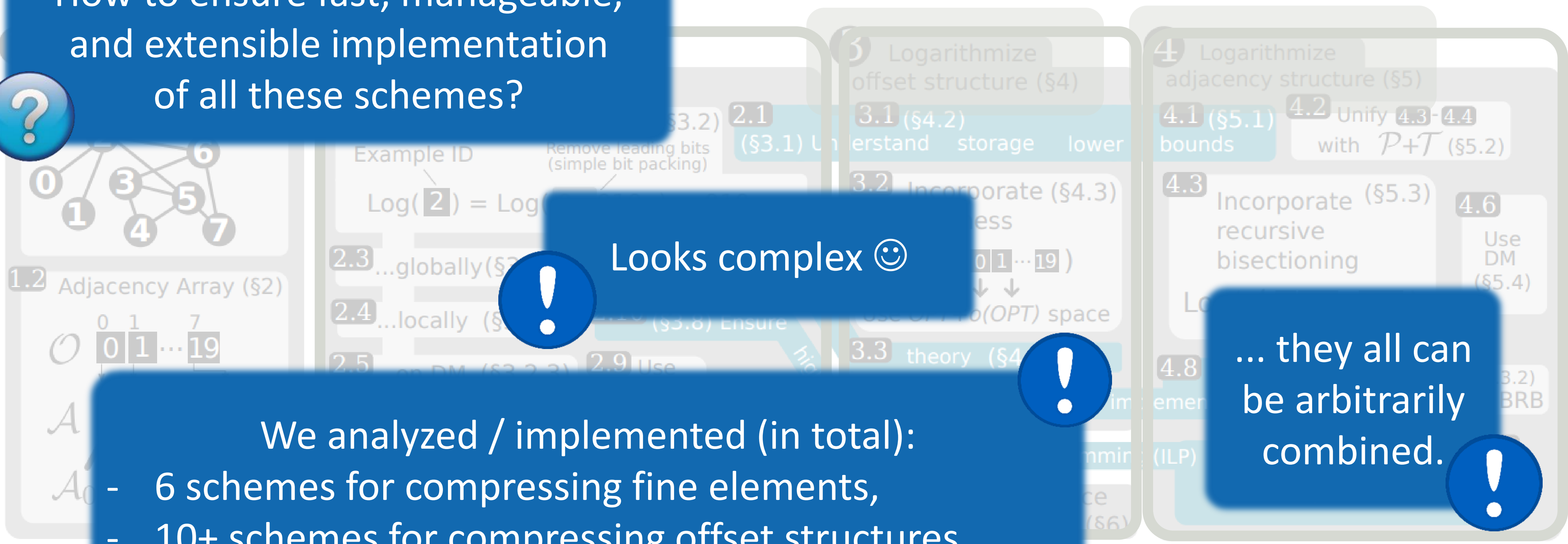

How to ensure fast, manageable, and extensible implementation of all these schemes?

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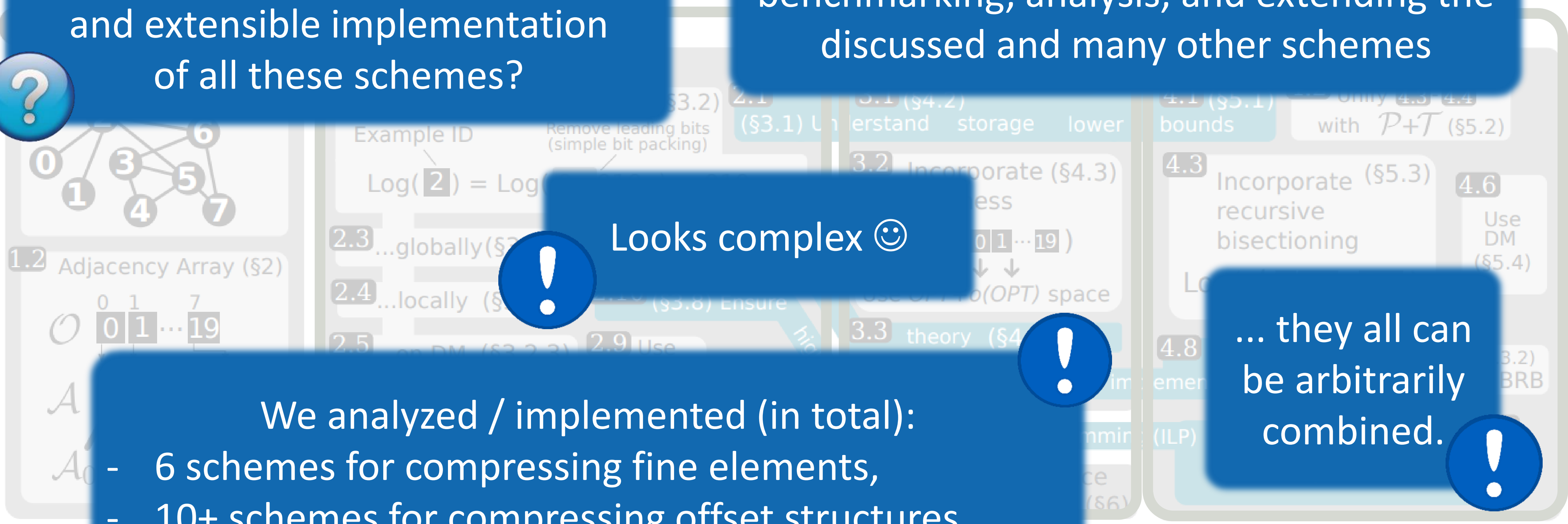

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COMING SOON

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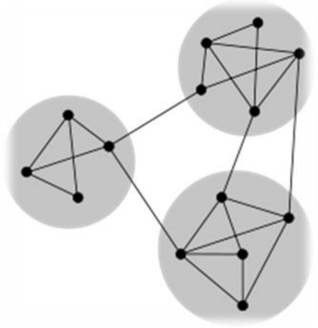
M. Besta et al.: "Log(Graph): A Near-Optimal High-Performance Graph Representation", PACT'18

1 Log (Vertex labels), Log (Edge weights) Storage, Performance

SSSP



Kronecker graphs
Number of vertices: 4M

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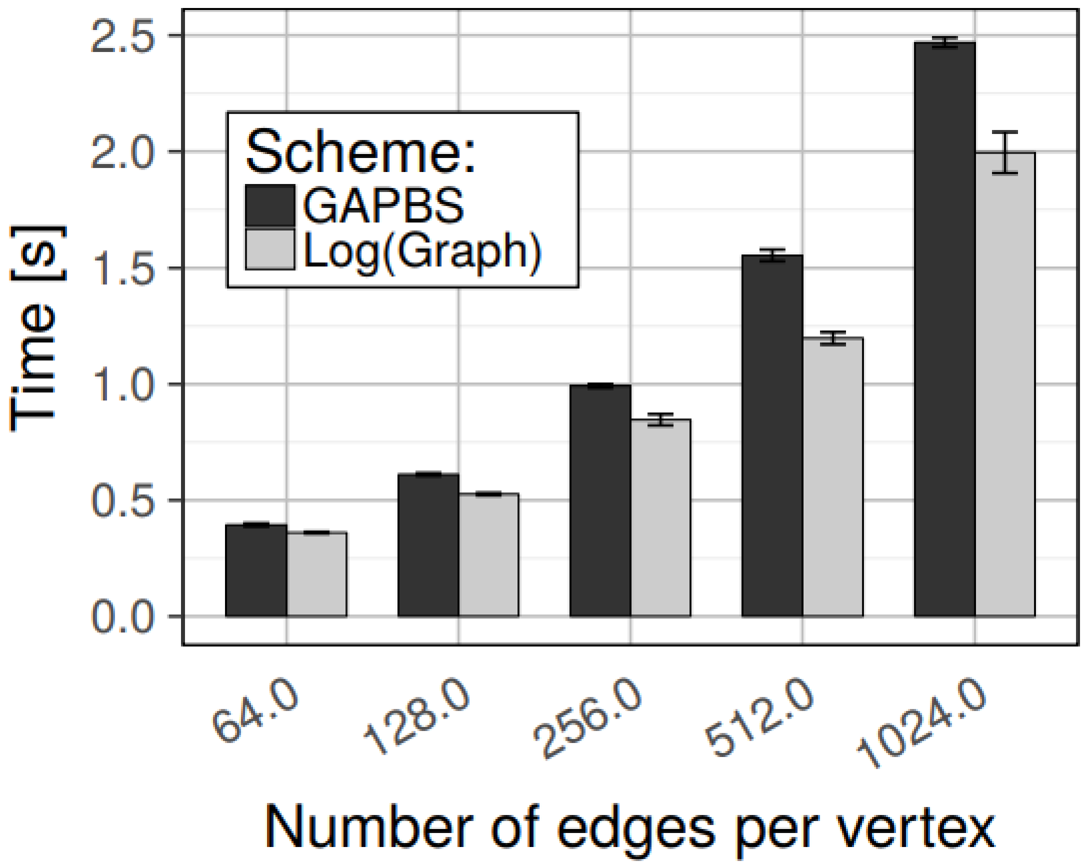

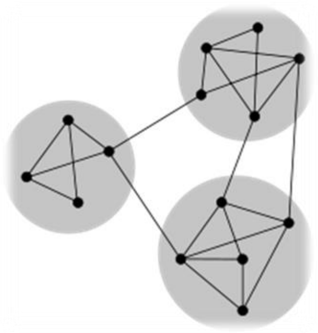

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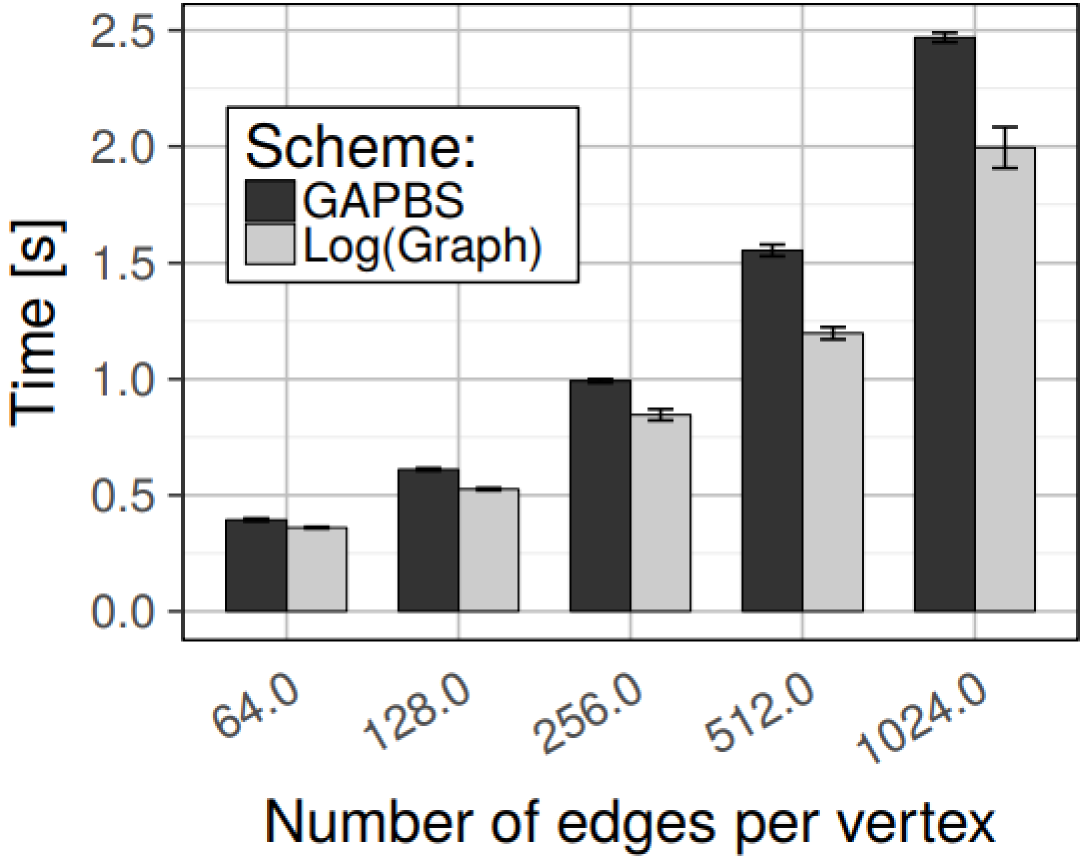

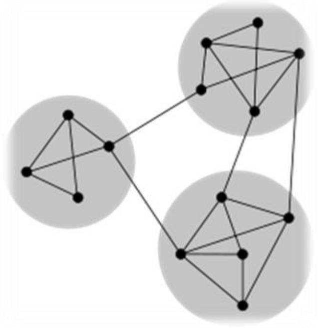

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Log(Graph) consistently reduces storage overhead (by 20-35%)

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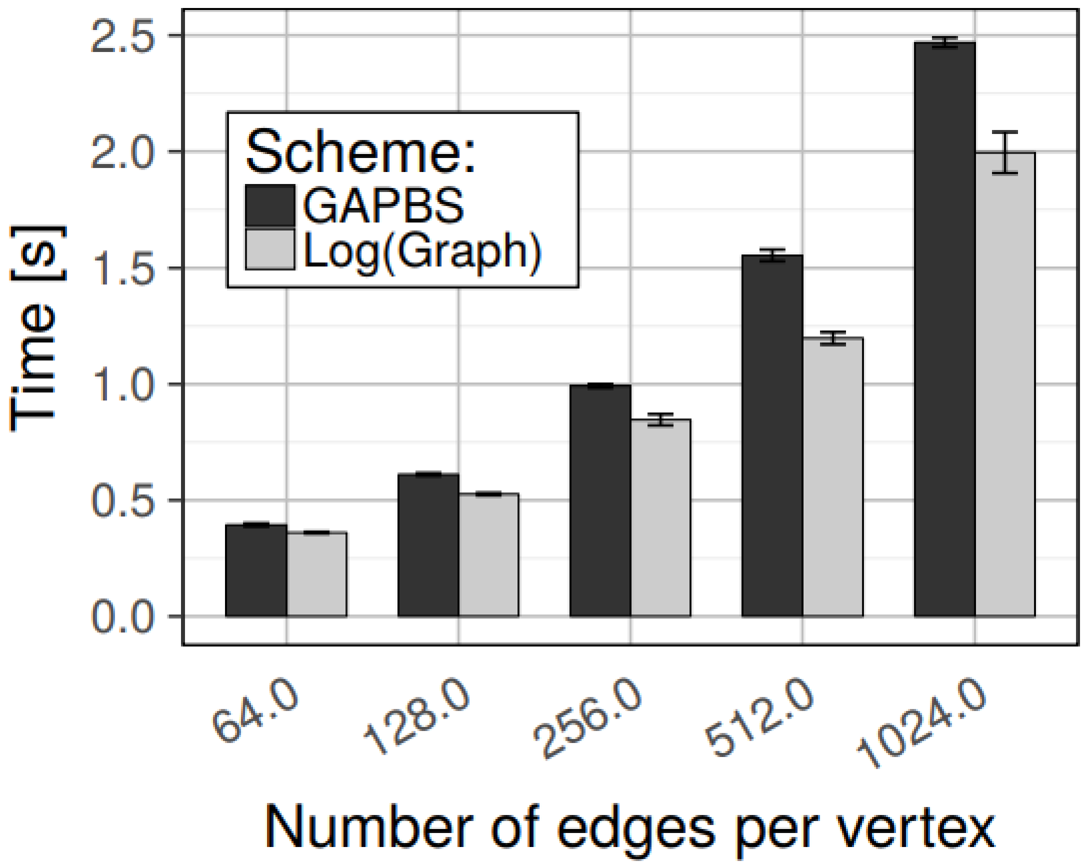

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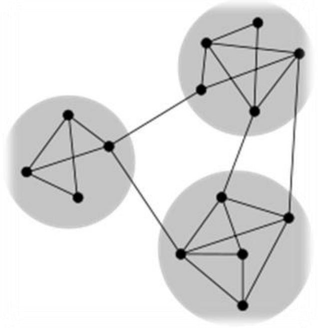

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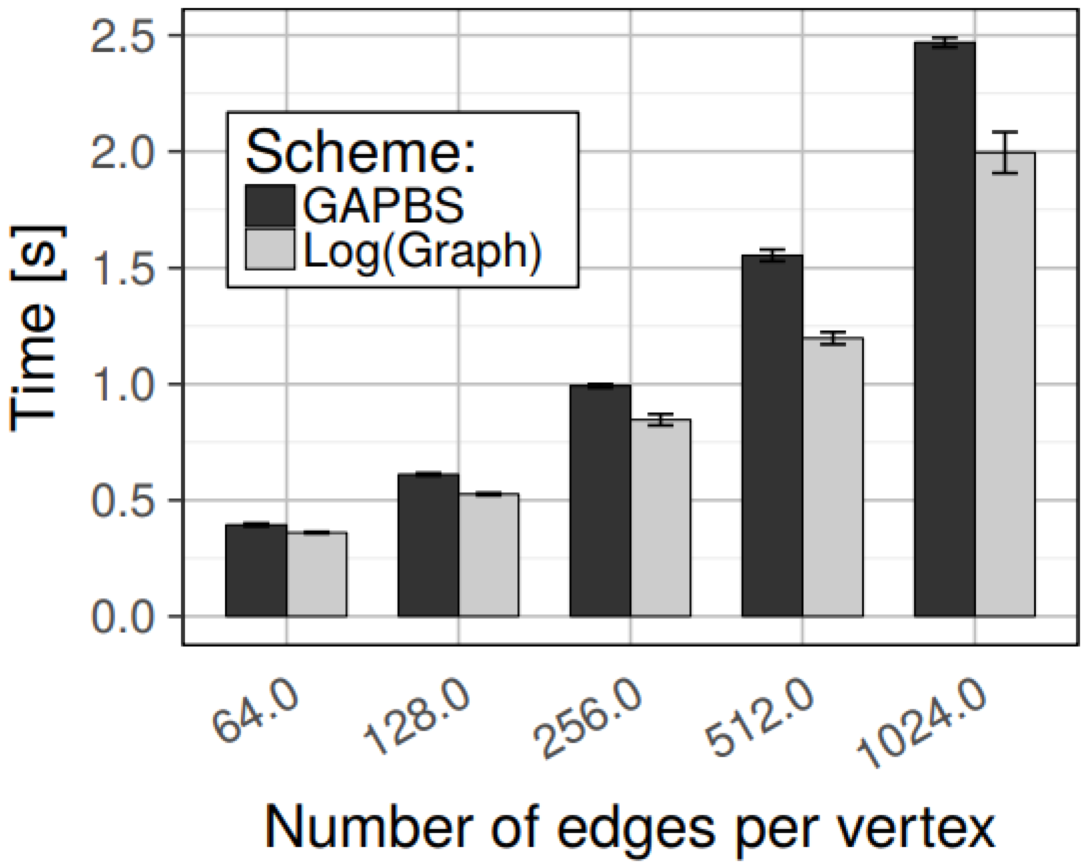

Log(Graph) **accelerates** GAPBS

Both storage and performance are improved **simultaneously**

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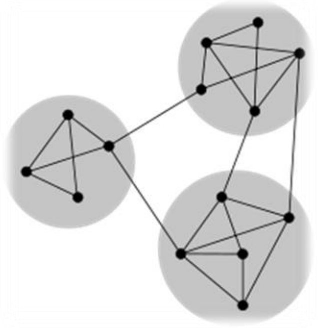

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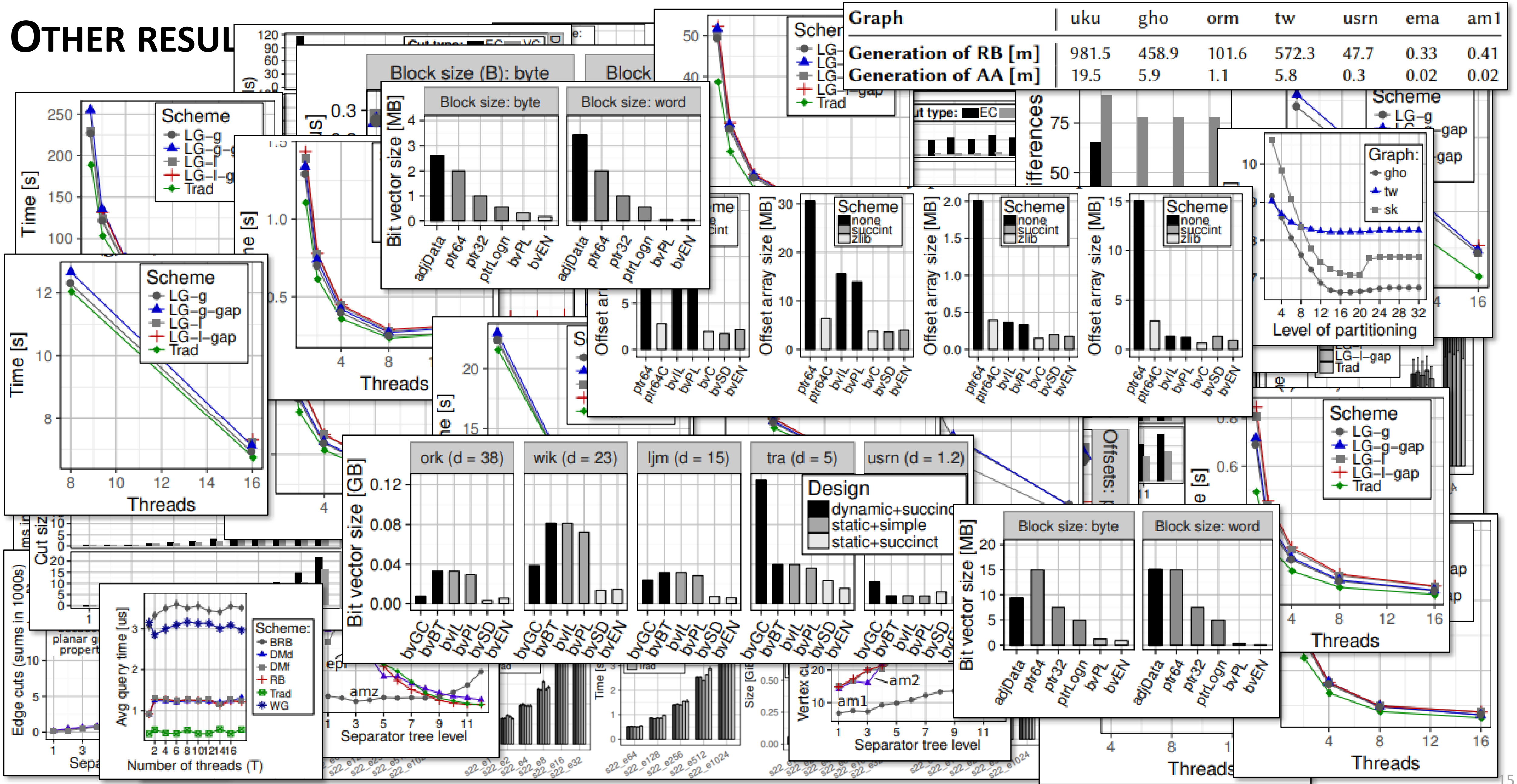
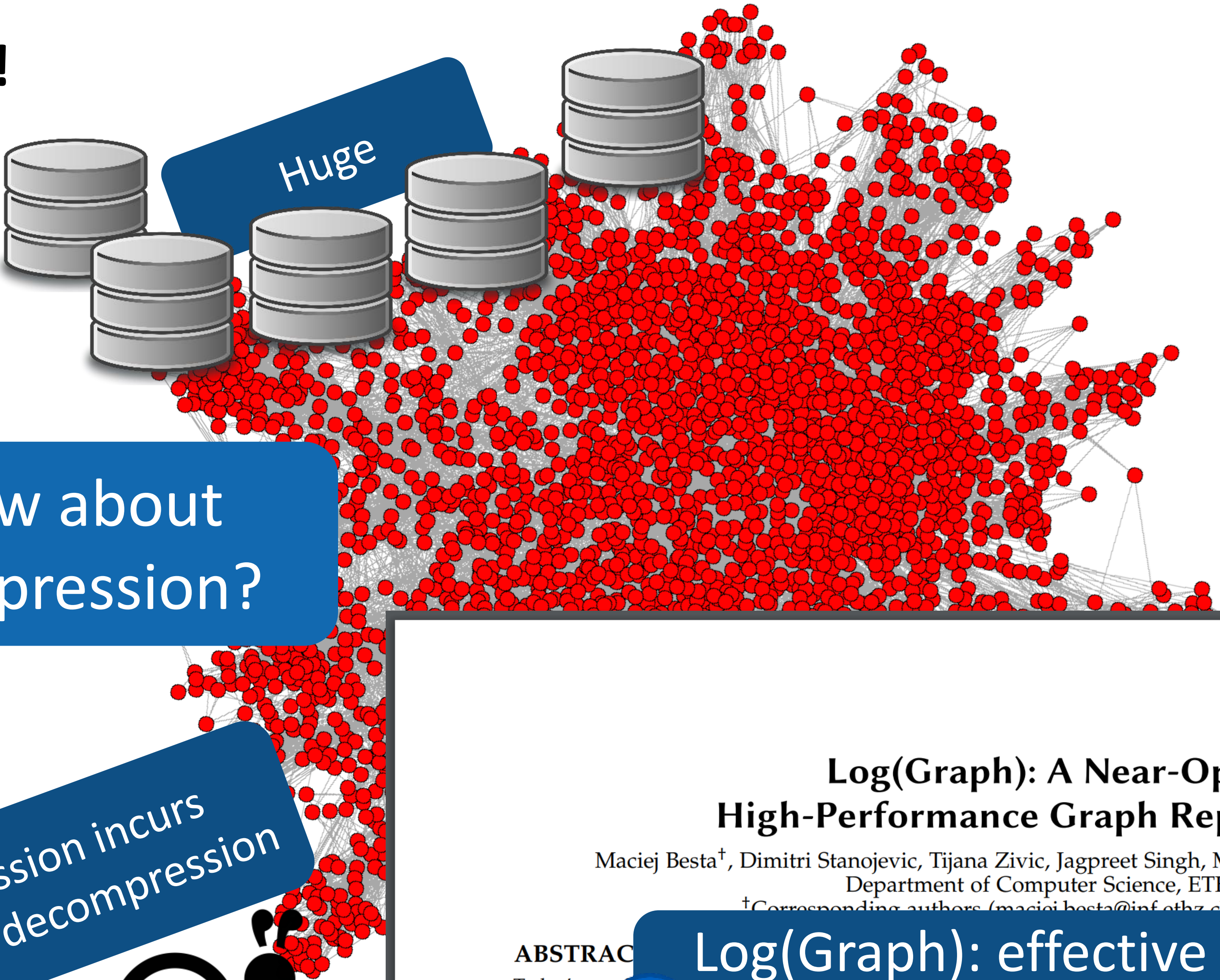
OTHER RESULTS

OTHER RESULTS

Graph	uku	gho	orm	tw	usrn	ema	am1
Generation of RB [m]	981.5	458.9	101.6	572.3	47.7	0.33	0.41
Generation of AA [m]	19.5	5.9	1.1	5.8	0.3	0.02	0.02



Problems!



Huge

How about compression?

Compression incurs expensive decompression

Log(Graph): A Near-Optimal High-Performance Graph Representation

Maciej Besta†, Dimitri Stanojevic, Tijana Zivic, Jagpreet Singh, Maurice Hoerold, Torsten Hoefler†
Department of Computer Science, ETH Zurich
†Corresponding authors (maciej.besta@inf.ethz.ch, htor@inf.ethz.ch)

ABSTRAC
Today's gra
social netw
edges. Yet
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significant number of bits while graph compression schemes
size of such graphs is becoming increasingly important.

Log(Graph): effective compression with low-overhead decompression!

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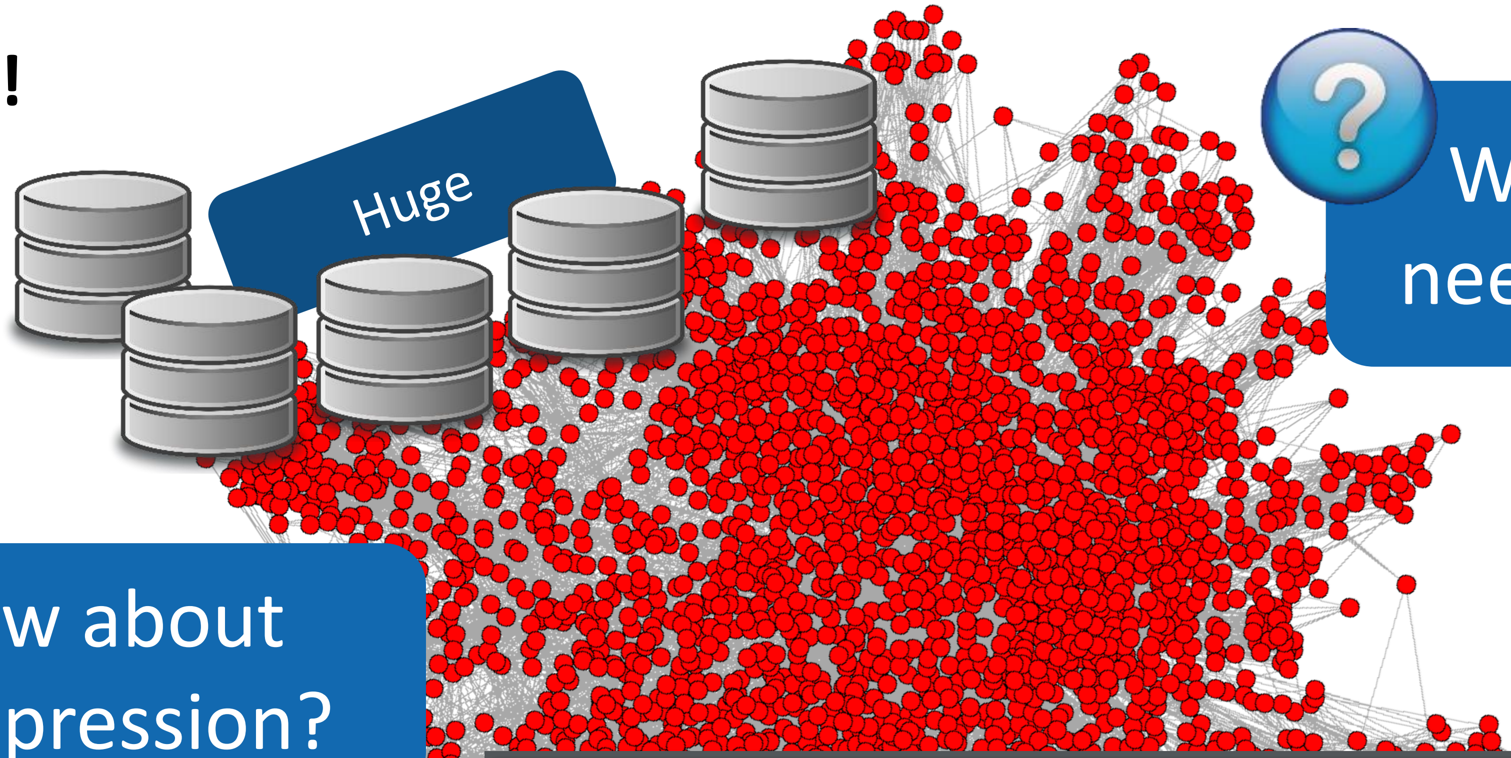

What if we don't need full precision?

How about compression?

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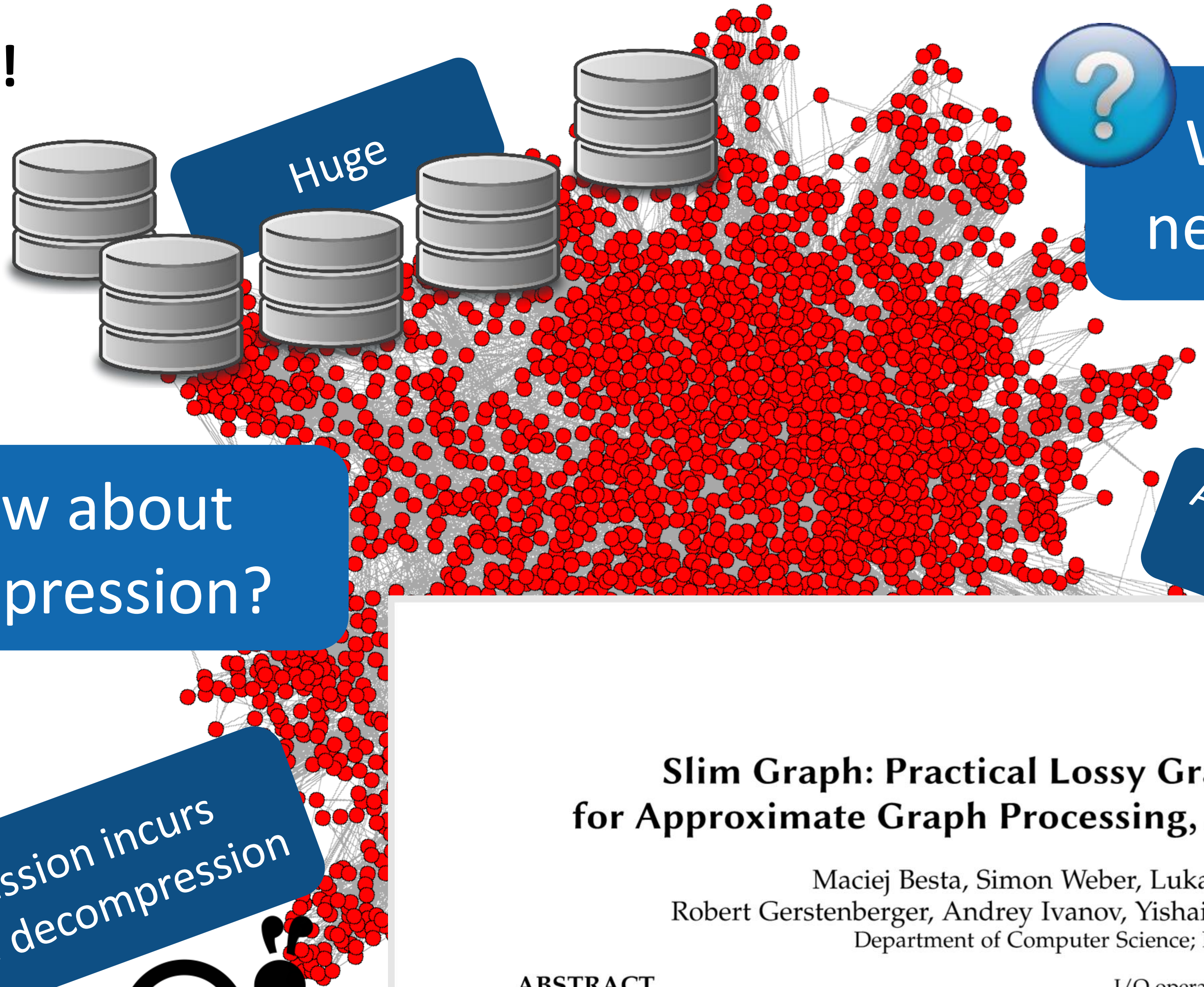

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Accepted @ ACM/IEEE SC19, Best paper finalist

Slim Graph: Practical Lossy Graph Compression for Approximate Graph Processing, Storage, and Analytics

Maciej Besta, Simon Weber, Lukas Gianinazzi,
Robert Gerstenberger, Andrey Ivanov, Yishai Oltchik, Torsten Hoefler
Department of Computer Science; ETH Zurich

ABSTRACT

We propose Slim Graph: the first programming model and framework for practical lossy graph compression that facilitates high-performance approximate graph processing, storage, and analytics. Slim Graph enables the developer to

I/O operations, the amount of data communicated over the network, and by storing a larger fraction of data in caches.

There exist many *lossless* schemes for graph compression, including WebGraph [21], k^2-trees [25], and others [17]. They provide various degrees of storage reductions. Unfortunately,

JPEG compression level: 0%

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JPEG compression level: 50%

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JPEG compression level: 90%

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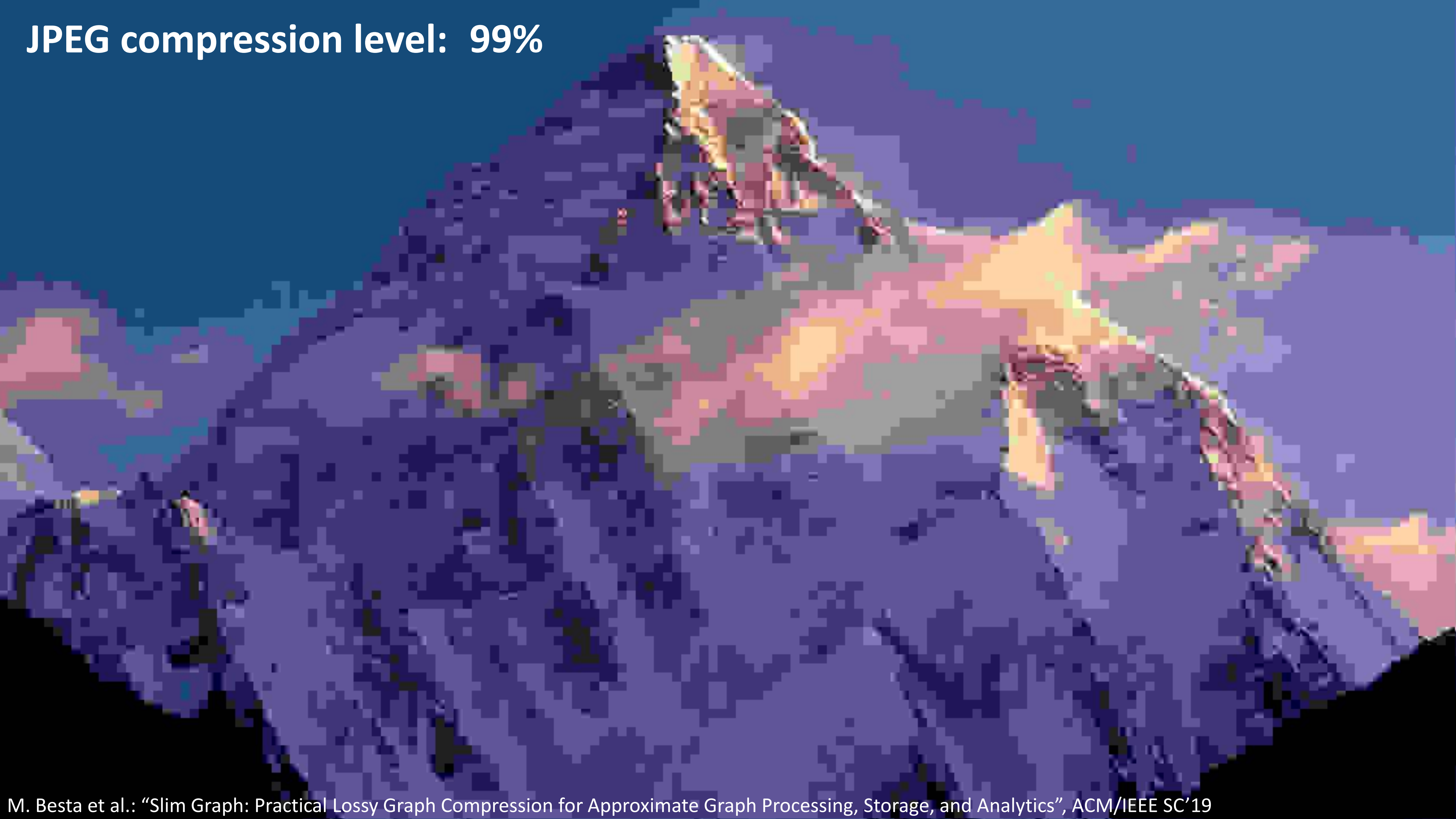
JPEG compression level: 99%



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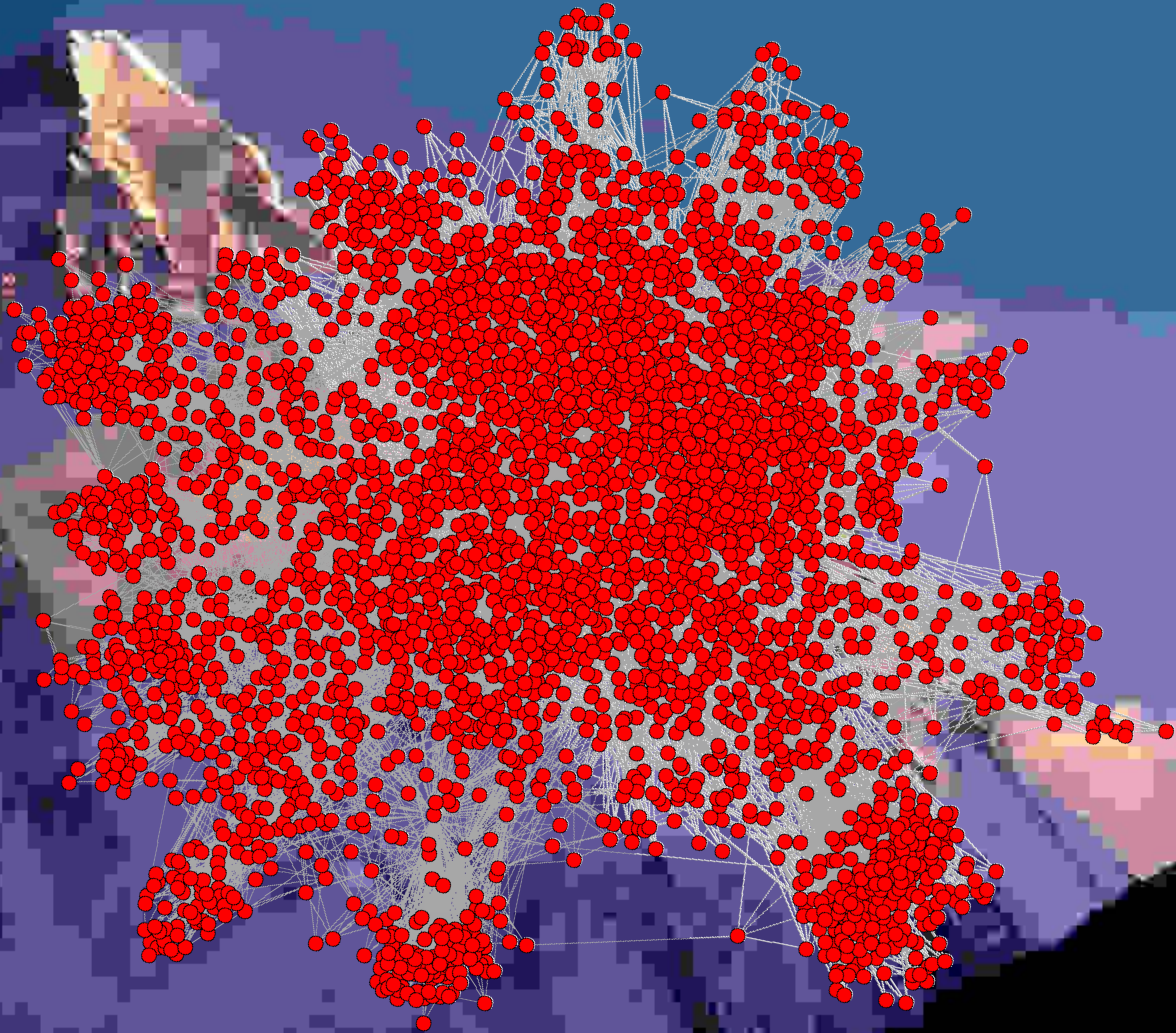
JPEG compression level: 99%

Can we apply a similar reasoning to graphs? How?



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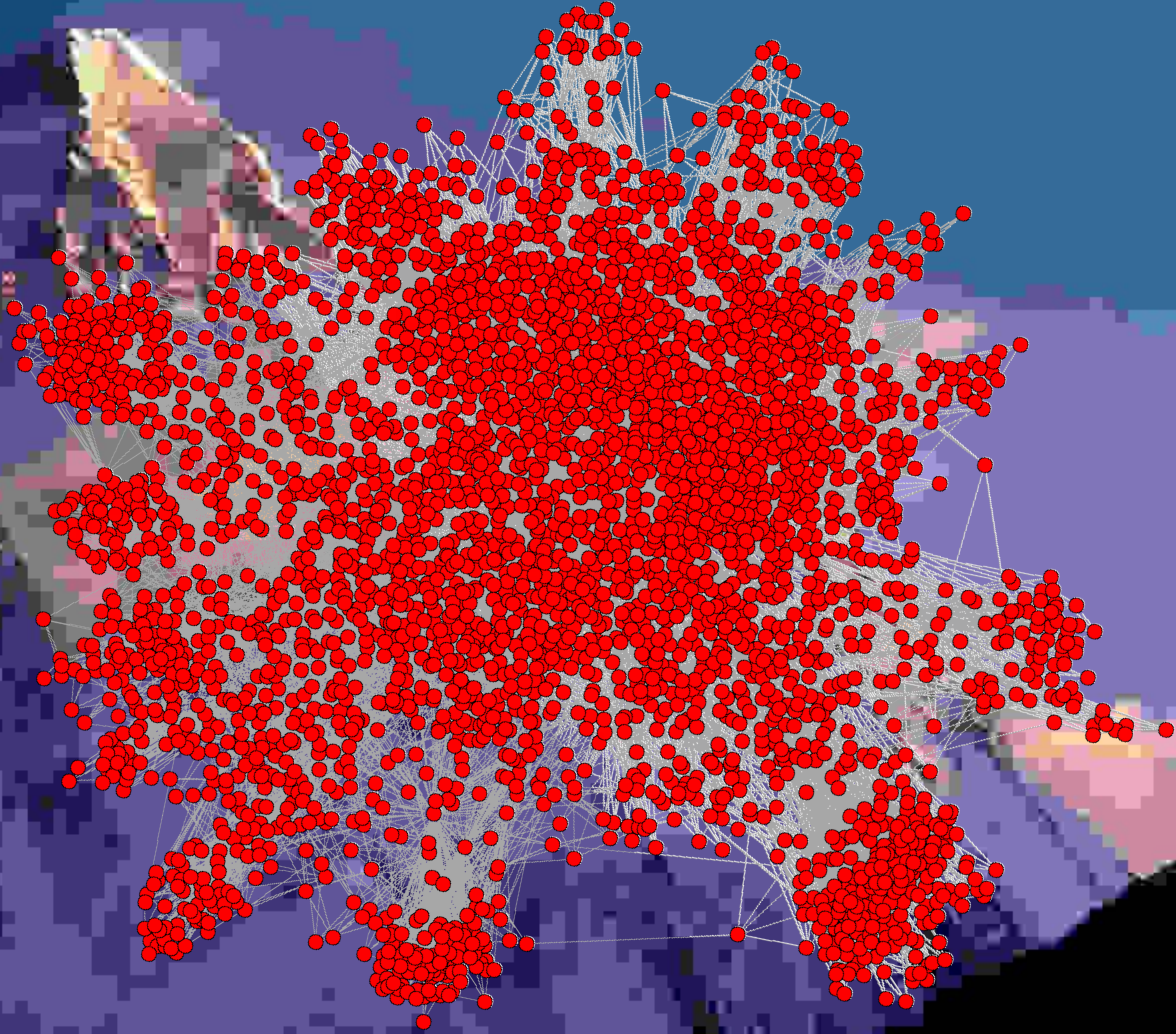

What should we pay attention to? (there is no “visual similarity” measure in this case…)

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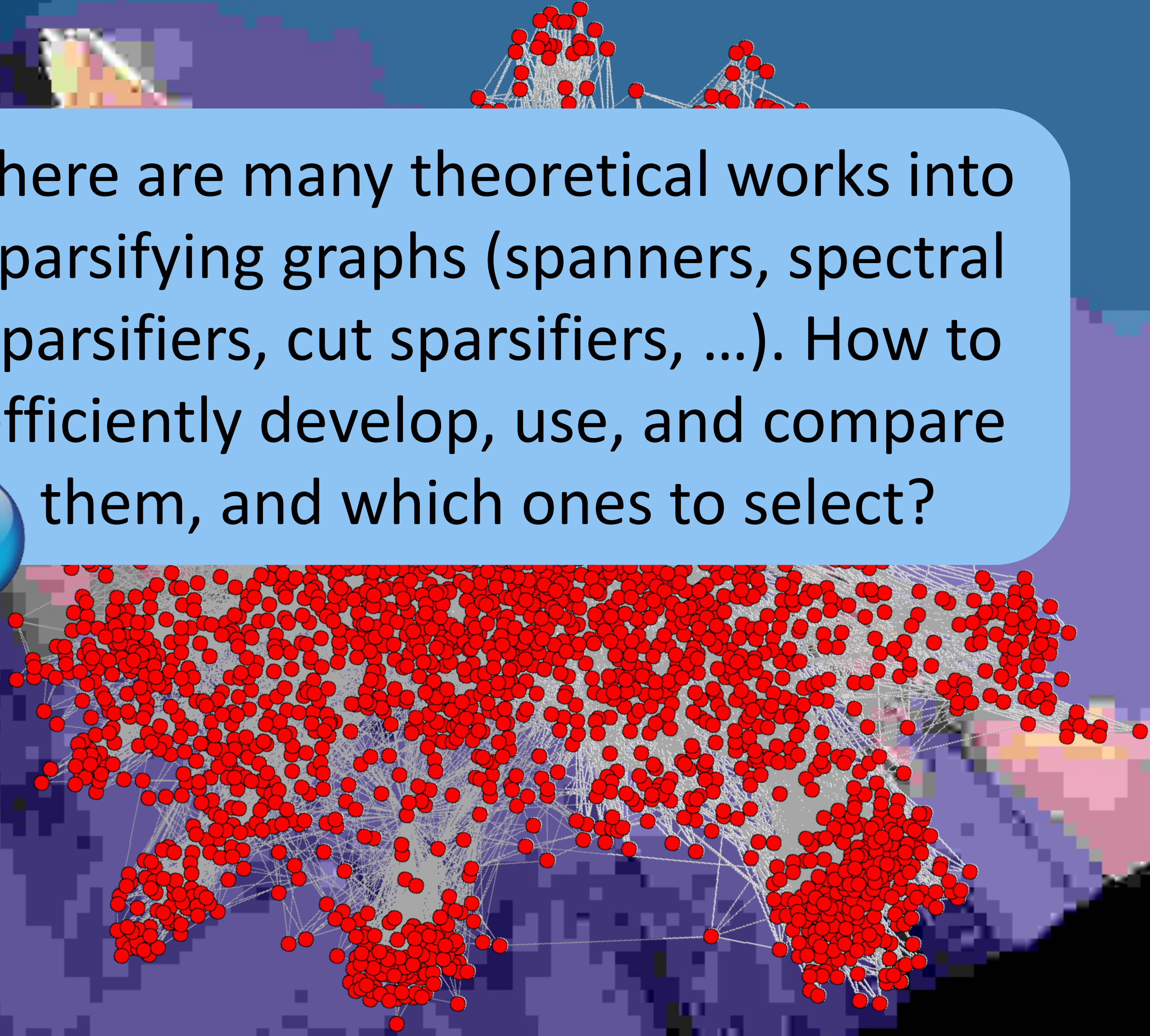
There are many theoretical works into sparsifying graphs (spanners, spectral sparsifiers, cut sparsifiers, …). How to efficiently develop, use, and compare them, and which ones to select?



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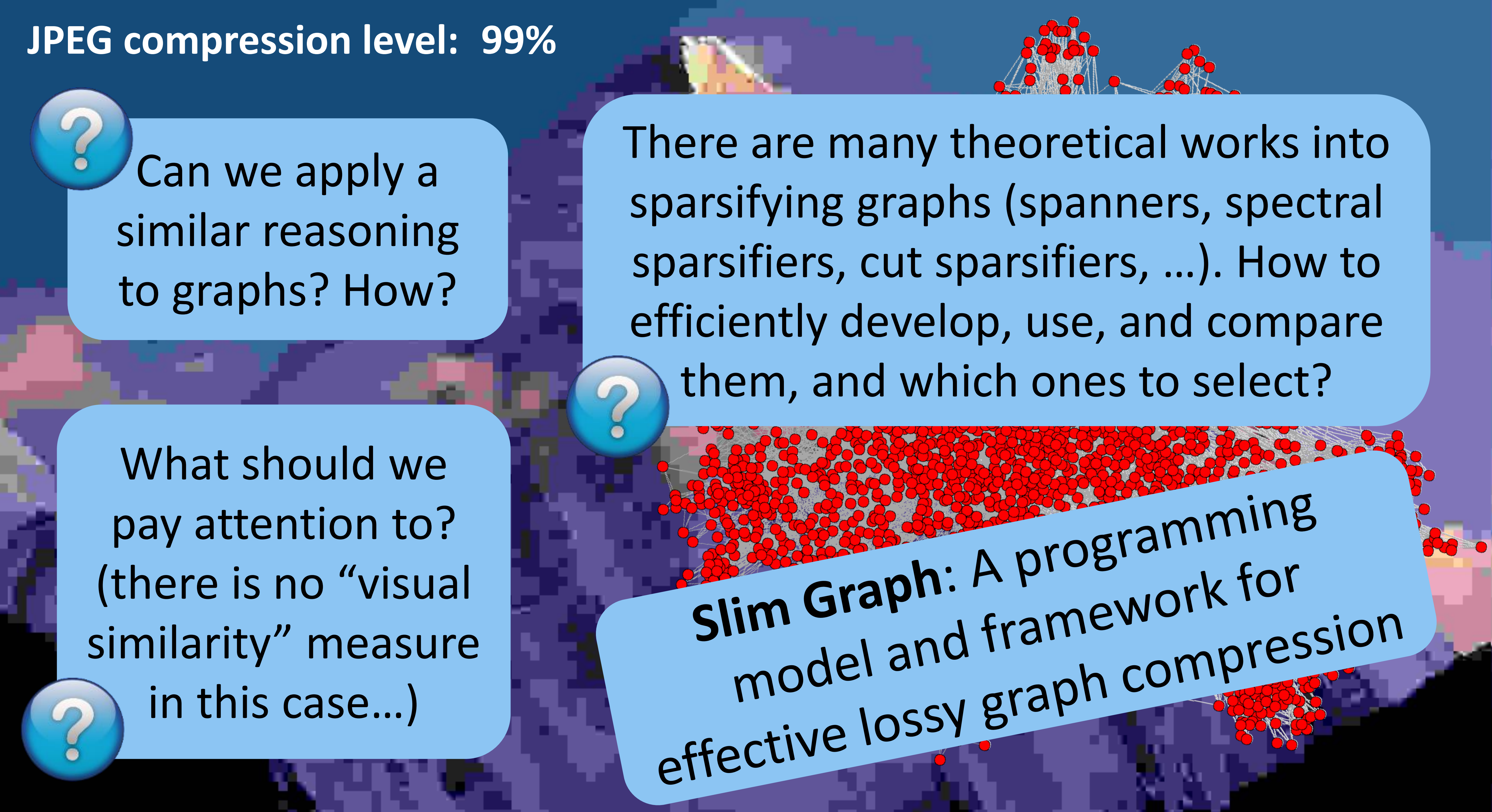

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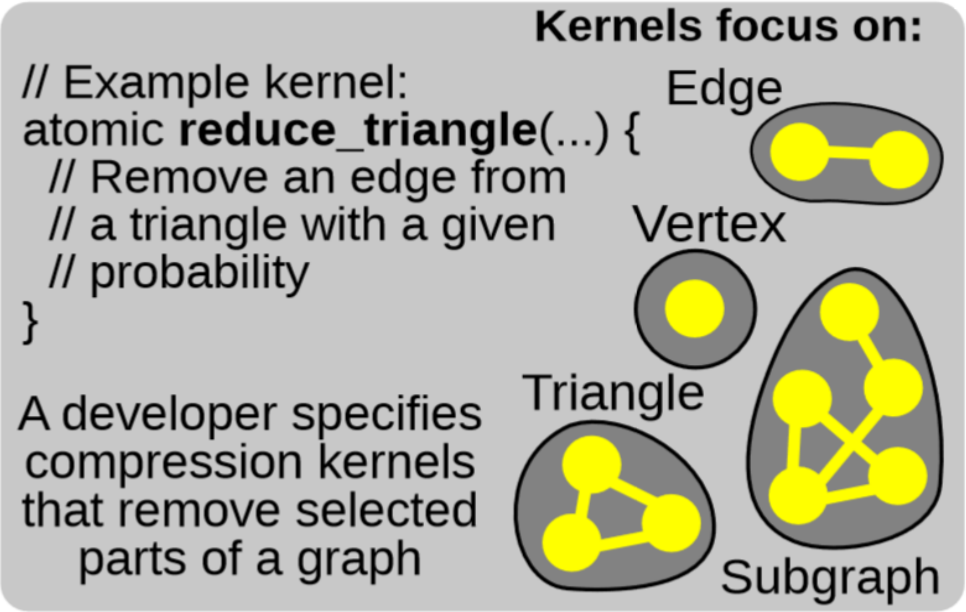
Slim Graph Overview

Programming Model



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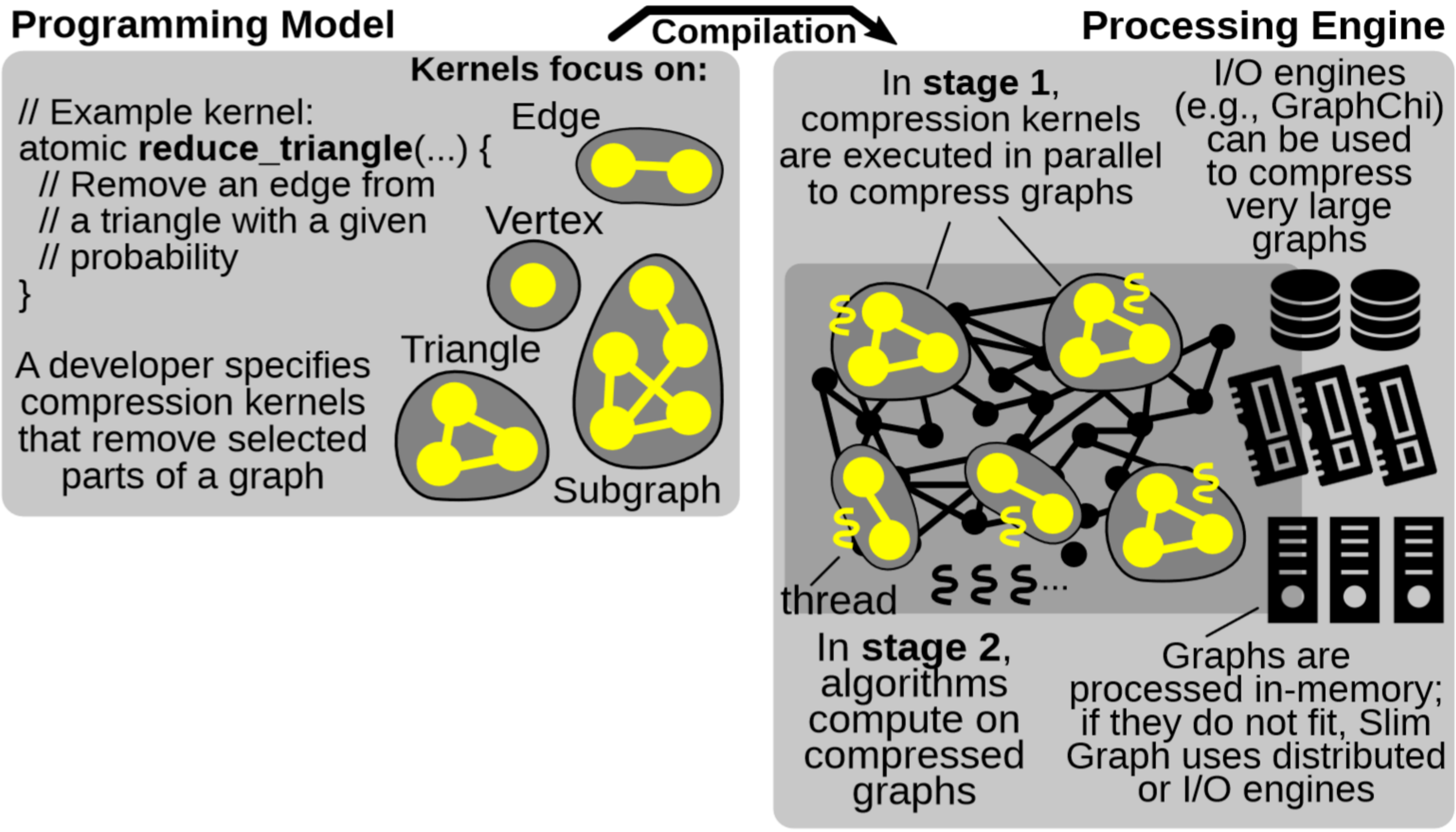

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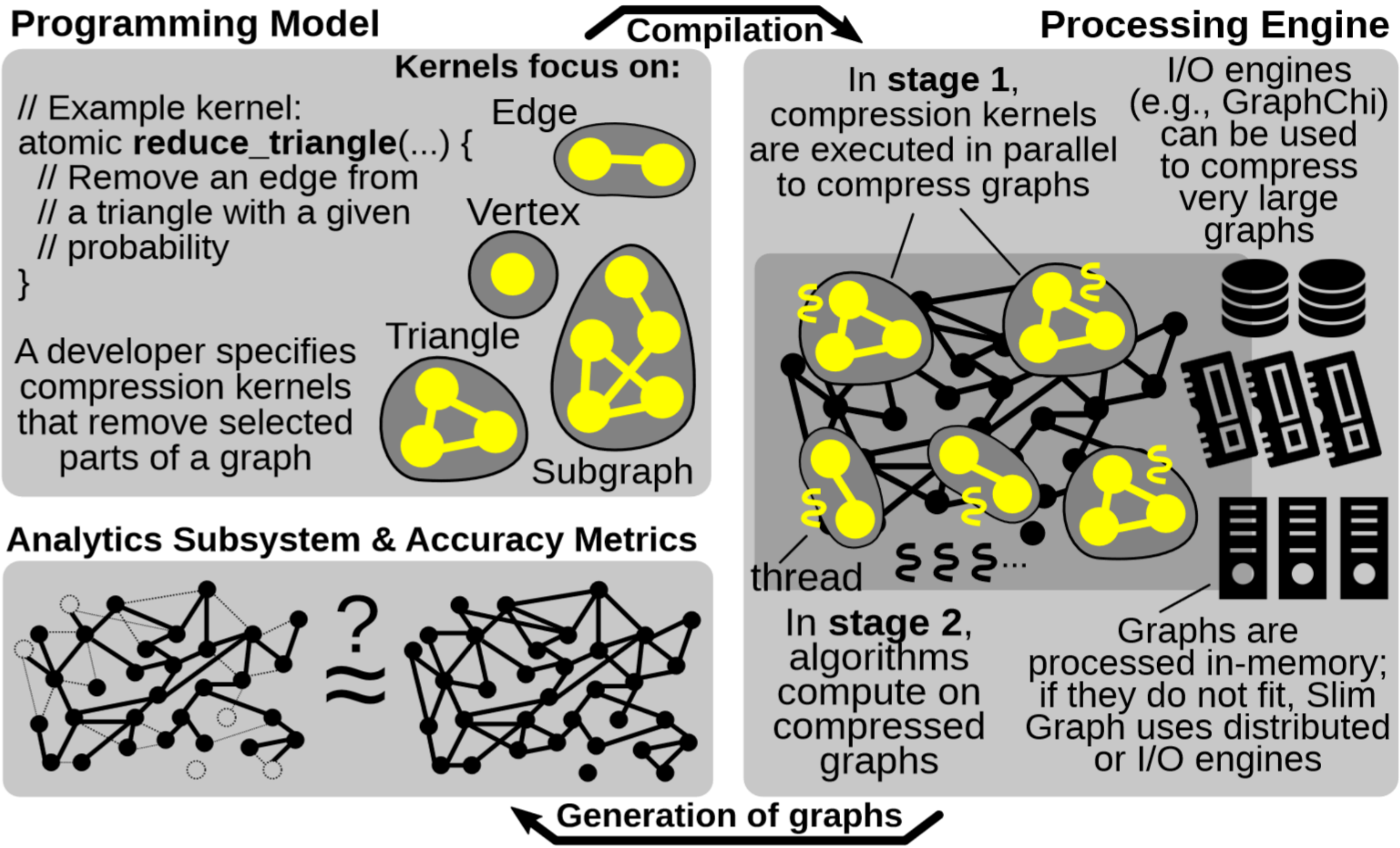

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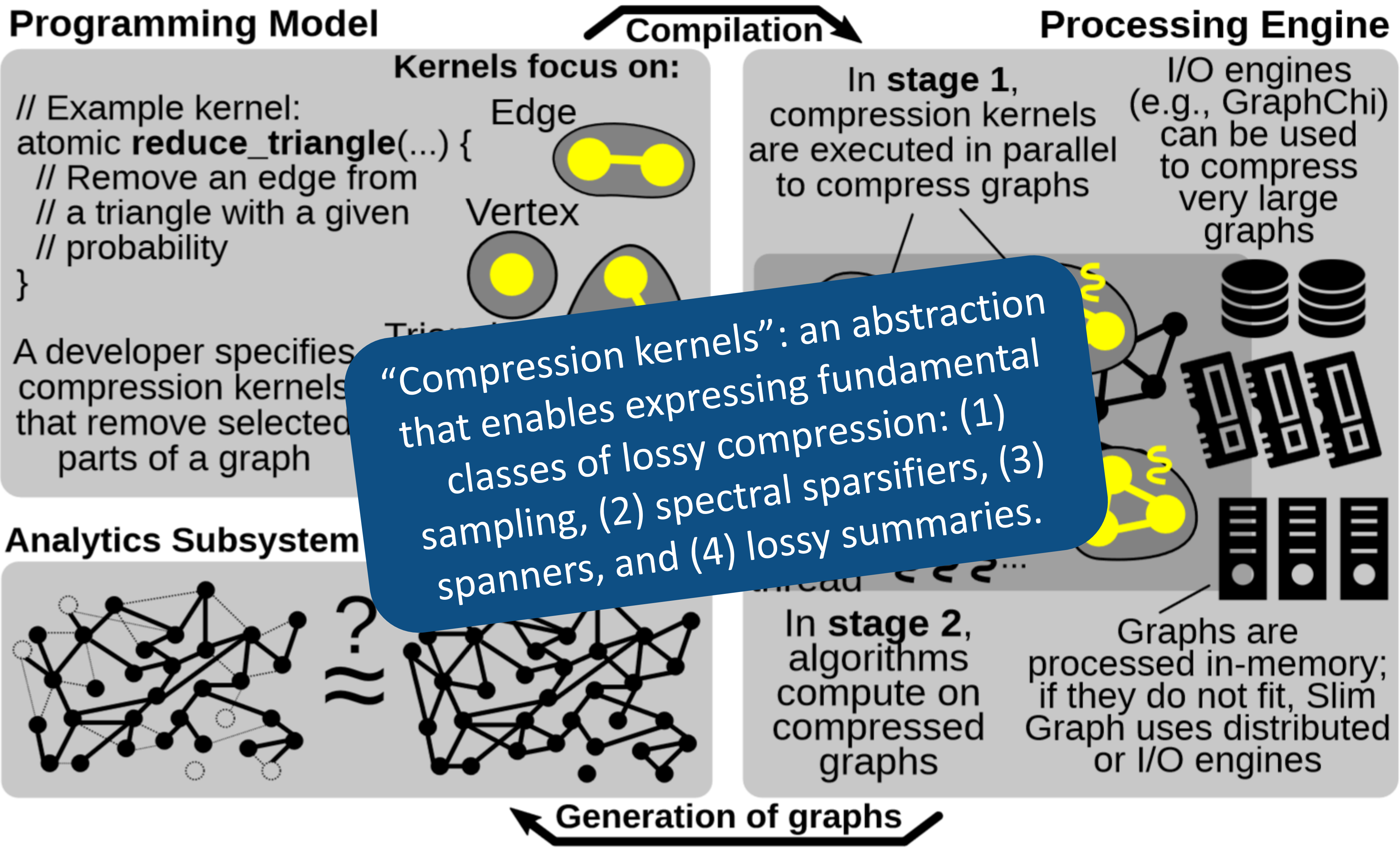

"Compression kernels": an abstraction that enables expressing fundamental classes of lossy compression: (1) sampling, (2) spectral sparsifiers, (3) spanners, and (4) lossy summaries.

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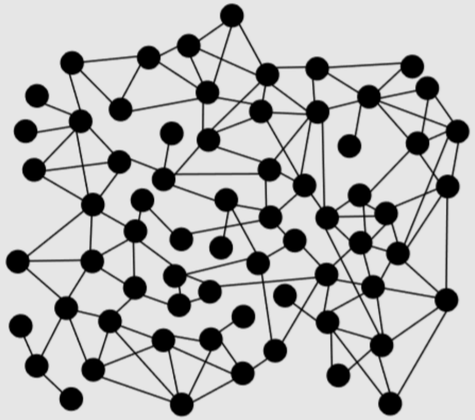
Example Compression Kernel: Triangle Reduction

Input graph:

n = 67
m = 132



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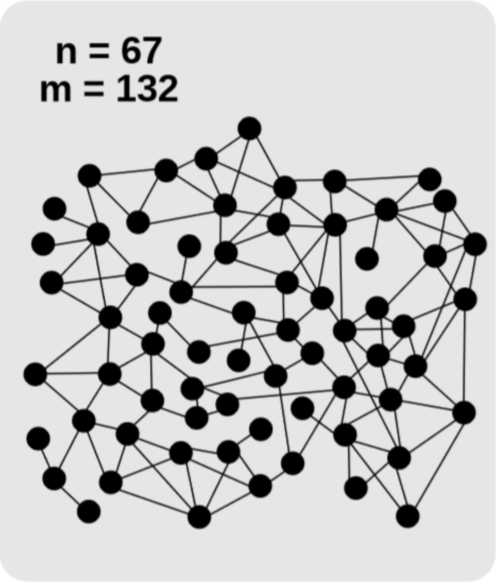

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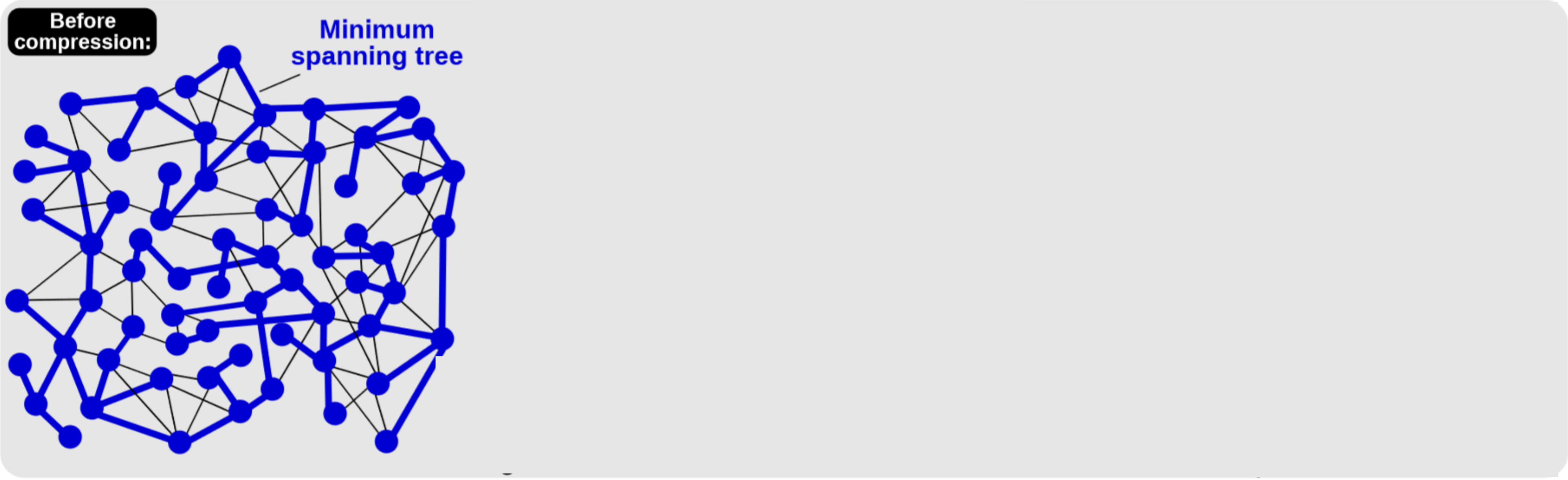

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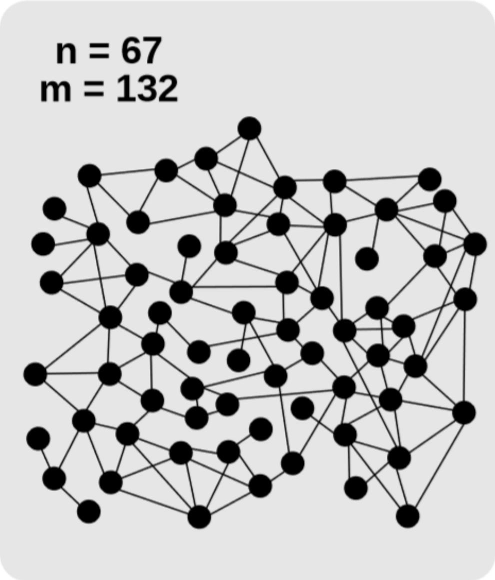

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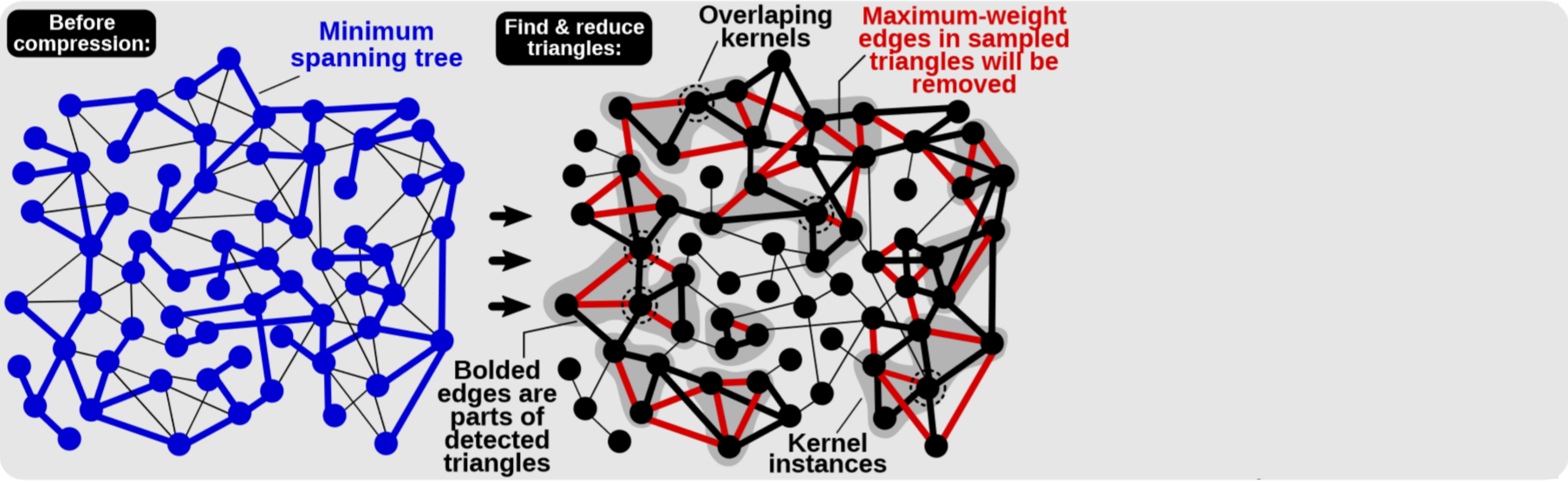

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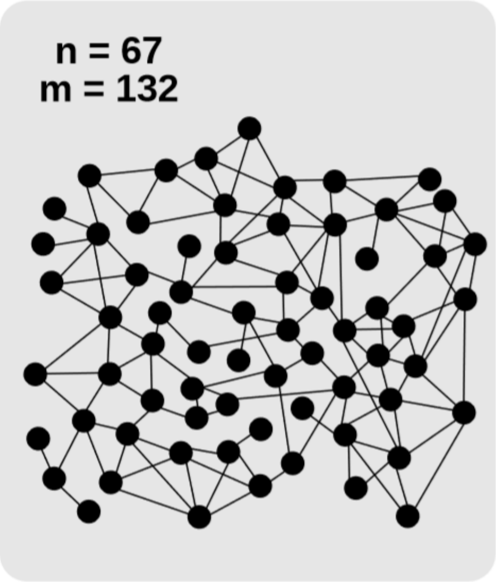

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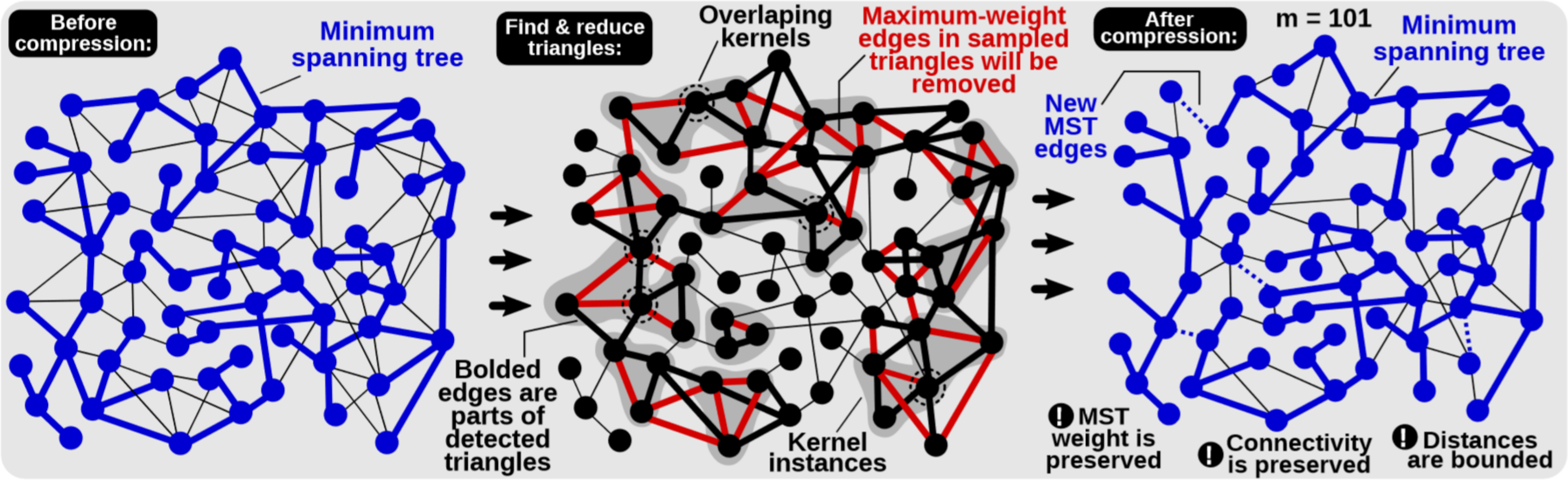

Other types of compression kernels enable expressing and implementing other lossy graph compression schemes

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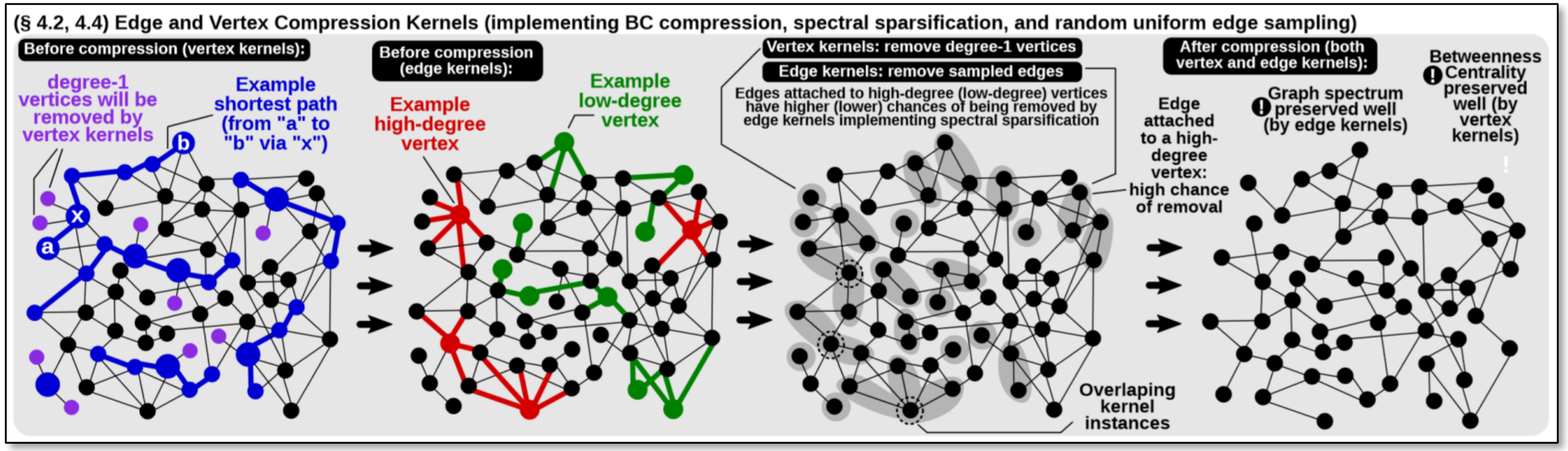

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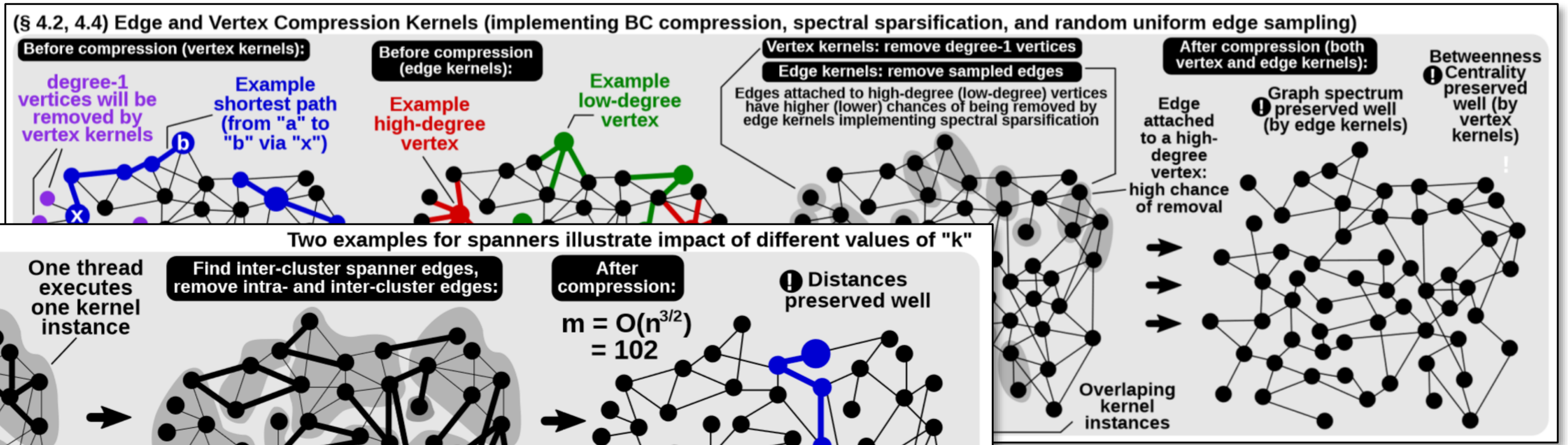

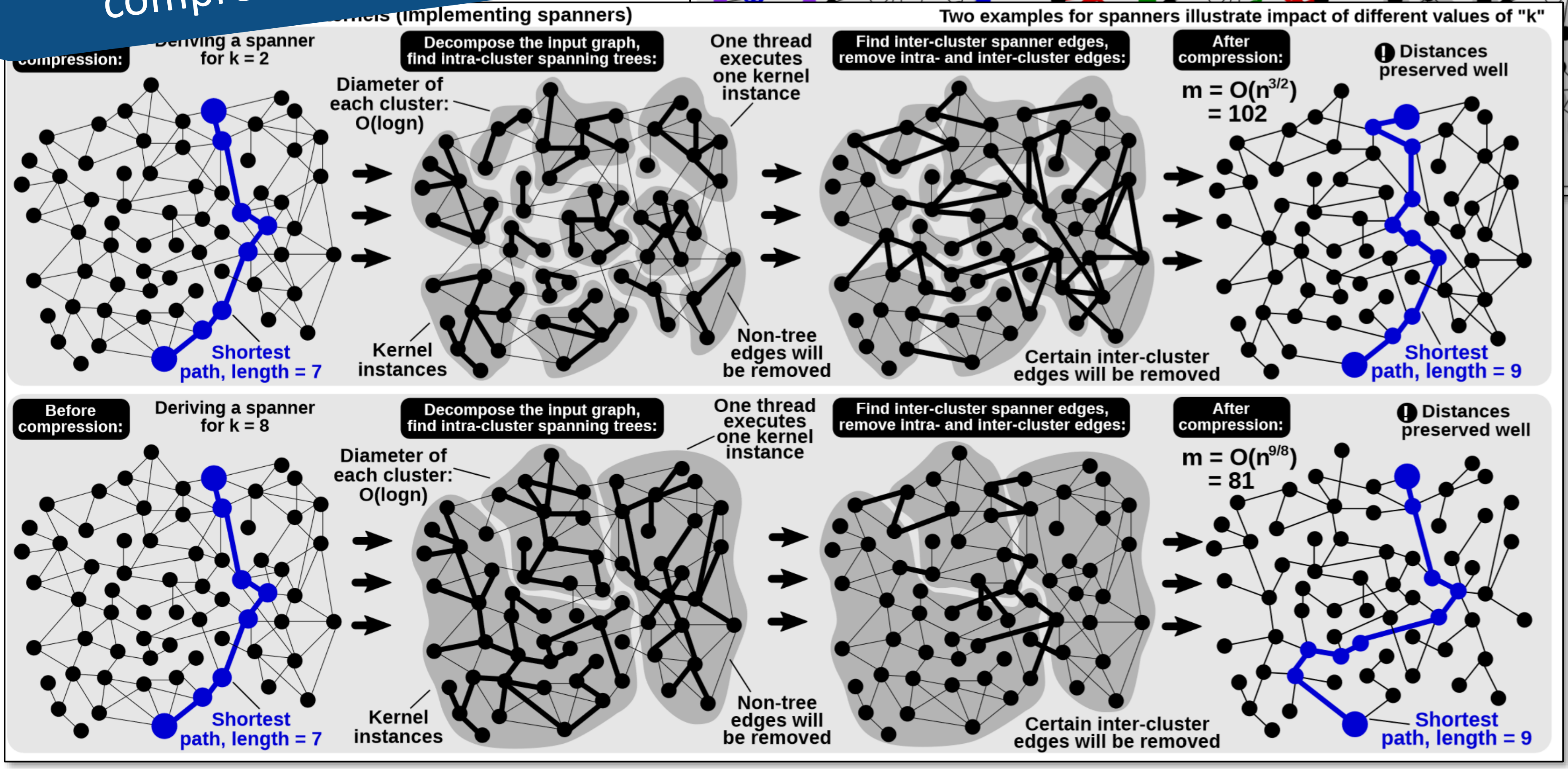

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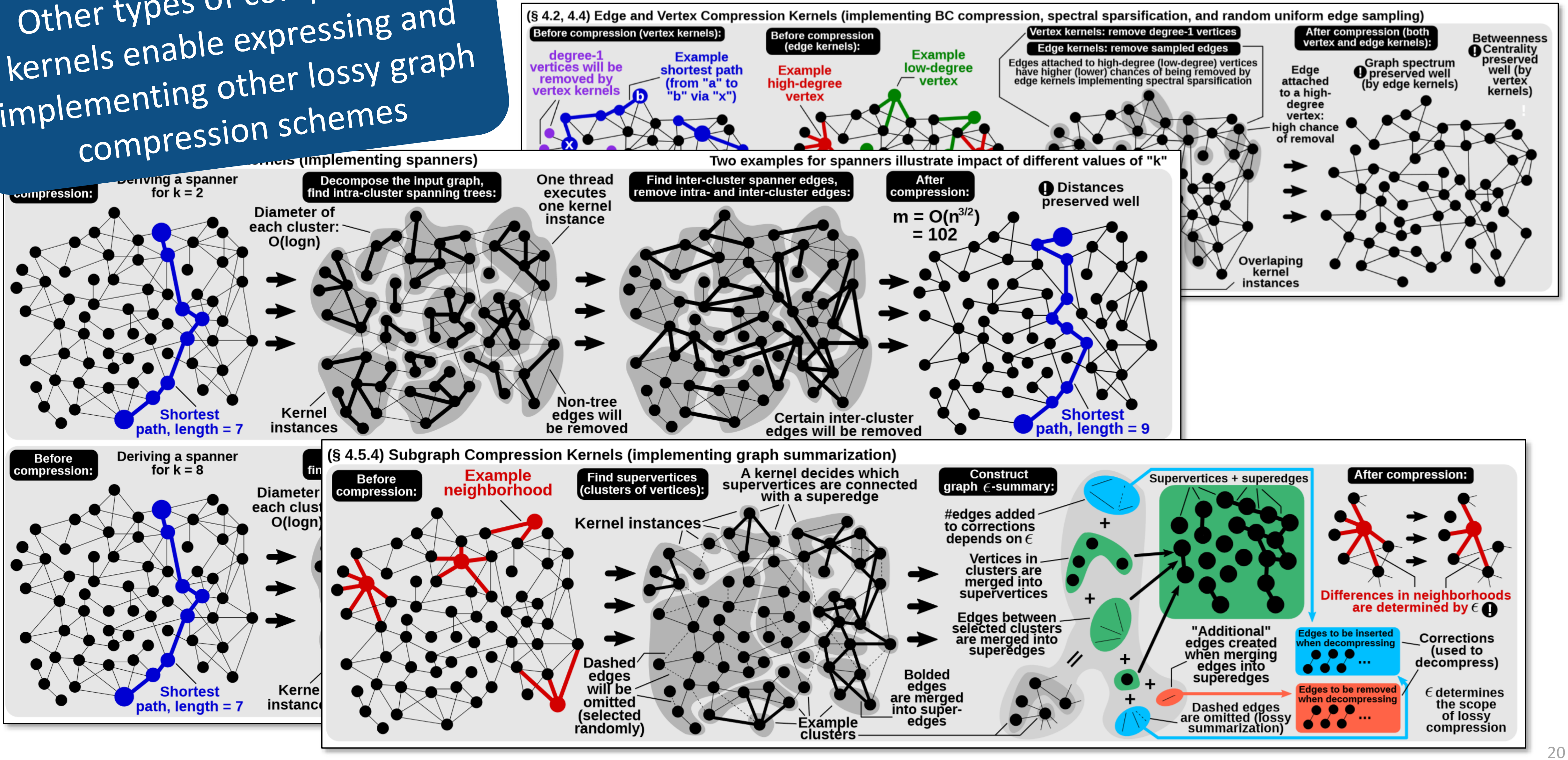

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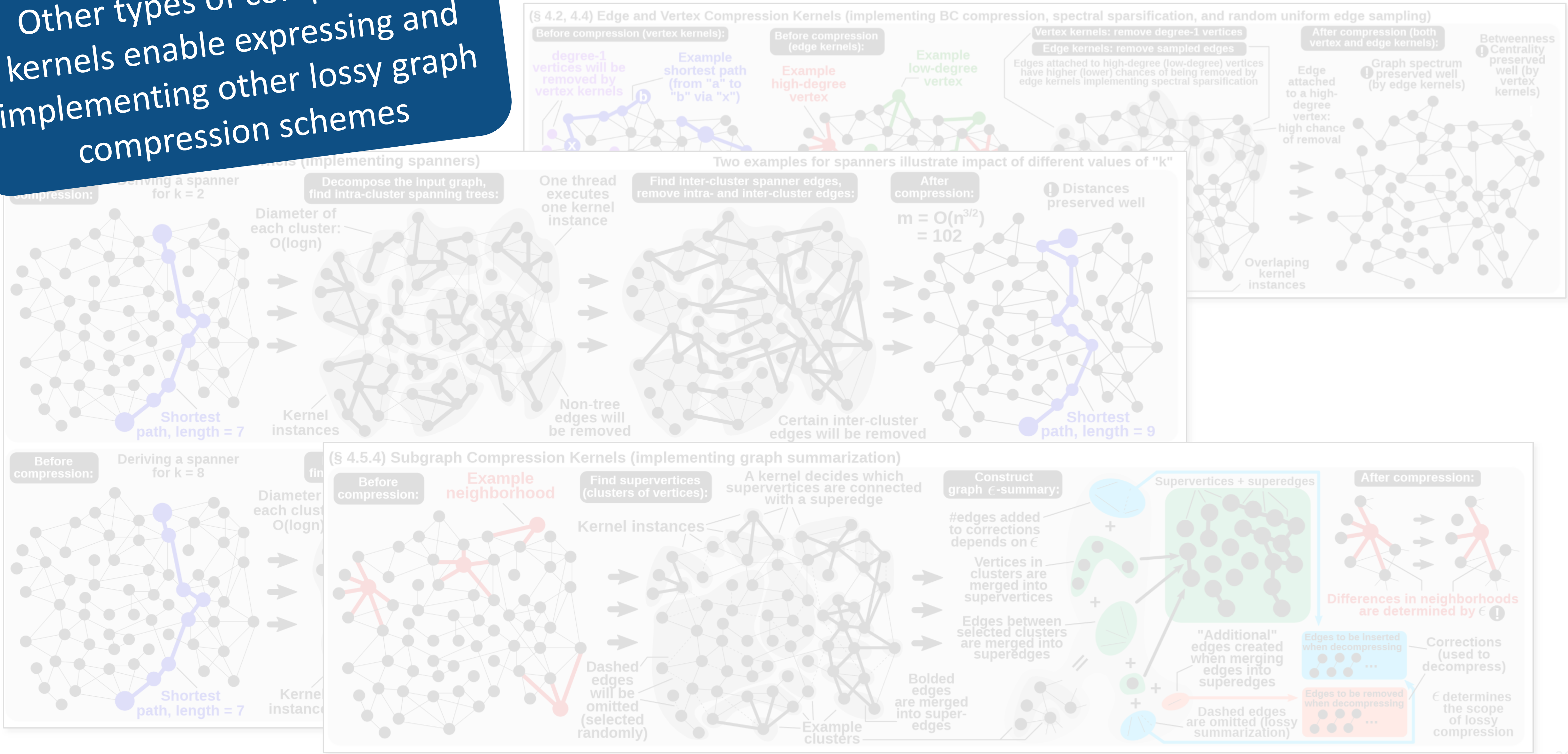

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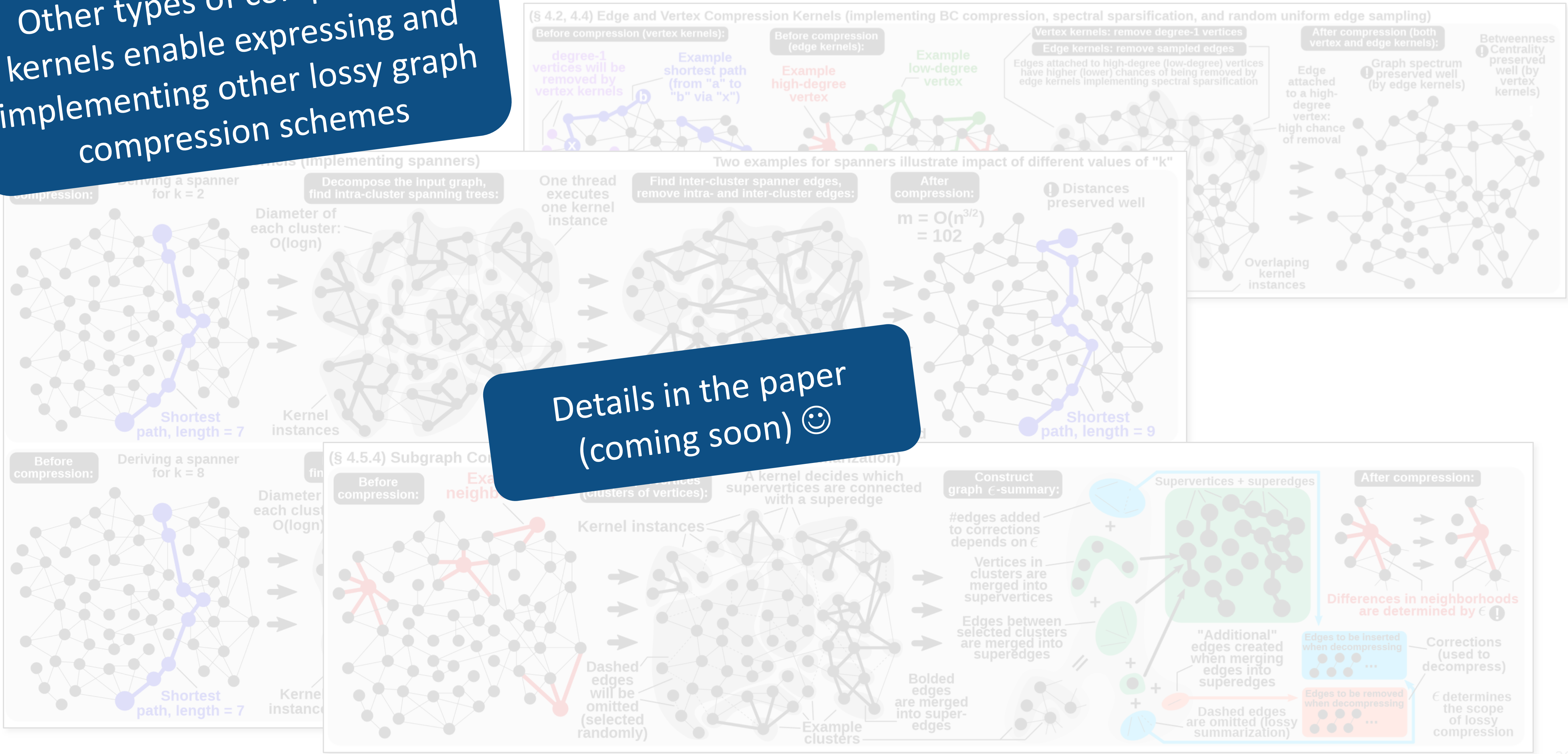

Details in the paper (coming soon) ☺

Storage Reductions vs. Speedups vs. Accuracy Loss [vs. Compression Overhead]

Various real-world graphs are used

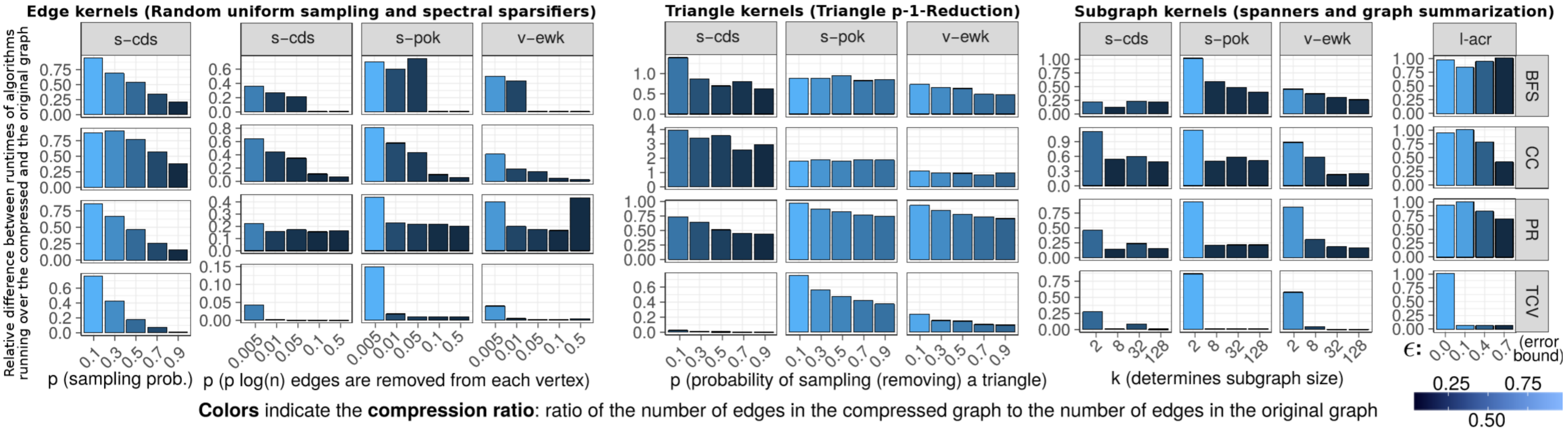

Figure 4: Analysis of **storage and performance** tradeoffs of various lossy compression schemes implemented in Slim Graph (**when varying compression parameters**).

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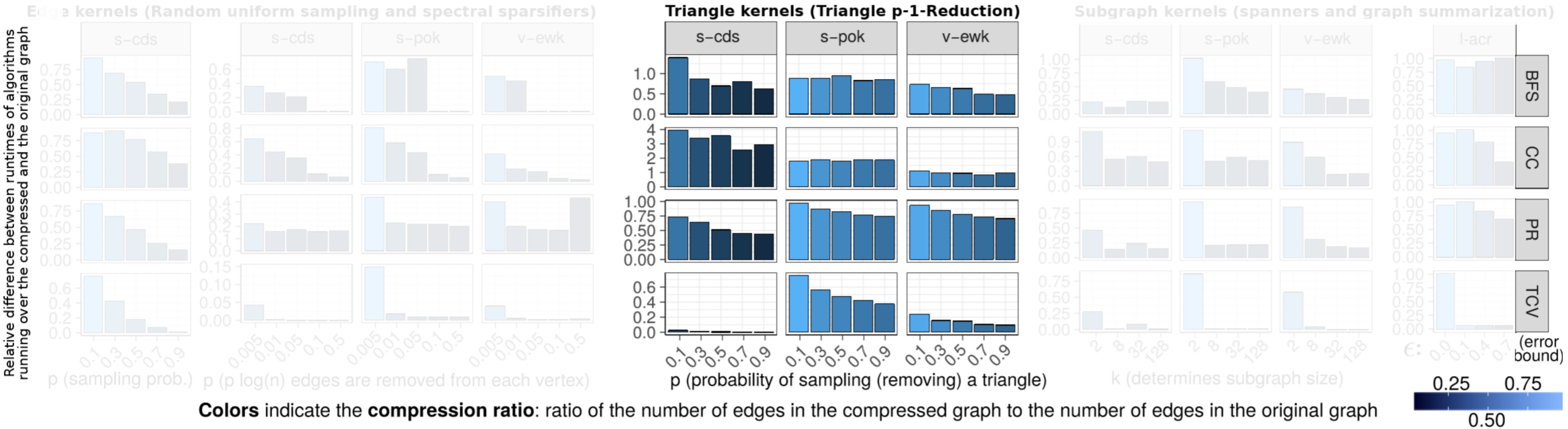

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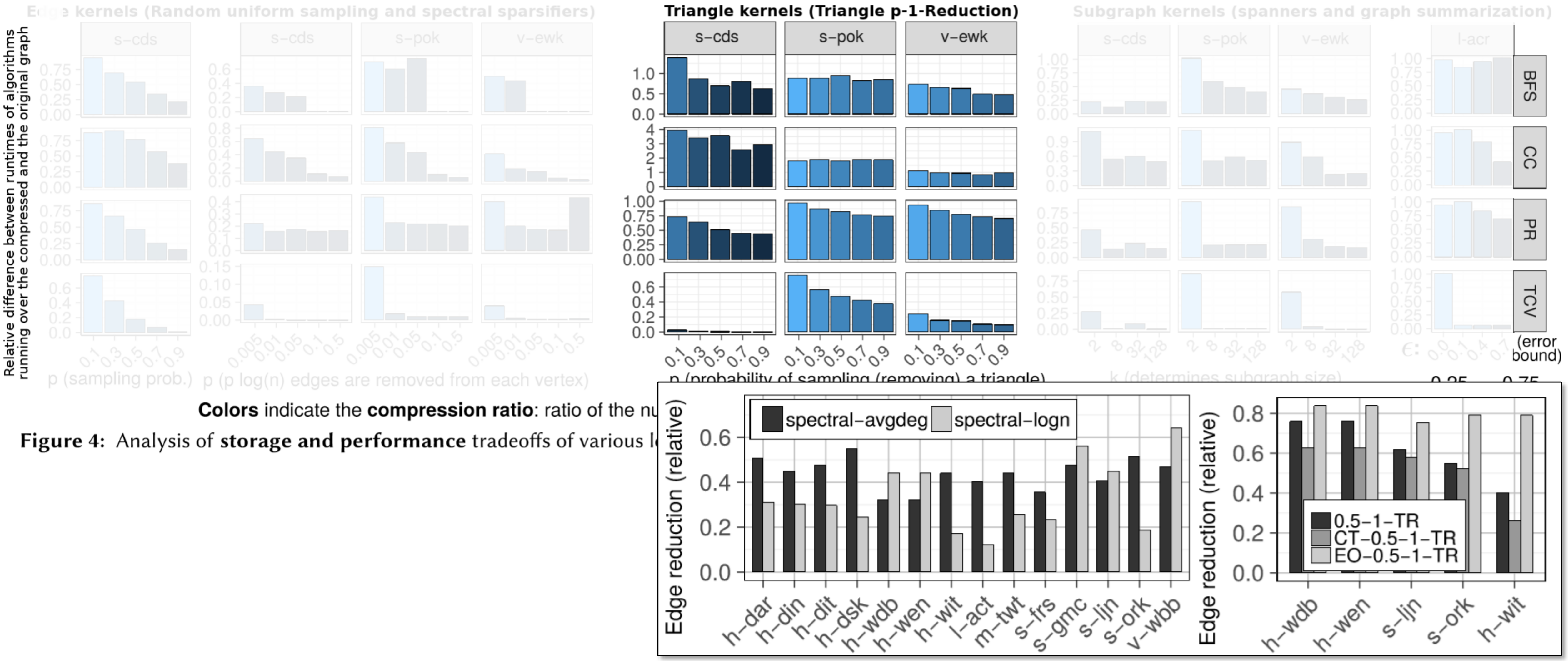

Colors indicate the **compression ratio**: ratio of the nu

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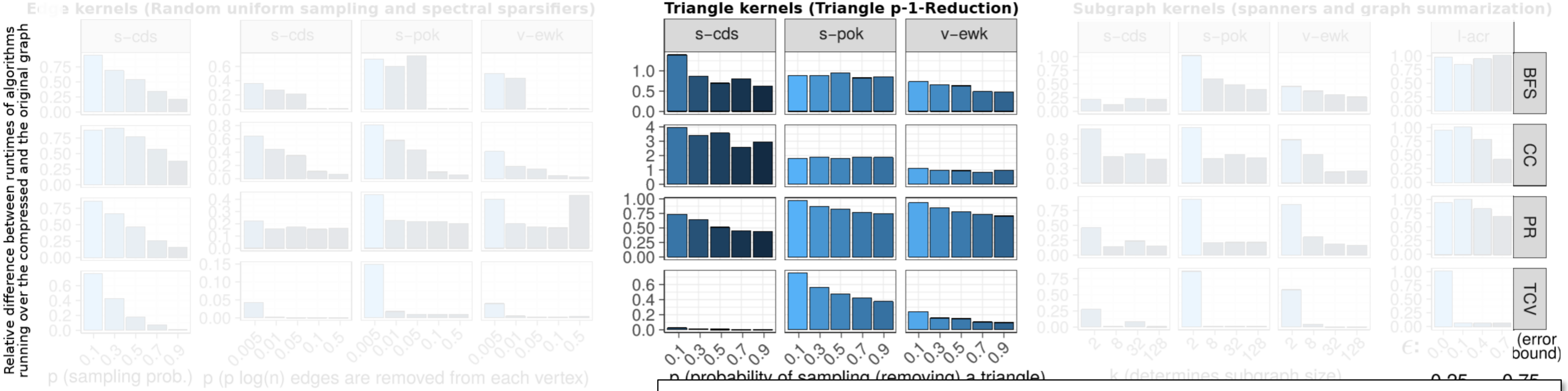

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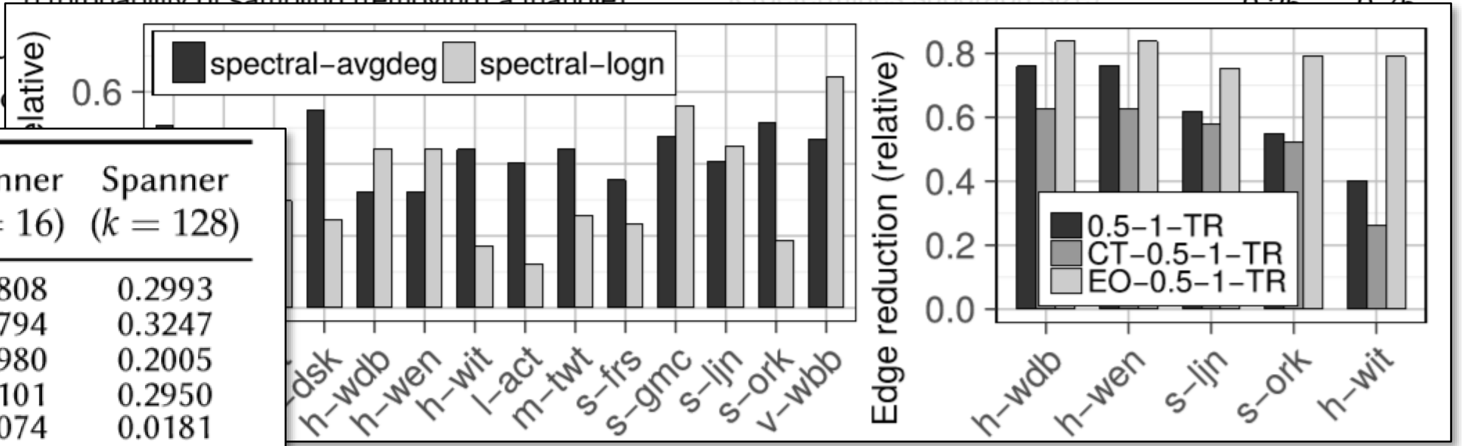
Figure 4: Analysis of **storage and performance** tradeoffs of various l

Graph	EO 0.8-1-TR	EO 1.0-1-TR	Uniform ($p = 0.2$)	Uniform ($p = 0.5$)	Spanner ($k = 2$)	Spanner ($k = 16$)	Spanner ($k = 128$)
s-you	0.0121	0.0167	0.1932	0.6019	0.0054	0.2808	0.2993
h-hud	0.0187	0.0271	0.0477	0.1633	0.0340	0.2794	0.3247
il-dbl	0.0459	0.0674	0.0749	0.2929	0.0080	0.1980	0.2005
v-skt	0.0410	0.0643	0.0674	0.2695	0.0311	0.1101	0.2950
v-usa	0.0089	0.0100	0.1392	0.5945	0.0000	0.0074	0.0181



Storage Reductions vs. Speedups vs. Accuracy Loss [vs. Compression Overhead]

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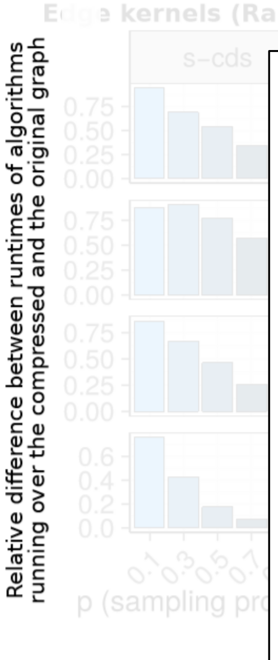
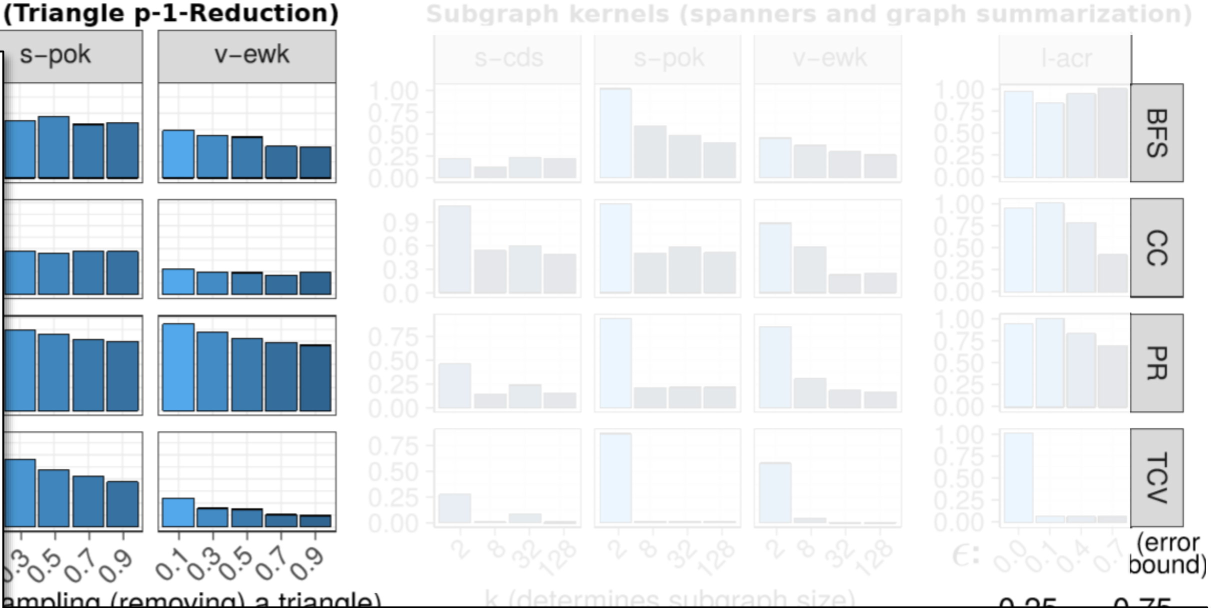



Figure 4: Analy

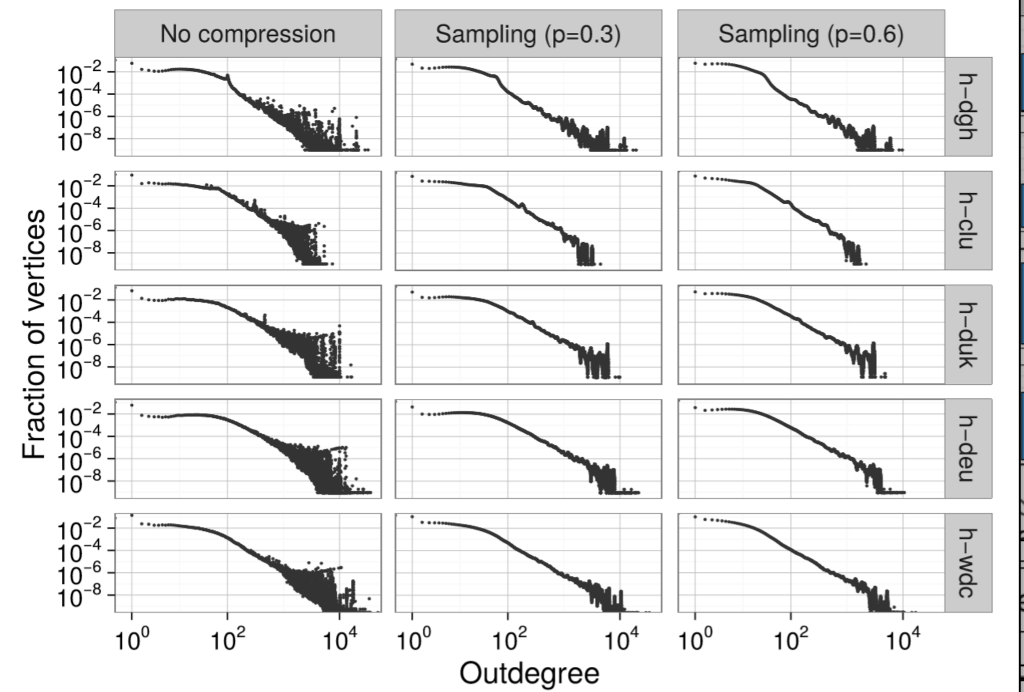
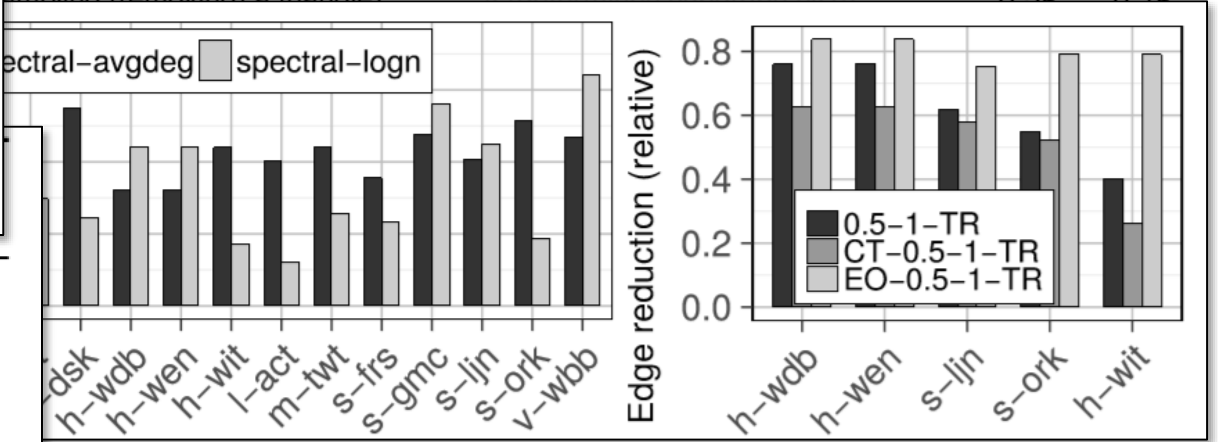


Figure 7: (Accuracy) Impact of random uniform sampling on the degree distribution of large graphs (the largest, h-wdc, has ≈128B edges).

Graph	EO 0.8-1-						
s-you	0.0121	0.0167	0.1932	0.6019	0.0054	0.2808	0.2993
h-hud	0.0187	0.0271	0.0477	0.1633	0.0340	0.2794	0.3247
il-dbl	0.0459	0.0674	0.0749	0.2929	0.0080	0.1980	0.2005
v-skt	0.0410	0.0643	0.0674	0.2695	0.0311	0.1101	0.2950
v-usa	0.0089	0.0100	0.1392	0.5945	0.0000	0.0074	0.0181



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	$\lvert V\rvert$	$\lvert E\rvert$	Shortest s-t path length	Average path length	Diameter	Average degree	Maximum degree	#Triangles	#Connected components	Chromatic number	Max. indep. set size	Max. cardinal. matching size
Original graph	n	m	$\mathcal{P}$	$\overline{\mathcal{P}}$	D	$\bar{d}$	d	T	$\mathcal{C}$	C_R	$\widehat{I}_S$	$\widehat{M}_C$
Lossy ϵ-summary	n	$m \pm 2\epsilon m$	$1,\dots,\infty$	$1,\dots,\infty$	$1,\dots,\infty$	$\bar{d} \pm \epsilon\bar{d}$	$d \pm \epsilon d$	$T \pm 2\epsilon m$	$\mathcal{C} \pm 2\epsilon m$	$C_R \pm 2\epsilon m$	$\widehat{I}_S \pm 2\epsilon m$	$\widehat{M}_C \pm 2\epsilon m$
Simple p–sampling	n	$(1-p)m$	∞	∞	∞	$(1-p)\bar{d}$	$(1-p)d$	$(1-p^3)T$	$\leq \mathcal{C} + pm$	$\geq C_R - pm$	$\leq \widehat{I}_S + pm$	$\geq \widehat{M}_C - pm$
Spectral ϵ-sparsifier	n	$\tilde{O}(n/\epsilon^2)$	$\leq n$	$\leq n$	$\leq n$	$\tilde{O}(1/\epsilon^2)$	$\geq d/2(1+\epsilon)$	$\tilde{O}(n^{3/2}/\epsilon^3)$	$\overset{w.h.p.}{=} \mathcal{C}$	$\leq d/2(1+\epsilon)$	$\geq 2(1+\epsilon)n/d$	≥ 0
$O(k)$–spanner	n	$O(n^{1+1/k})$	$O(k\mathcal{P})$	$O(k\overline{\mathcal{P}})$	$O(kD)$	$O(n^{1/k})$	$\leq d$	$O(n^{1+2/k})$	$\mathcal{C}$	$O(n^{1/k}\log n)$	$\Omega\left(\frac{n^{1-1/k}}{\log n}\right)$	≥ 0
EO p–1–Triangle Red.	n	$\leq m - \frac{pT}{3d}$	$\overset{w.h.p.}{\leq} \mathcal{P} + p\mathcal{P}$	$\leq \overline{\mathcal{P}} + \frac{pT}{n(n-1)}$	$\overset{w.h.p.}{\leq} D + pD$	$\leq \bar{d} - \frac{pT}{dn}$	$\geq d/2$	$\leq (1 - \frac{p}{d})T$	$\mathcal{C}$	$\geq C_R - pT$	$\leq \widehat{I}_S + pT$	$\geq \widehat{M}_C/2$
remove k deg-1 vertices	$n-k$	$m-k$	$\mathcal{P}$	$\geq \overline{\mathcal{P}} - \frac{kD}{n}$	$\geq D-2$	$\geq \bar{d} - \frac{k}{n}$	d	T	$\mathcal{C}$	C_R	$\geq \widehat{I}_S - k$	$\geq \widehat{M}_C - k$

Table 3: The impact of various compression schemes on the outcome of selected graph algorithms. Bounds that do not include inequalities hold deterministically. If not otherwise stated, the other bounds hold in expectation. Bounds annotated with w.h.p. hold w.h.p. (if the involved quantities are large enough). Note that since the listed compression schemes (except the scheme where we remove the degree 1 vertices) return a subgraph of the original graph, m, C_R, $\bar{d}$, d, T, and $\widehat{M}_C$ never increase. Moreover, $\mathcal{P}$, $\overline{\mathcal{P}}$, D, $\mathcal{C}$, and $\widehat{I}_S$ never decrease during compression. ϵ is a parameter that controls how well a spectral sparsifier approximates the original graph spectrum.

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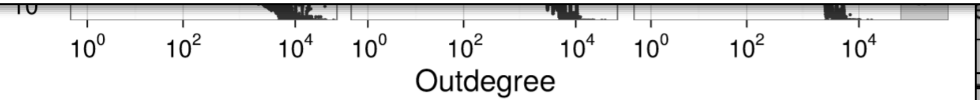

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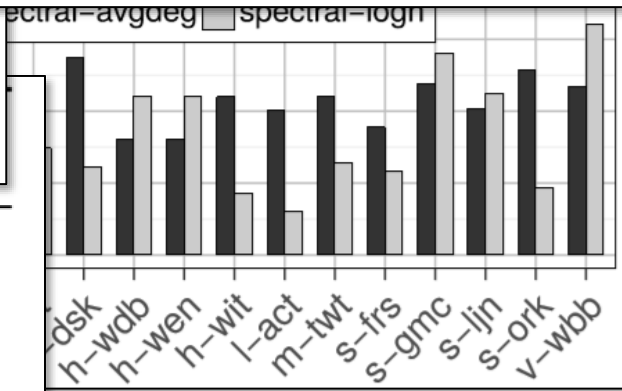

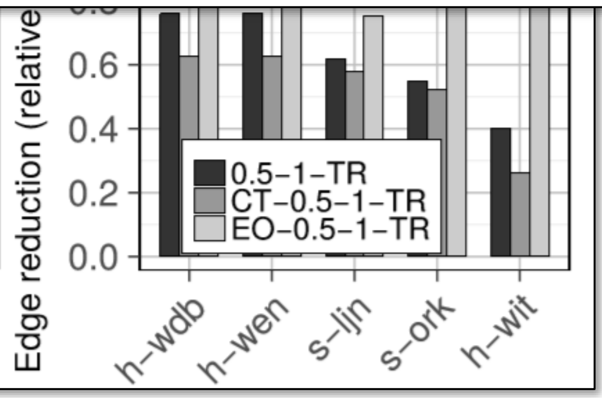

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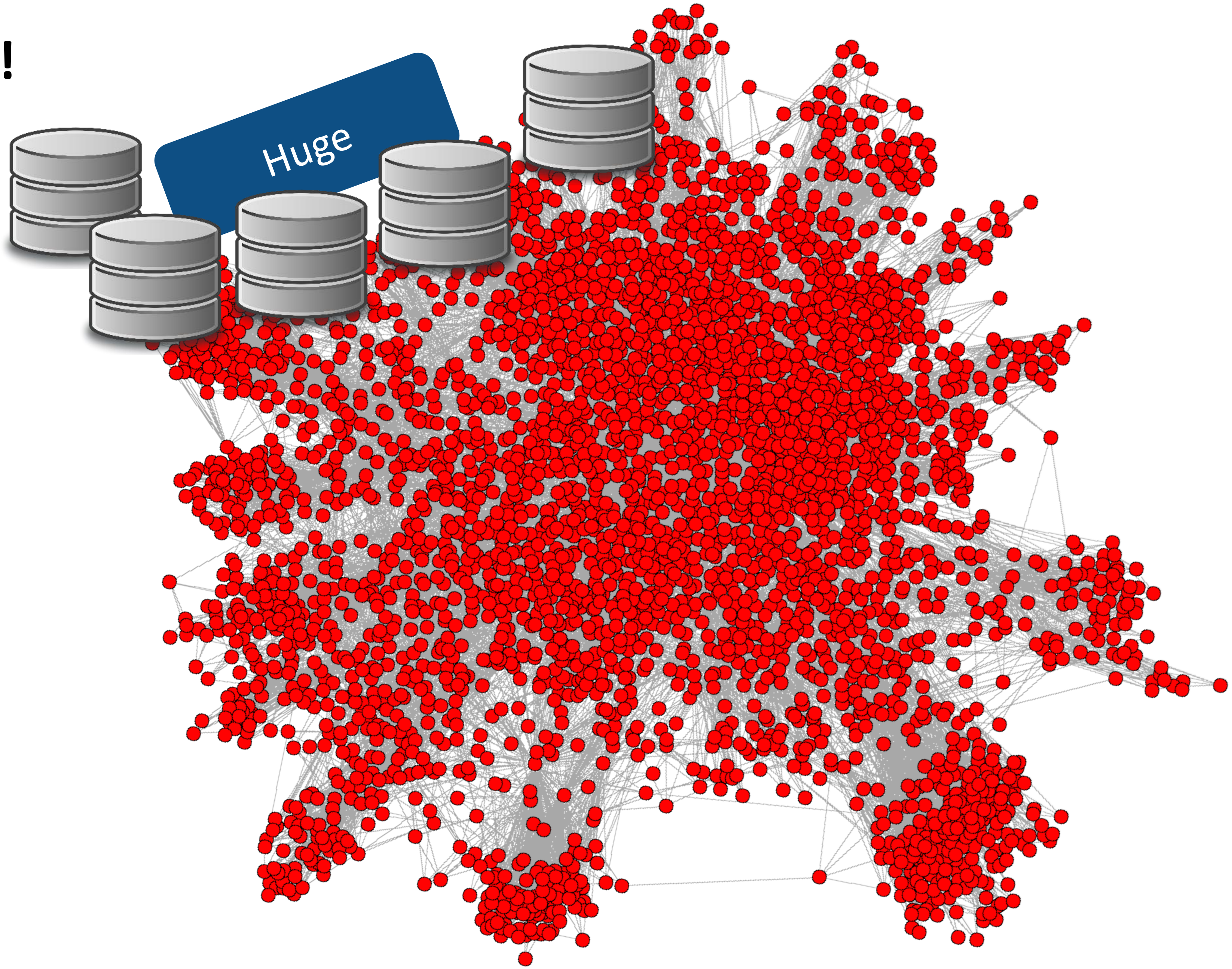

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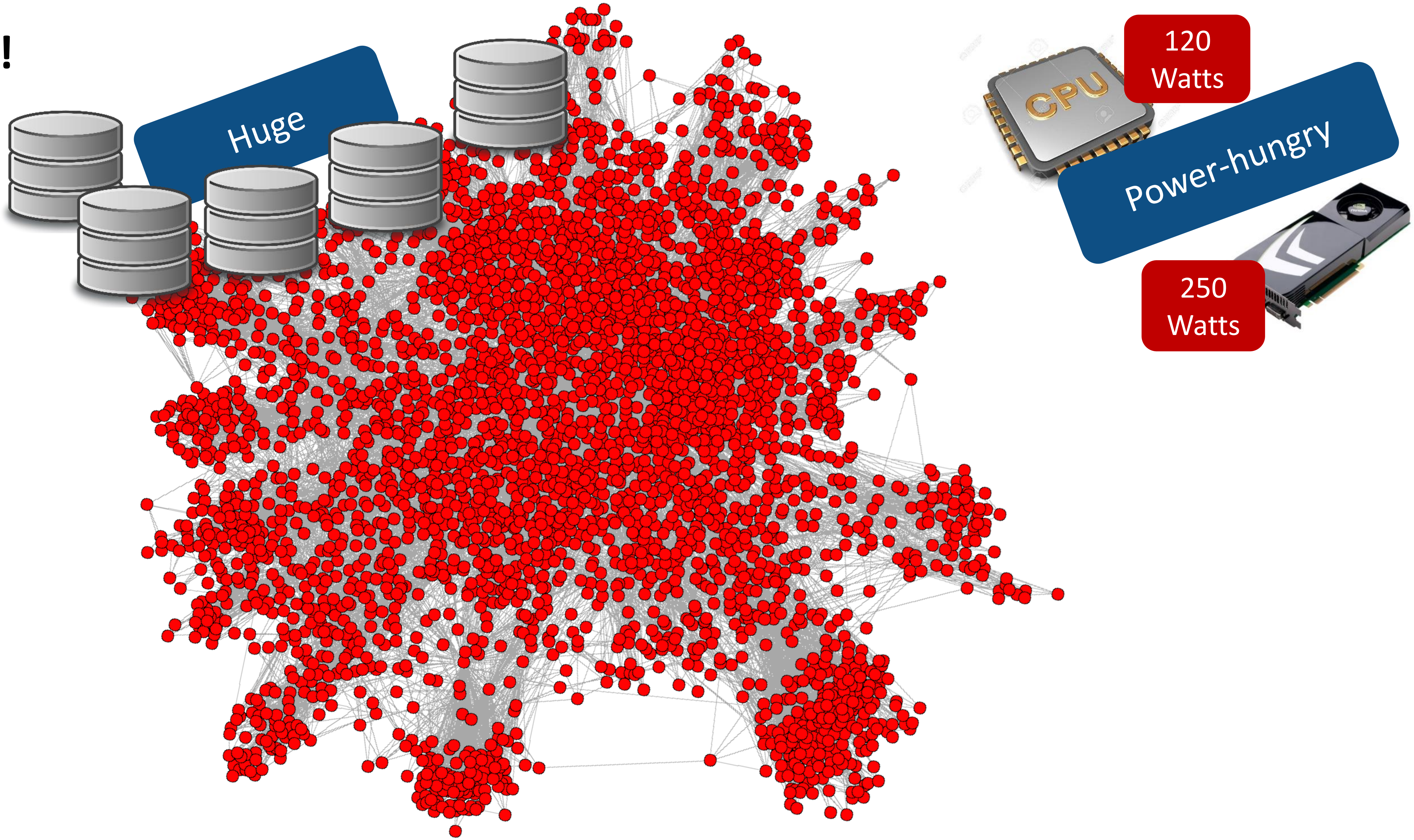

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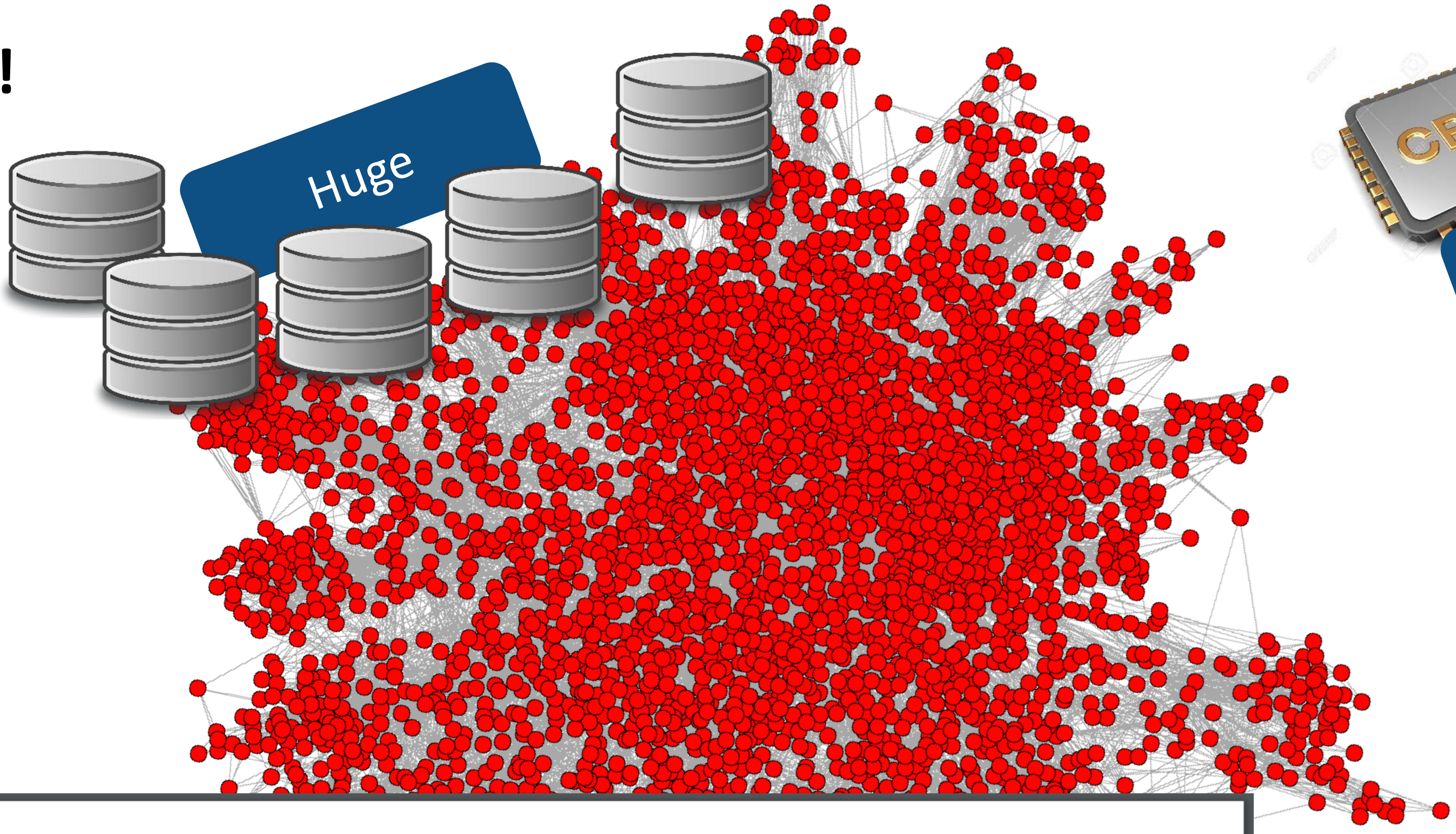
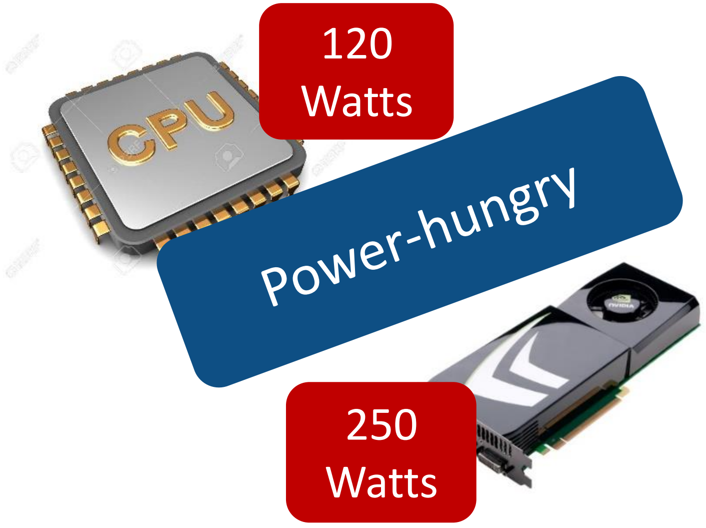




Substream-Centric Maximum Matchings on FPGA

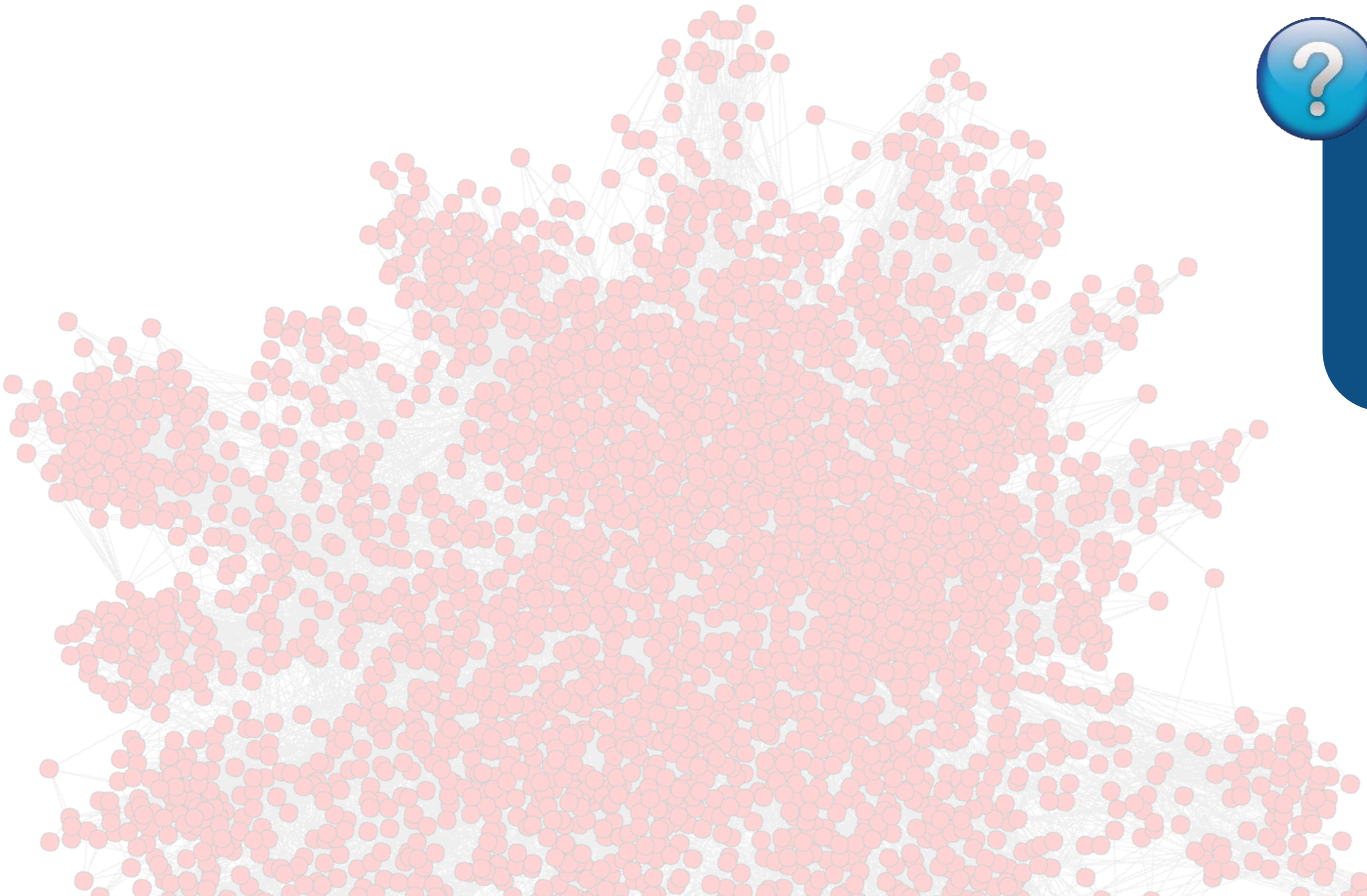
Maciej Besta, Marc Fischer, Tal Ben-Nun, Johannes De Fine Licht, Torsten Hoefler
Department of Computer Science, ETH Zurich

ABSTRACT

Developing high-performance and energy-efficient algorithms for maximum matchings is becoming increasingly important in social network analysis, computational sciences, schedul-

compound can be used to show the locations of double bonds in the chemical structure [59]. As deriving the exact MM is usually computationally expensive, significant focus has been placed on developing fast approximate solutions [17].

What graph programming paradigm for FPGAs and why?



M. Besta et al.: “Substream-Centric Maximum Matchings on FPGA”, ACM FPGA’19

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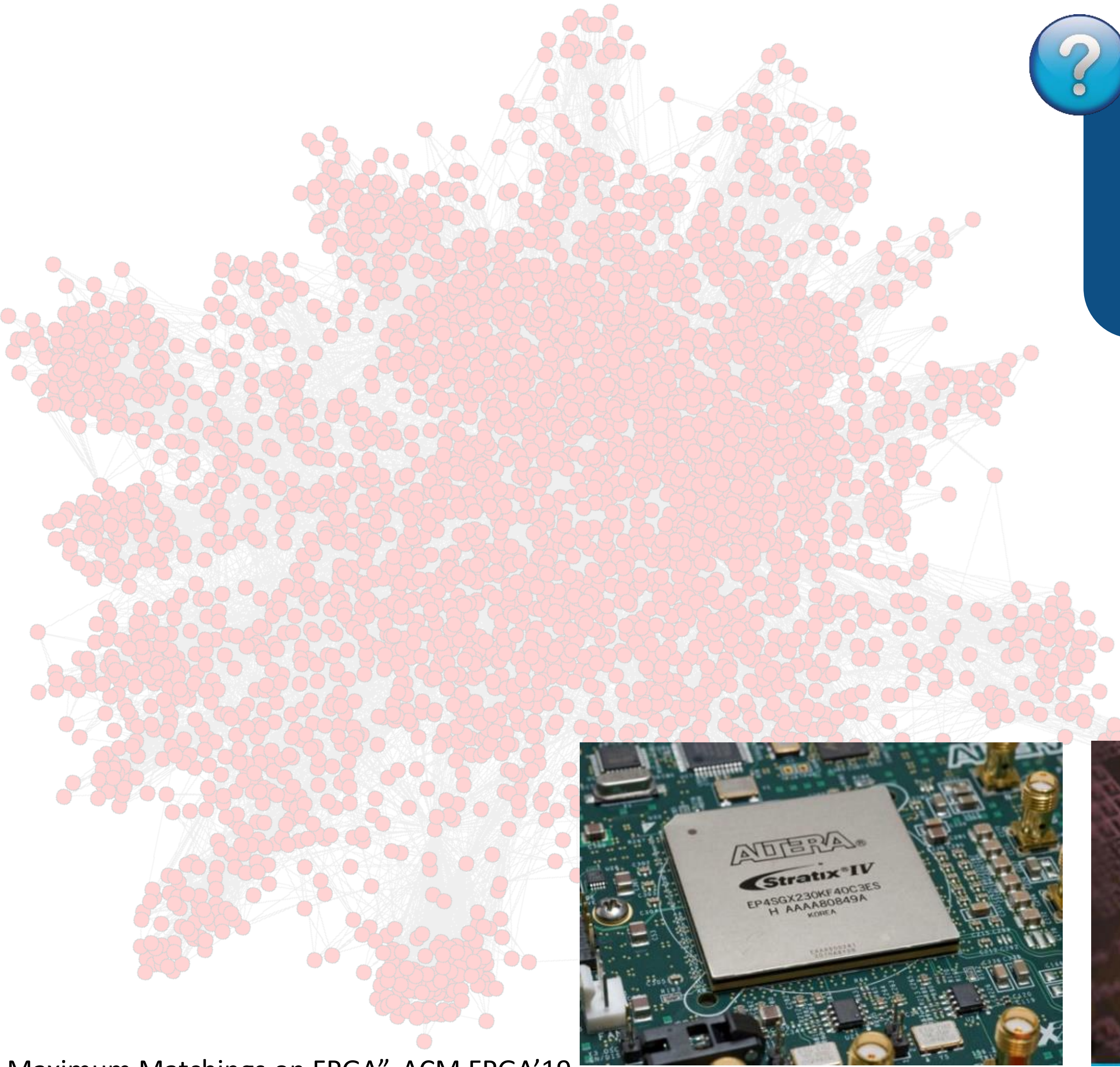

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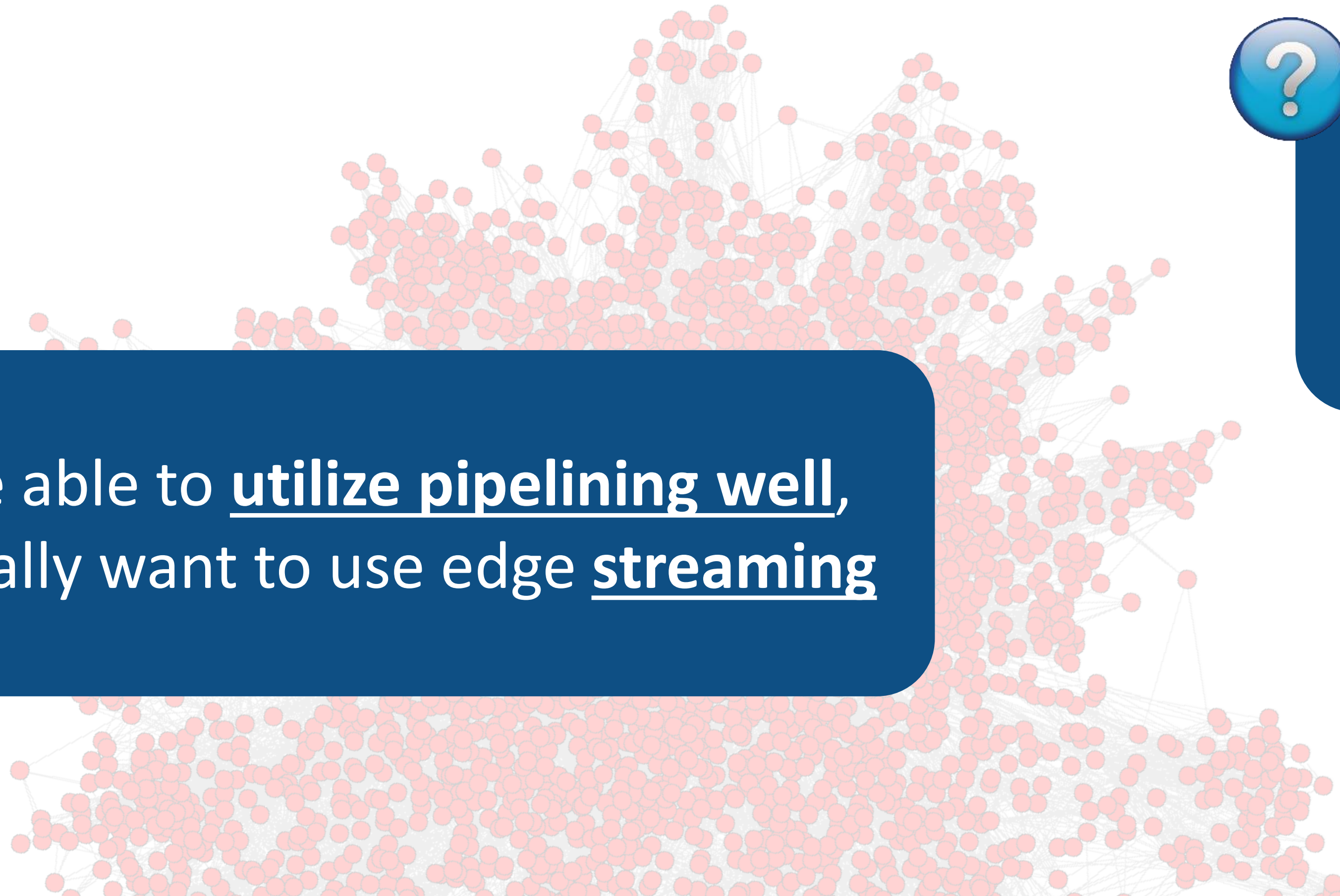
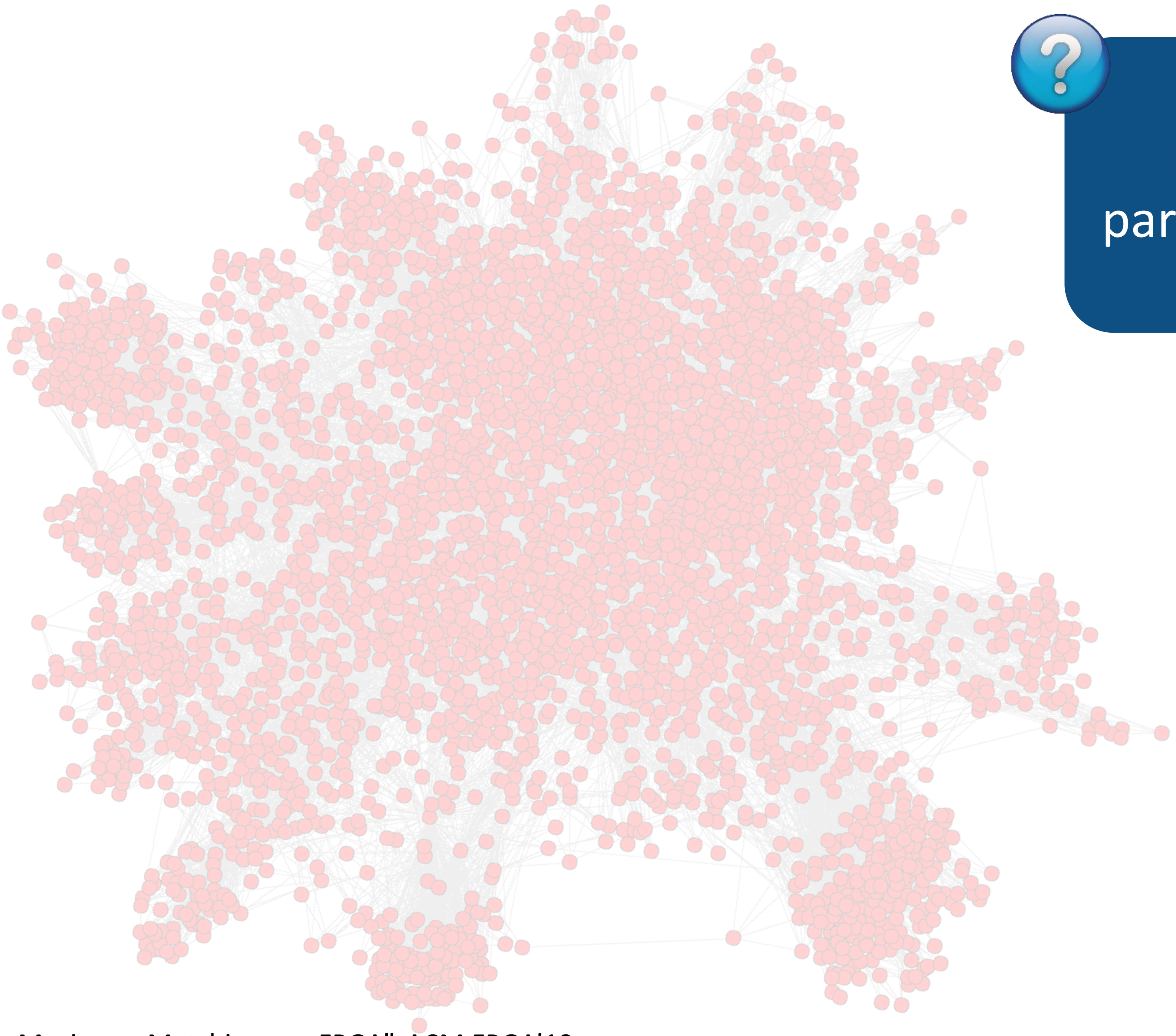


To be able to **utilize pipelining well**, we really want to use edge **streaming**

M. Besta et al.: "Substream-Centric Maximum Matchings on FPGA", ACM FPGA'19



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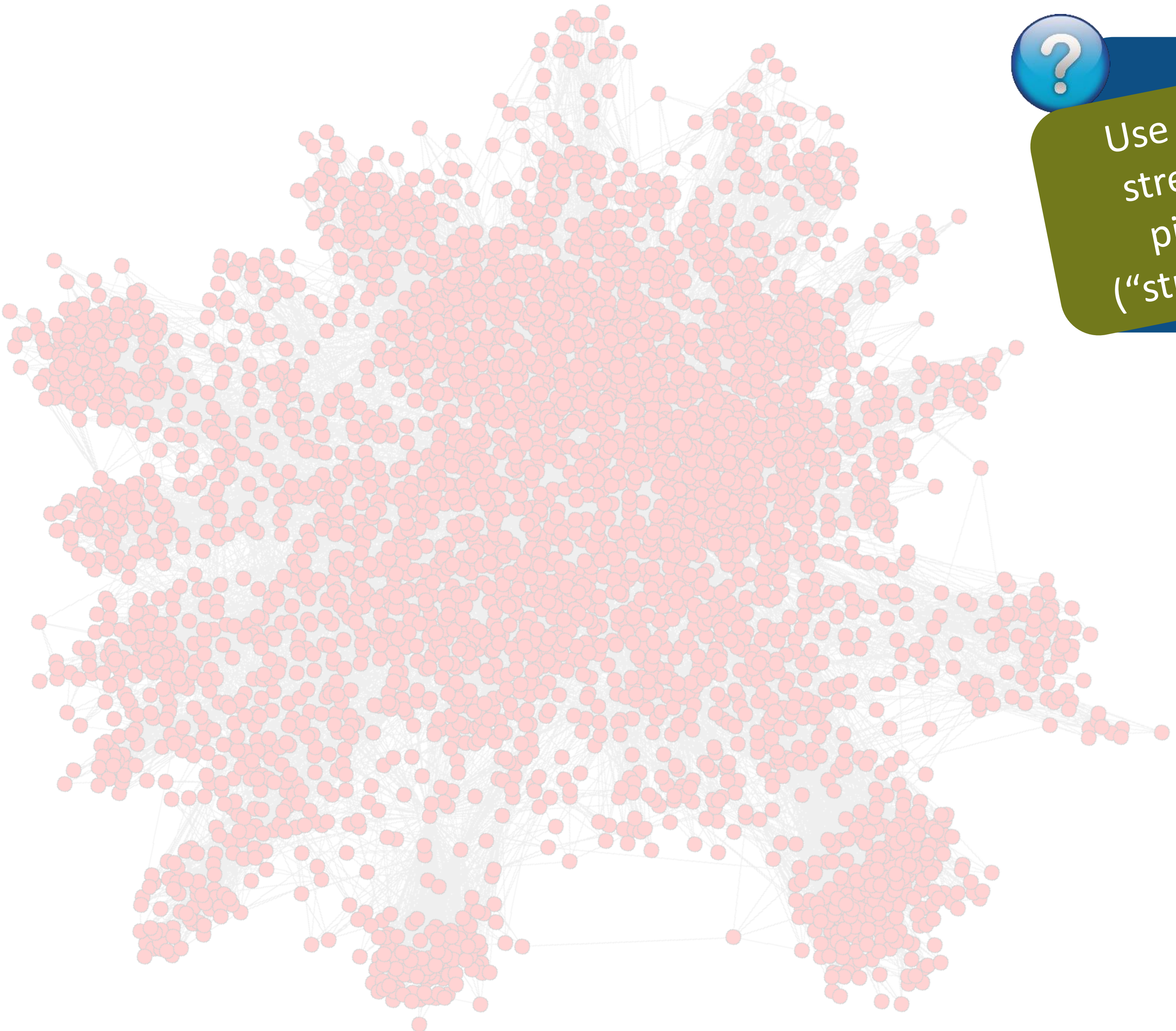

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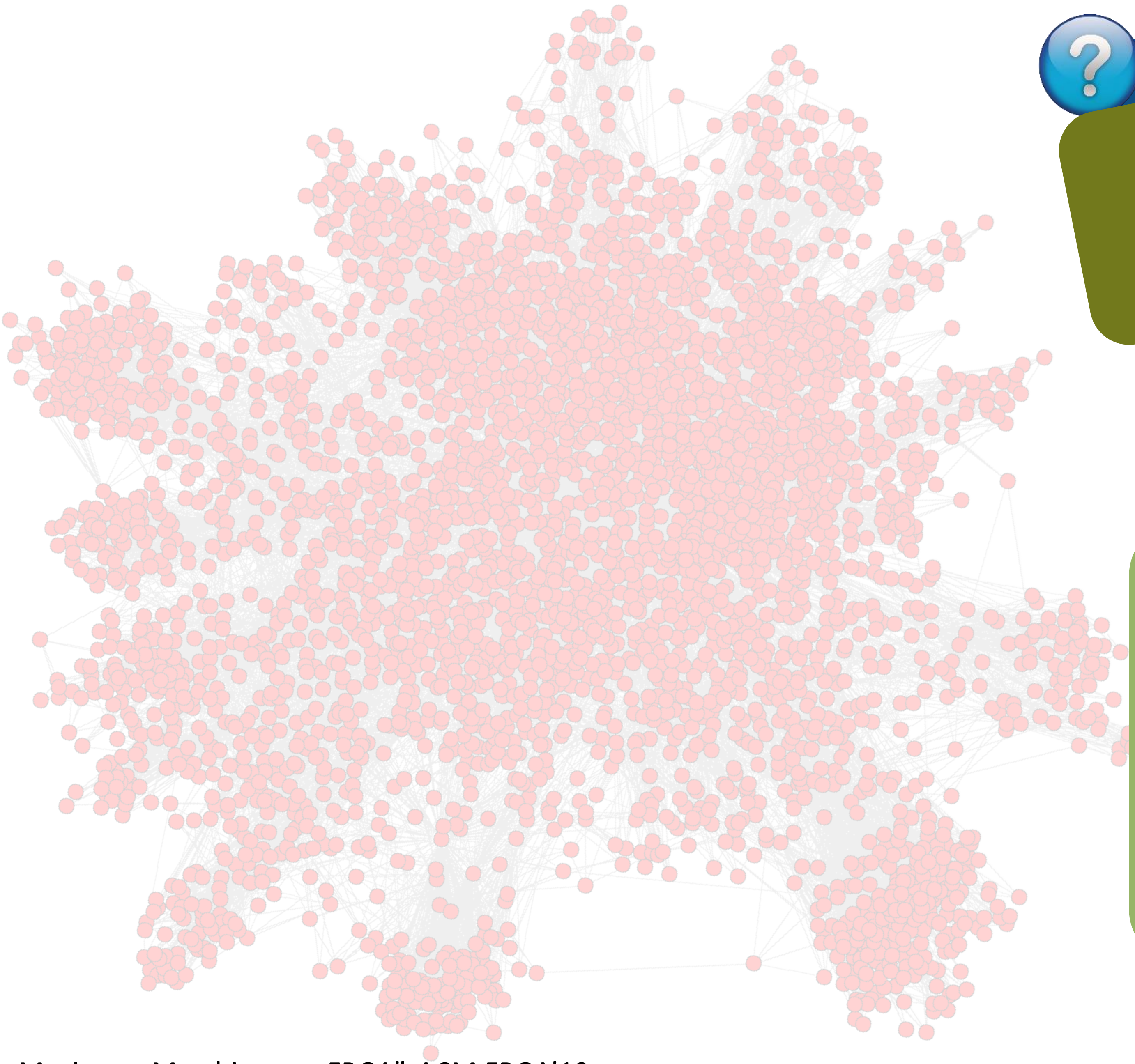

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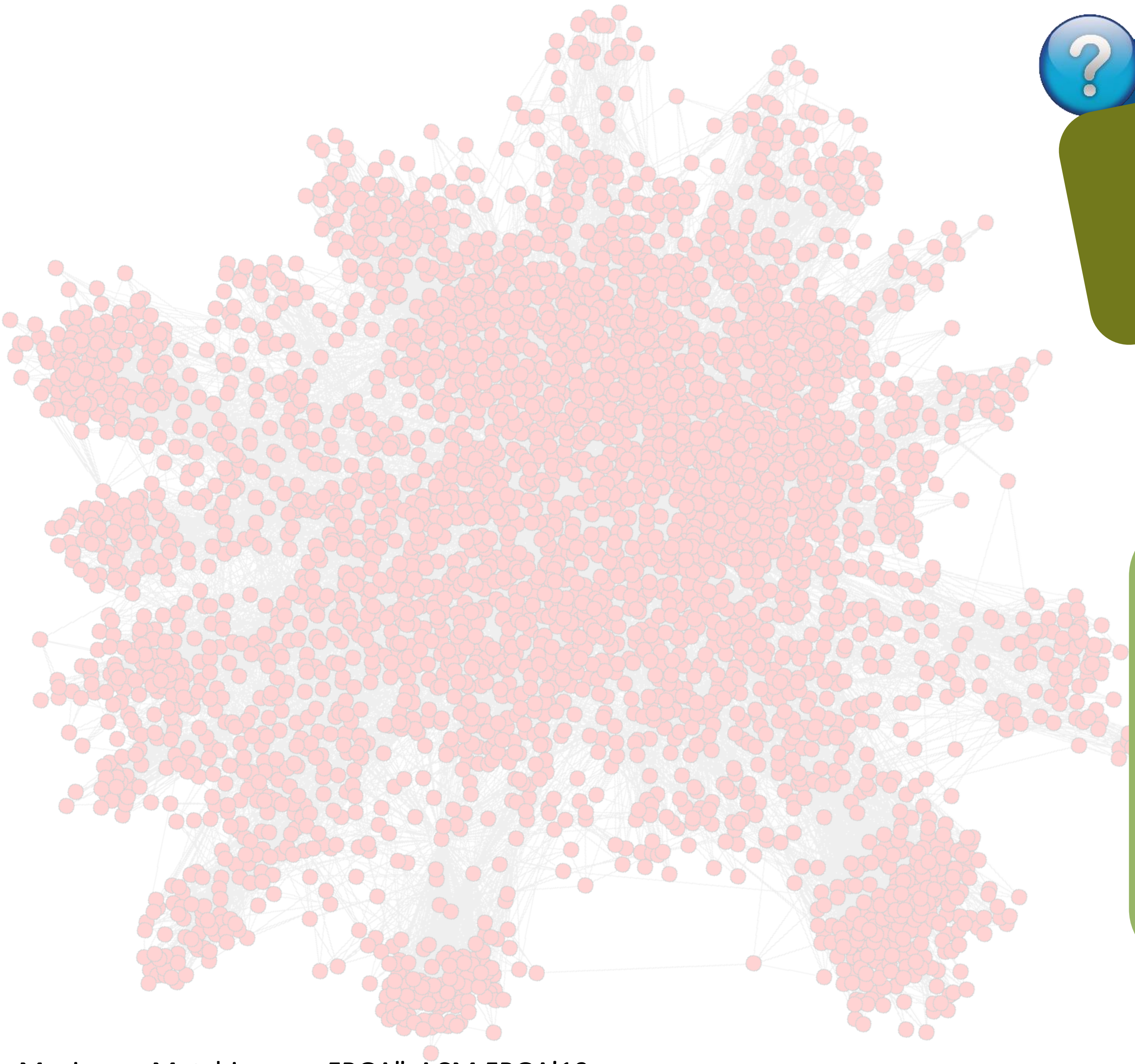

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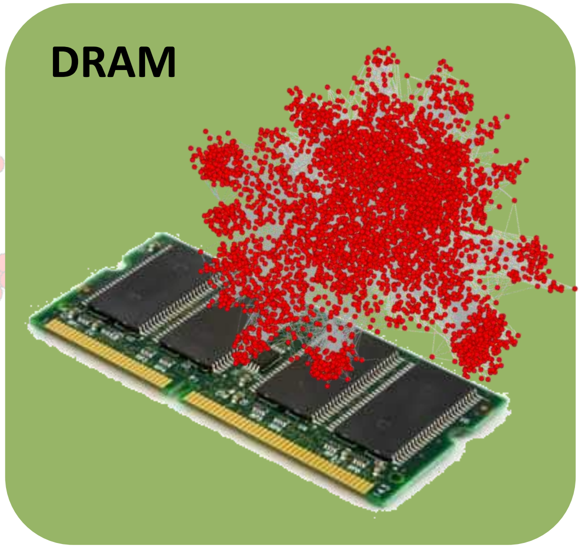

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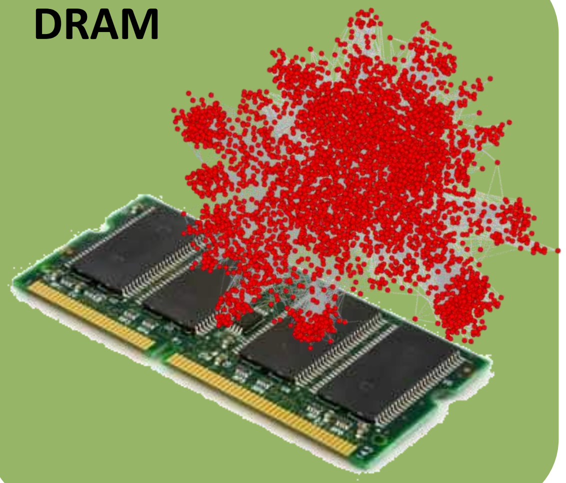

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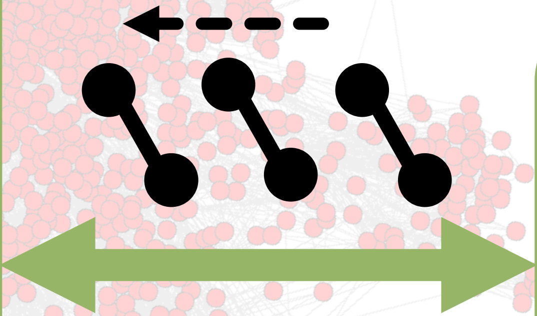

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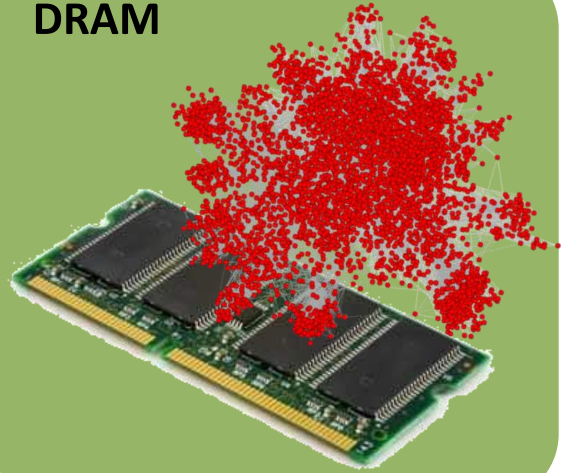

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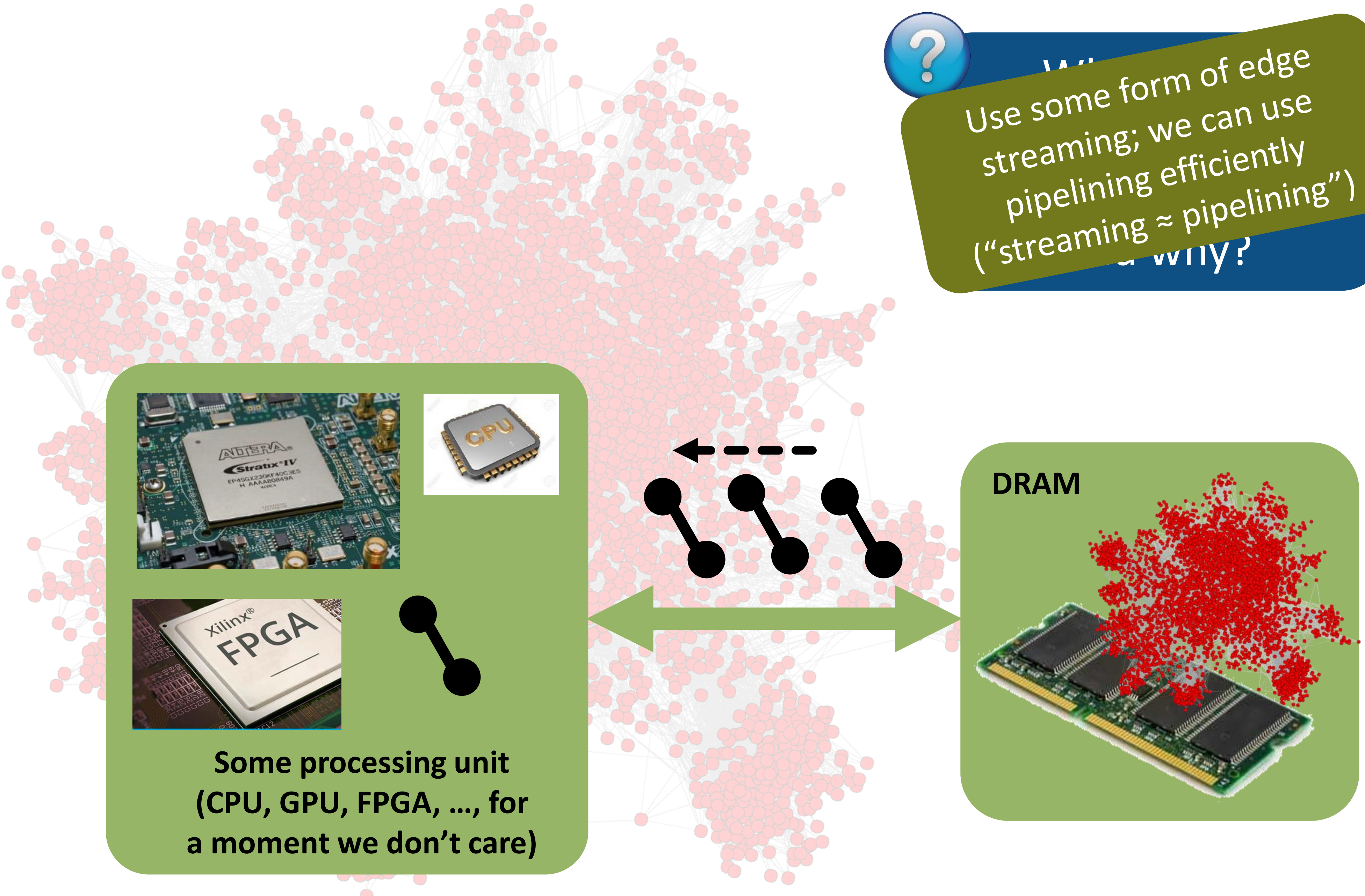

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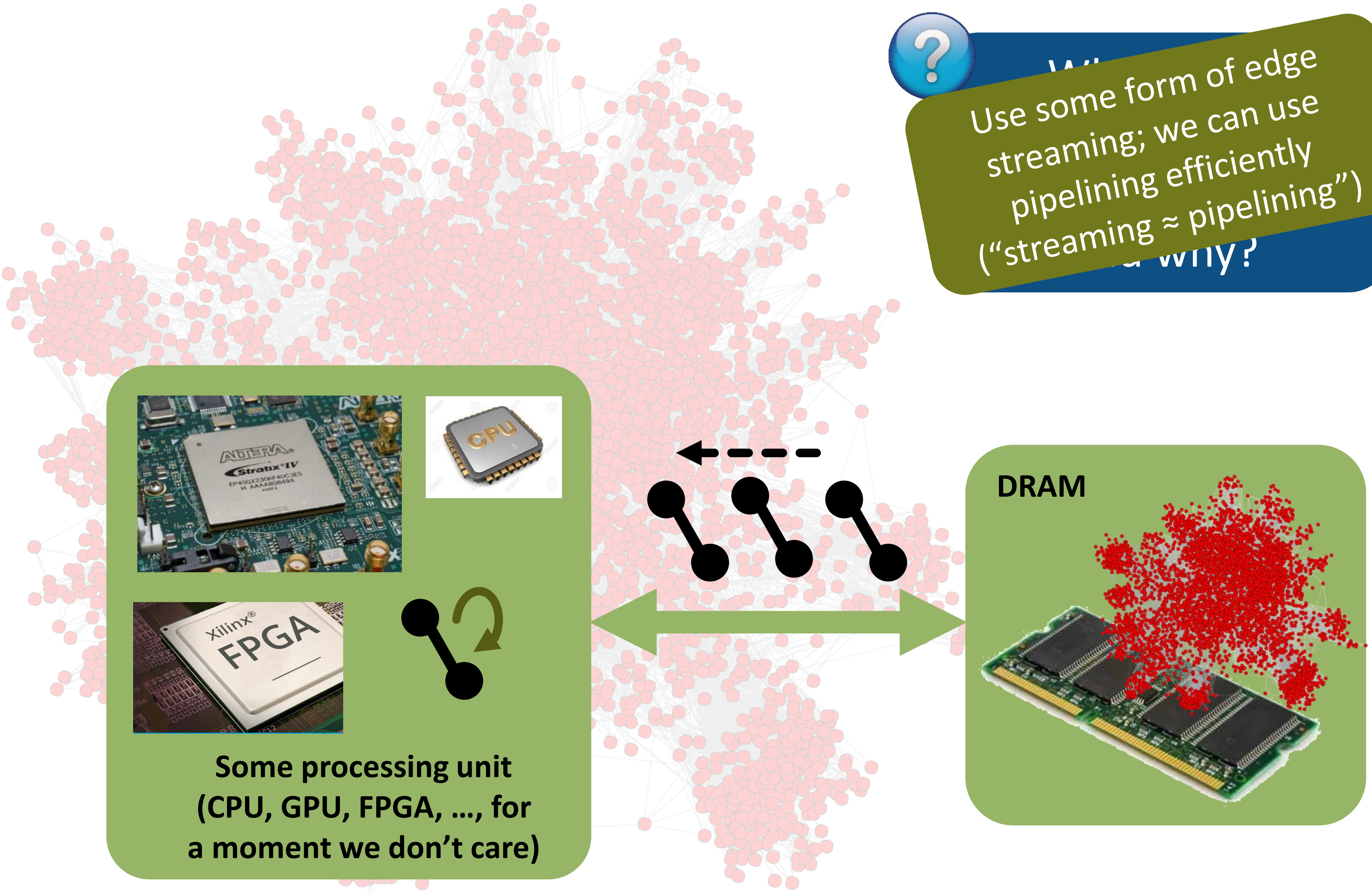

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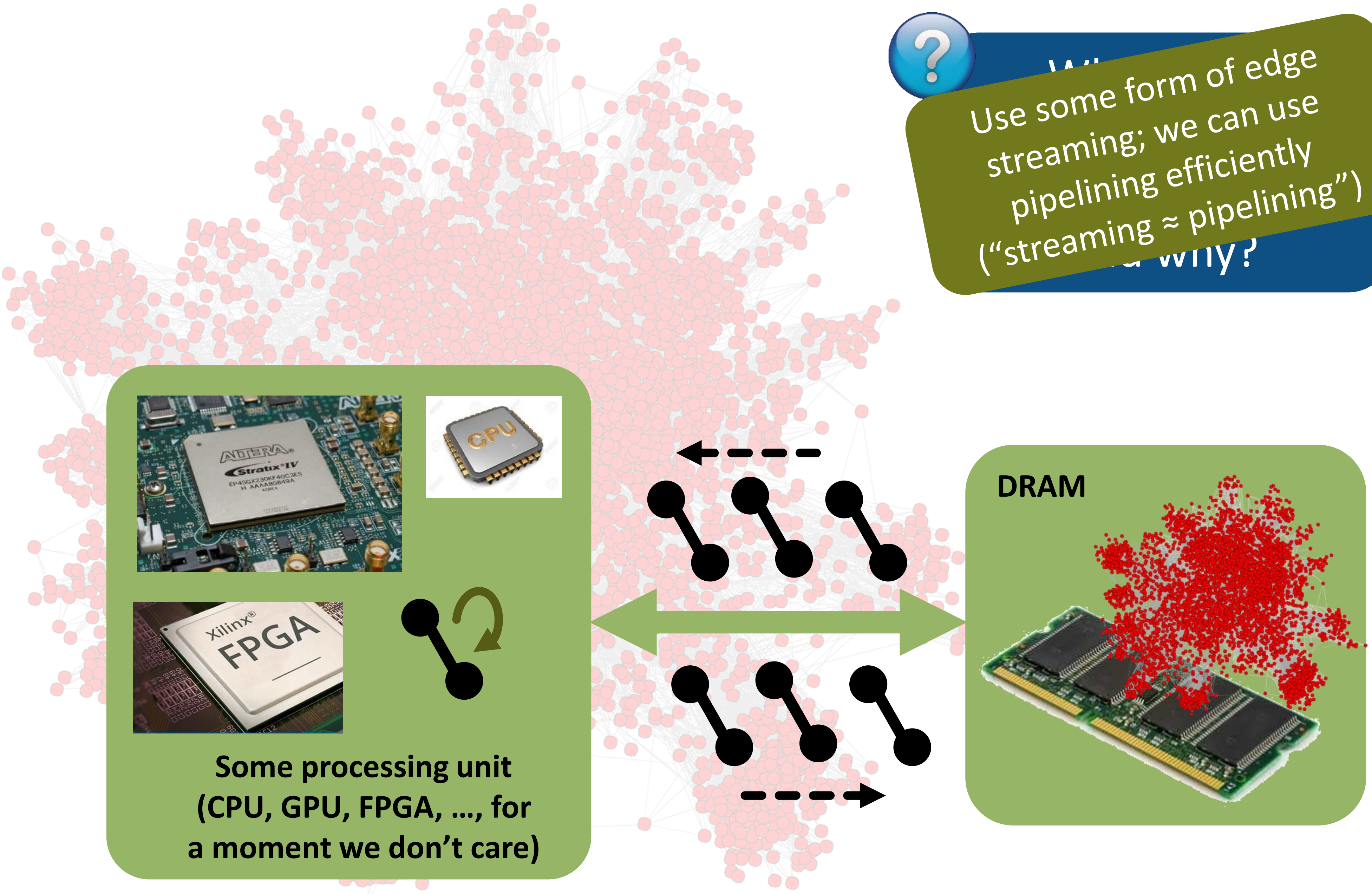

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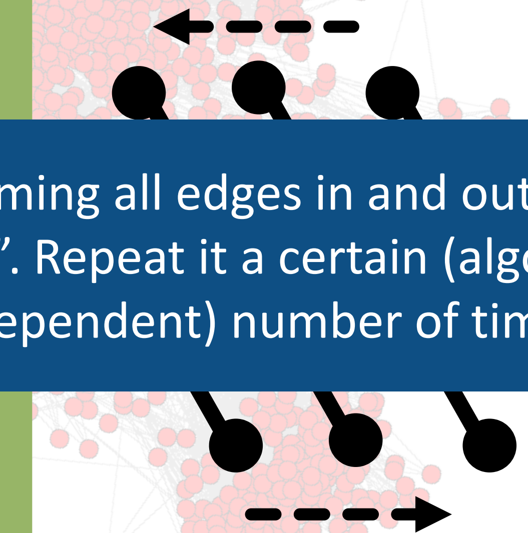

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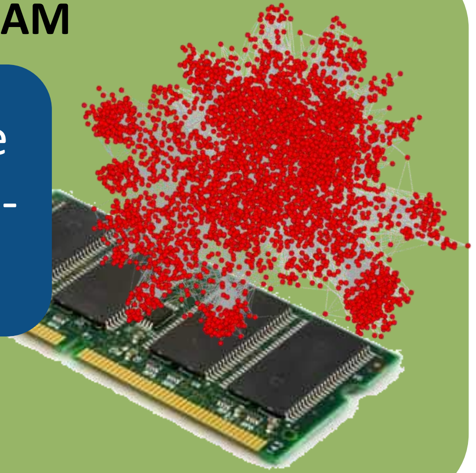

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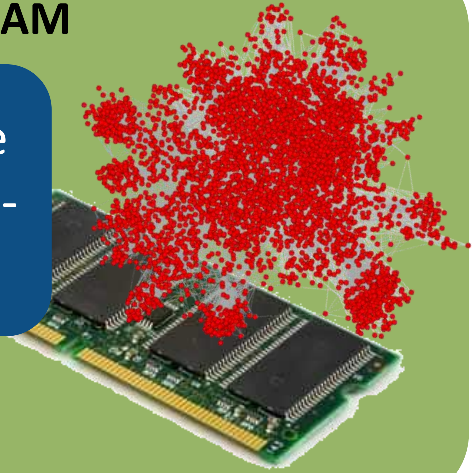

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…How to minimize the number of "passes" over edges? This can get **really** bad in the "traditional" edge-centric approach (e.g., BFS needs *D* passes; *D* = diameter [1]).

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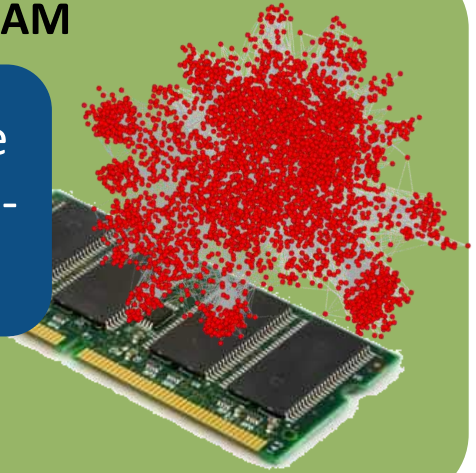

[1] A. Roy et al. X-stream: Edge-Centric Graph Processing using Streaming Partitions. SOSP. 2013.

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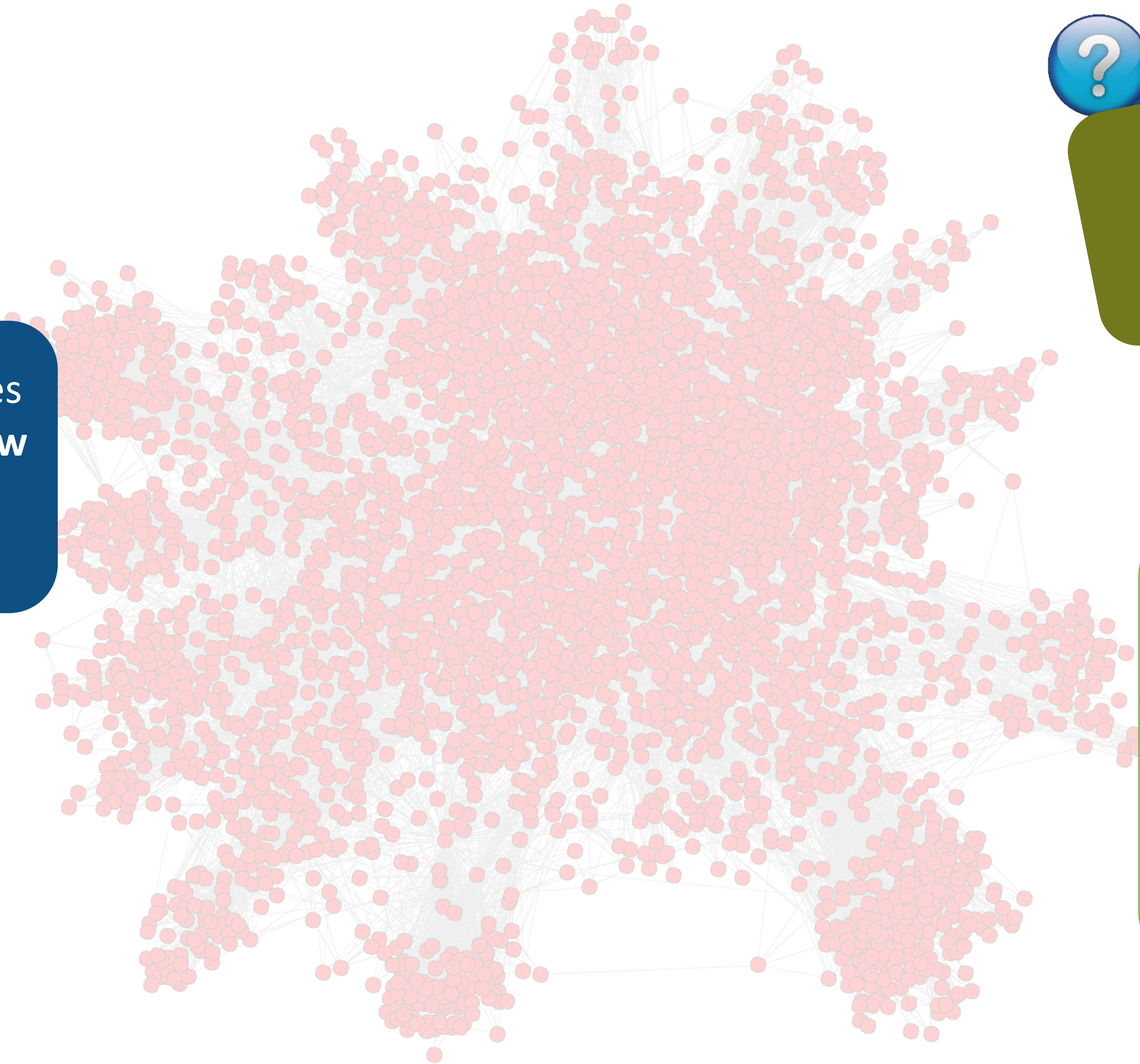

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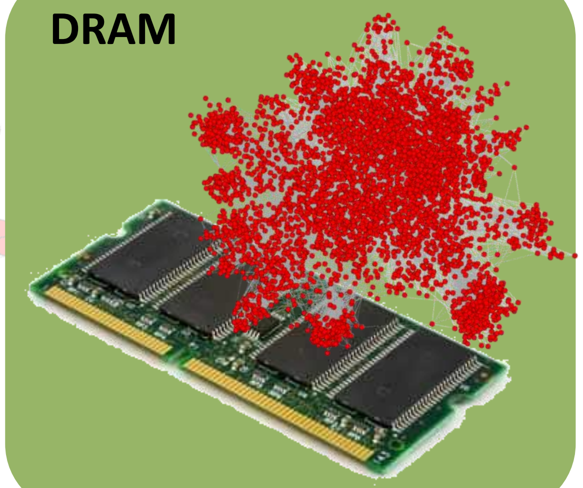

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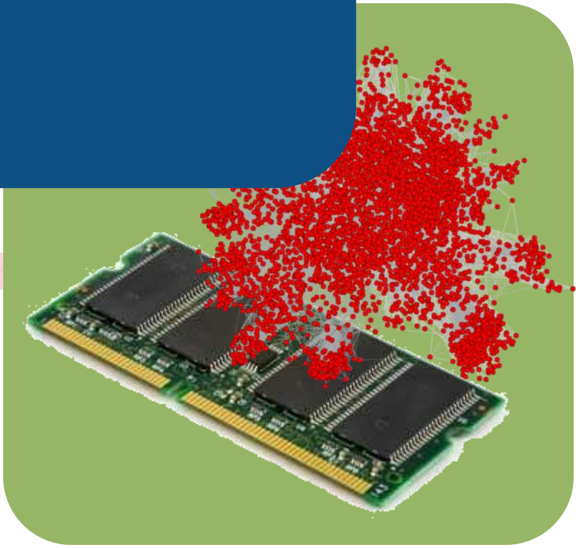
Substream-Centric: A new paradigm for processing graphs



Substream-Centric Graph Processing

A new paradigm for processing graphs

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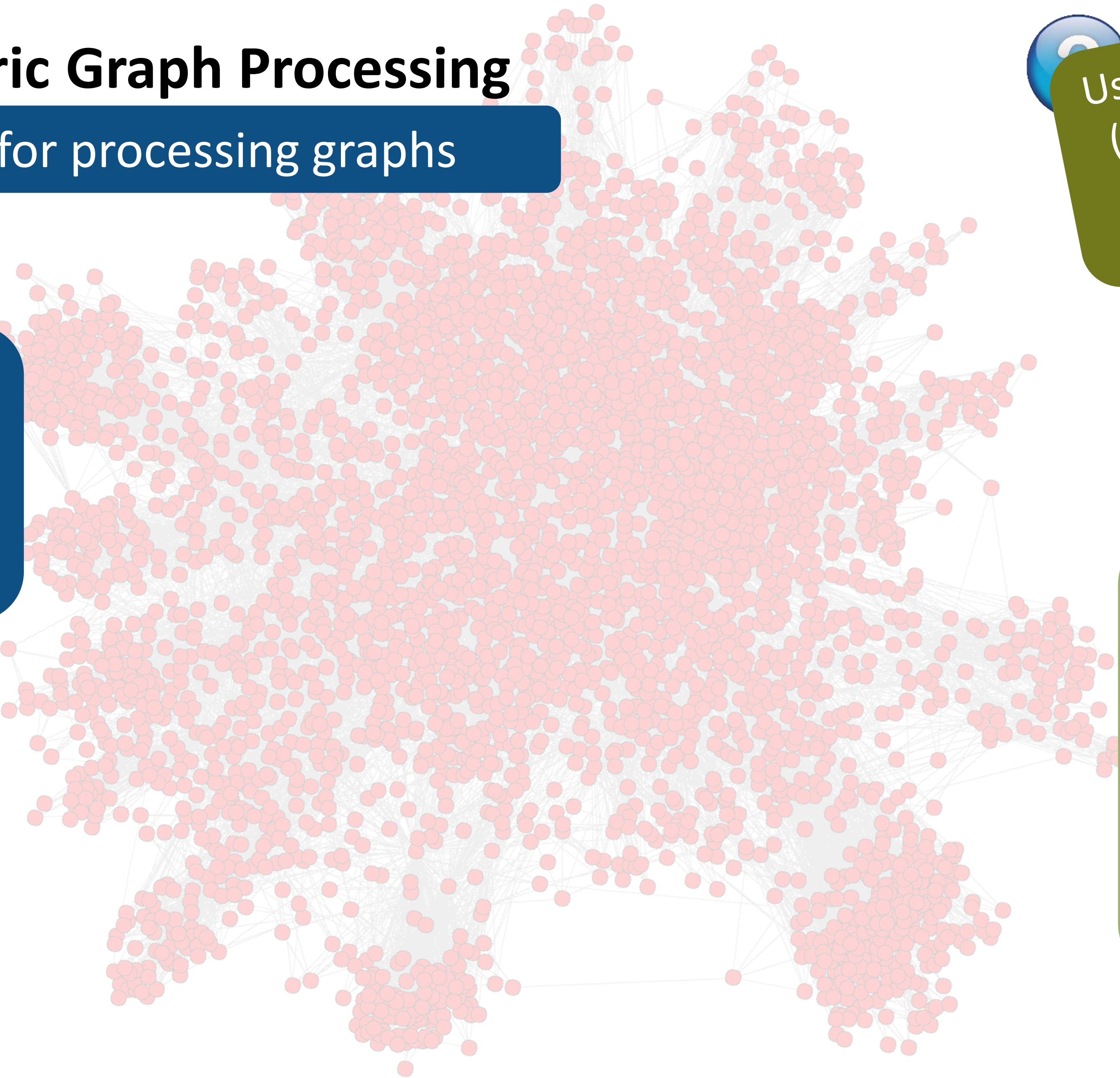

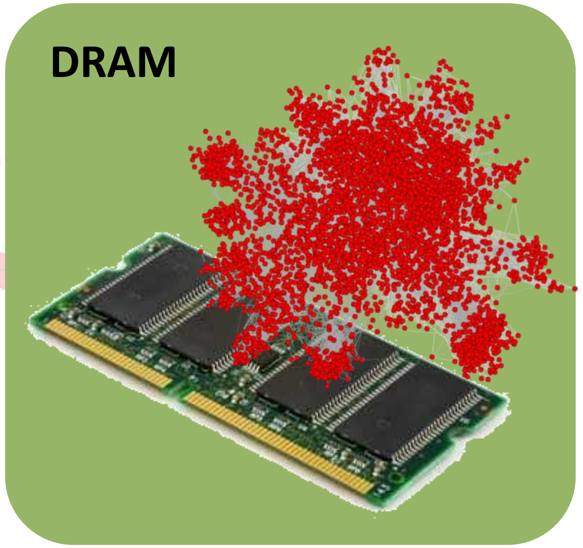

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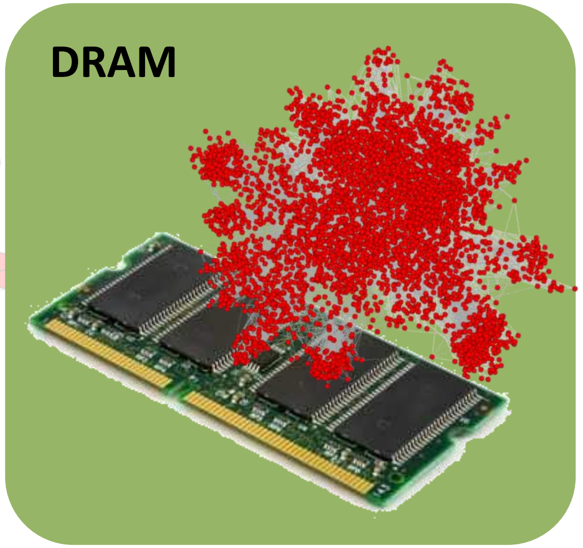

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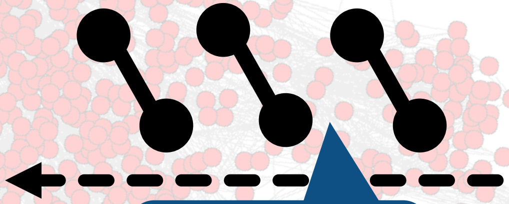
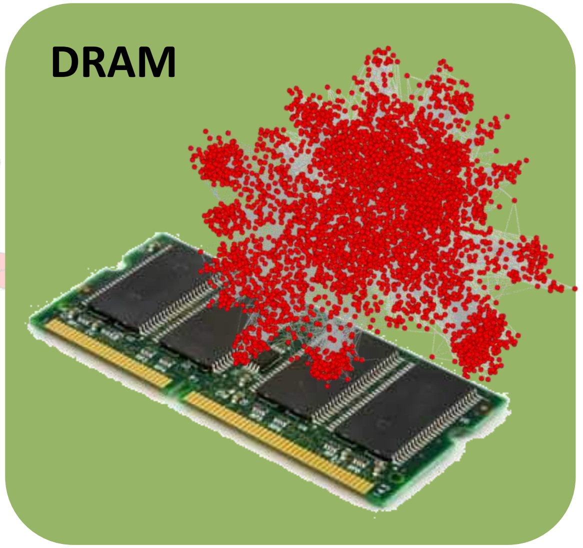


Weighted edges



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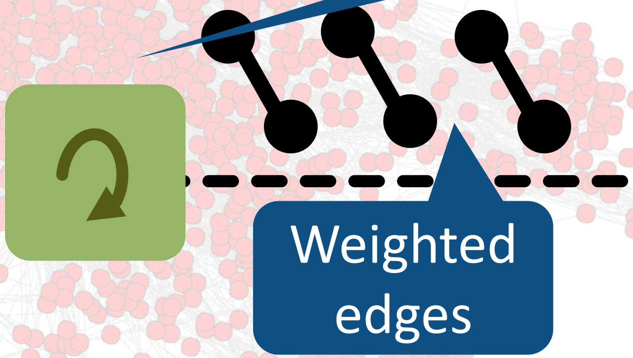

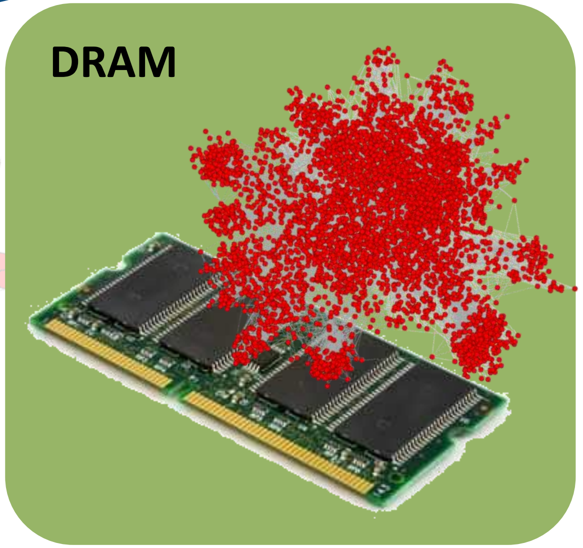

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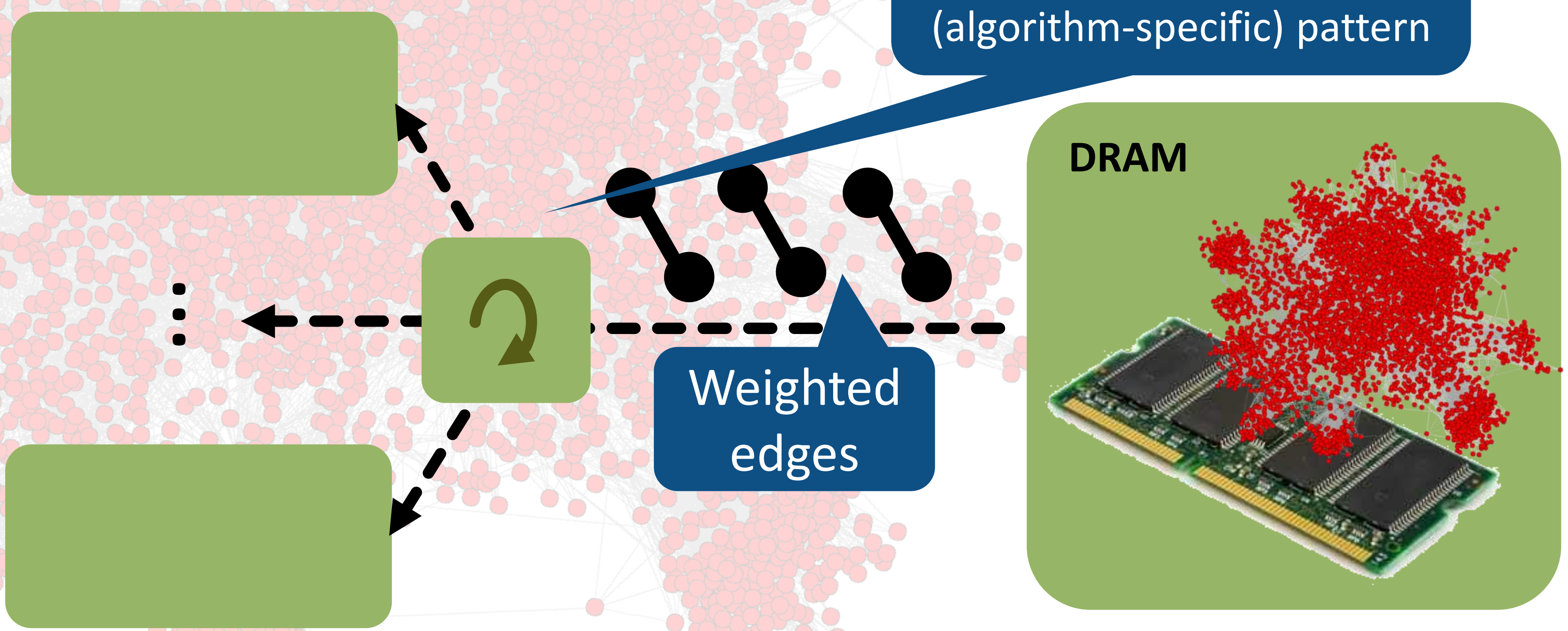

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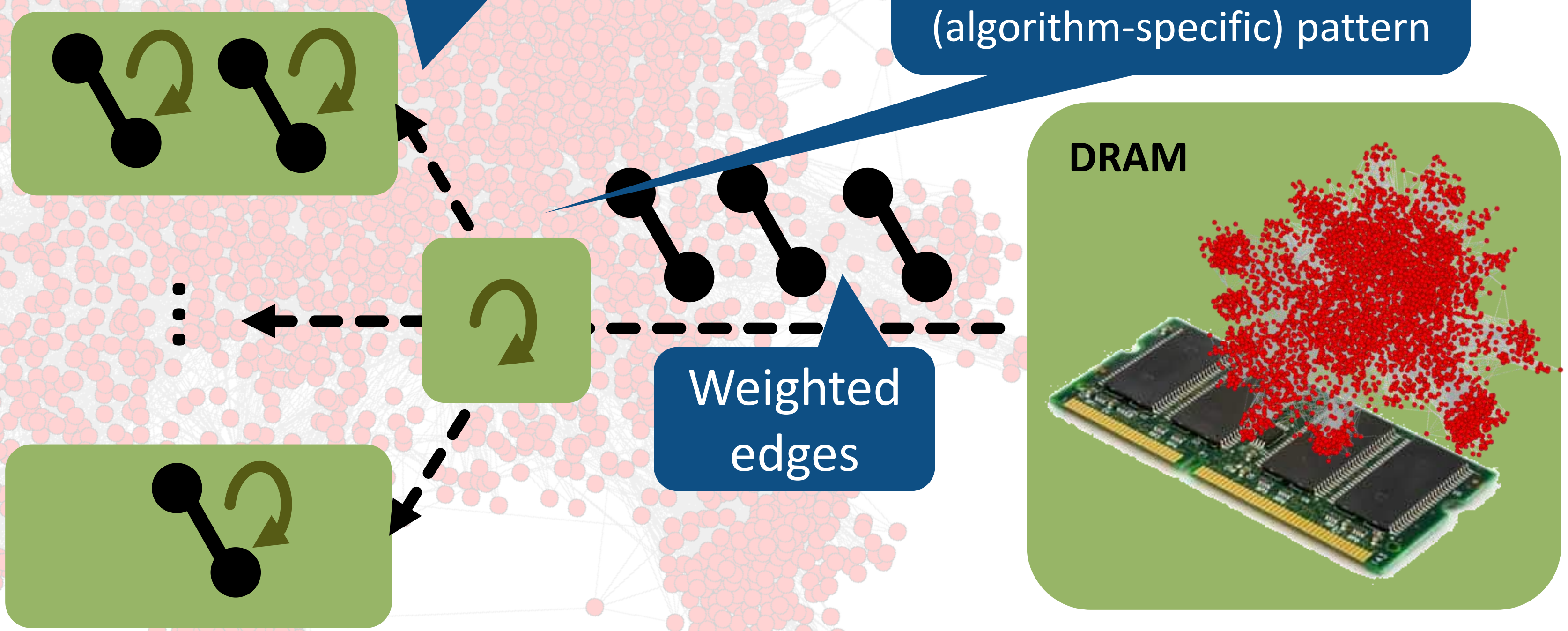

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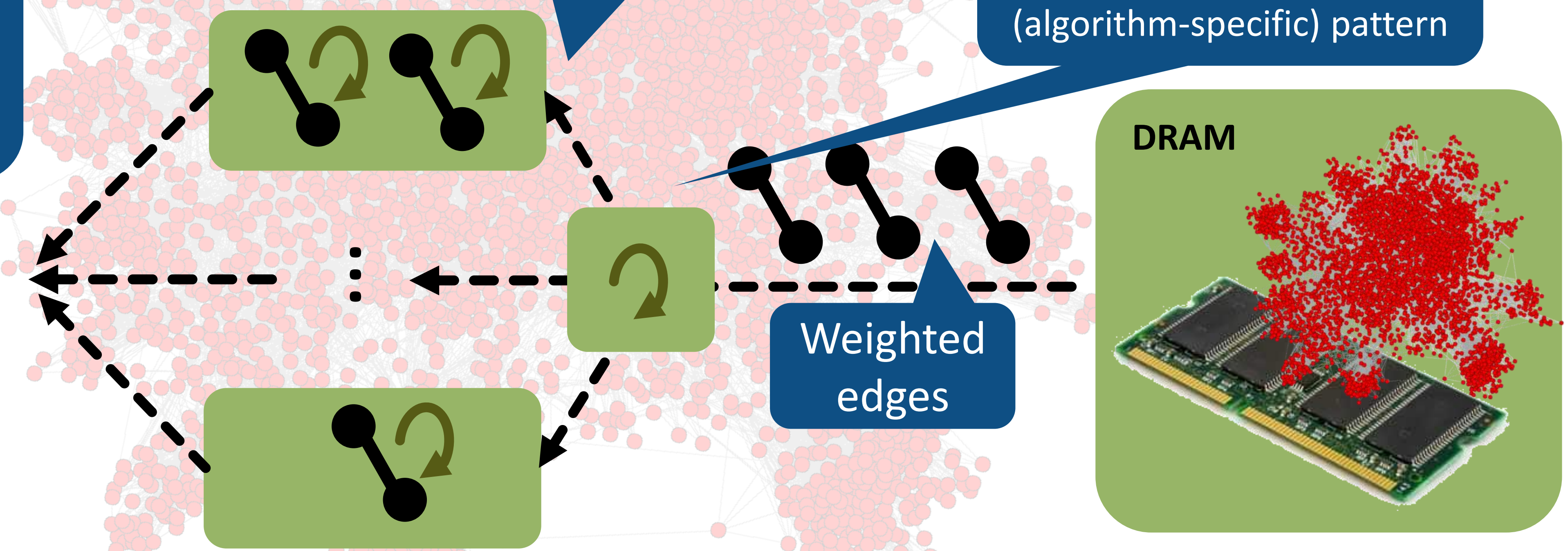

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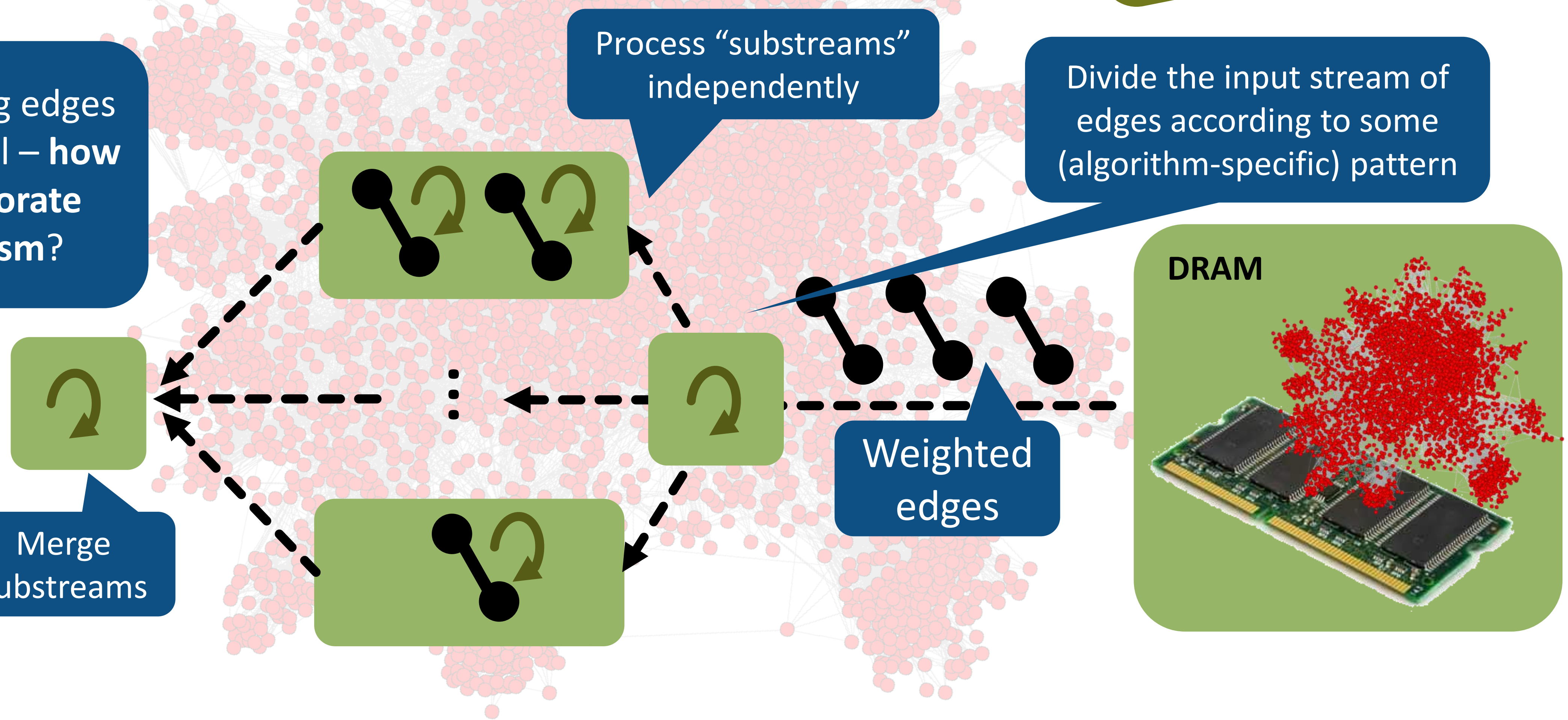

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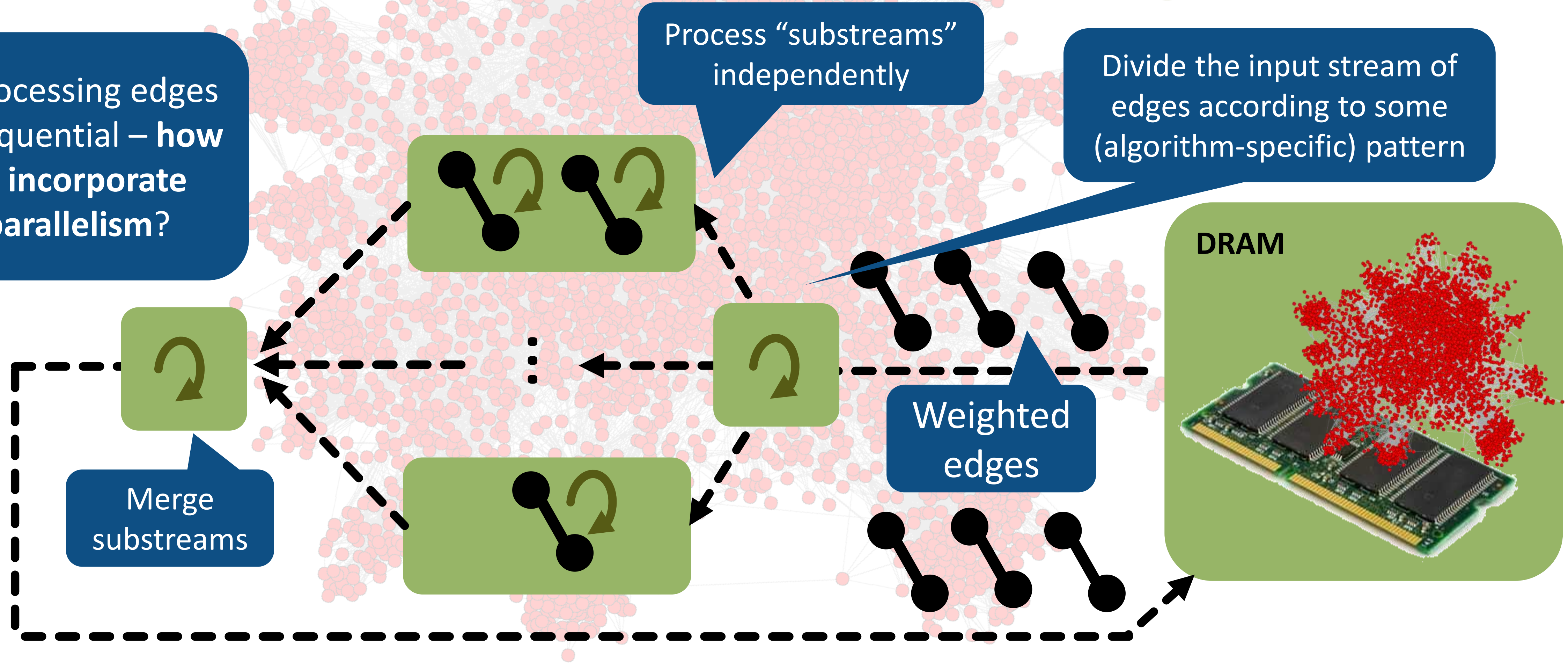


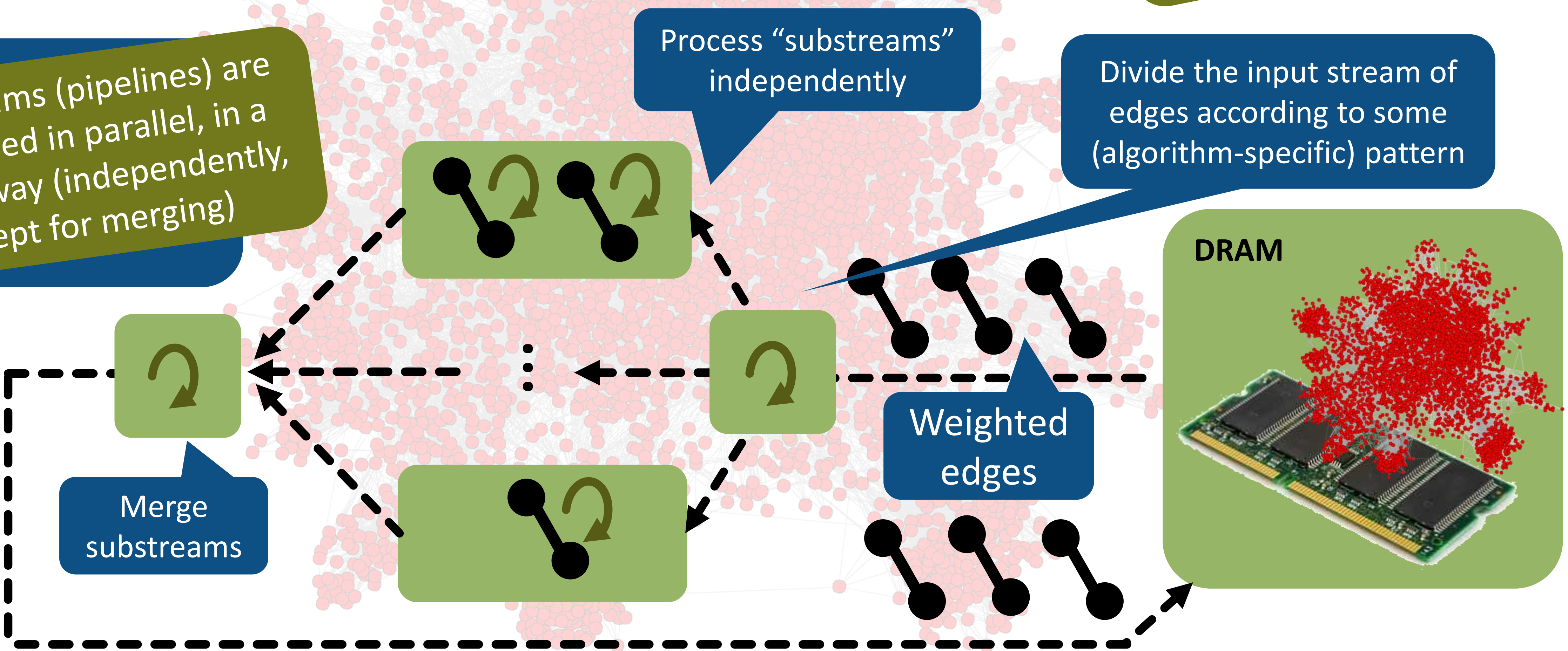
Substream-Centric Graph Processing

A new paradigm for processing graphs

It enhances edge-centric streaming approaches

Use some form of streaming (aka **edge-centric**); we can use pipelining efficiently ("streaming ≈ pipelining")

Substreams (pipelines) are processed in parallel, in a simple way (independently, except for merging)



Substream-Centric Graph Processing

A new paradigm for processing graphs

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Use some form of streaming (aka **edge-centric**); we can use pipelining efficiently (“streaming ≈ pipelining”)

Also, it enables (tunable) approximation and a (tunable) number of passes

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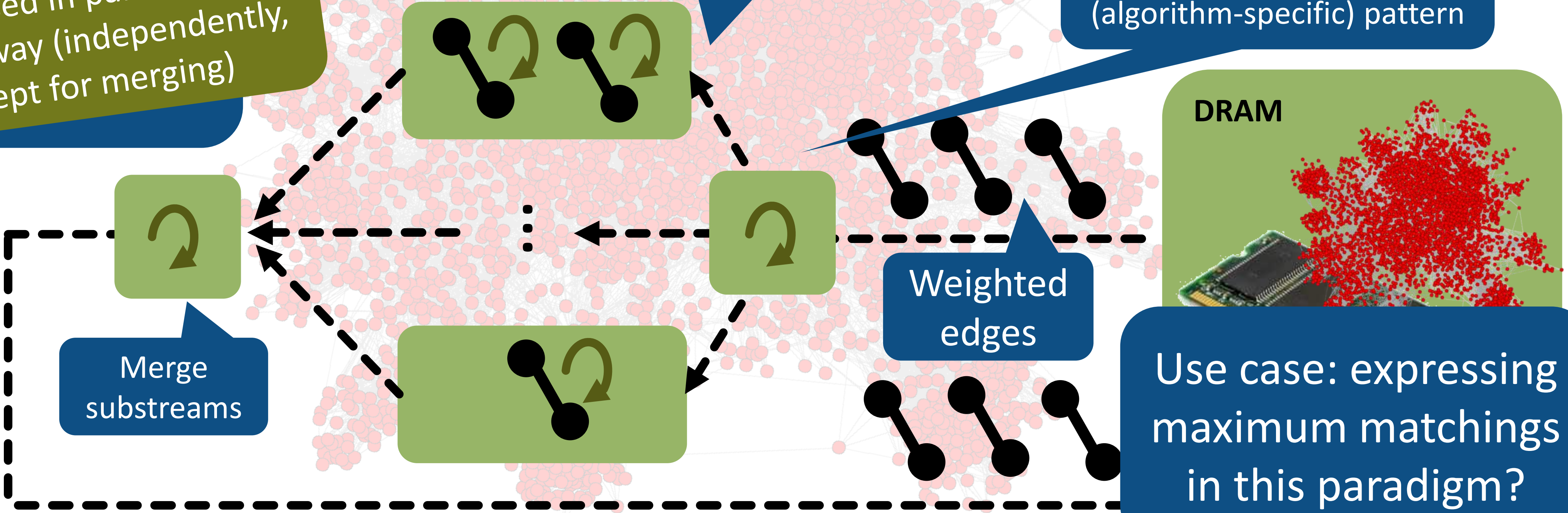
Divide the input stream of edges according to some (algorithm-specific) pattern



Use case: expressing maximum matchings in this paradigm?

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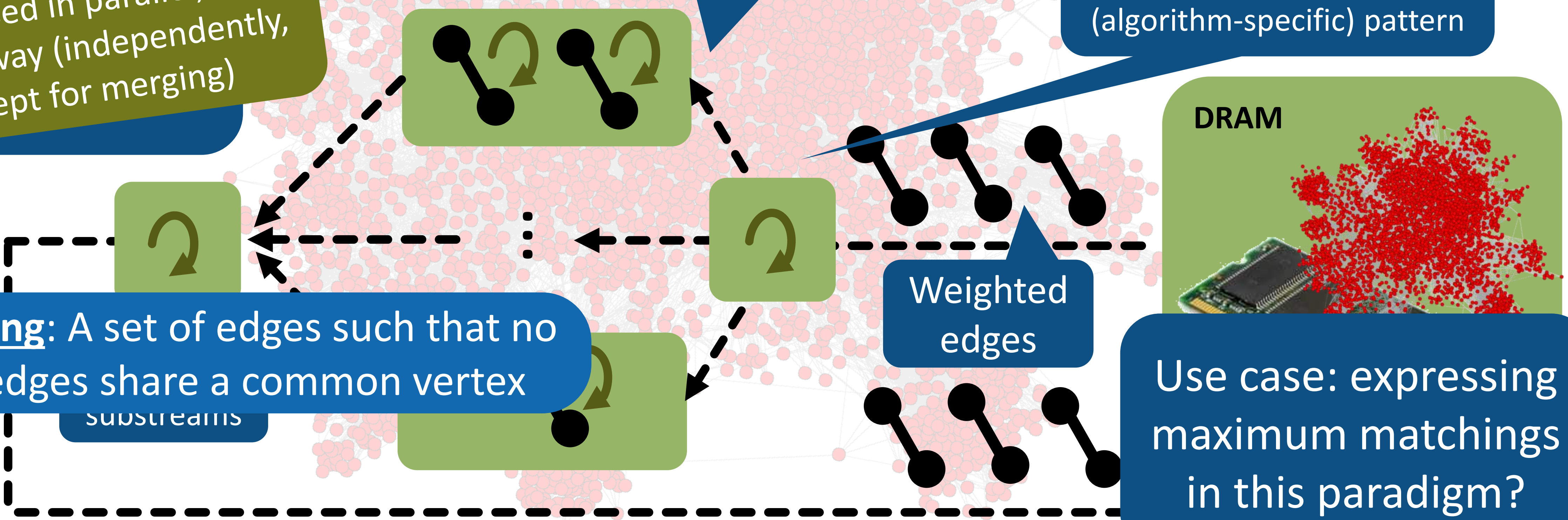

Matching: A set of edges such that no two edges share a common vertex

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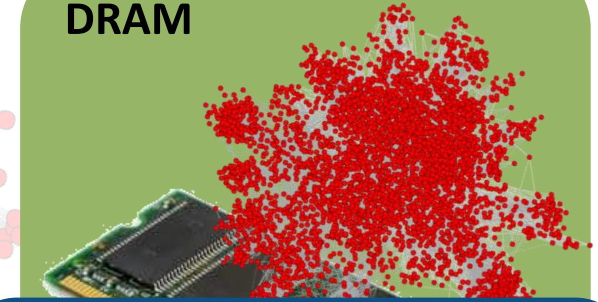
DRAM



Weighted edges

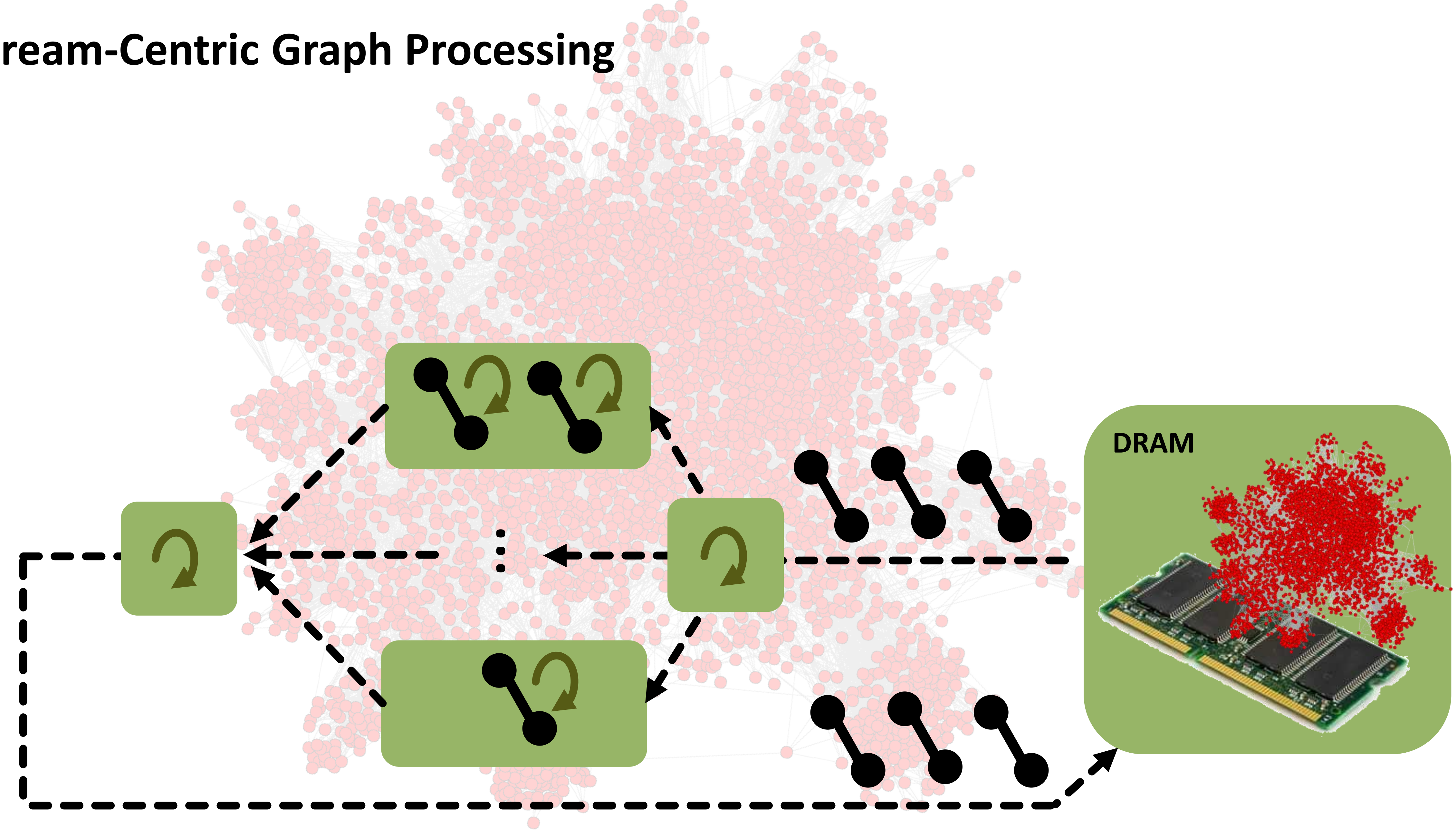
Matching: A set of edges such that no two edges share a common vertex

substreams

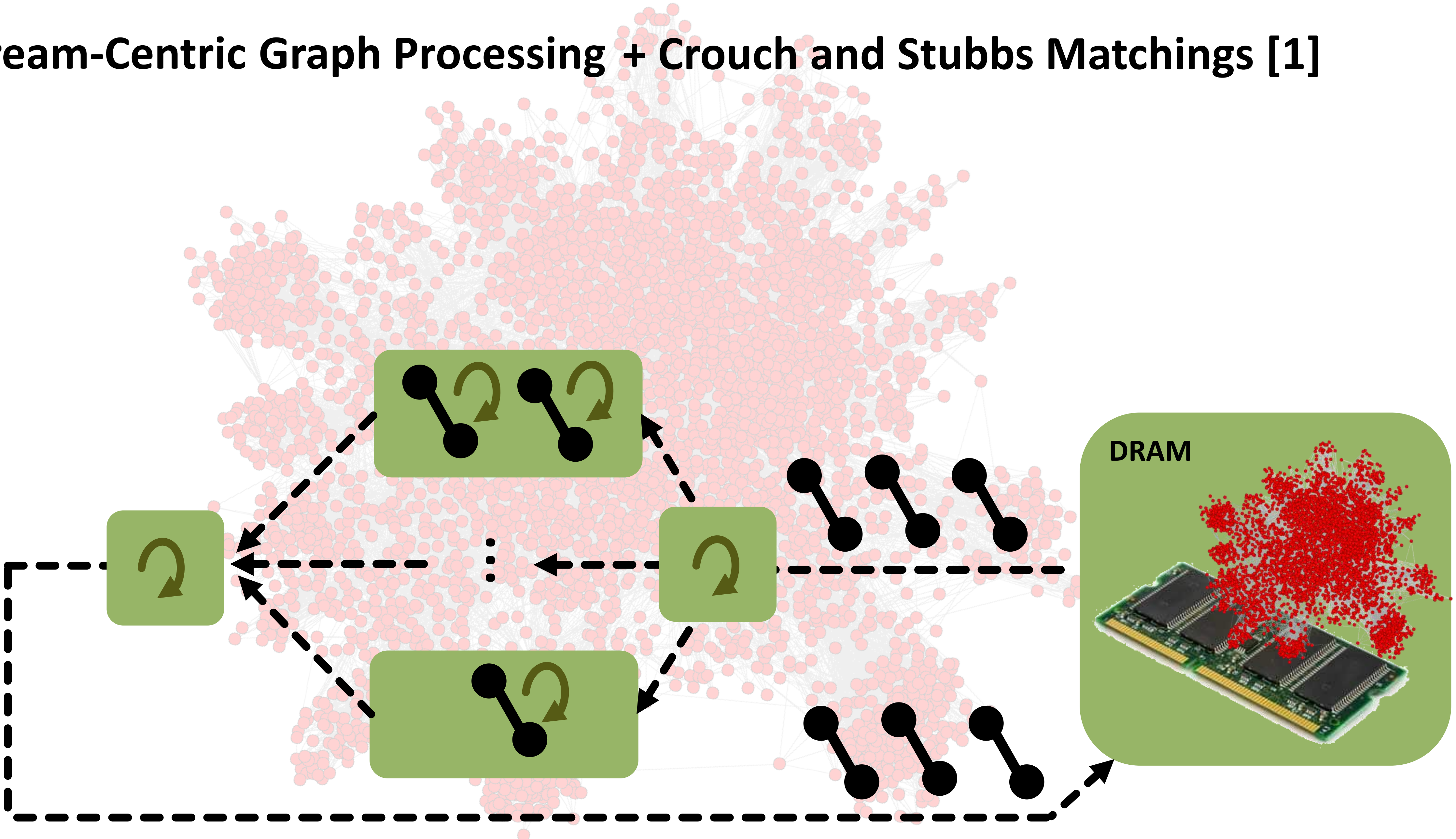
Maximum Weighted Matching (MWM): A matching such that the sum of the edge weights is maximized

Use case: expressing maximum matchings in this paradigm?

Substream-Centric Graph Processing



Substream-Centric Graph Processing + Crouch and Stubbs Matchings [1]



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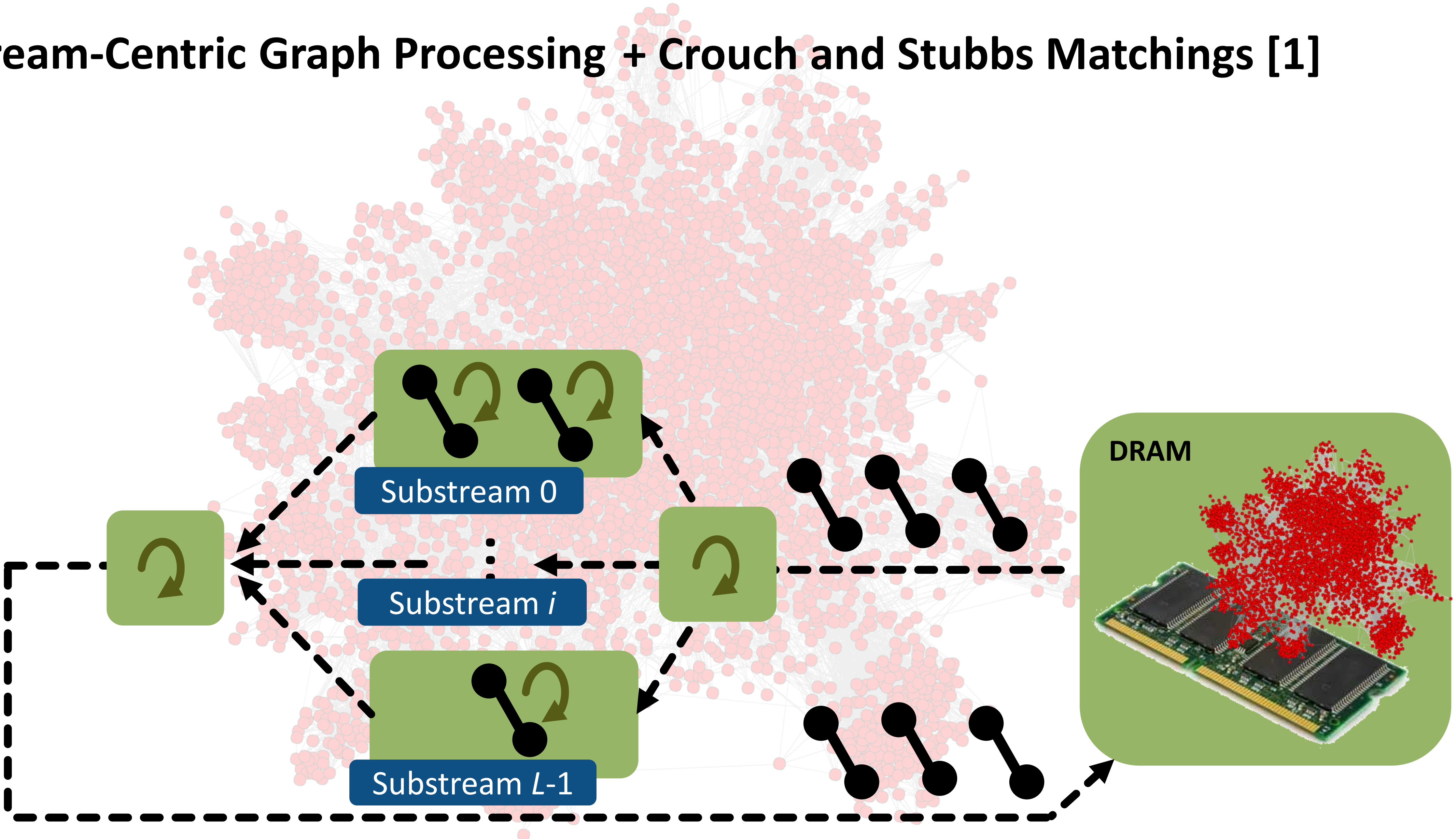

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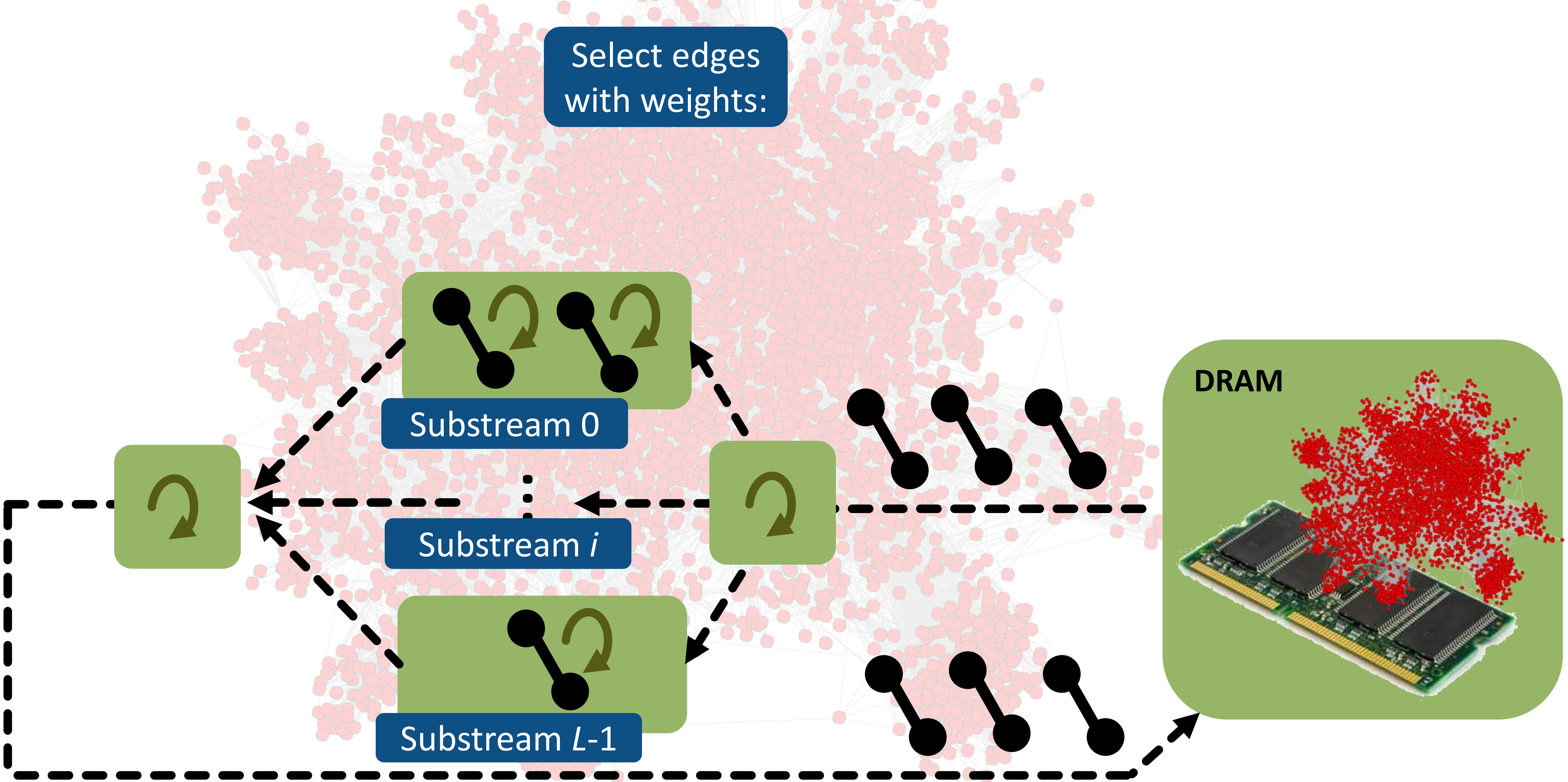

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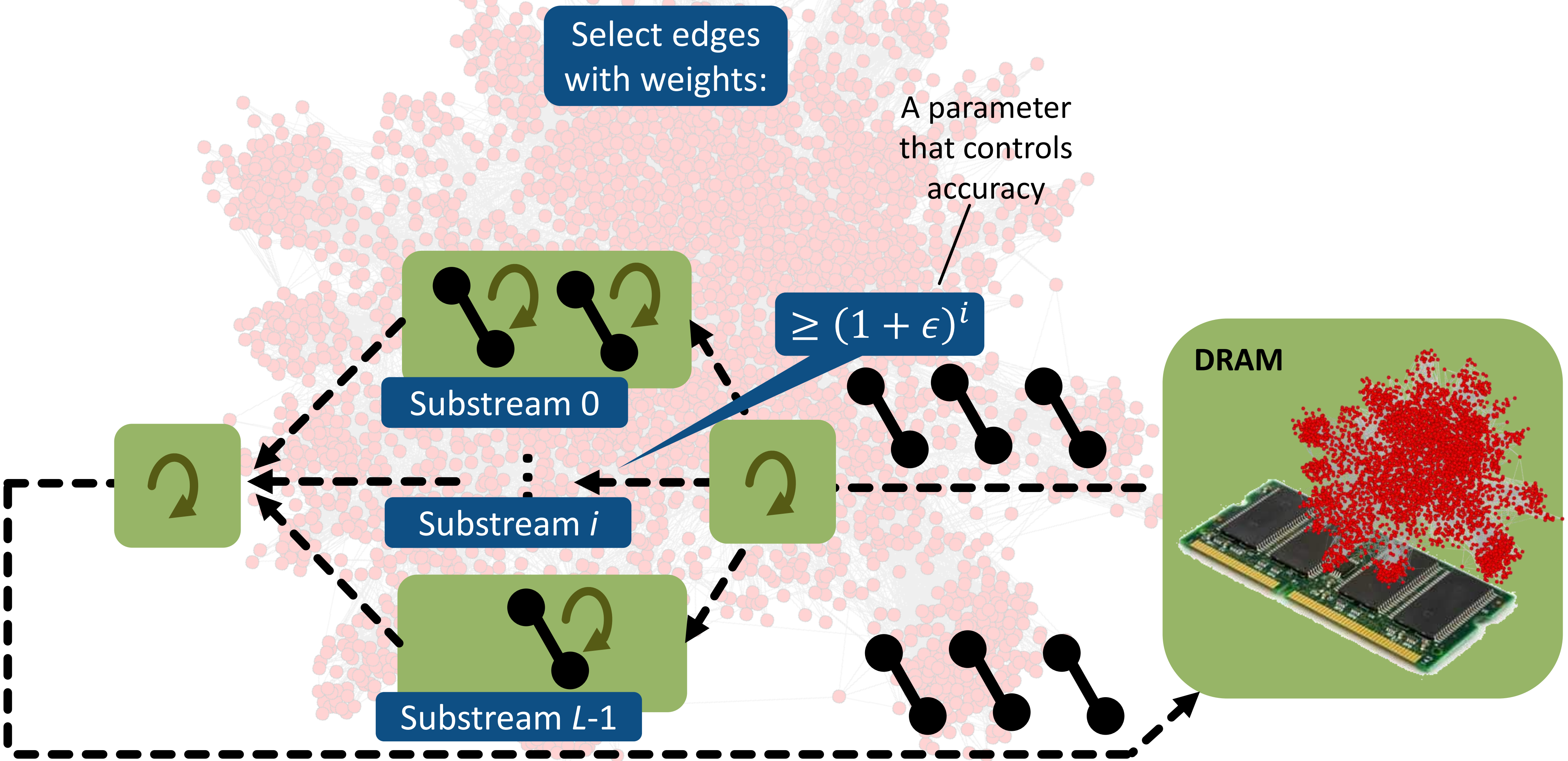

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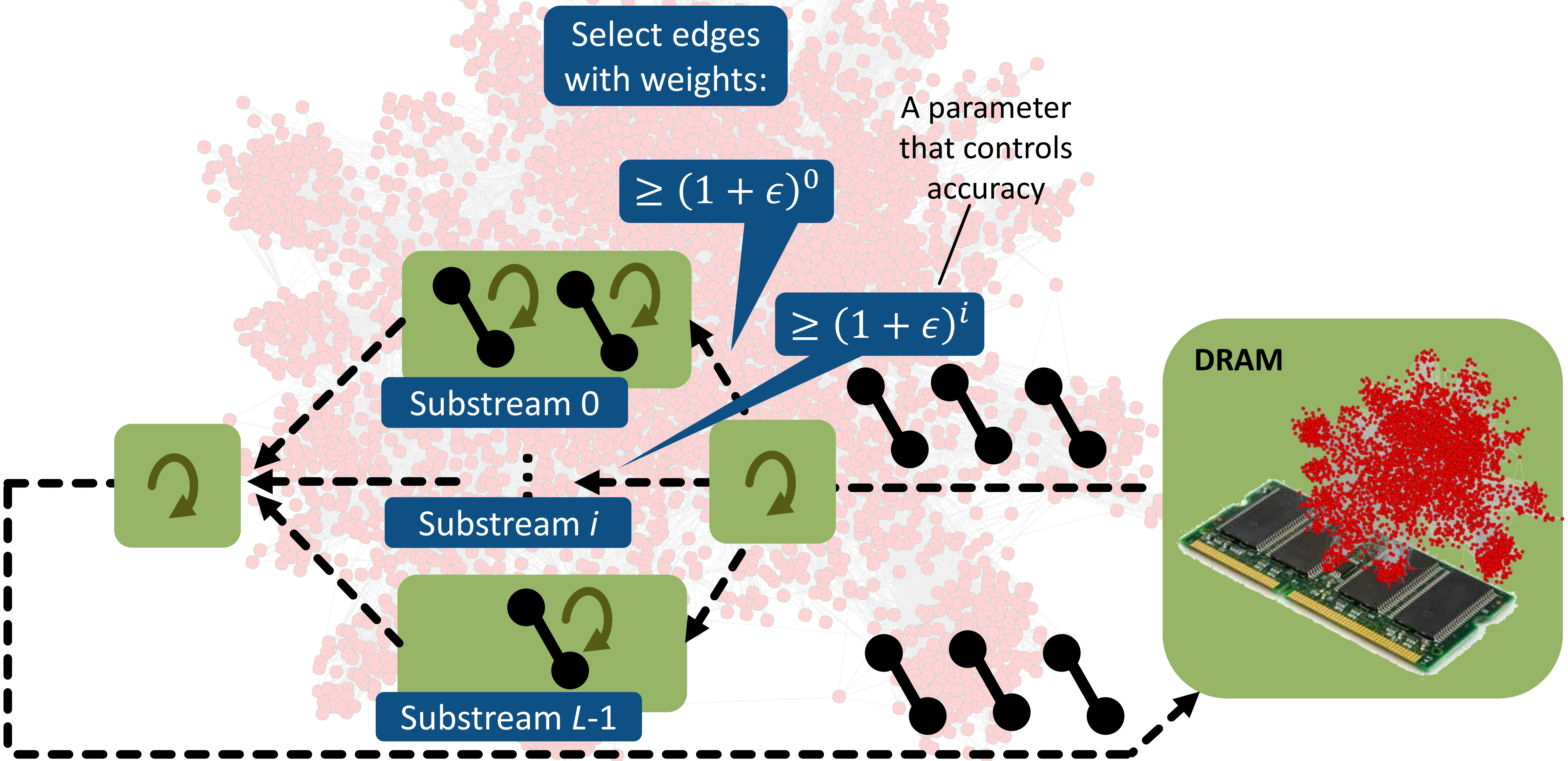

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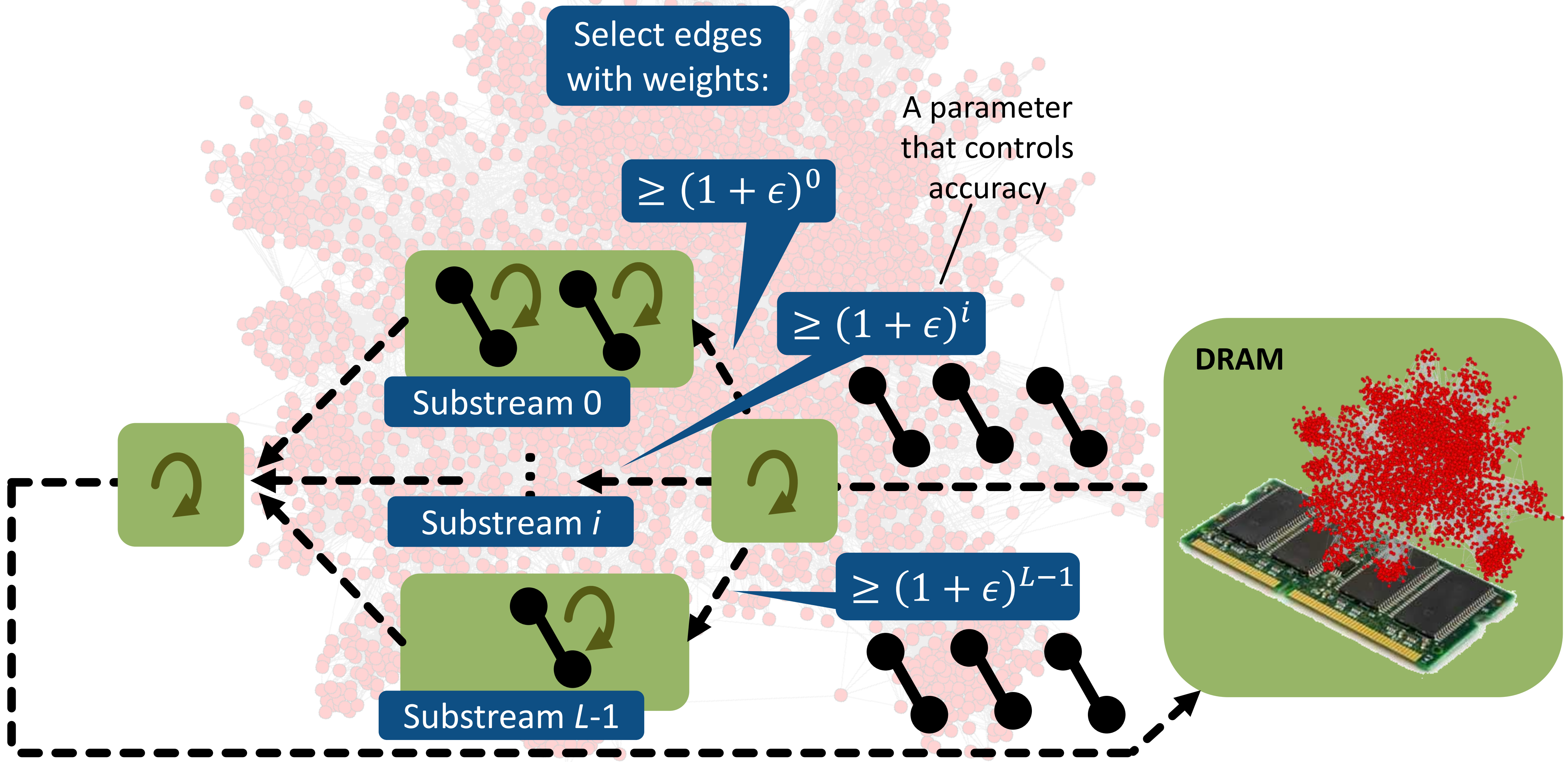

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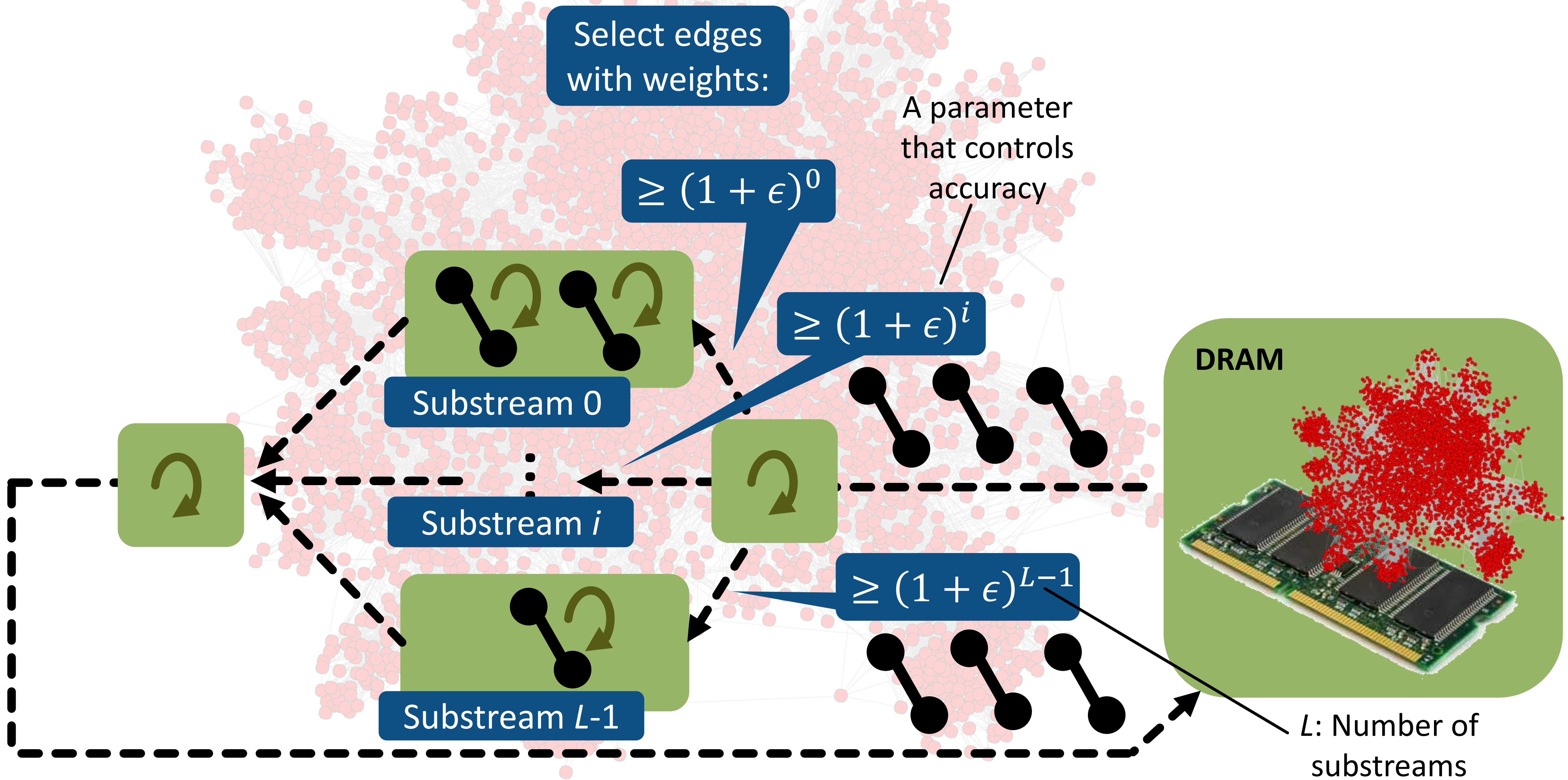

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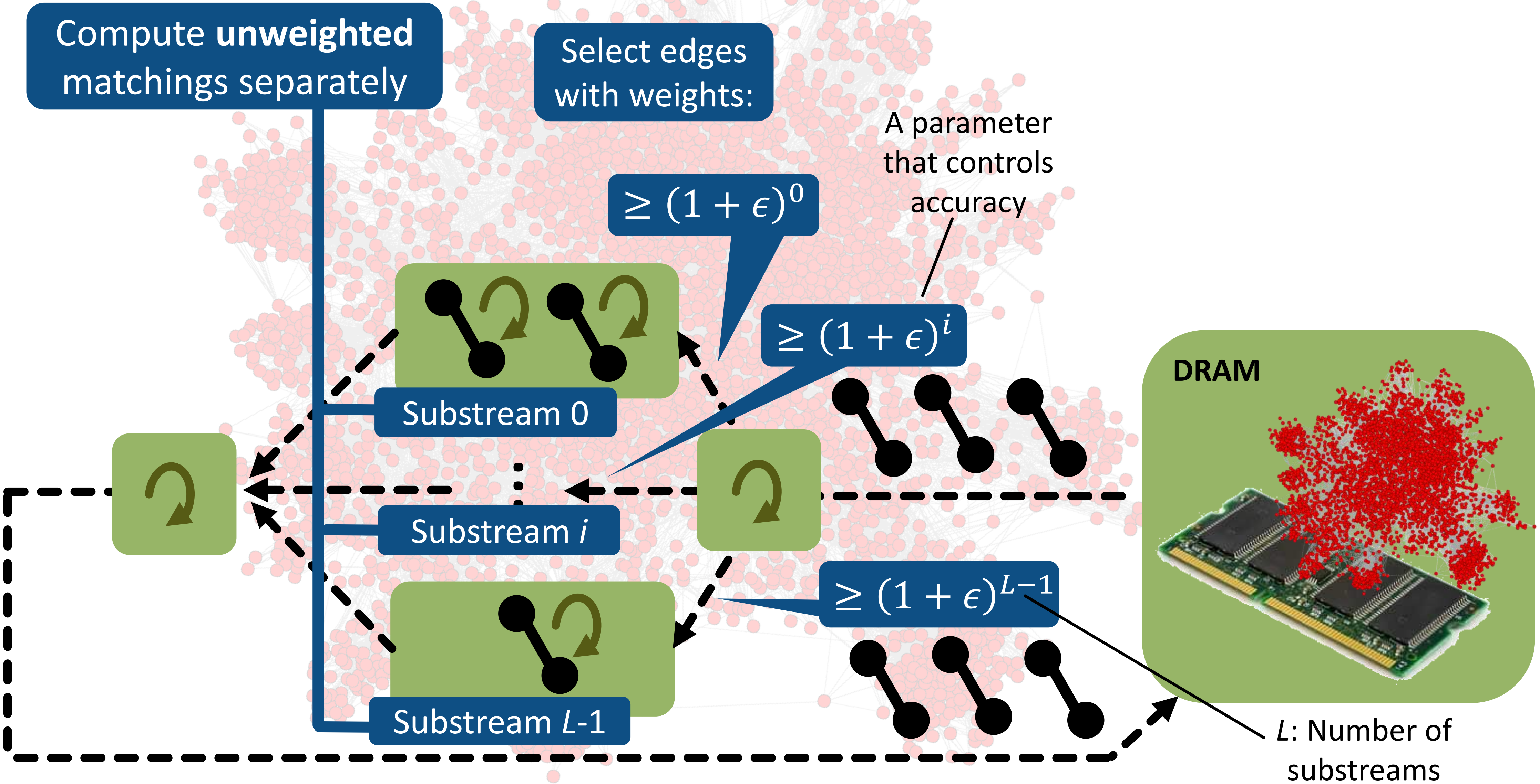

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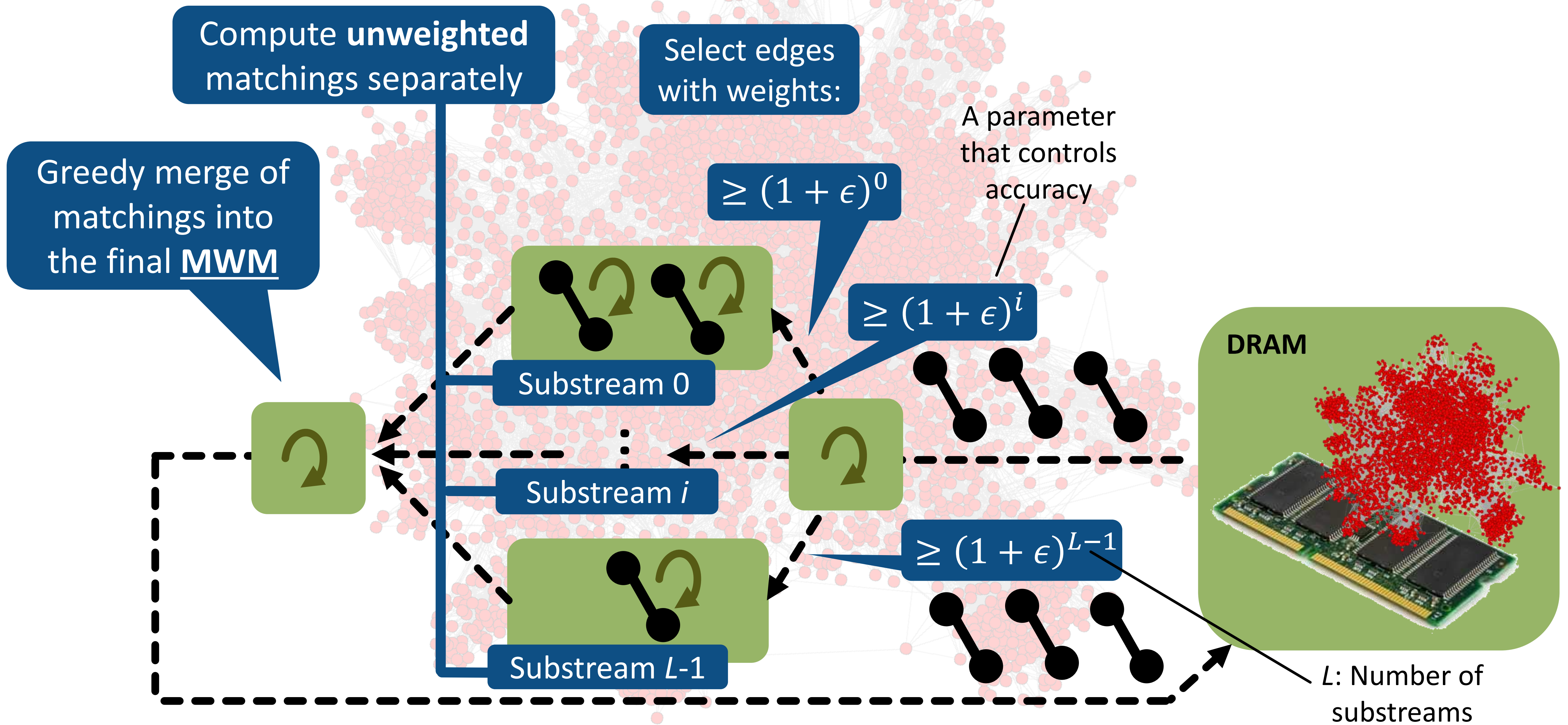

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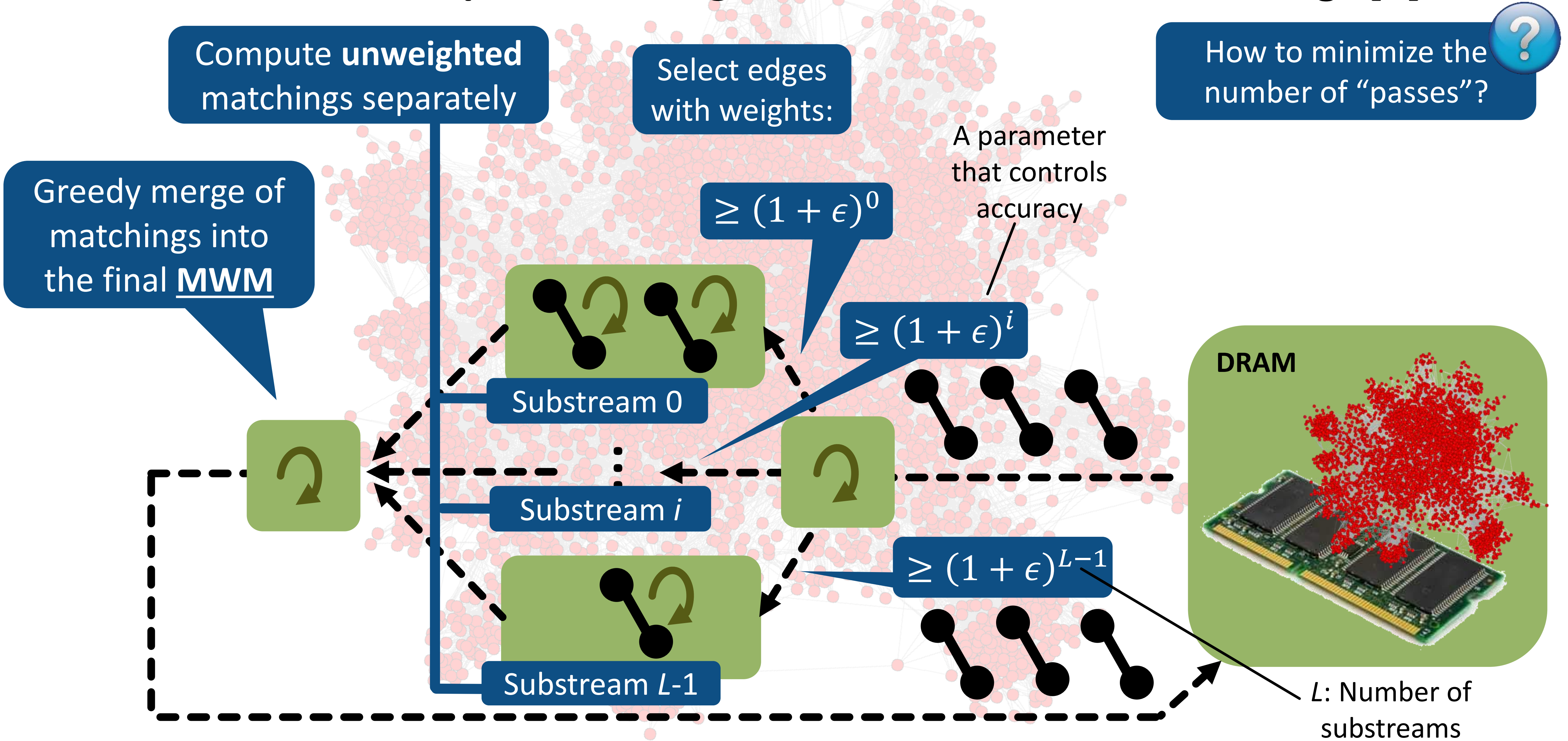

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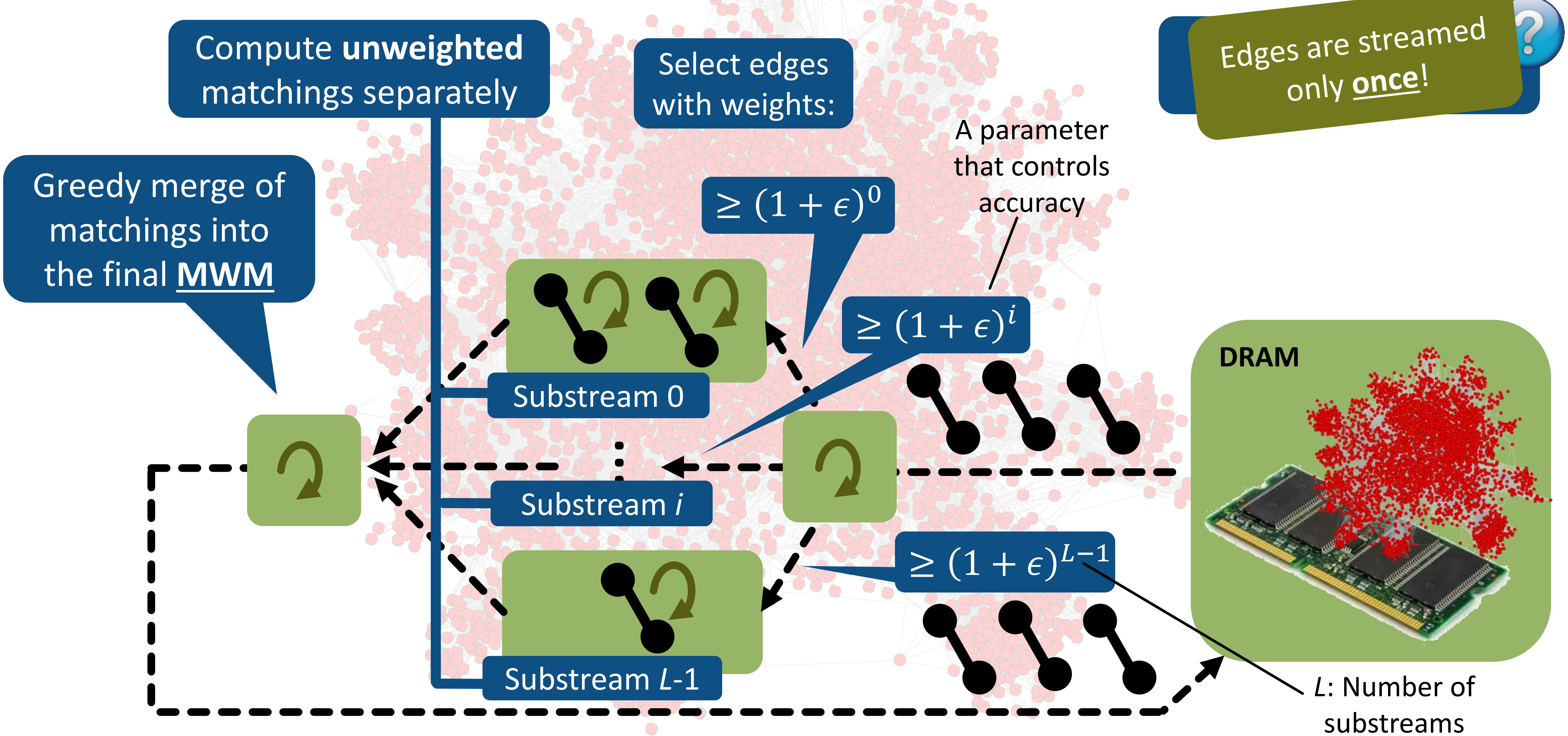

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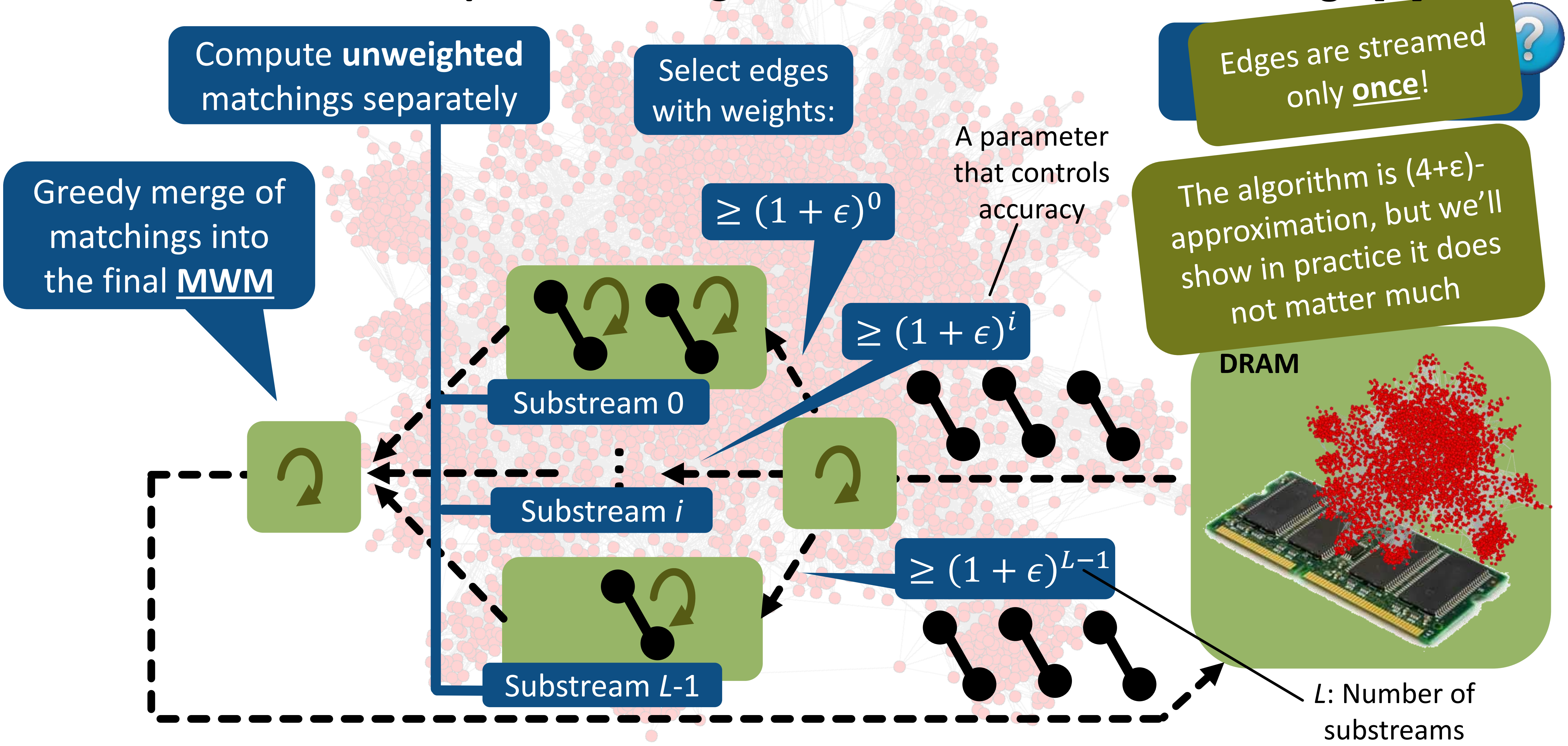

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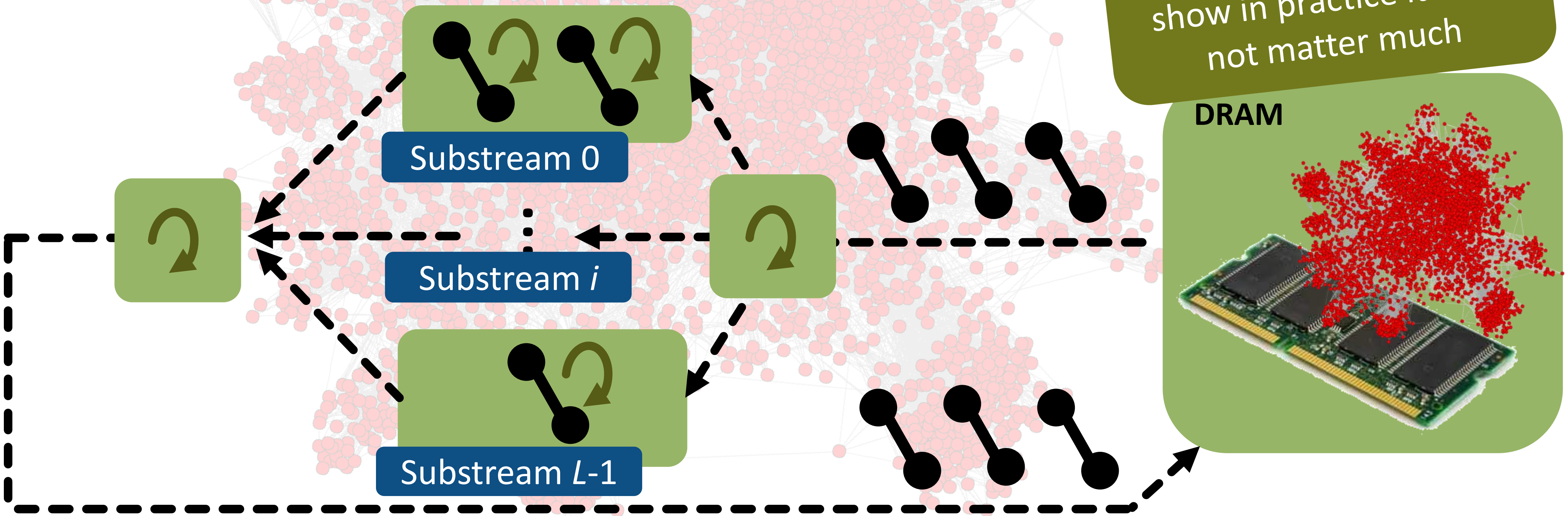
Edges are streamed only **once**!

The algorithm is (4+ε)-approximation, but we'll show in practice it does not matter much



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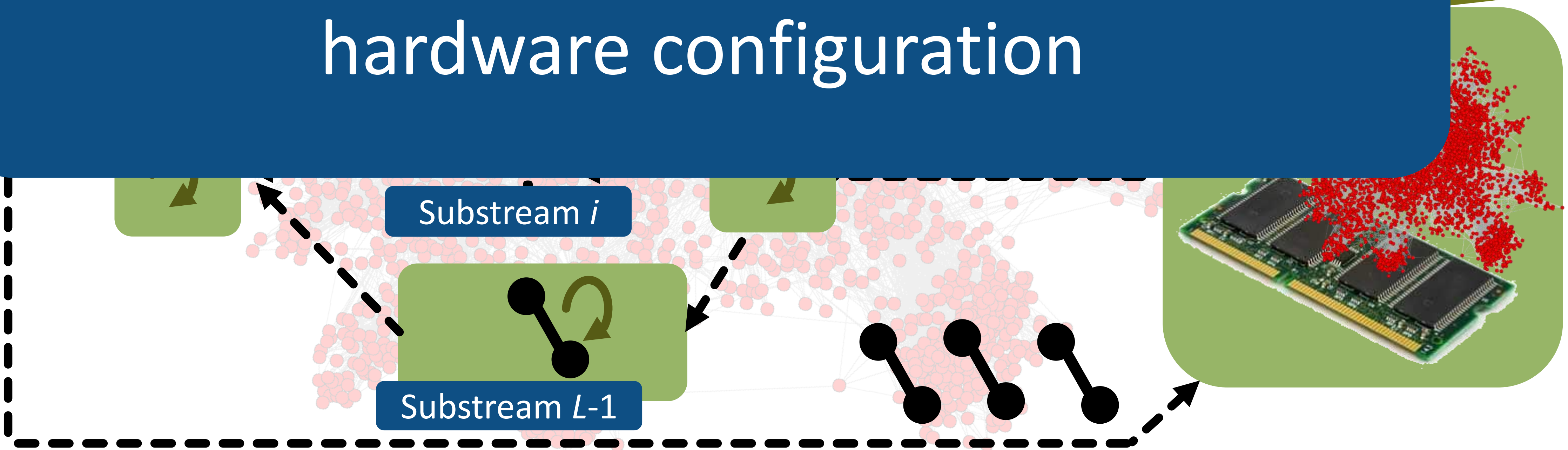
Mapping the algorithm to the „right” hardware configuration



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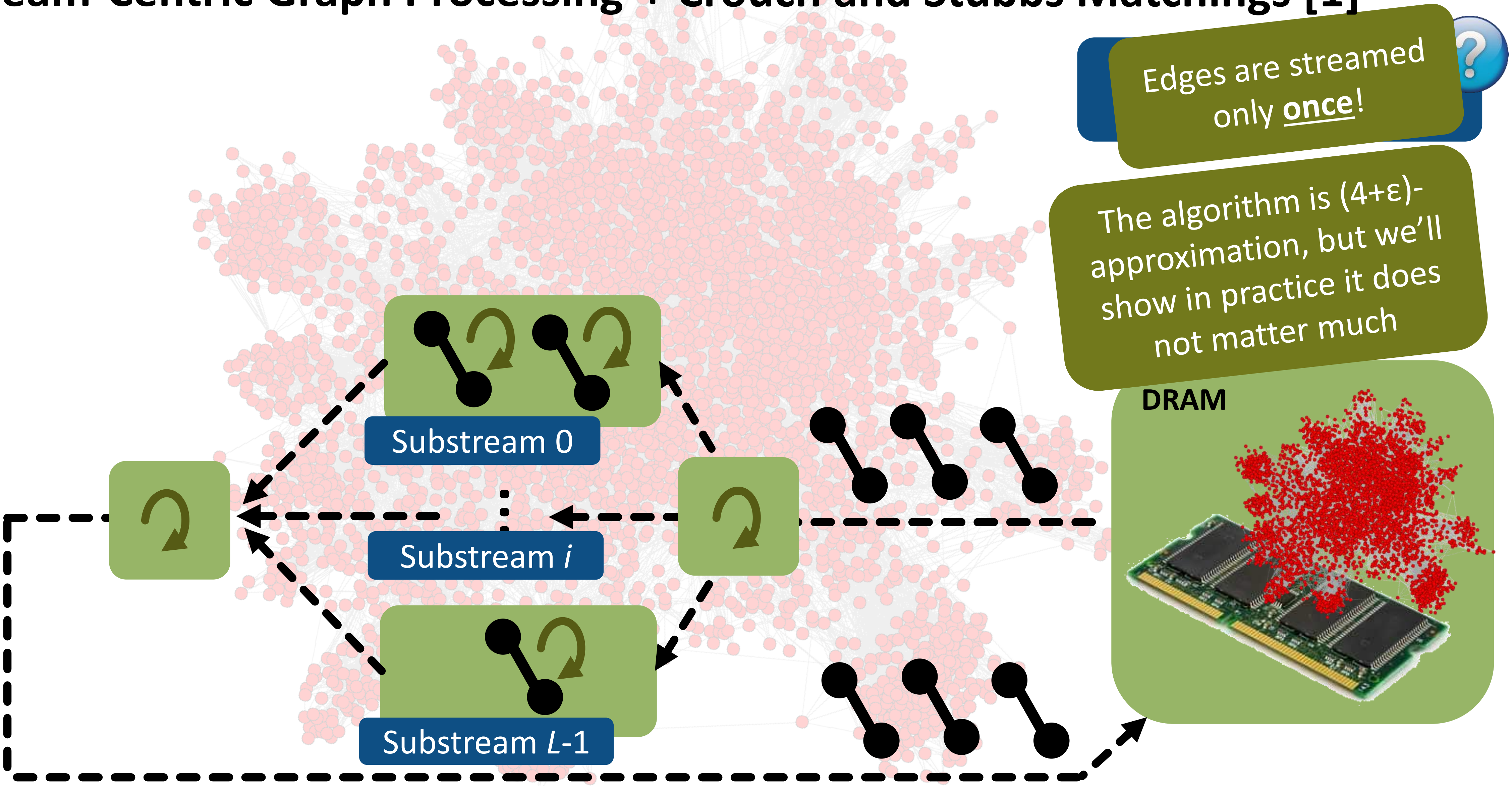

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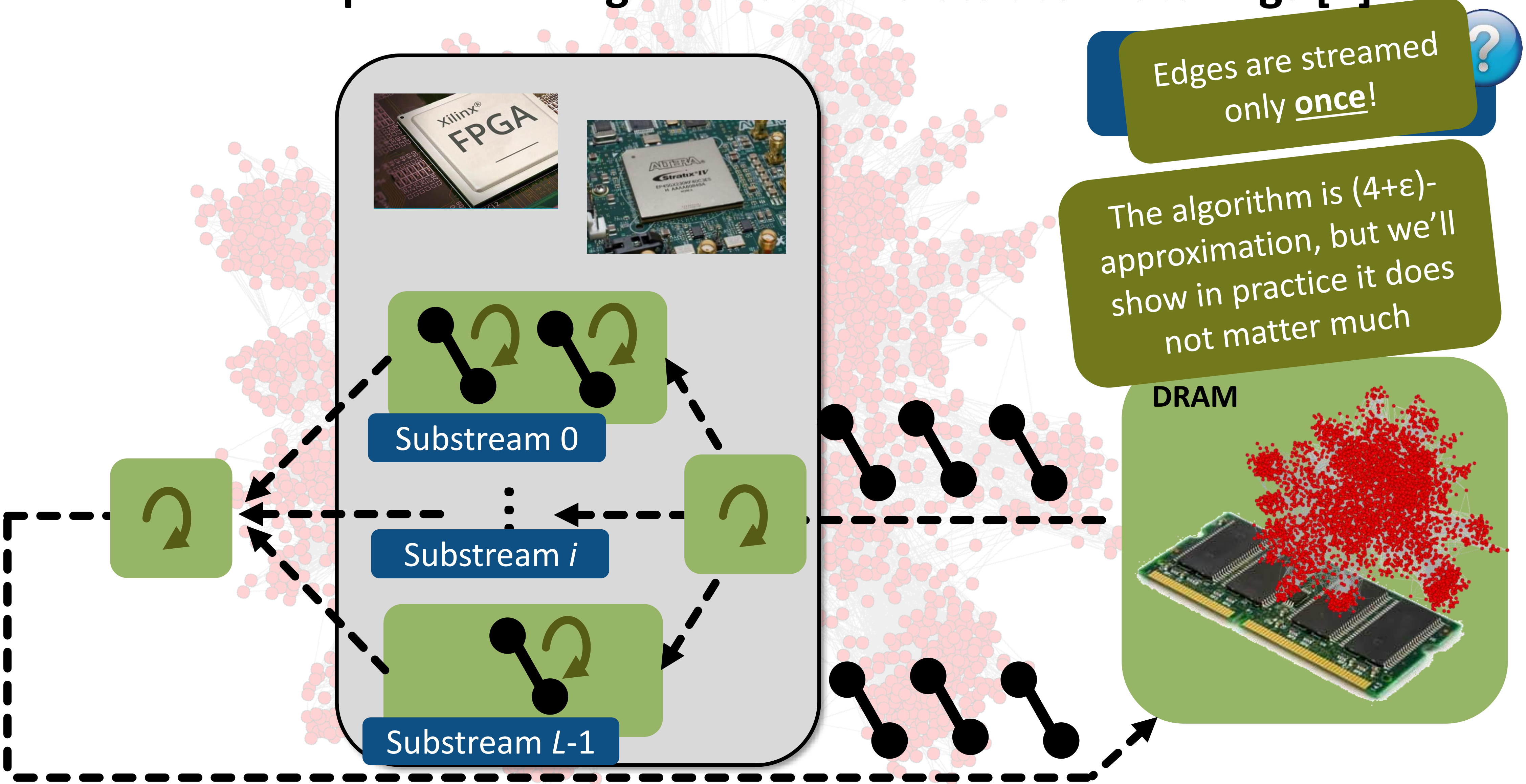

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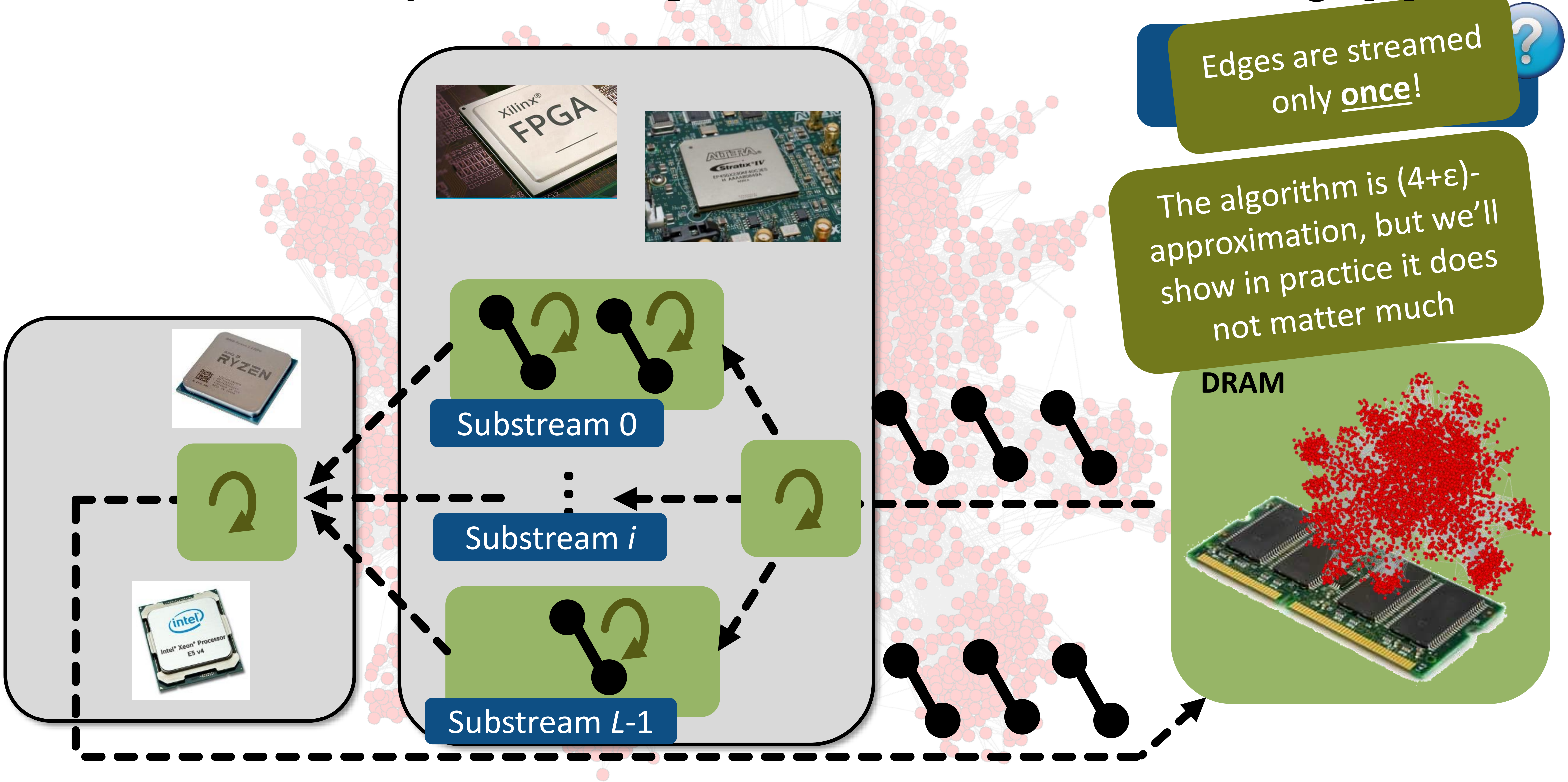

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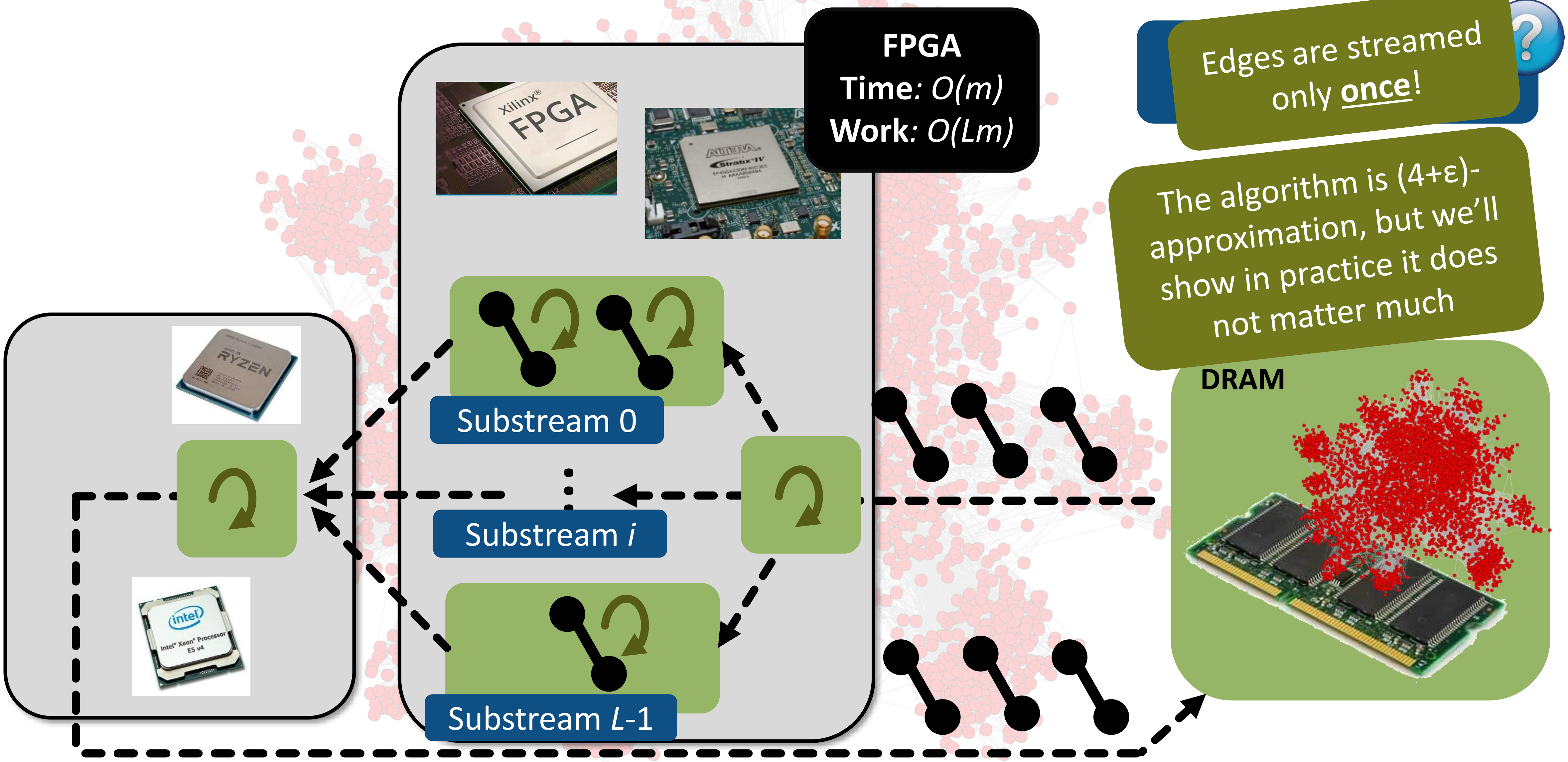

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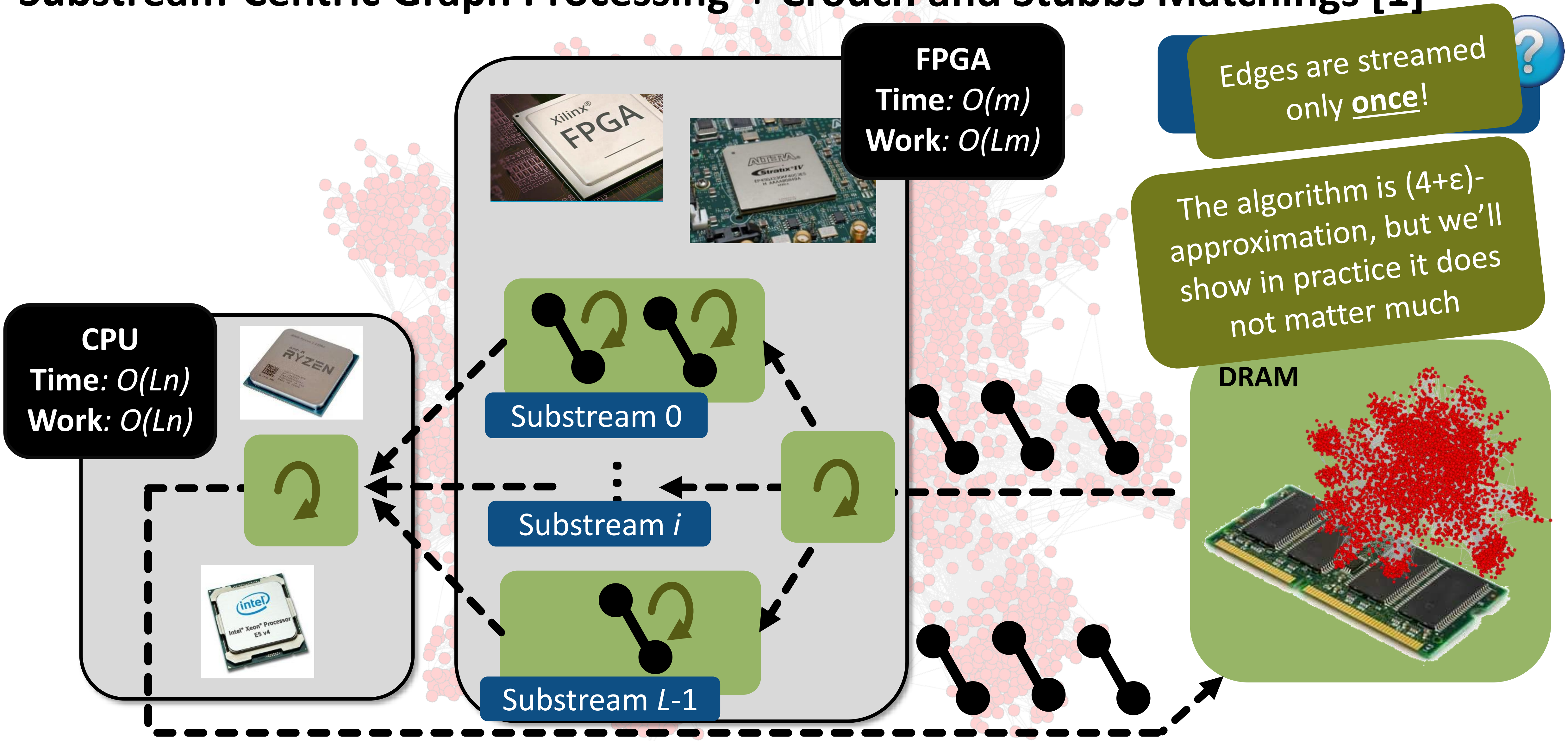

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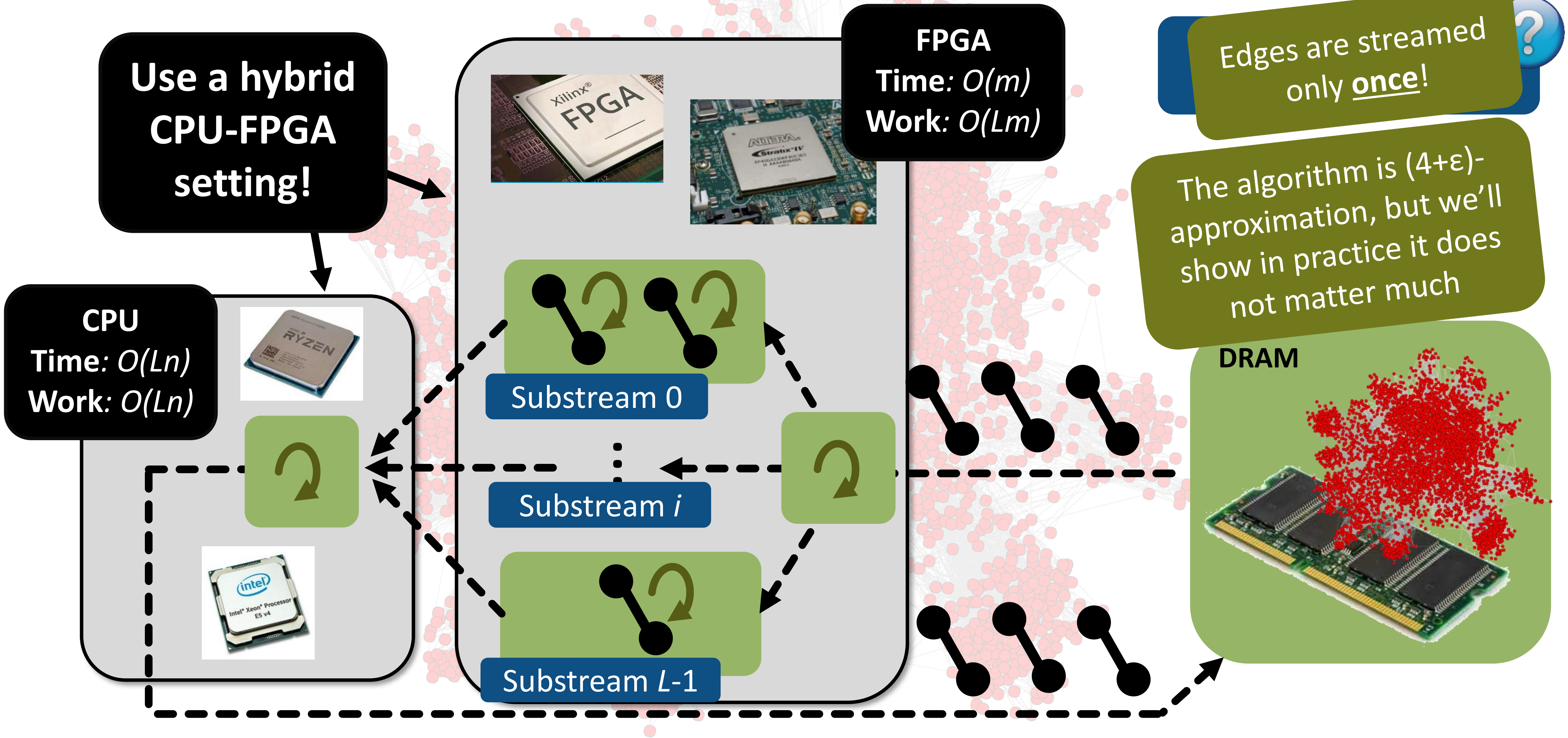

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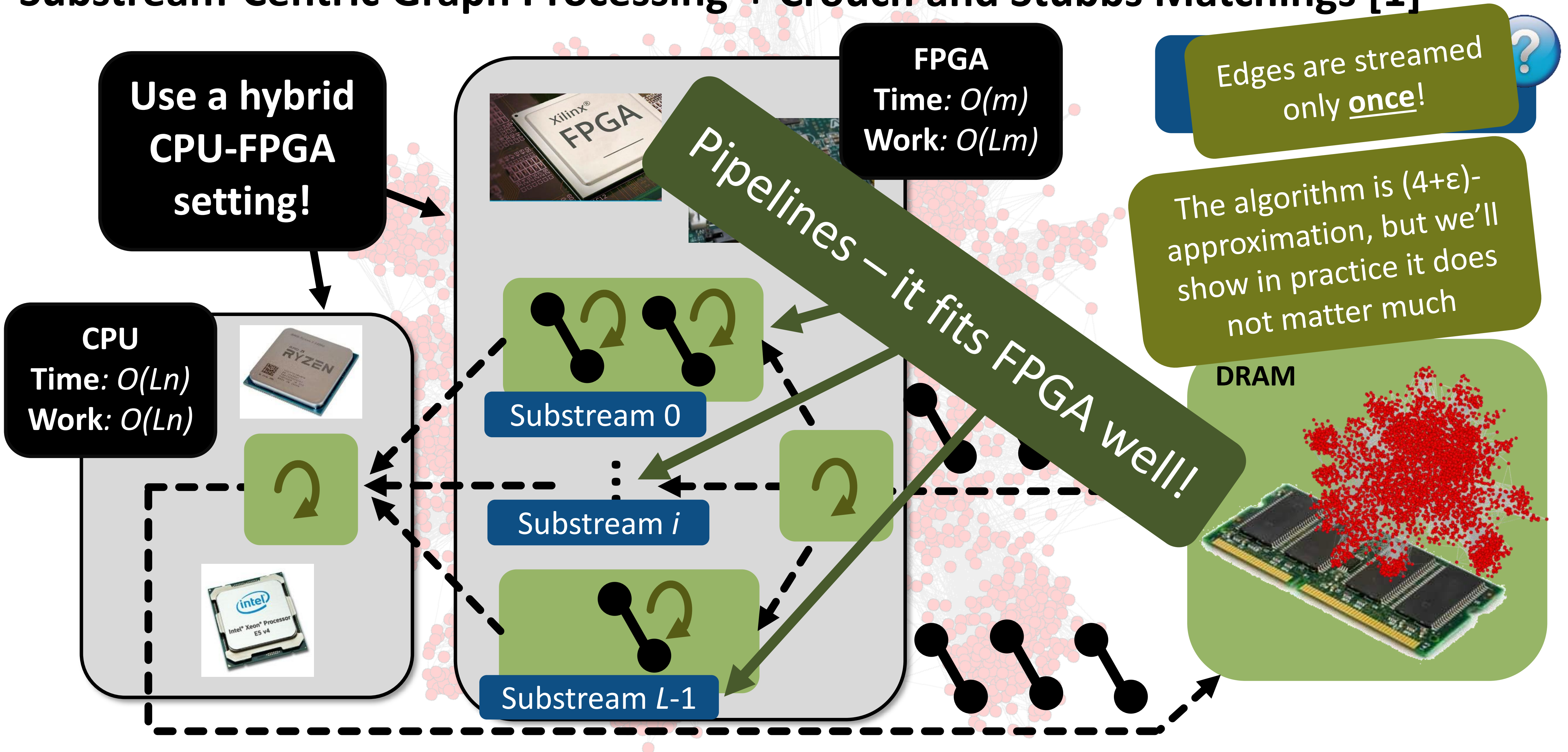

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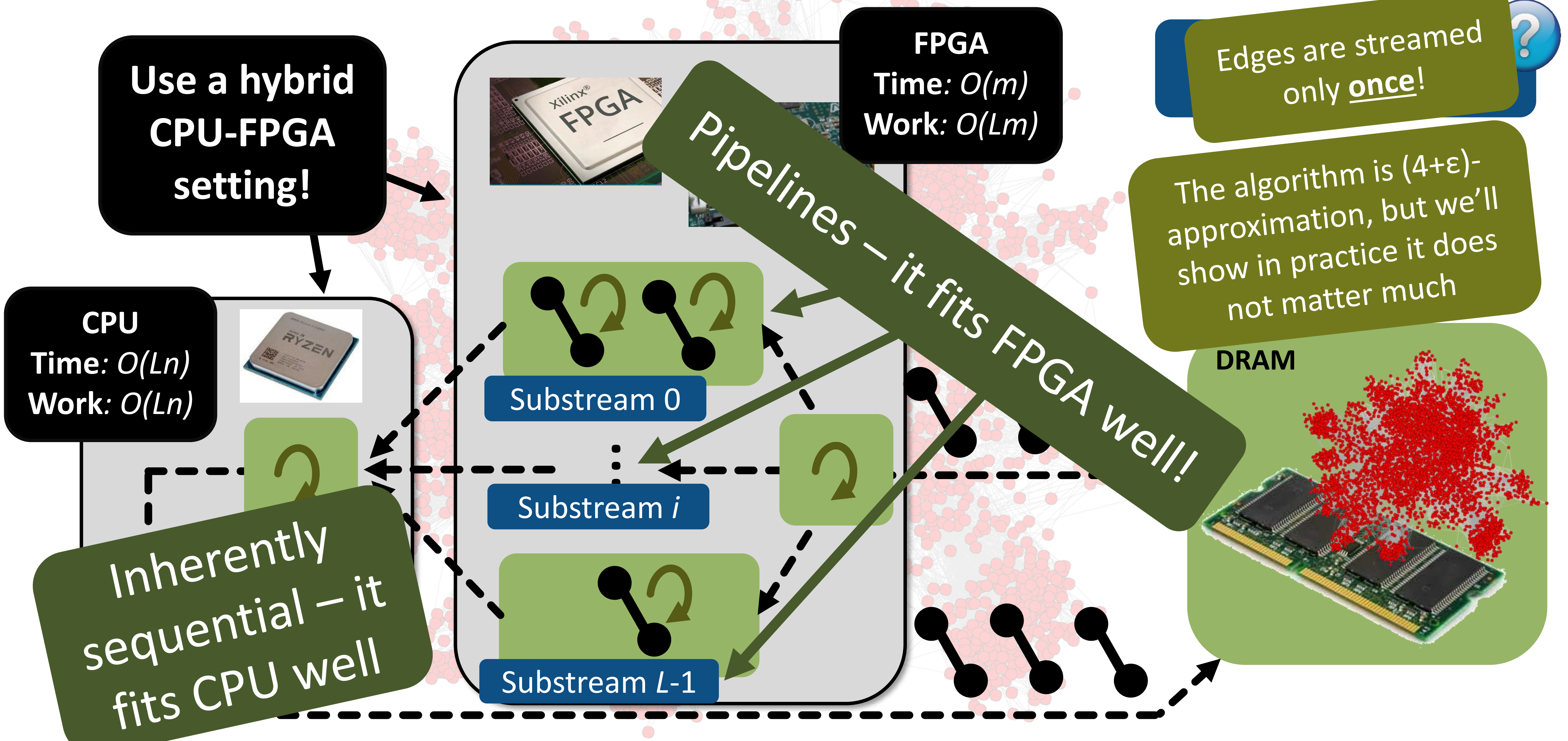

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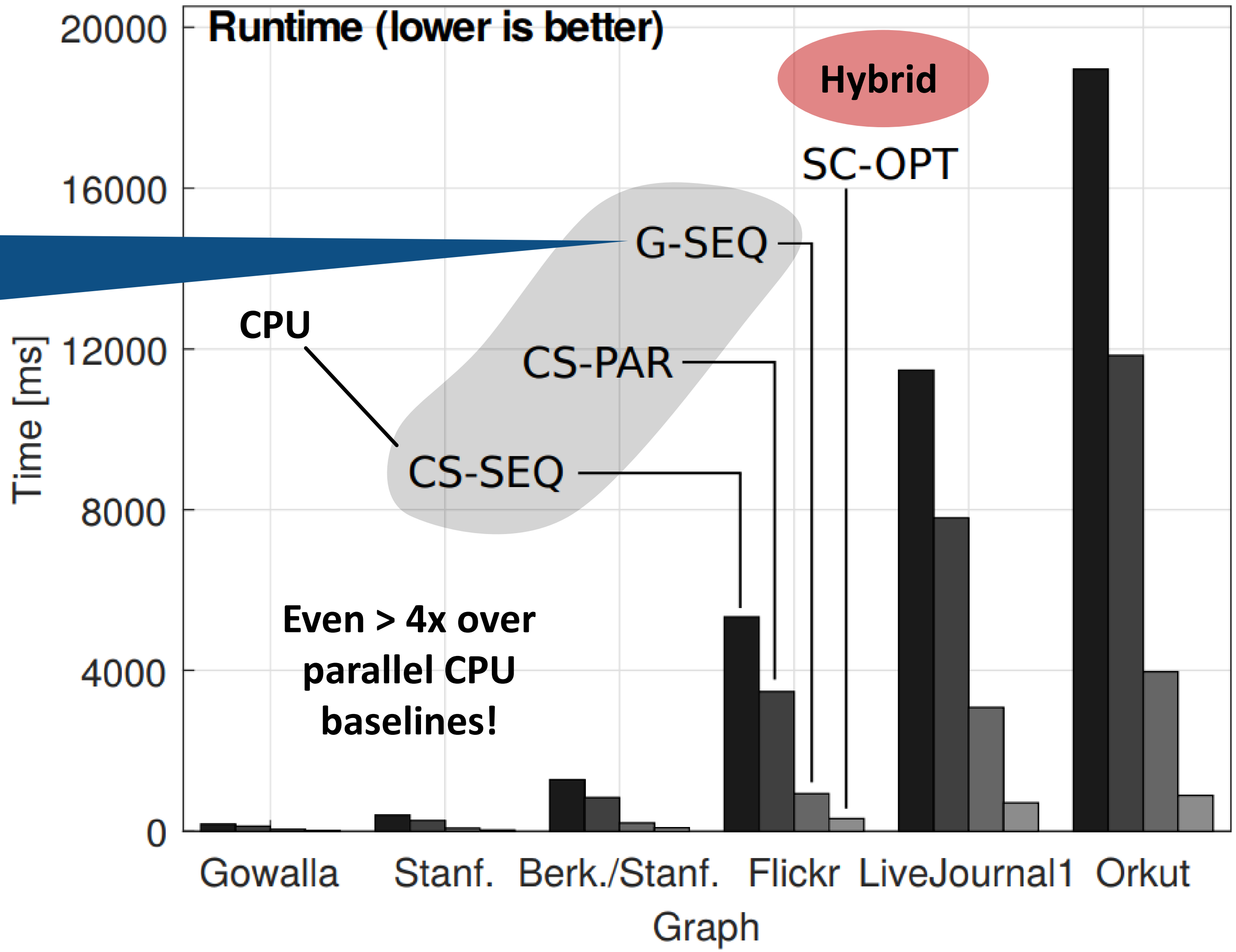
Performance Analysis
Various Graphs

Parameters:
#Substreams = 64, #Threads = 4

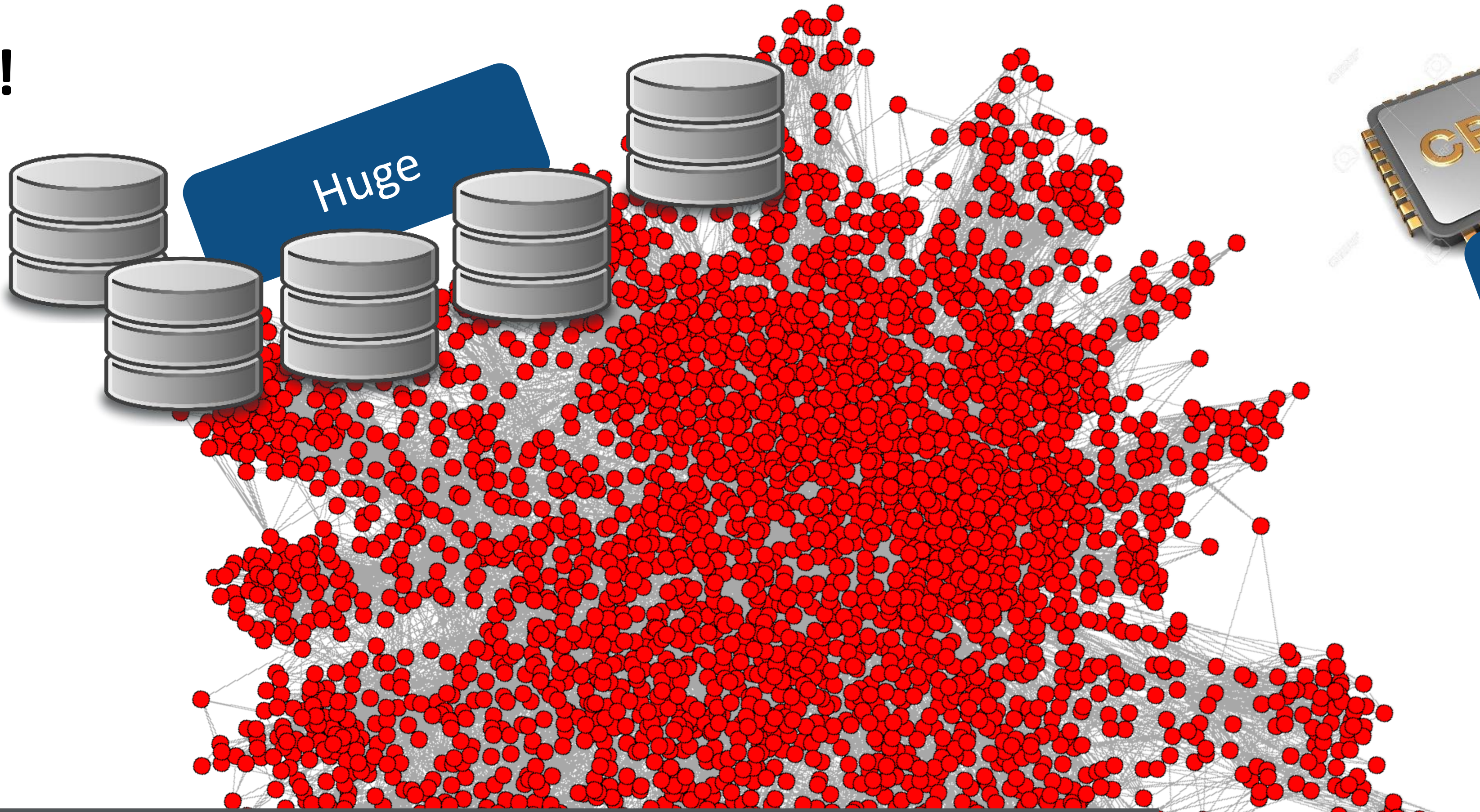
SC-OPT secures highest performance

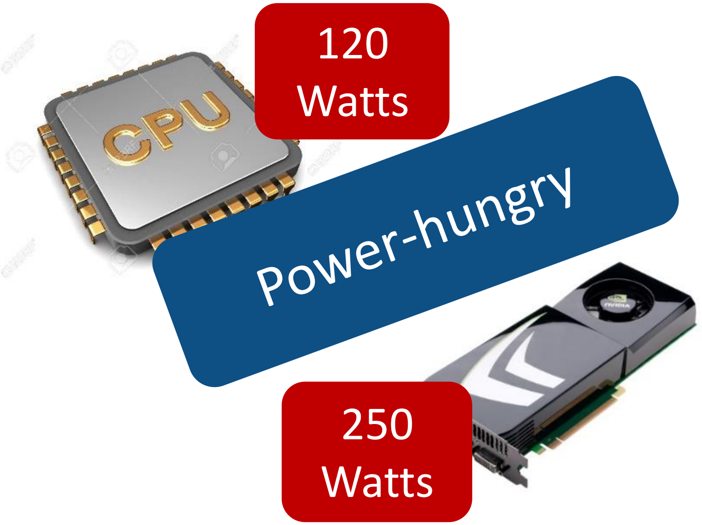
State-of-the-art MWM algorithm, space-optimal, **time-optimal (O(m))**, (2+ε)-approximation



Graph	Type	m	n
Kronecker	Synthetic power-law	$\approx 48n$	$2^k; k = 16, \ldots, 21$
Gowalla	Social network	950,327	196,591
Flickr	Social network	33,140,017	2,302,925
LiveJournal1	Social network	68,993,773	4,847,571
Orkut	Social network	117,184,899	3,072,441
Stanford	Hyperlink graph	2,312,497	281,903
Berkeley	Hyperlink graph	7,600,595	685,230
arXiv hep-th	Citation graph	352,807	27,770

Problems!





Substream-Centric Maximum Matchings on FPGA

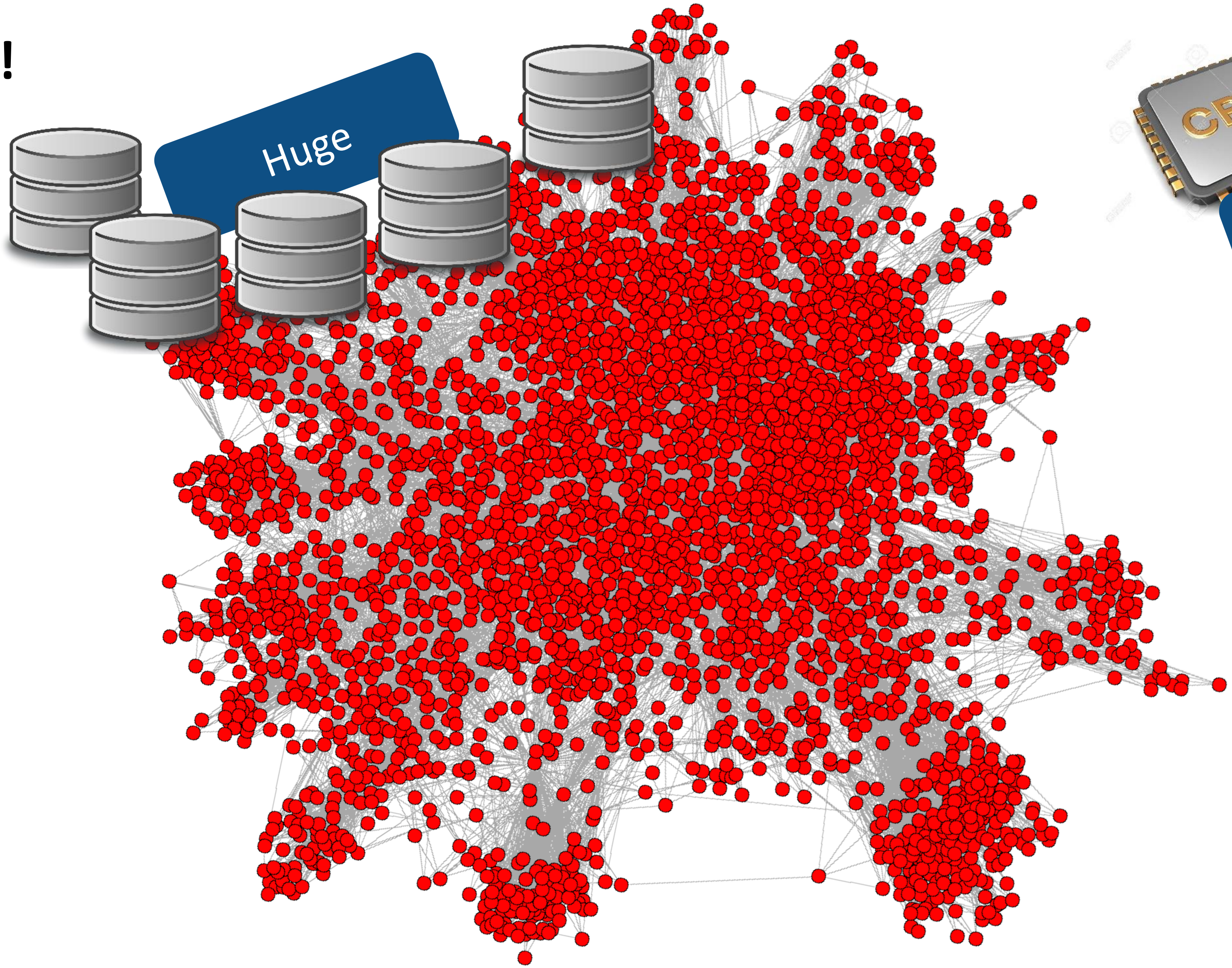
Maciej Besta, Marc Fischer, Tal Ben-Nun, Johannes De Fine Licht, Torsten Hoefler
Department of Computer Science, ETH Zurich

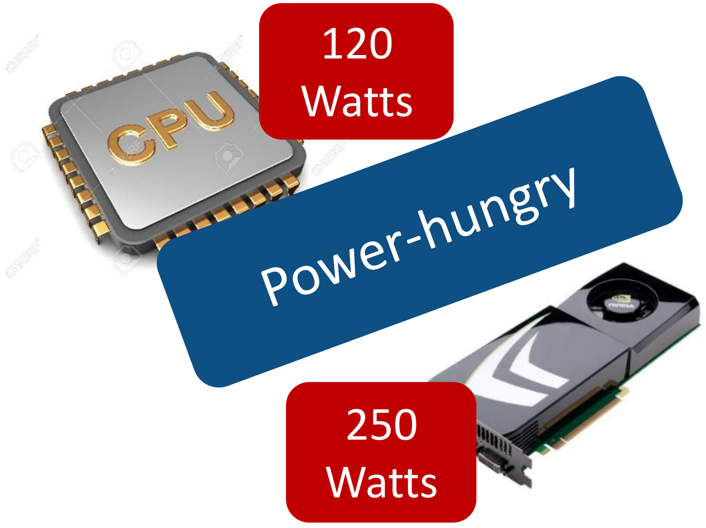
ABSTRACT

Developing high-performance and energy-efficient algorithms for maximum matchings is becoming increasingly important in social network analysis, computational sciences, schedul-

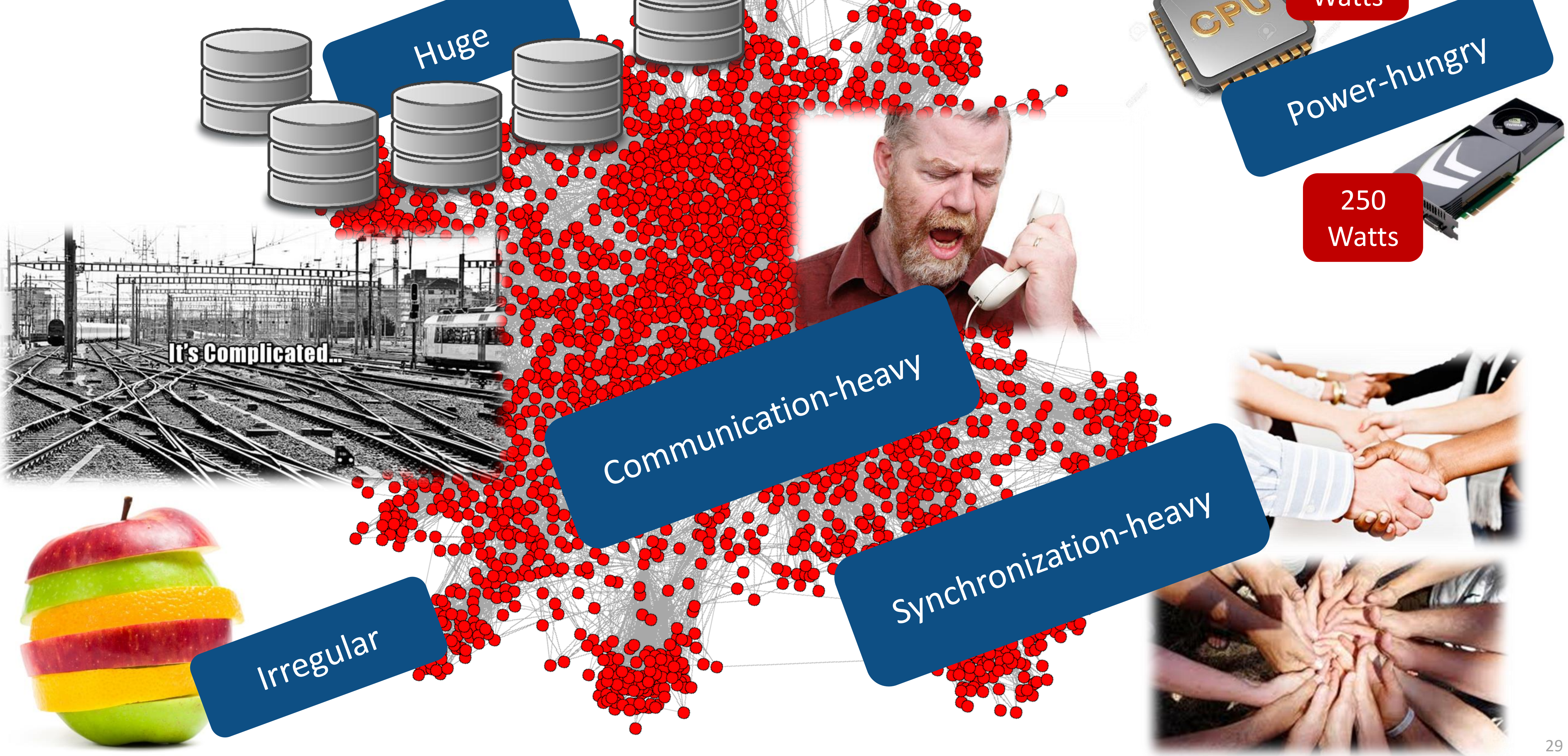
compound can be used to show the locations of double bonds in the chemical structure [59]. As deriving the exact MM is usually computationally expensive, significant focus has been placed on developing fast approximate solutions [17].

Problems!

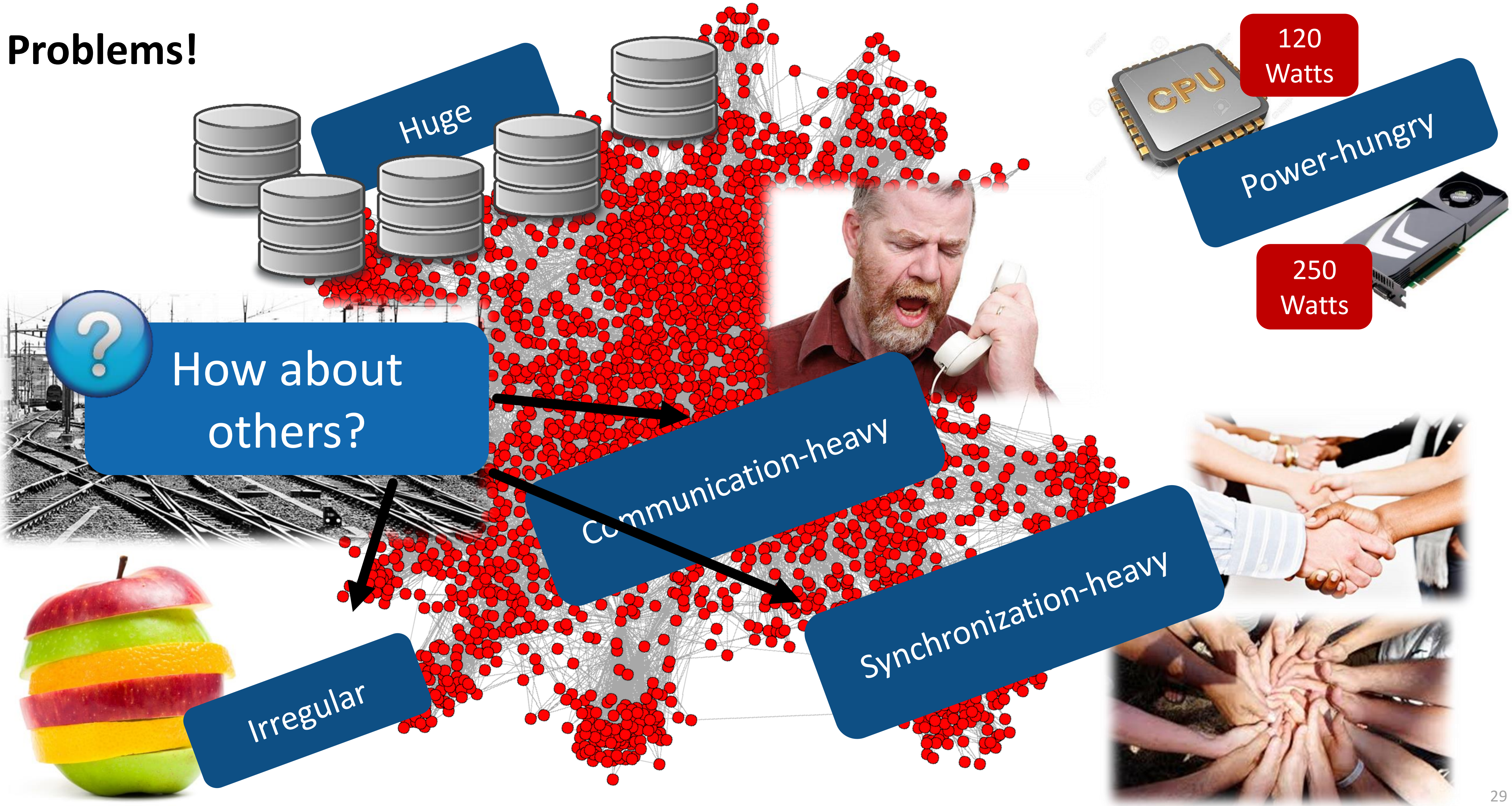




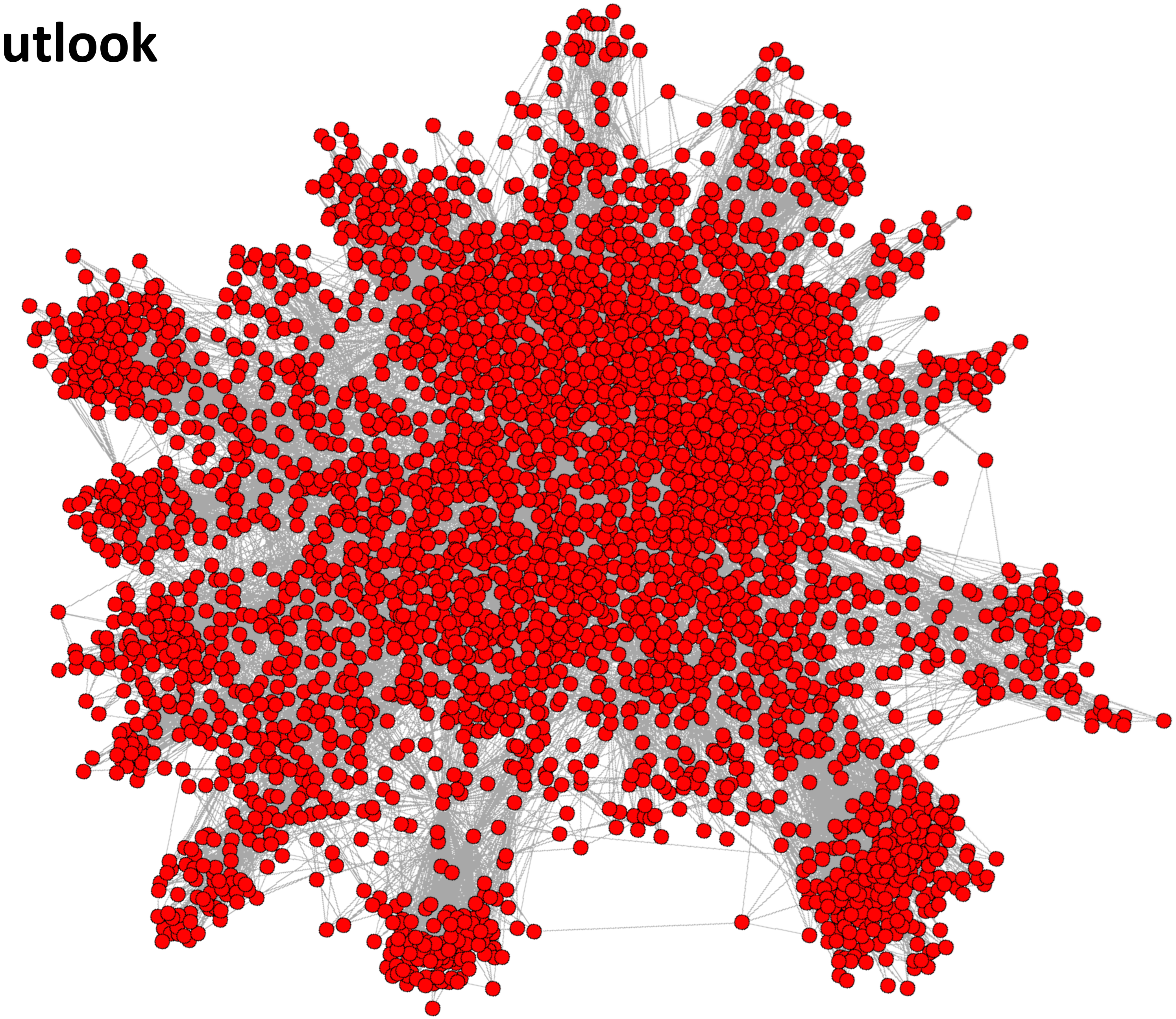
Problems!


Huge
120
Watts
CPU
Power-hungry
250
Watts
It's Complicated...
Communication-heavy
Synchronization-heavy
Irregular

Problems!


Huge
120
Watts
Power-hungry
250
Watts
How about
others?
Communication-heavy
Synchronization-heavy
Irregular

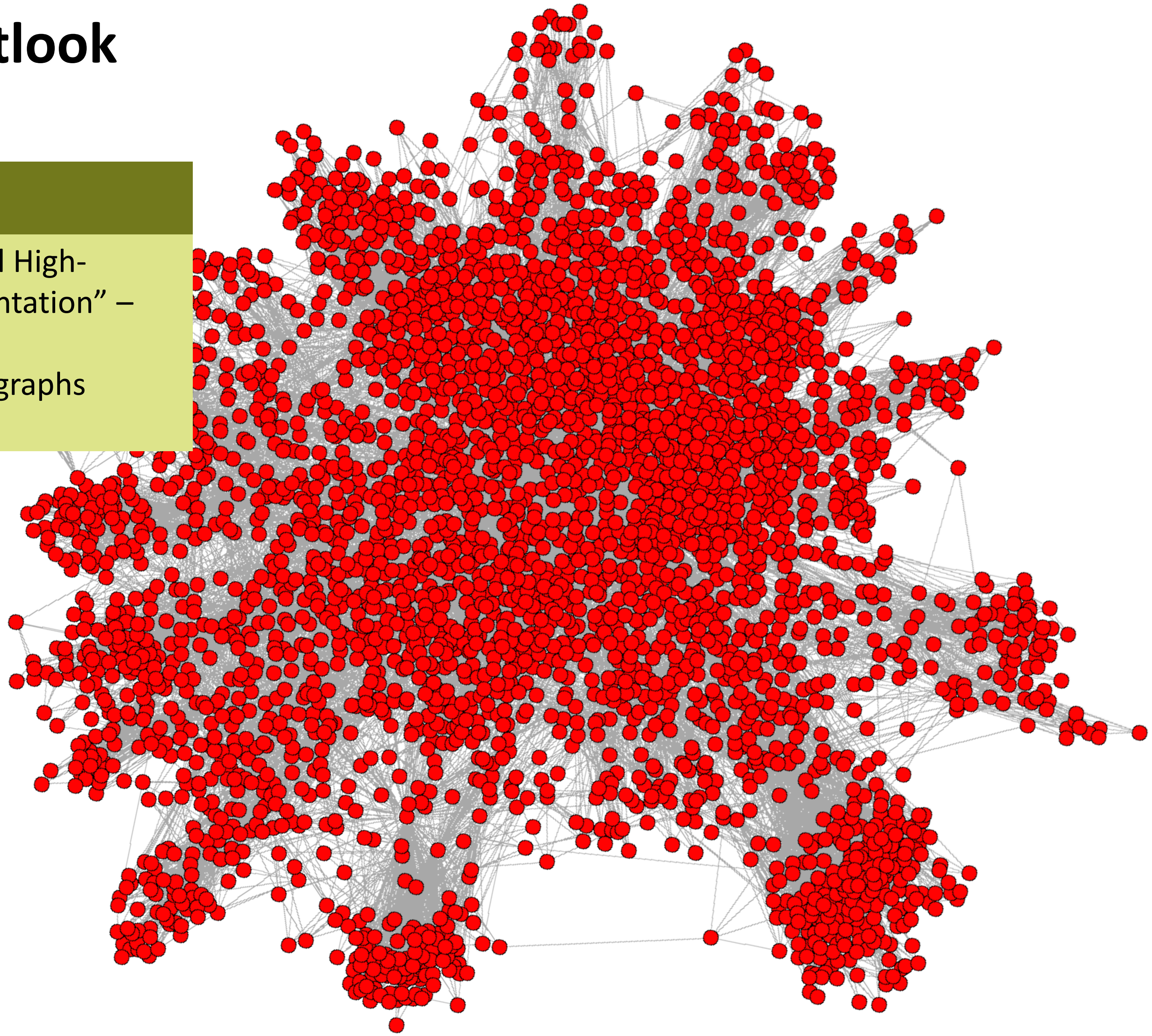
Summary and outlook



Summary and outlook

Log(Graph)

- "Log(Graph): A Near-Optimal High-Performance Graph Representation" – ACM PACT'18
- Minimal storage bounds for graphs during processing



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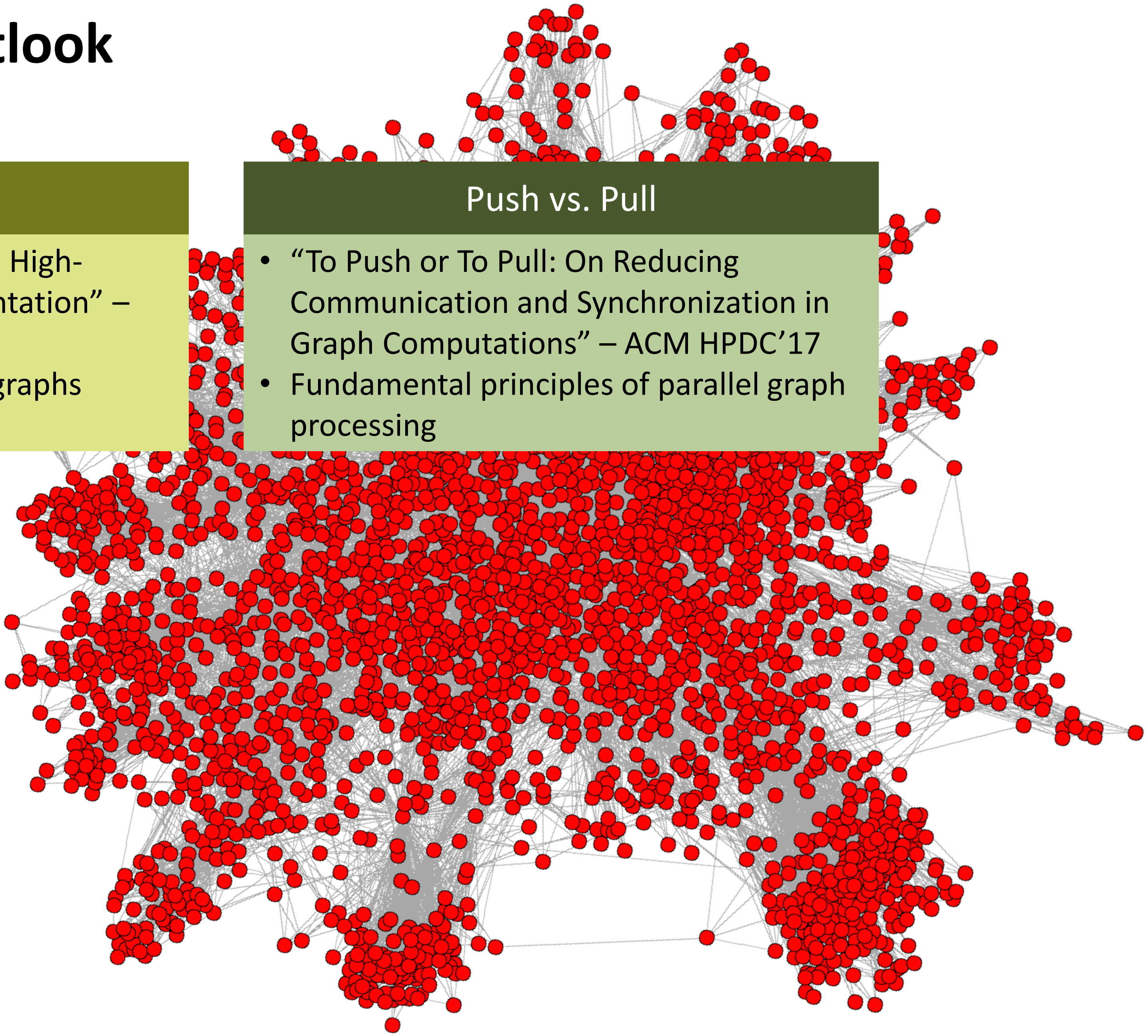

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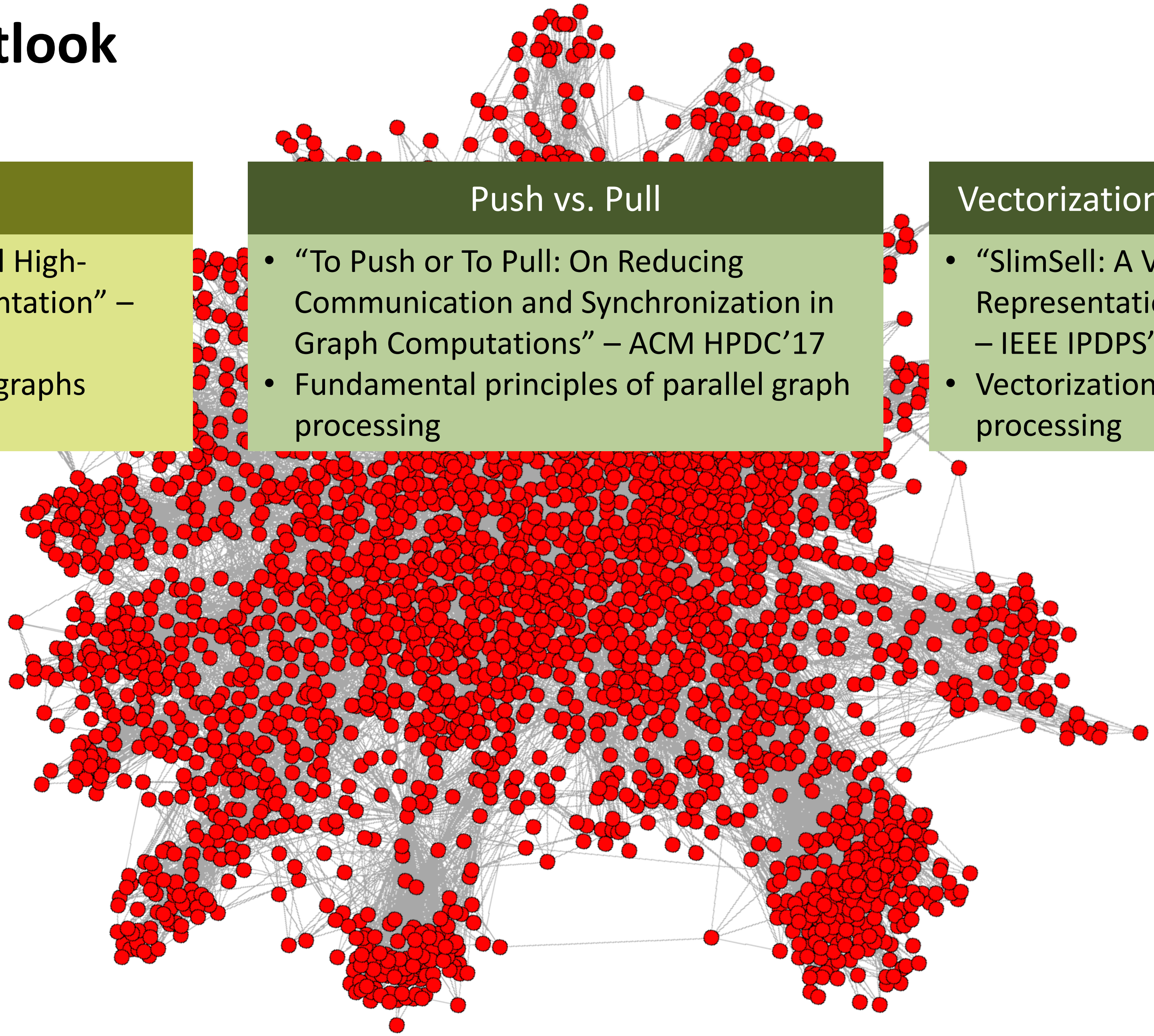

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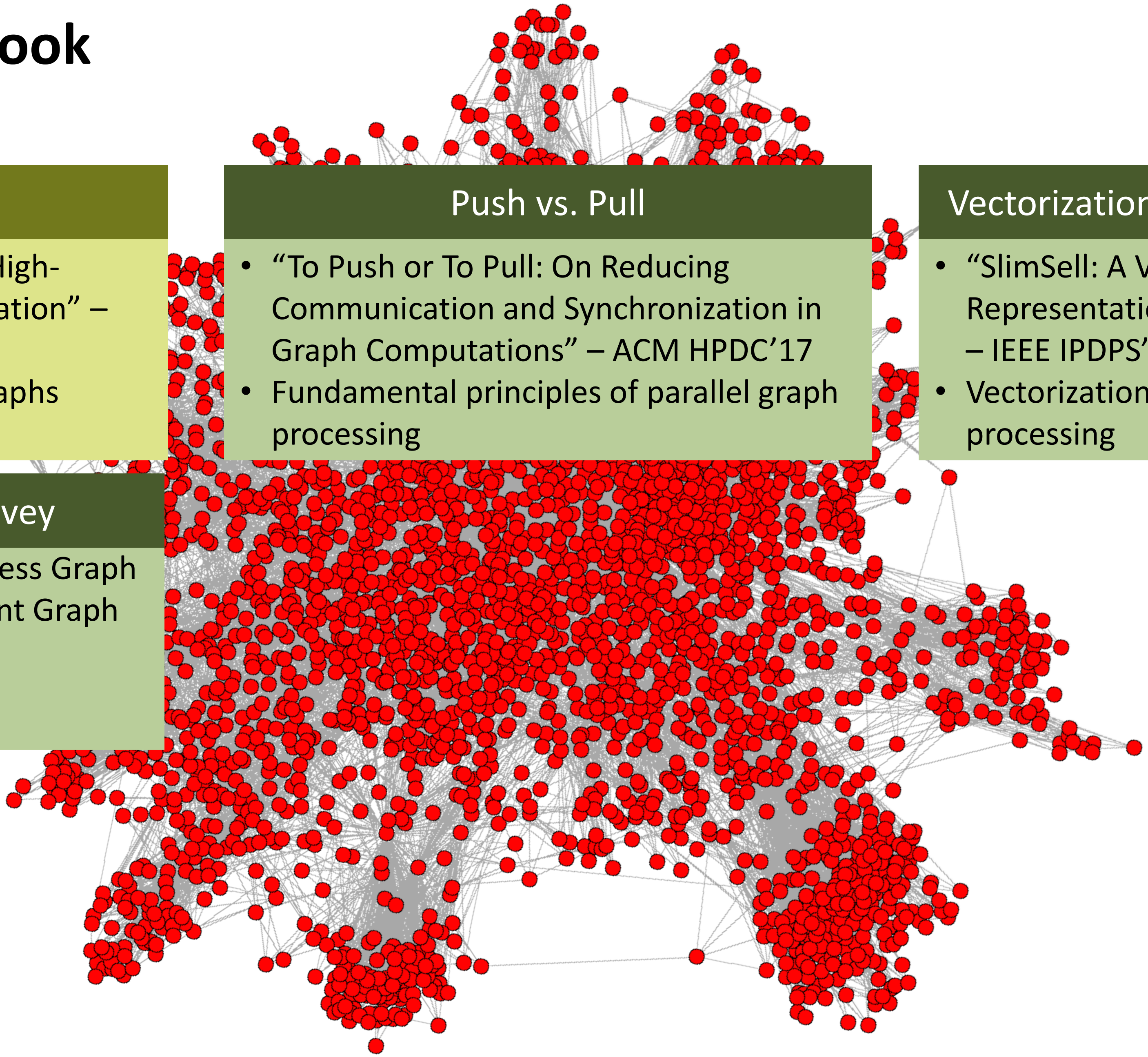

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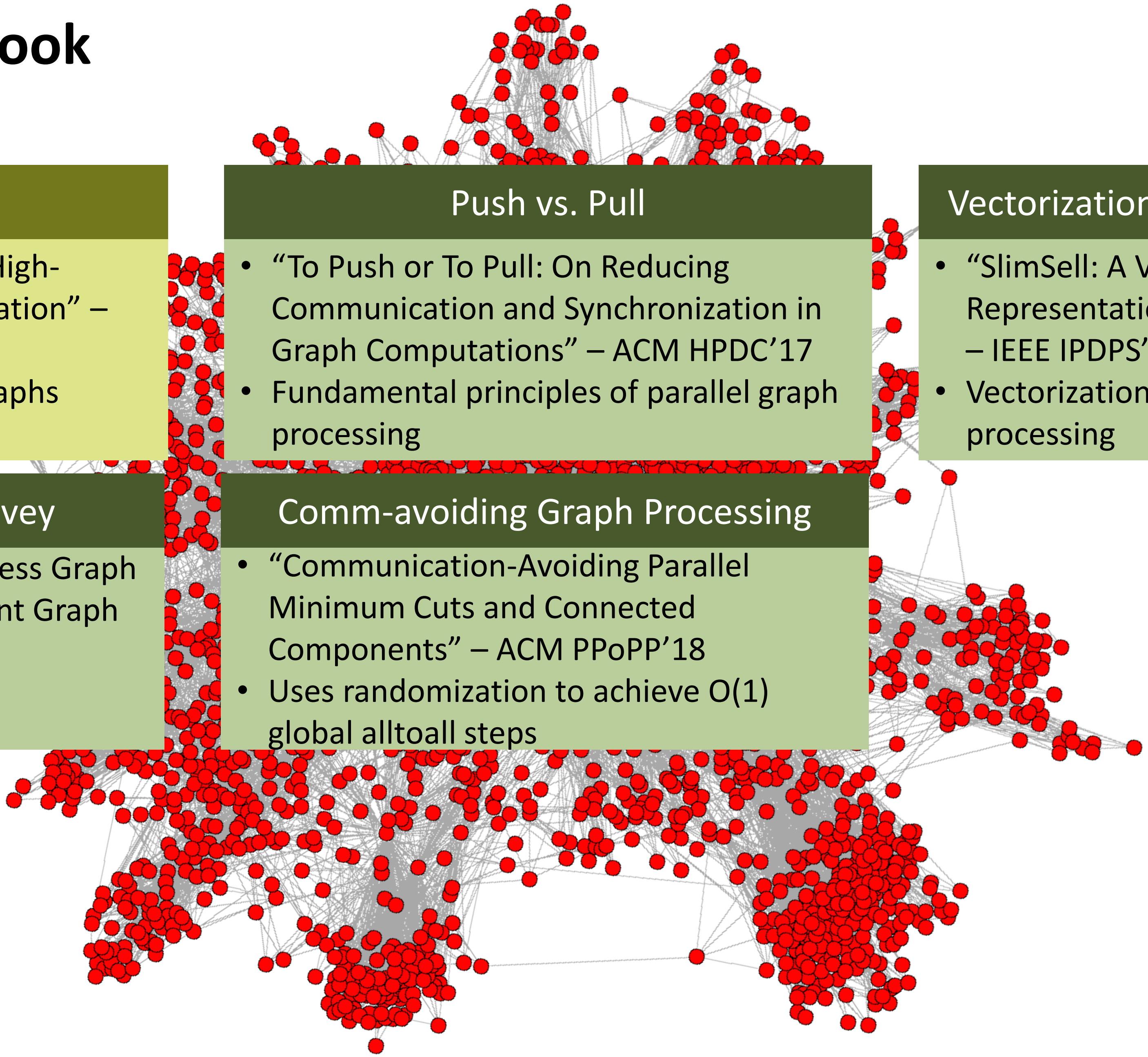

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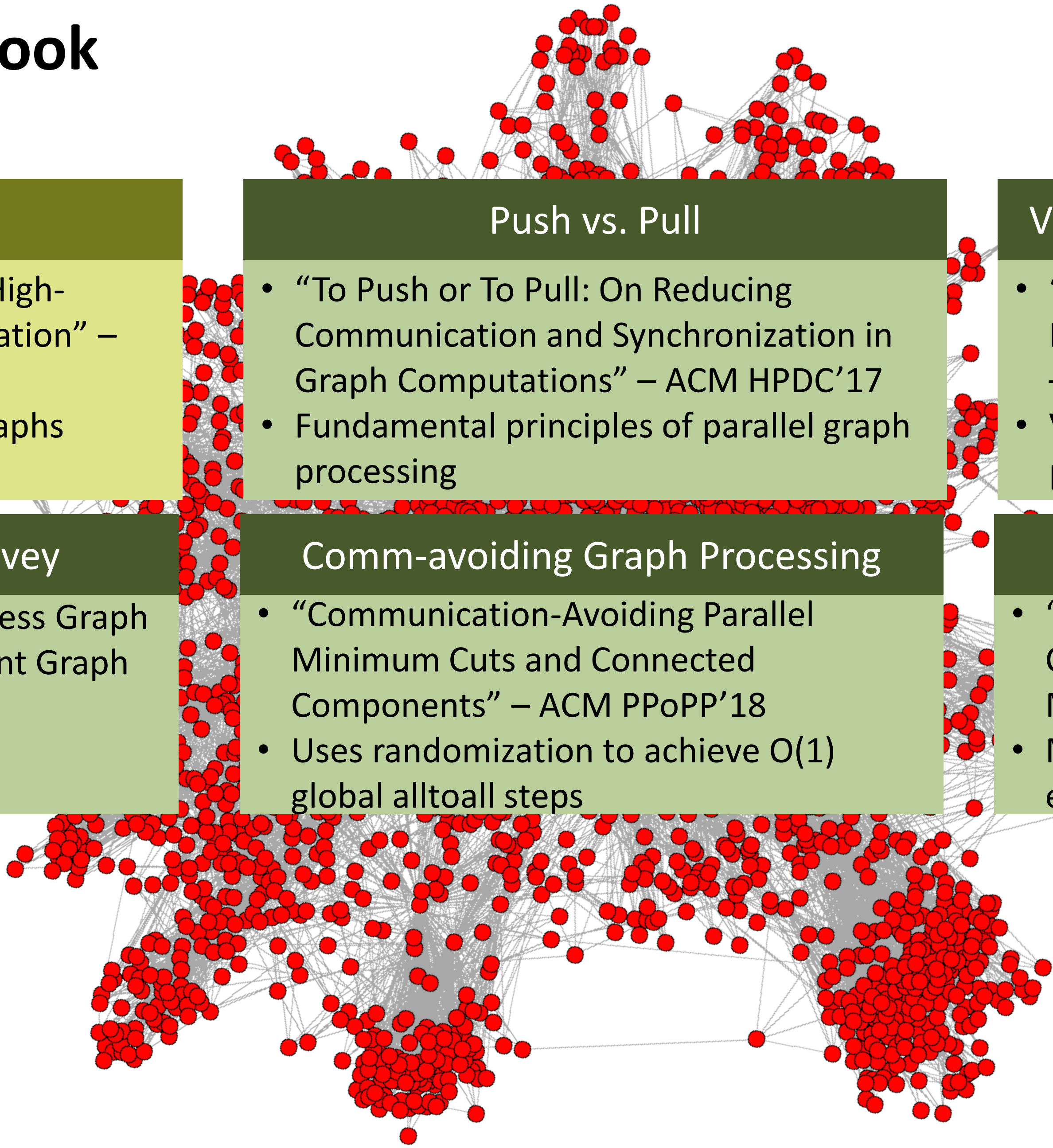

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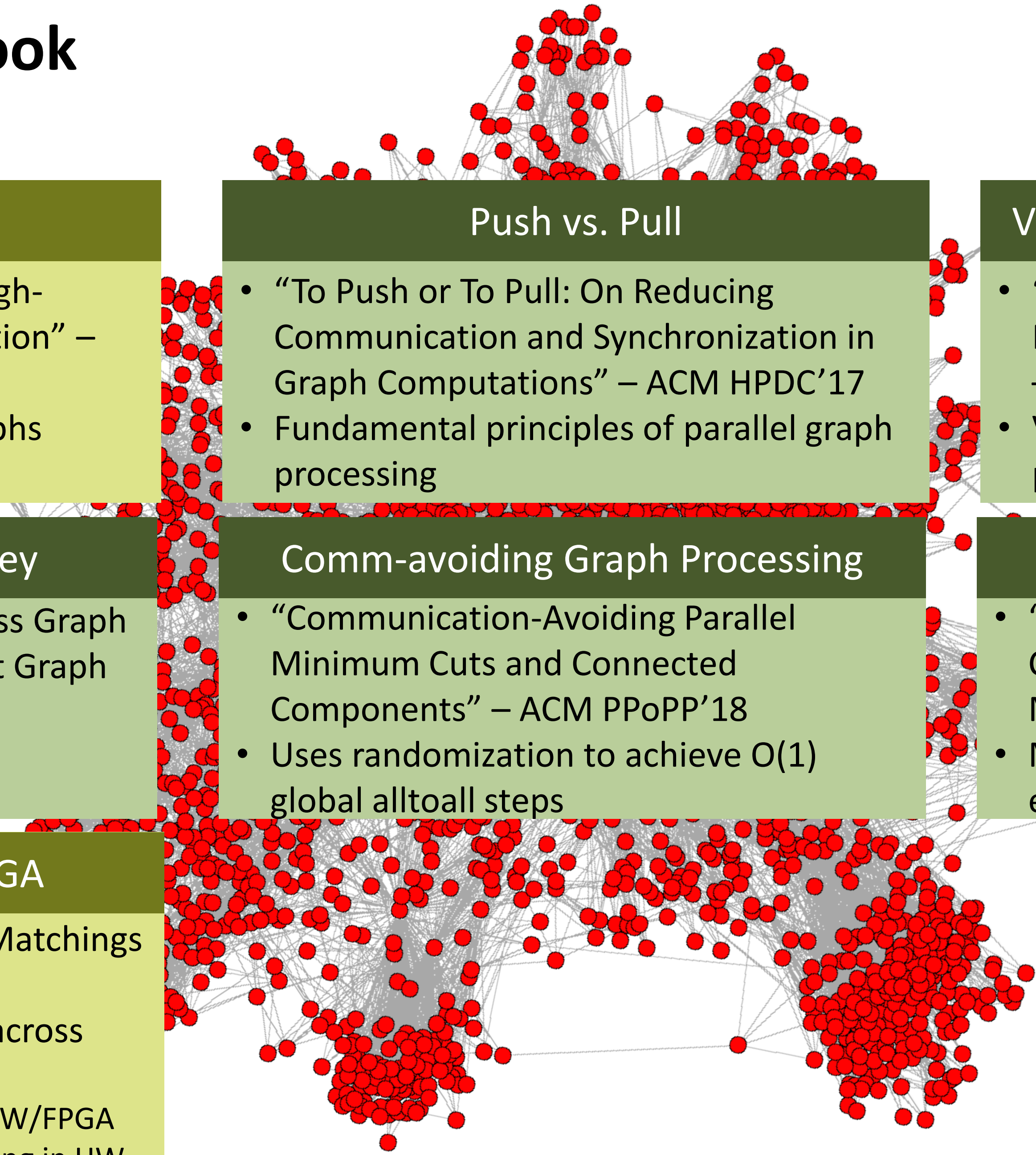
Algebraic Betweenness Centrality

- “Scaling Betweenness Centrality using Communication-Efficient Sparse Matrix Multiplication” – ACM SC’17
- More on the algebraic view – complex example, large-scale sparse matrices

Summary and outlook



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A big challenges ahead: develop a framework to integrate all techniques!

SPCL's approach: *stateful dataflow graphs*

We're always hiring excellent PhD students and postdocs at SPCL/ETH at spcl.inf.ethz.ch/Jobs

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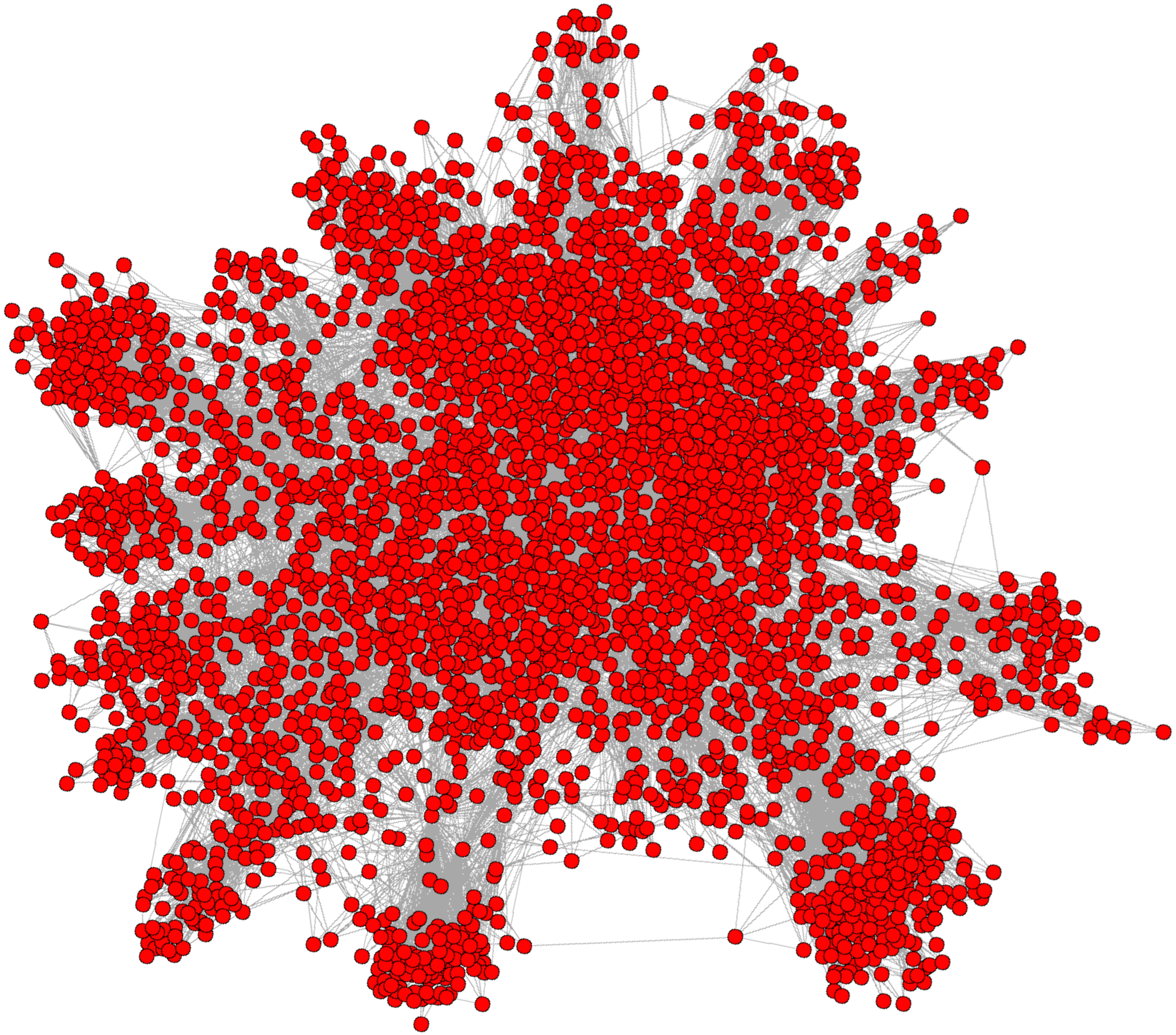
Backup

Problems!



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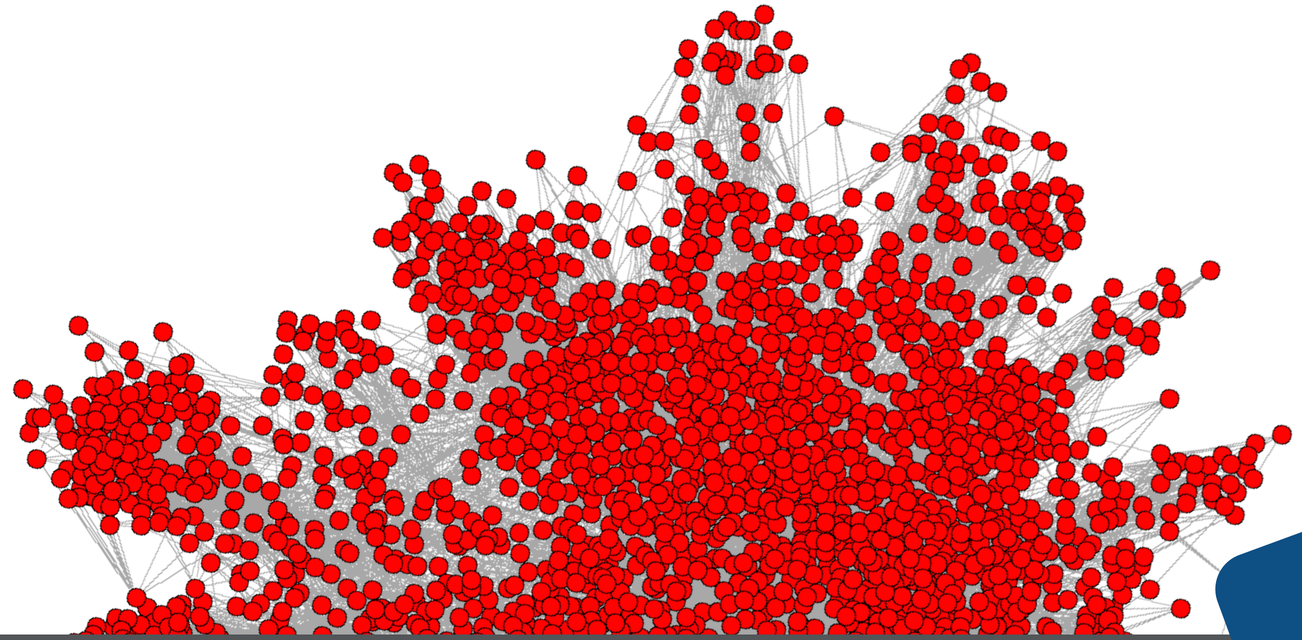
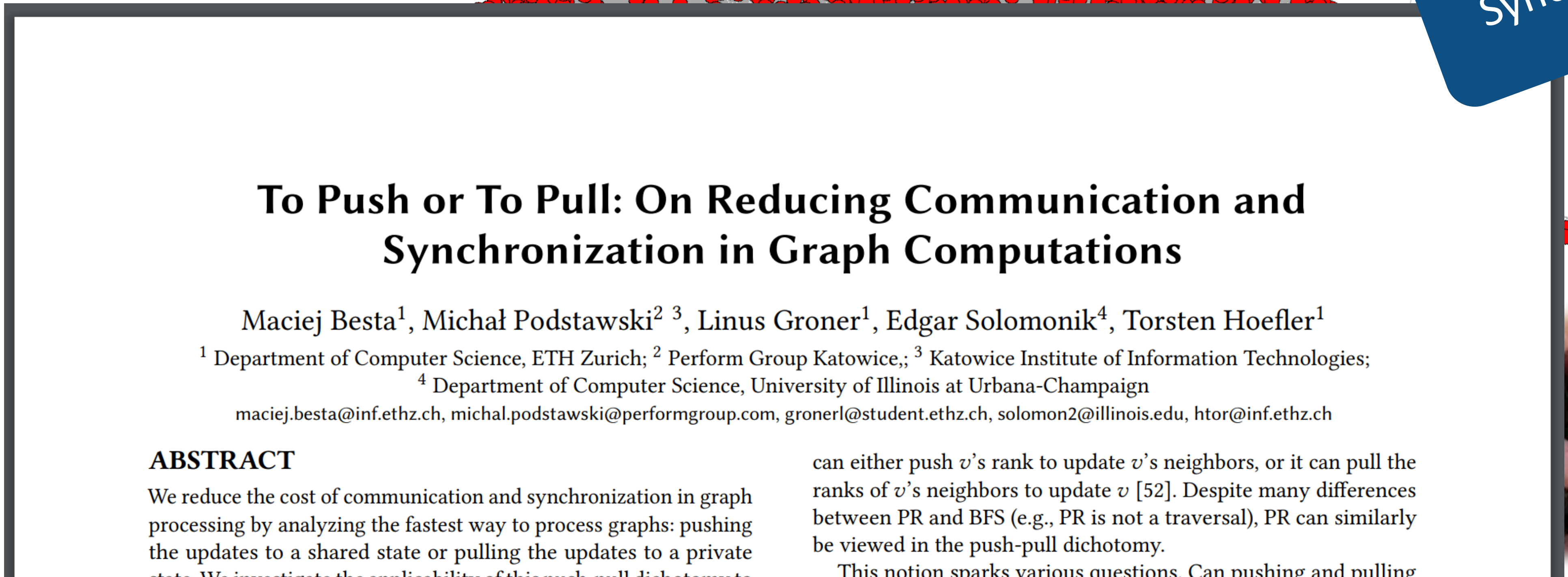


Synchronization-heavy



To Push or To Pull: On Reducing Communication and Synchronization in Graph Computations

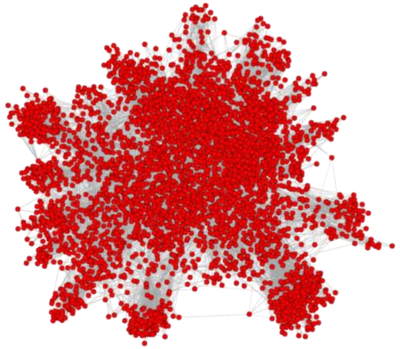
Maciej Besta[1], Michał Podstawski[2 3], Linus Groner[1], Edgar Solomonik[4], Torsten Hoefler[1]
[1] Department of Computer Science, ETH Zurich; [2] Perform Group Katowice,; [3] Katowice Institute of Information Technologies;
[4] Department of Computer Science, University of Illinois at Urbana-Champaign
maciej.besta@inf.ethz.ch, michal.podstawski@performgroup.com, gronerl@student.ethz.ch, solomon2@illinois.edu, htor@inf.ethz.ch

ABSTRACT
We reduce the cost of communication and synchronization in graph processing by analyzing the fastest way to process graphs: pushing the updates to a shared state or pulling the updates to a private

can either push v's rank to update v's neighbors, or it can pull the ranks of v's neighbors to update v [52]. Despite many differences between PR and BFS (e.g., PR is not a traversal), PR can similarly be viewed in the push-pull dichotomy.

PAGERANK

PAGERANK



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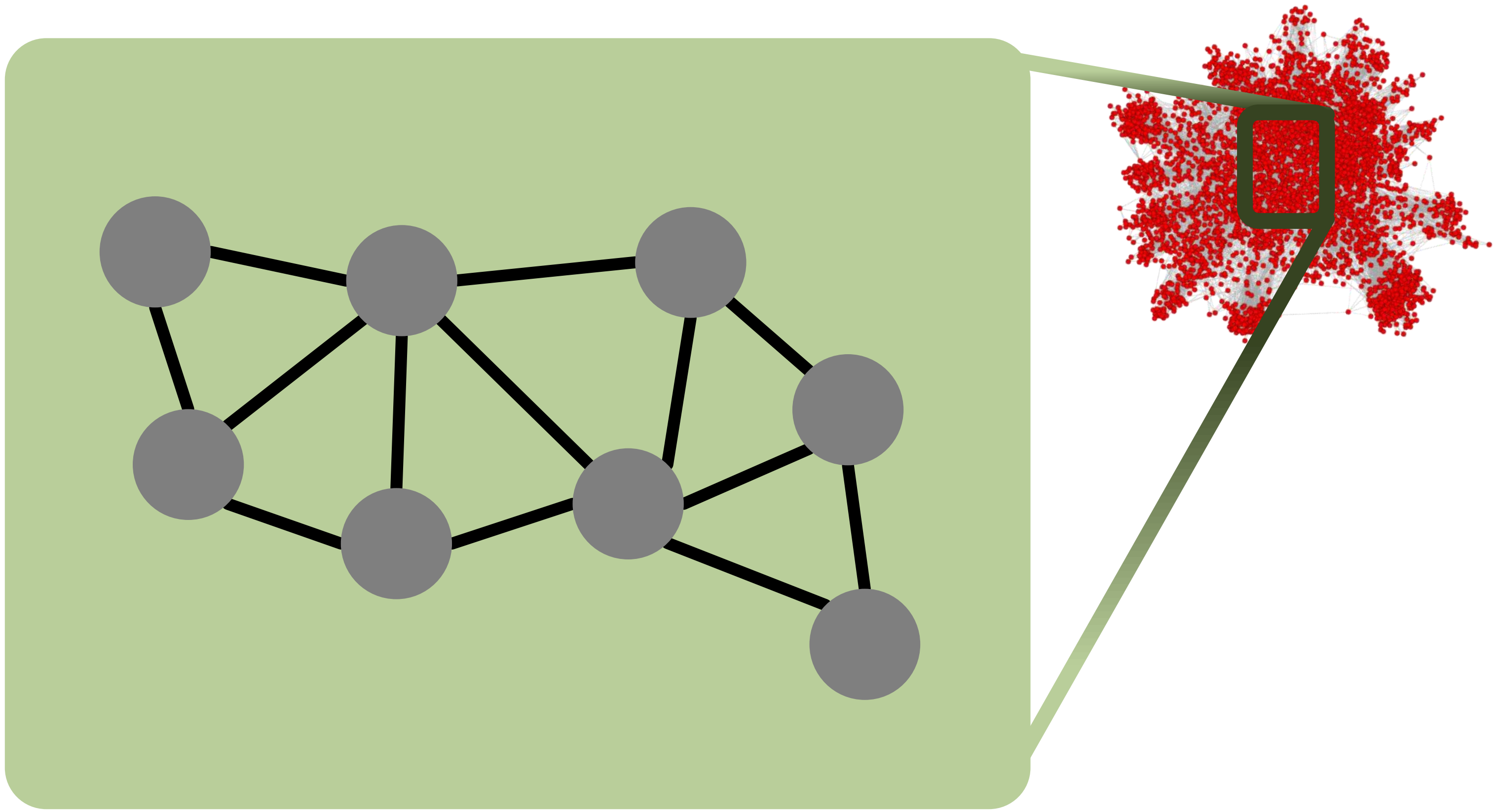
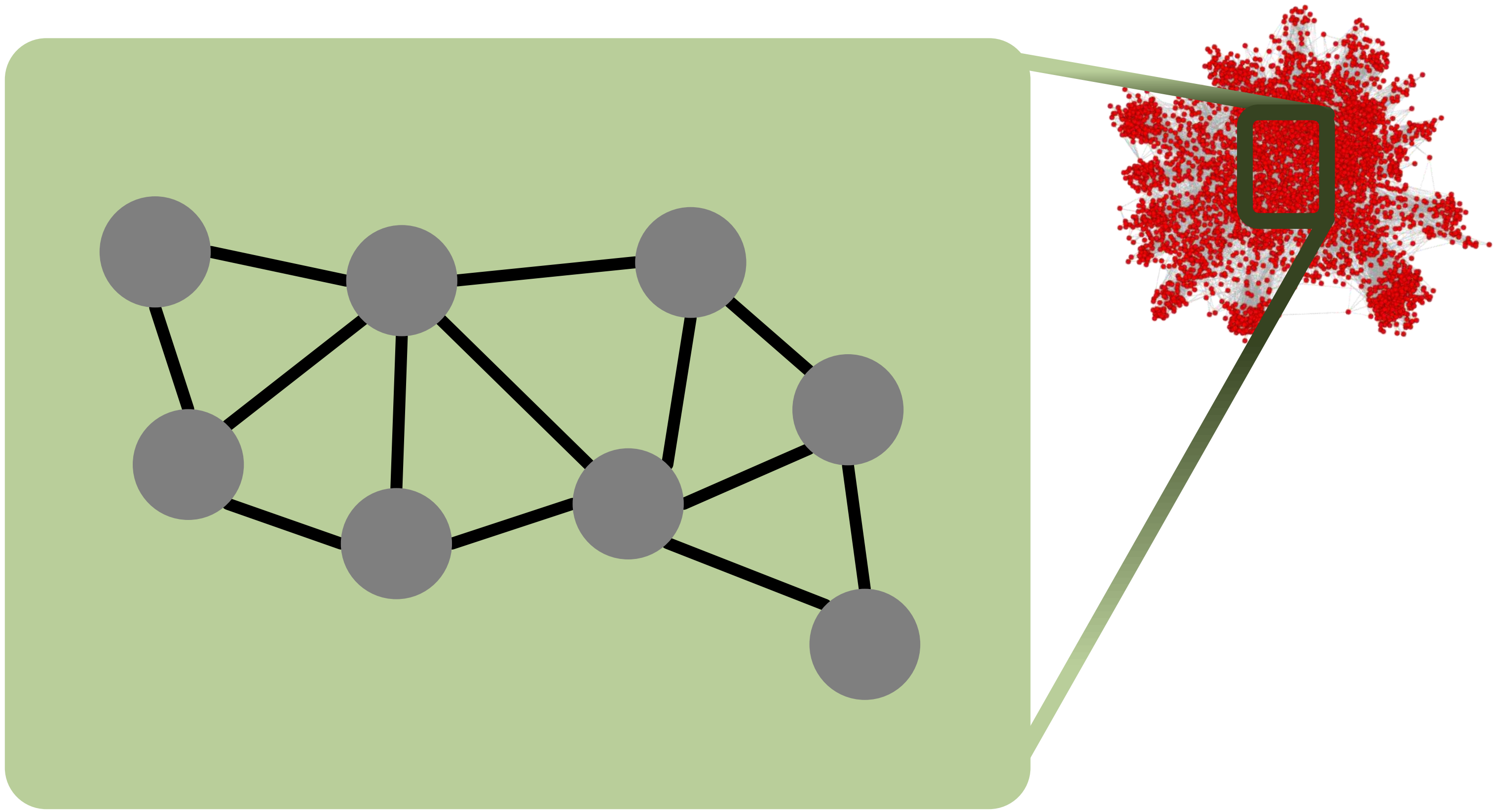


PageRank

P threads are used



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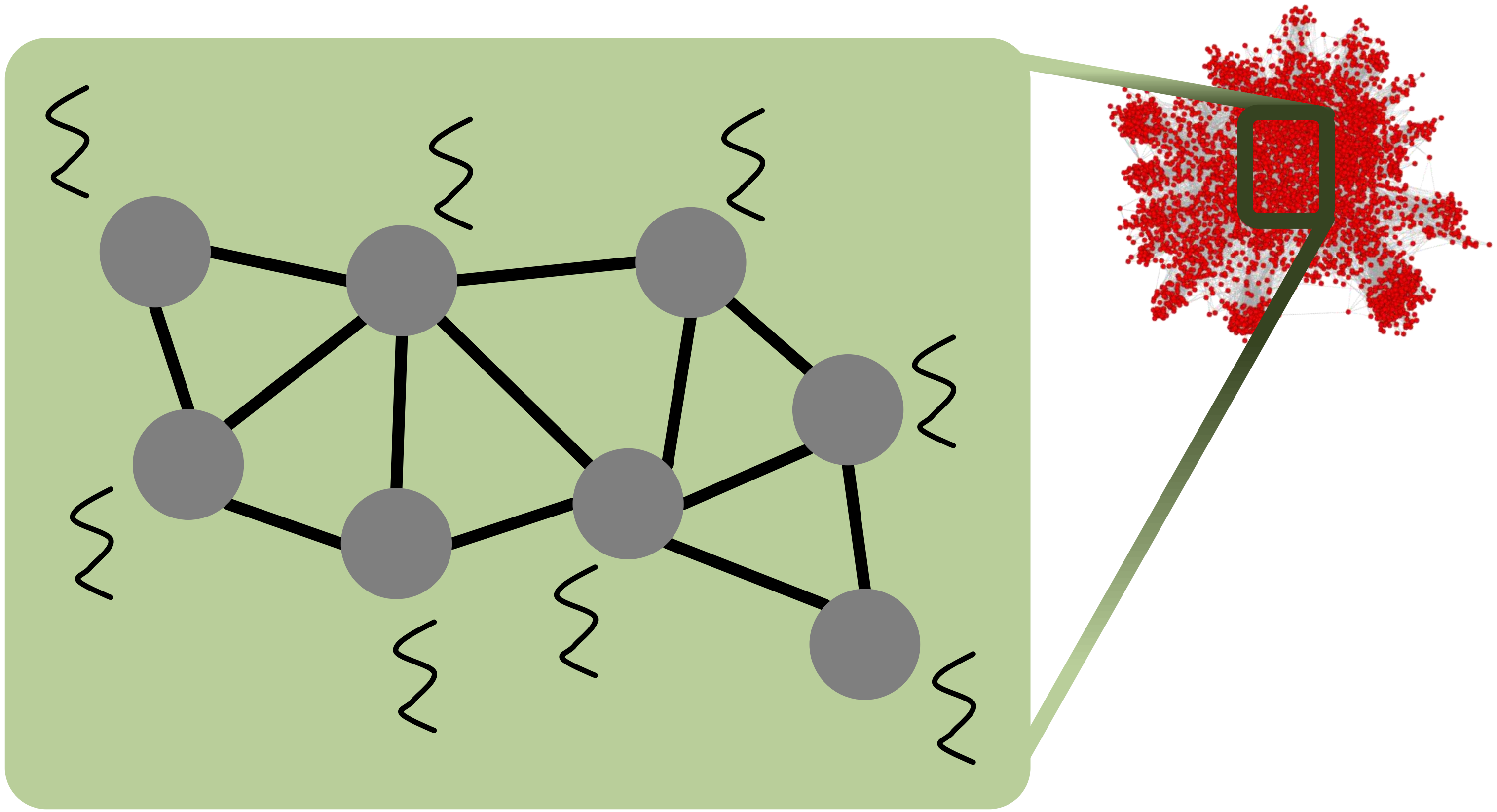

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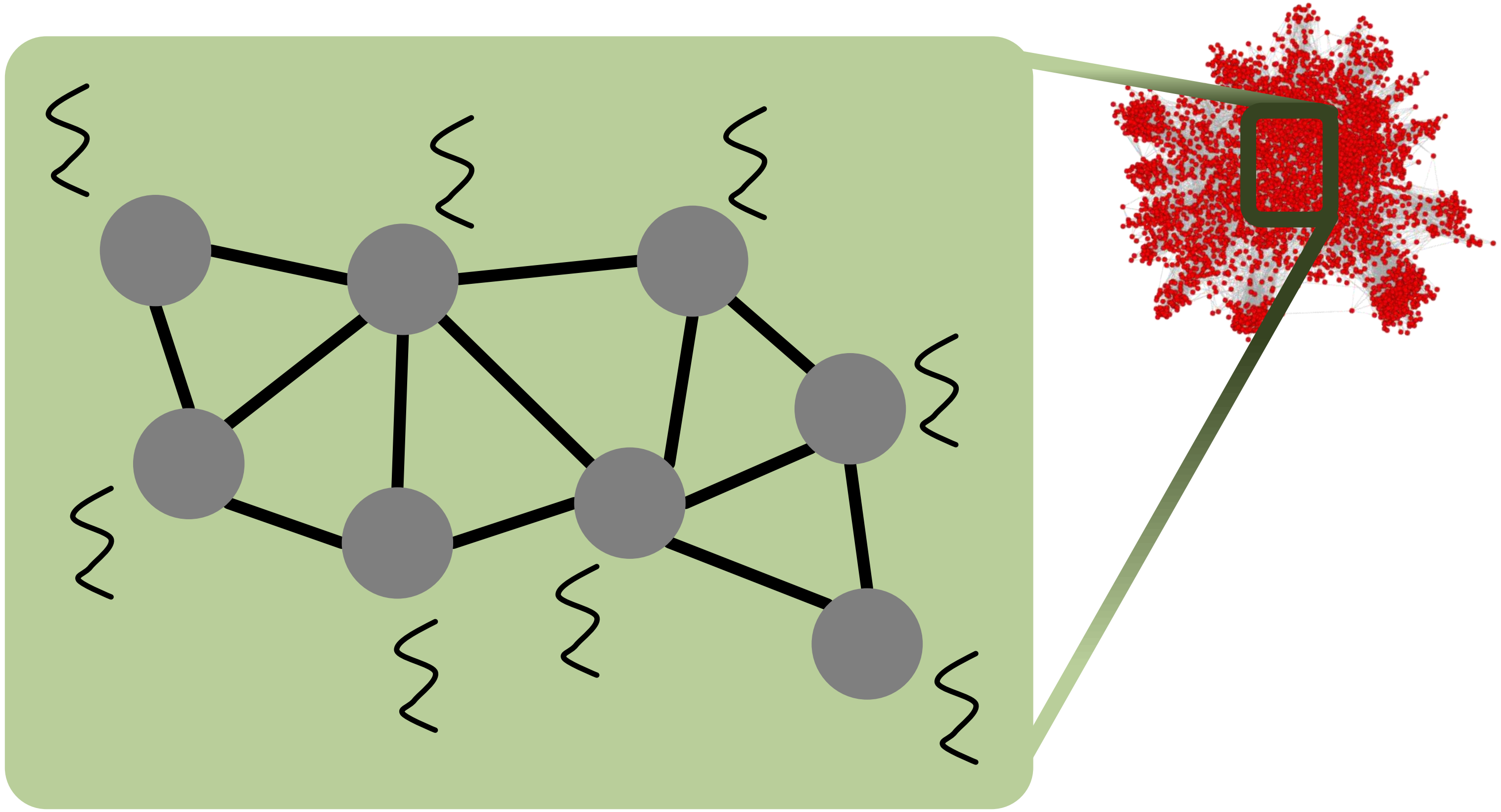

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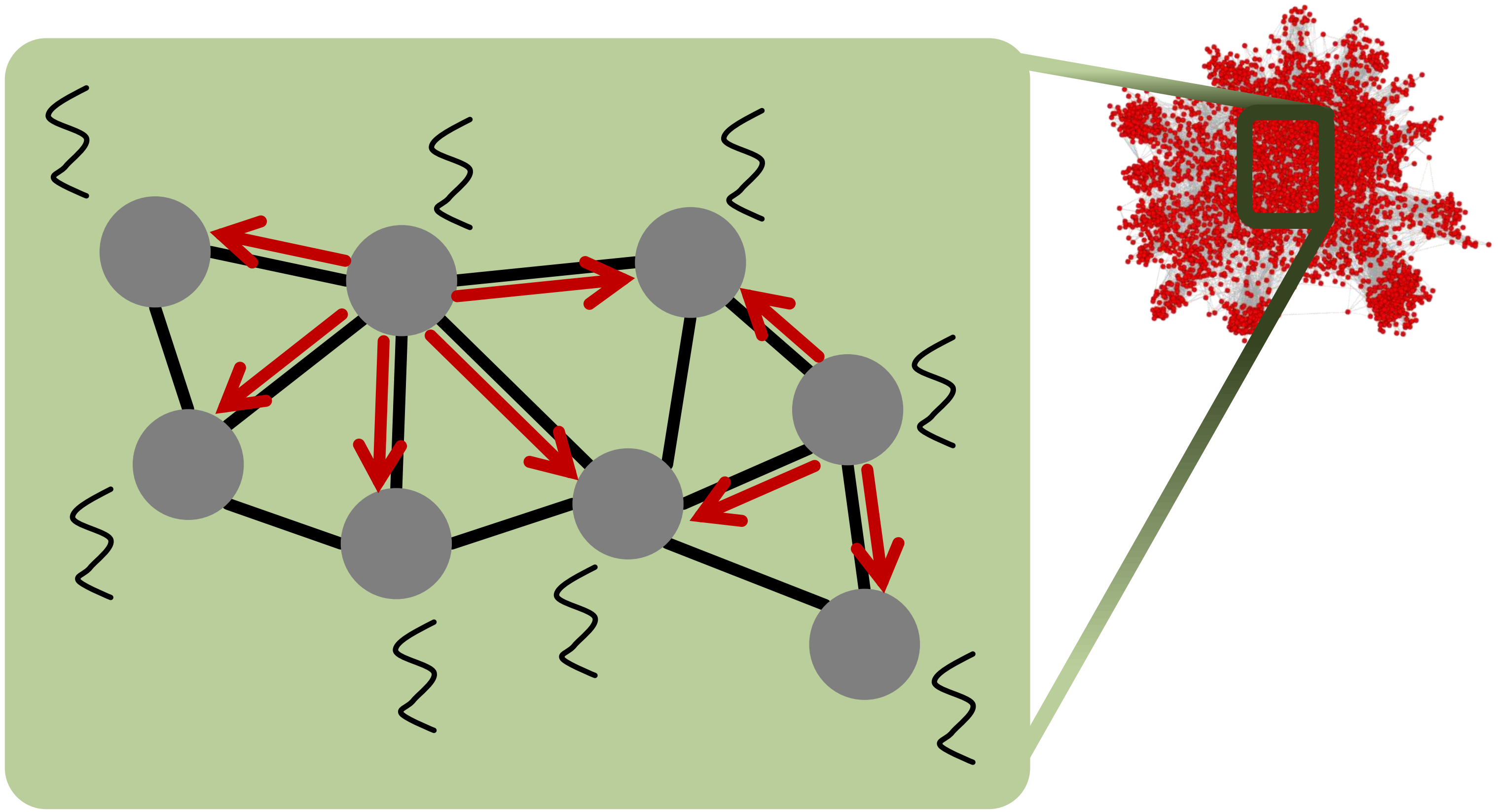

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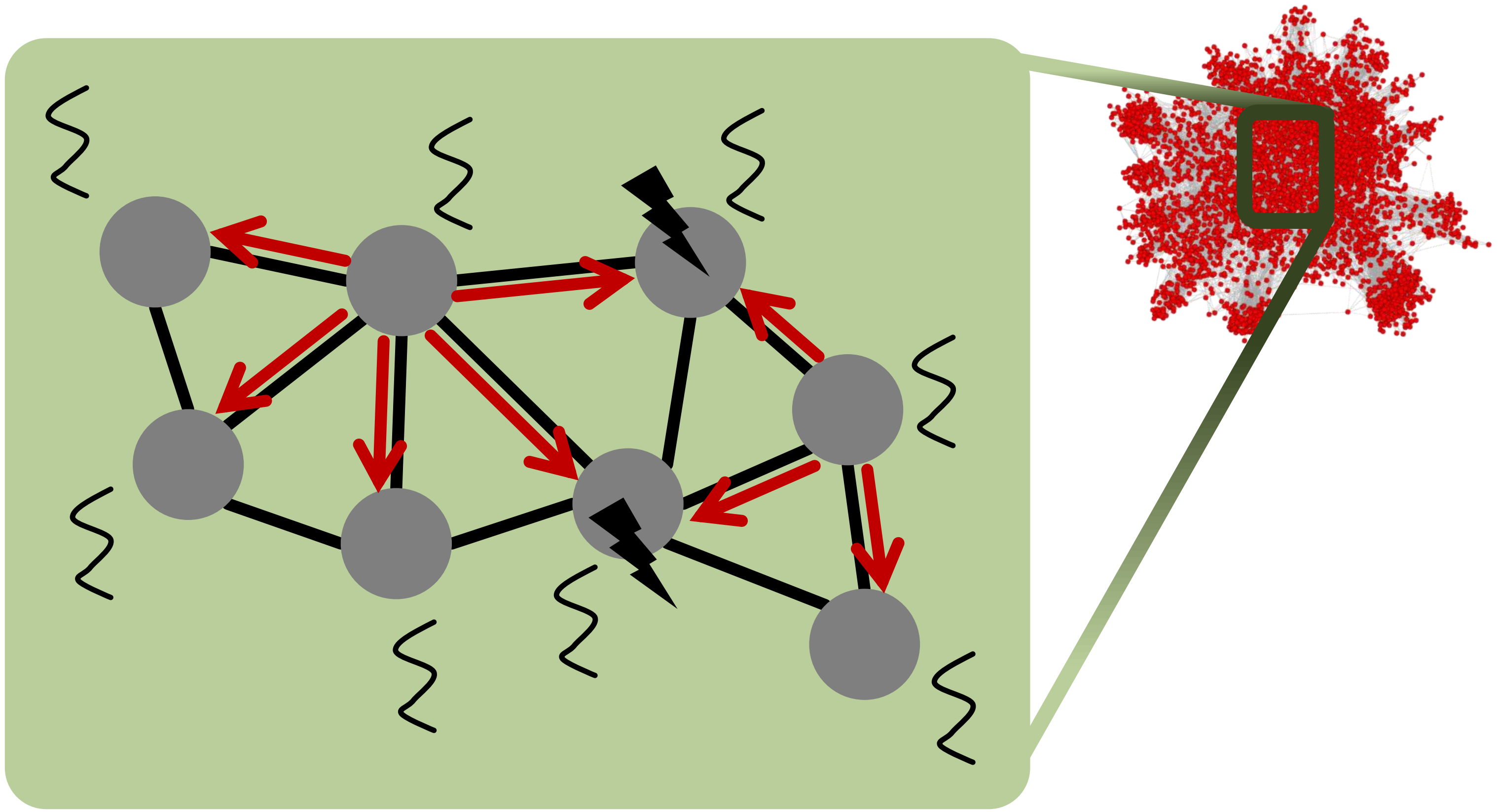

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Write conflicts

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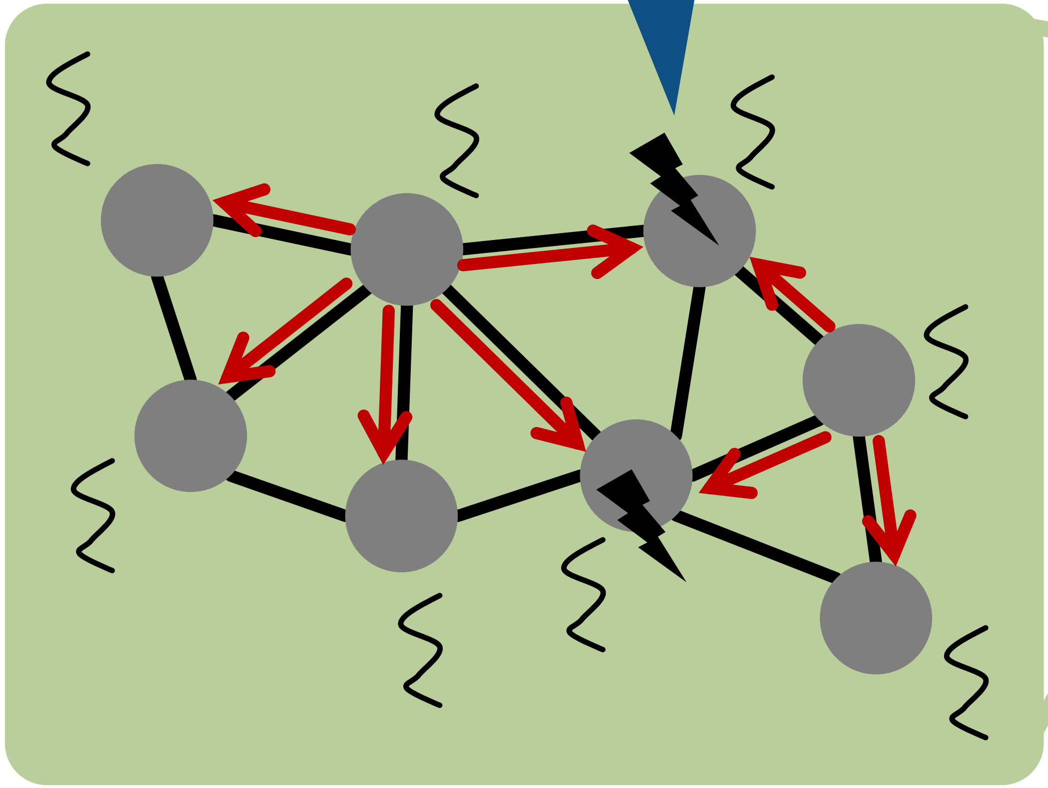

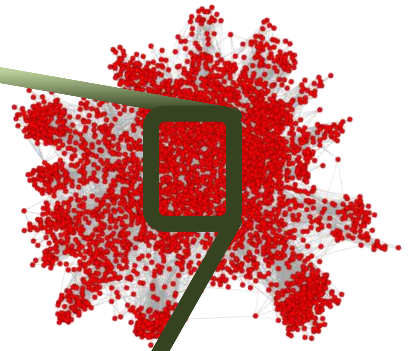

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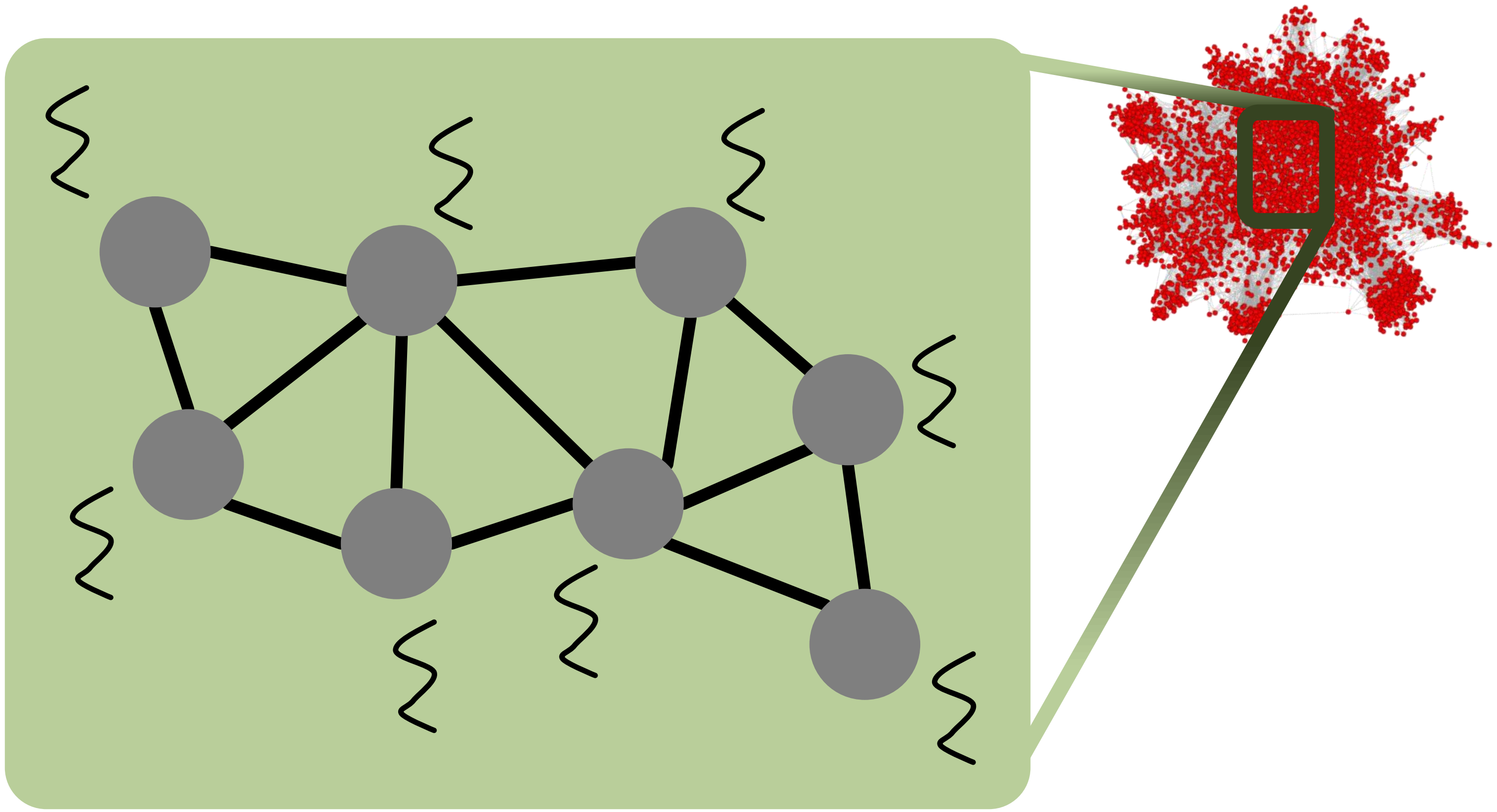

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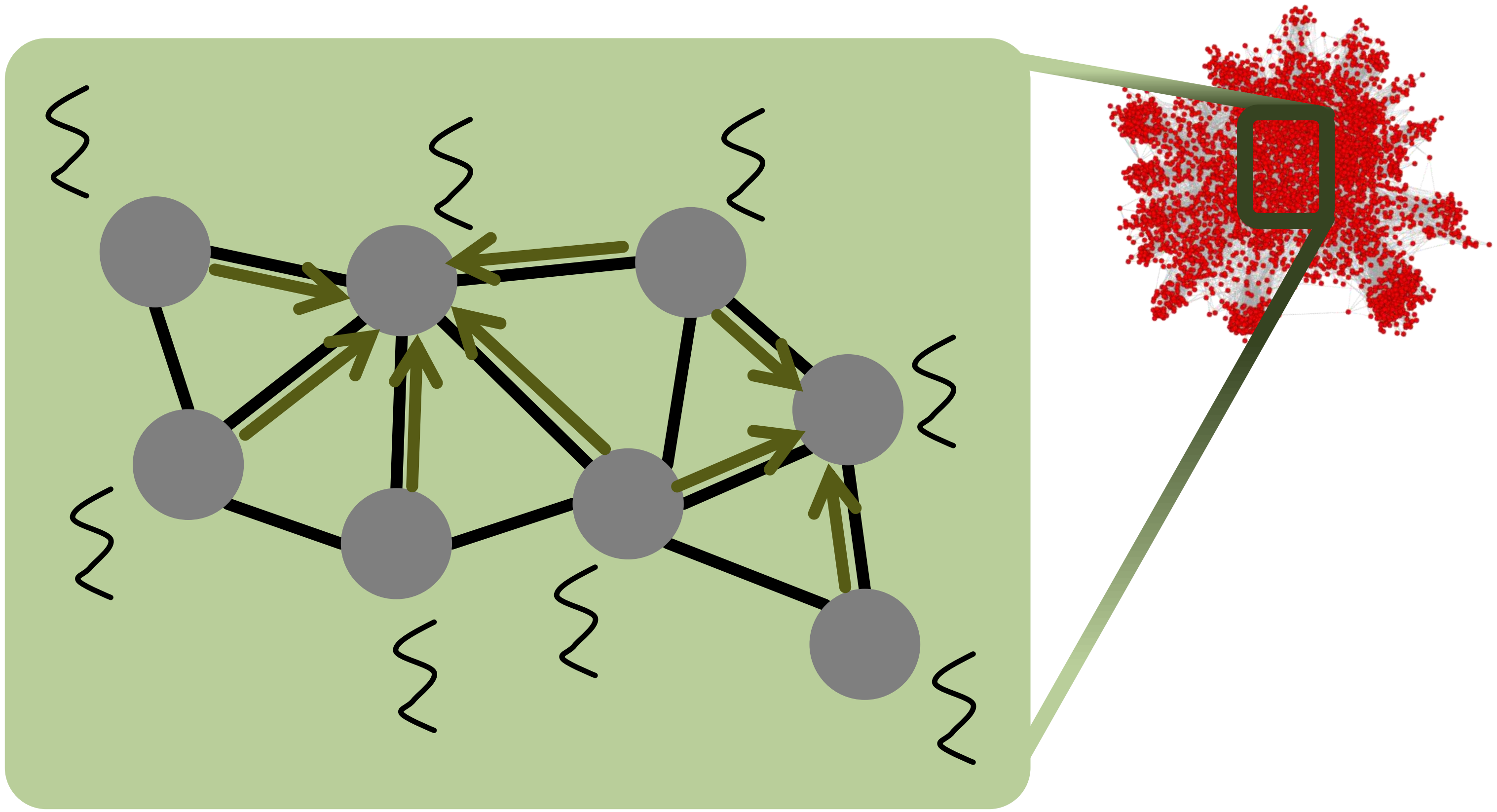

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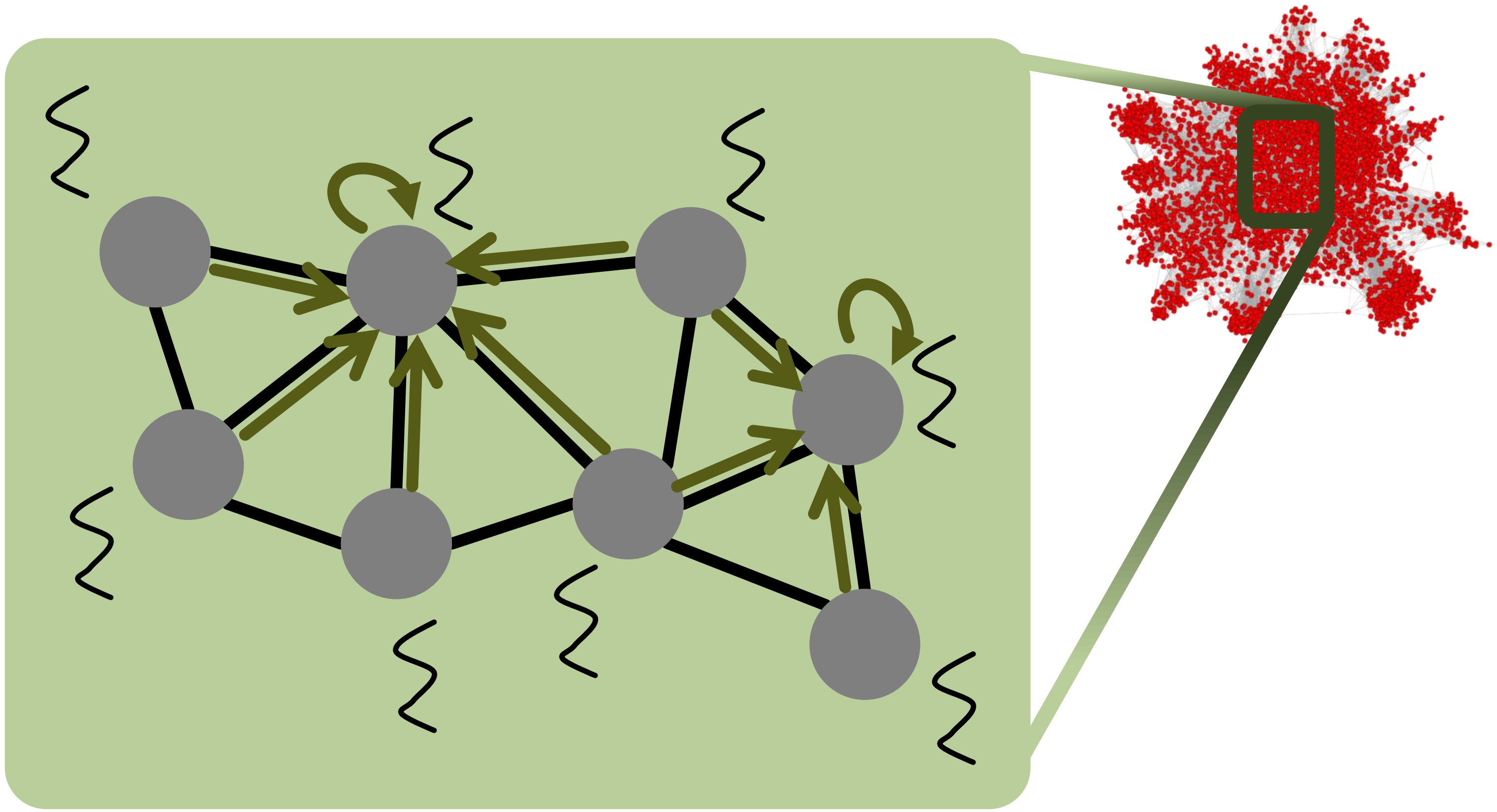

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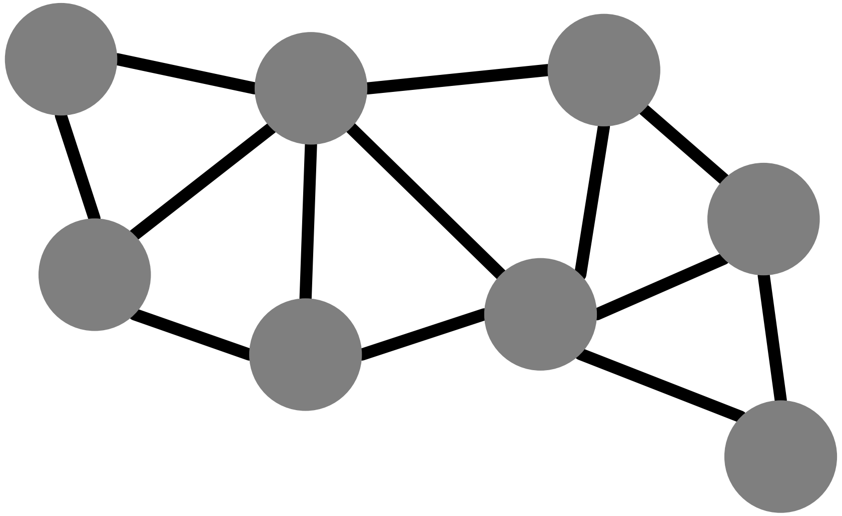

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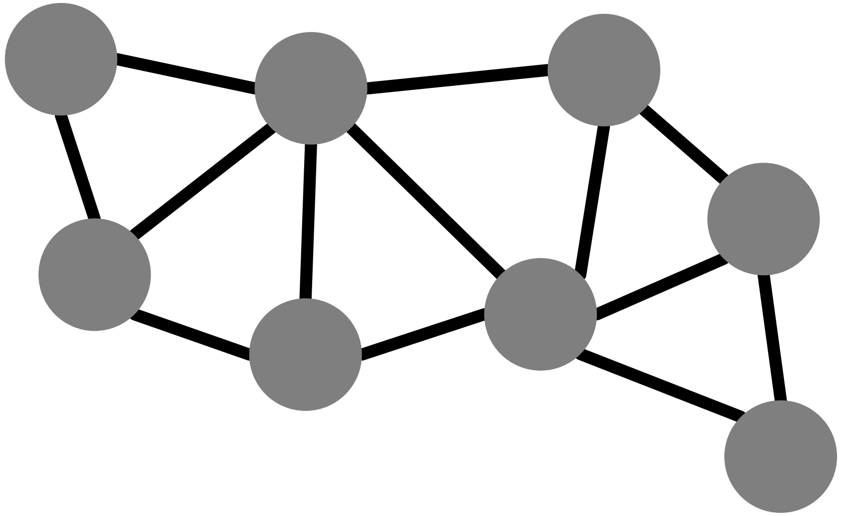

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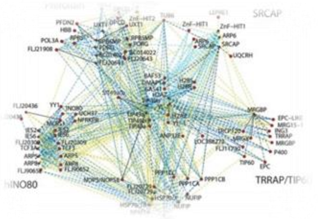

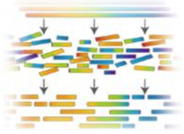

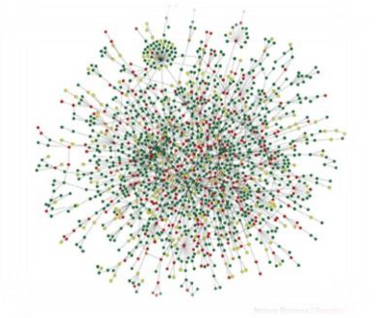

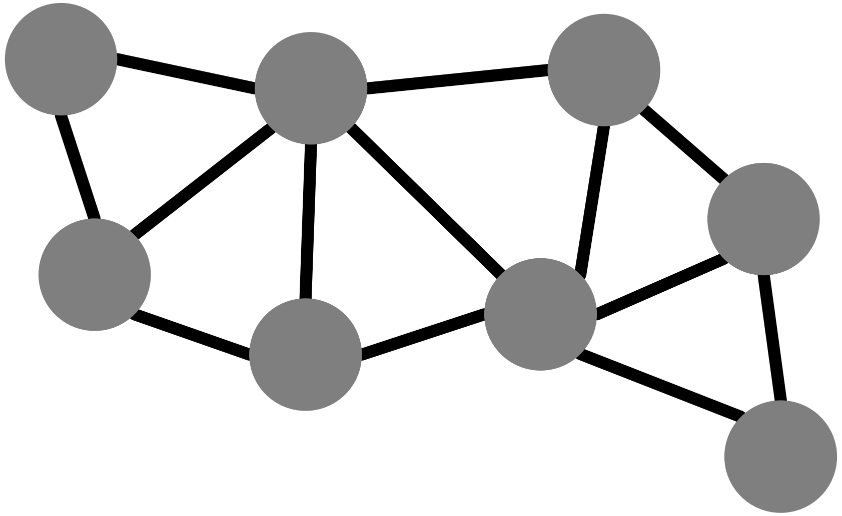

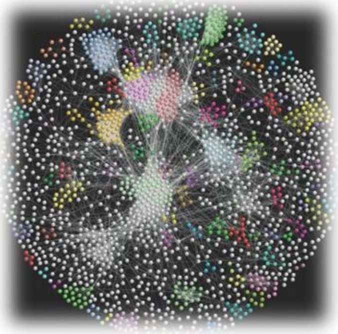

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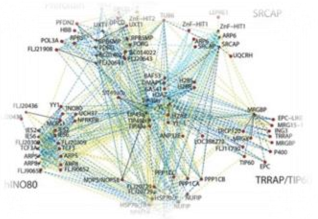

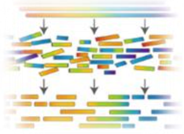

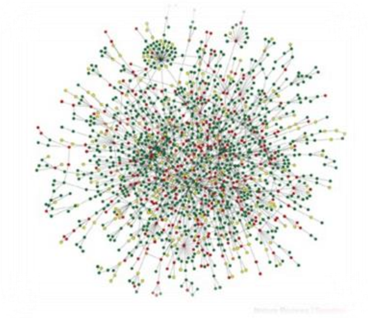

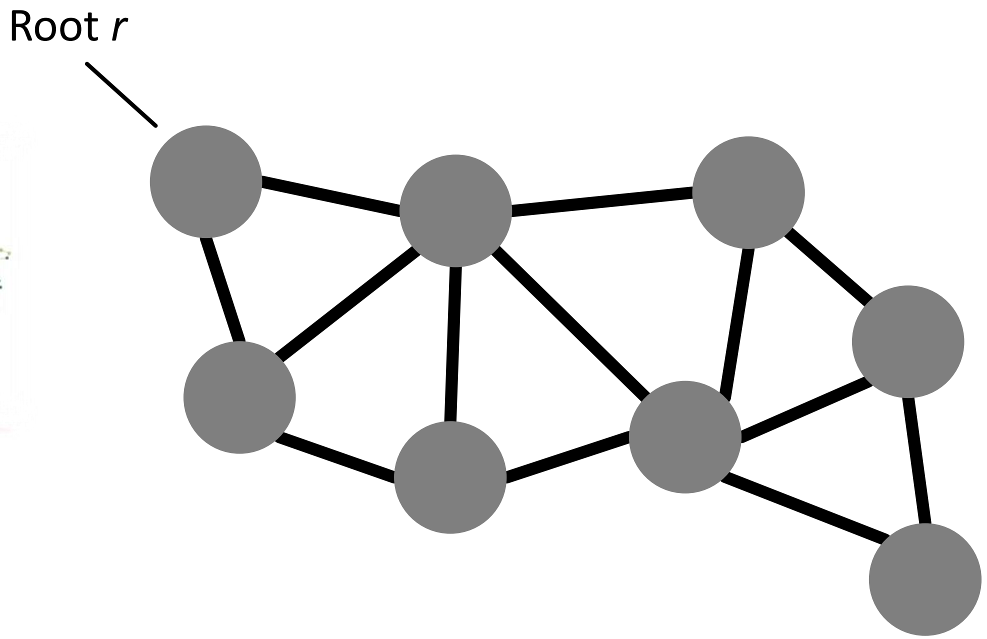

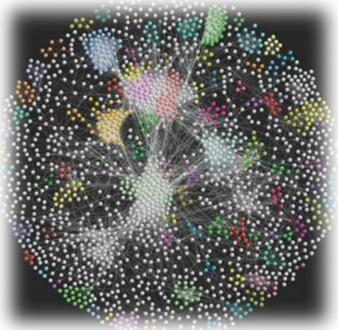

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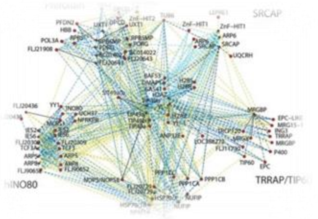

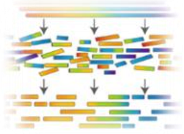

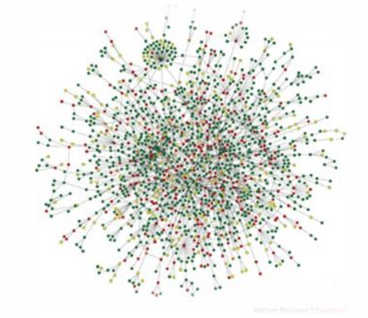

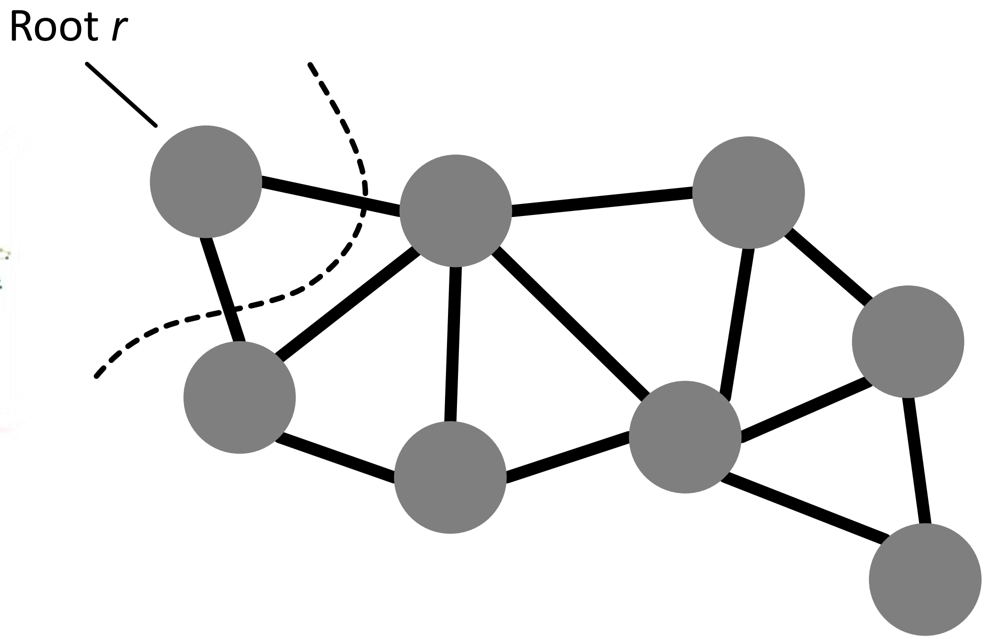

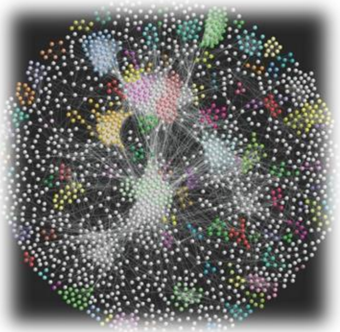

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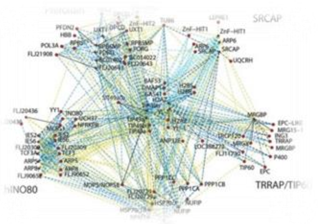
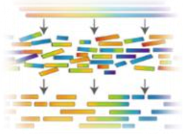
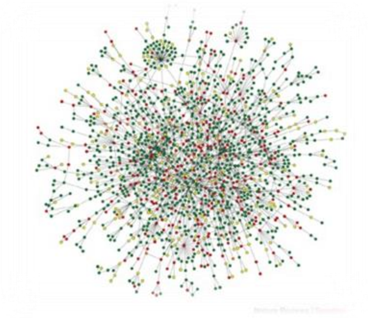
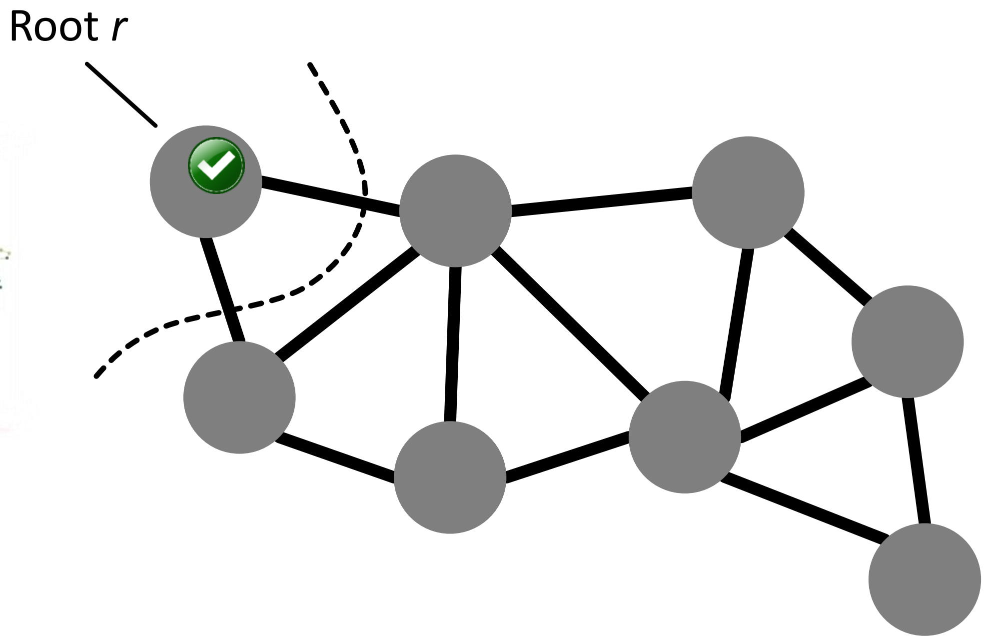
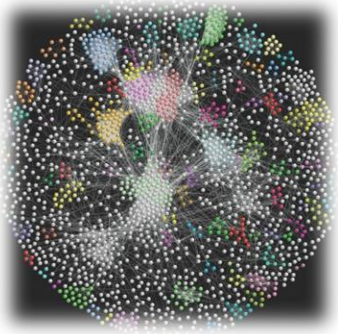


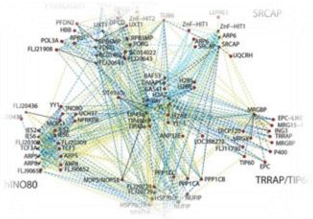


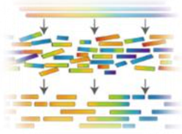


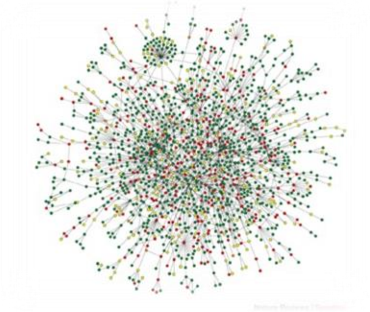


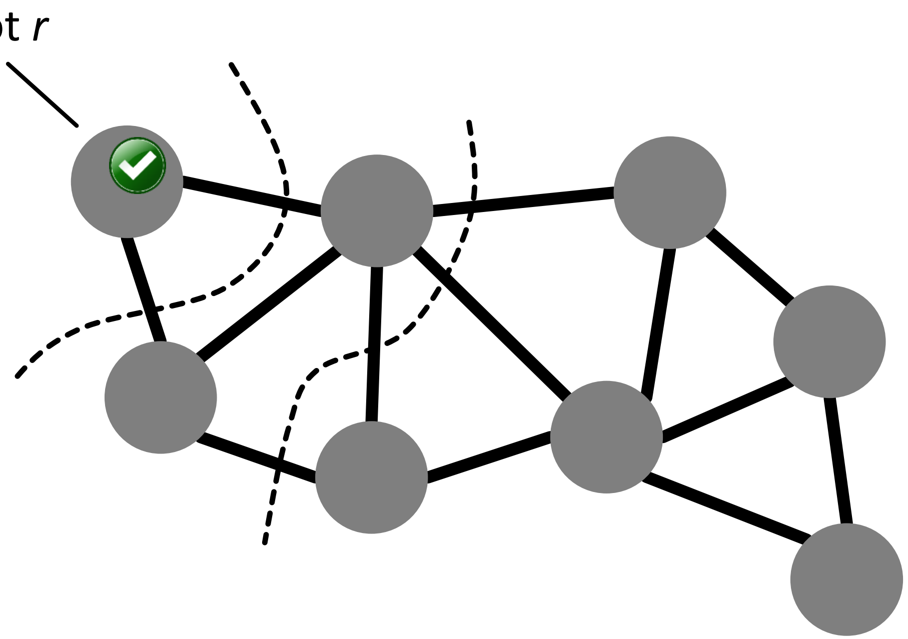


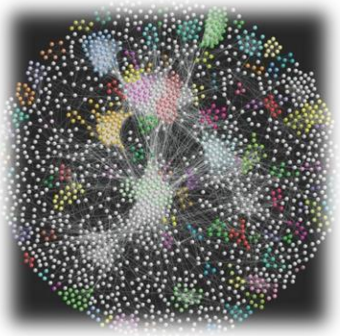
BFS
Top-Down vs. Bottom-Up [1]







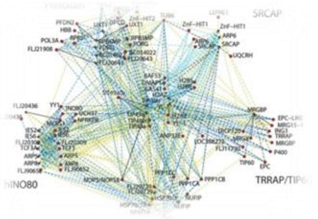


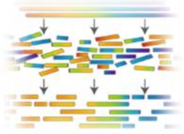


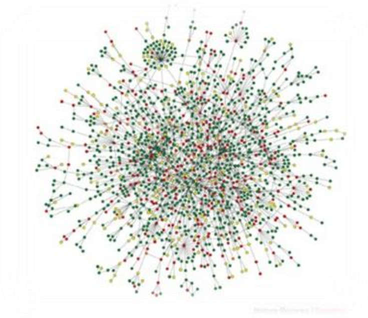
GRAPH 500

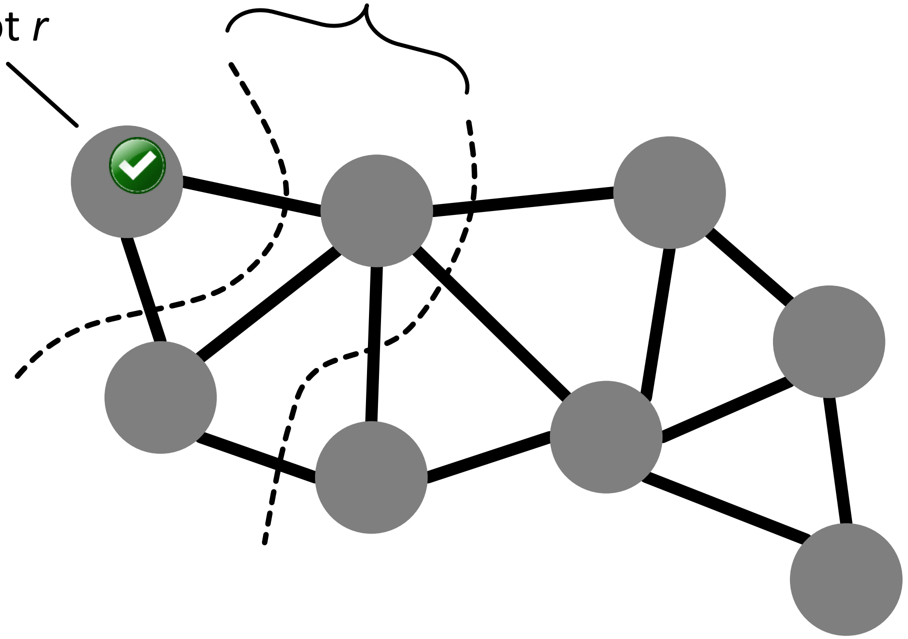
BFS

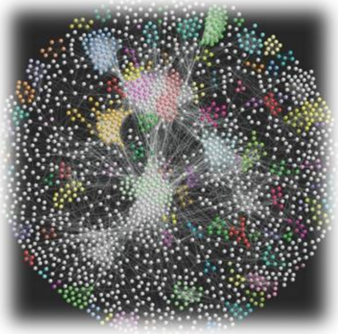
Top-Down vs. Bottom-Up [1]



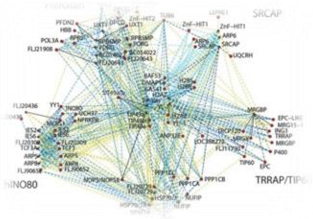


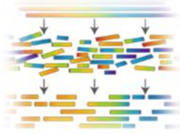


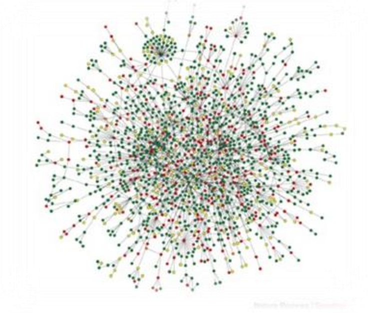


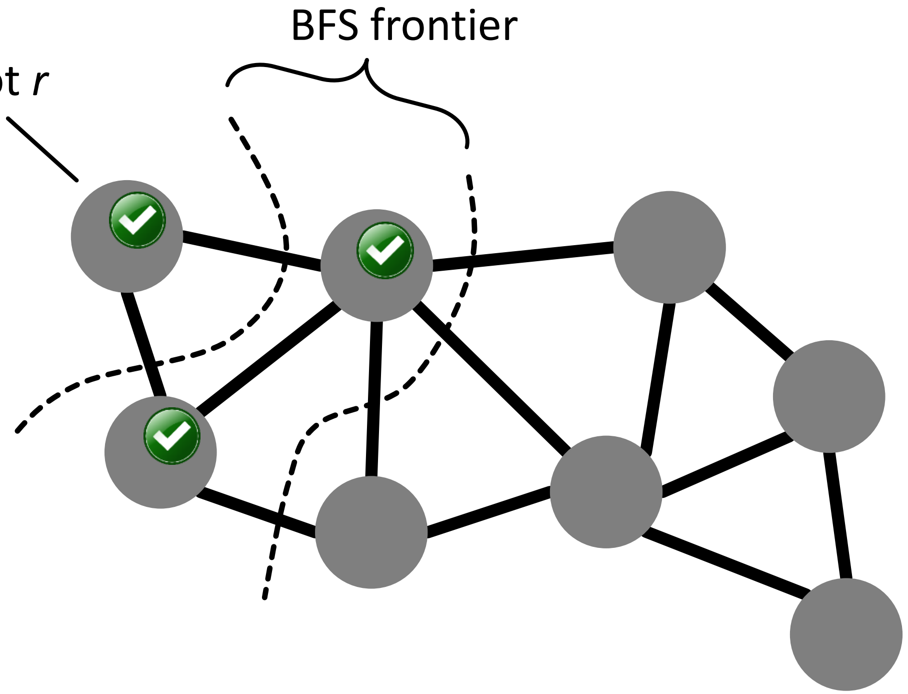


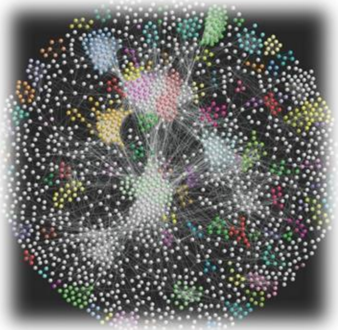
BFS
Top-Down vs. Bottom-Up [1]



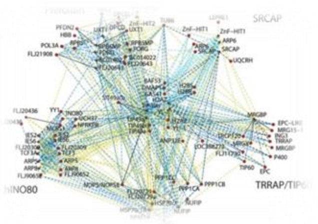


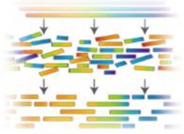


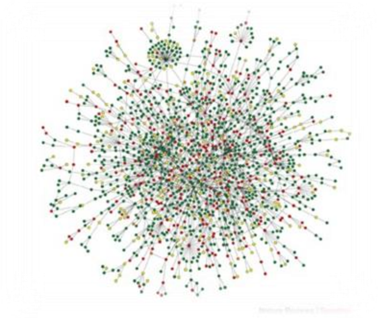


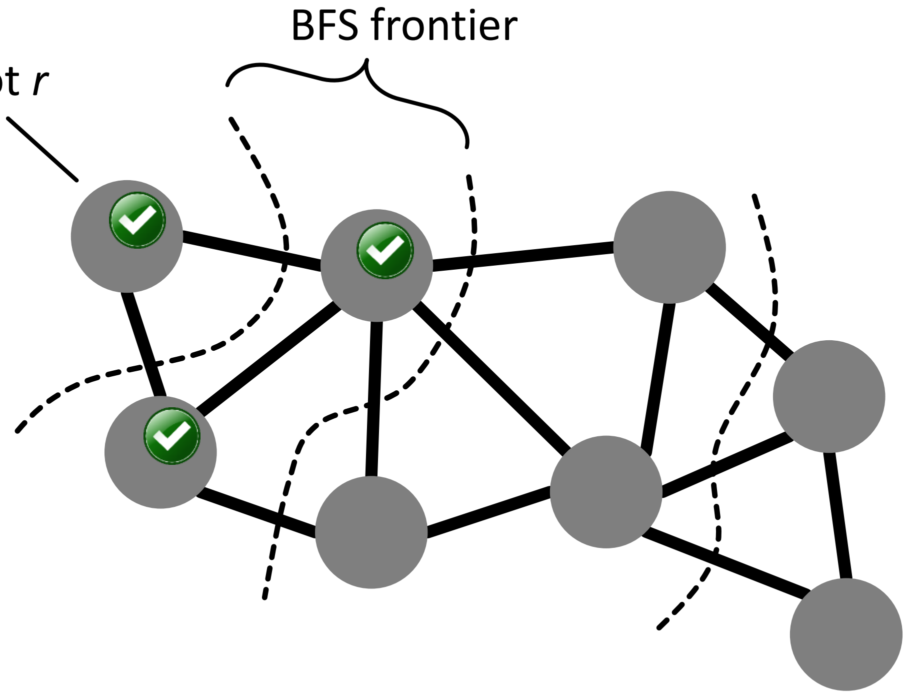


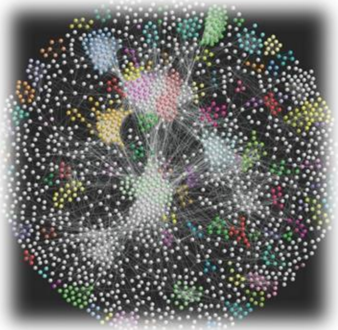
BFS
TOP-DOWN VS. BOTTOM-UP [1]



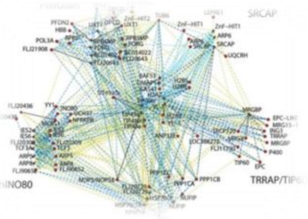


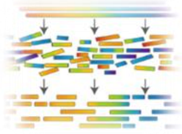


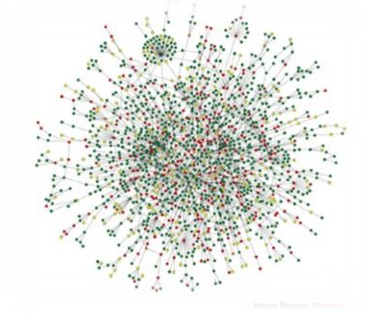


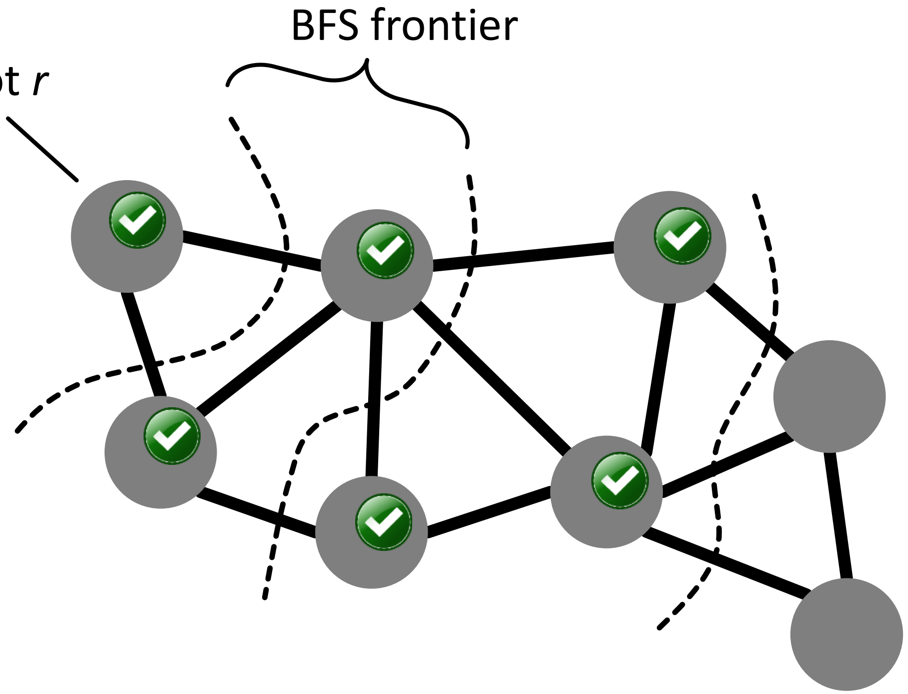


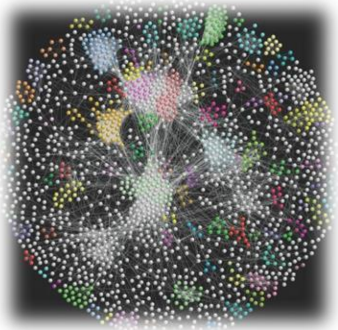
BFS
Top-Down vs. Bottom-Up [1]





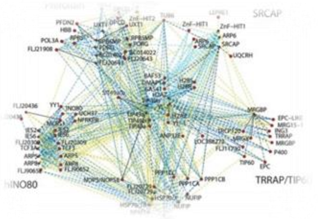


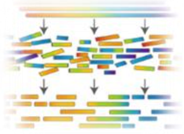


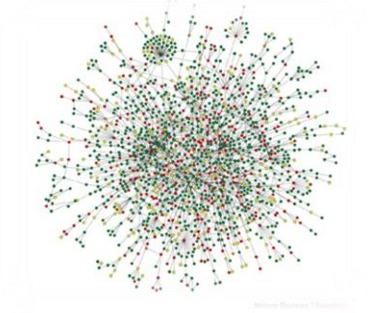


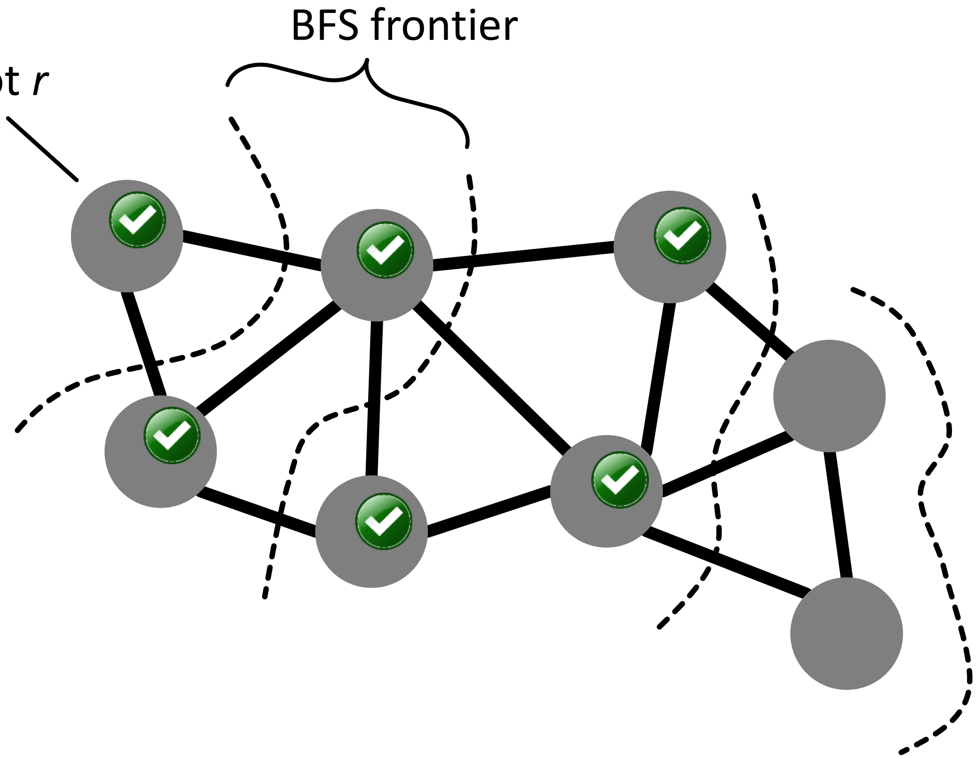
BFS

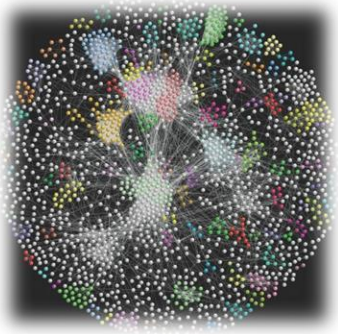
TOP-DOWN VS. BOTTOM-UP [1]



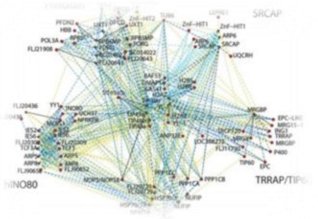


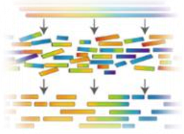


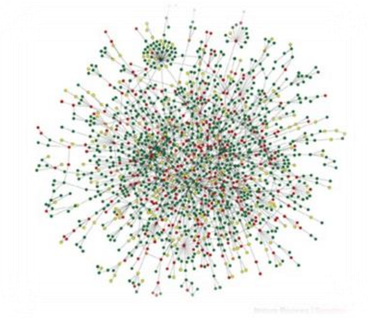


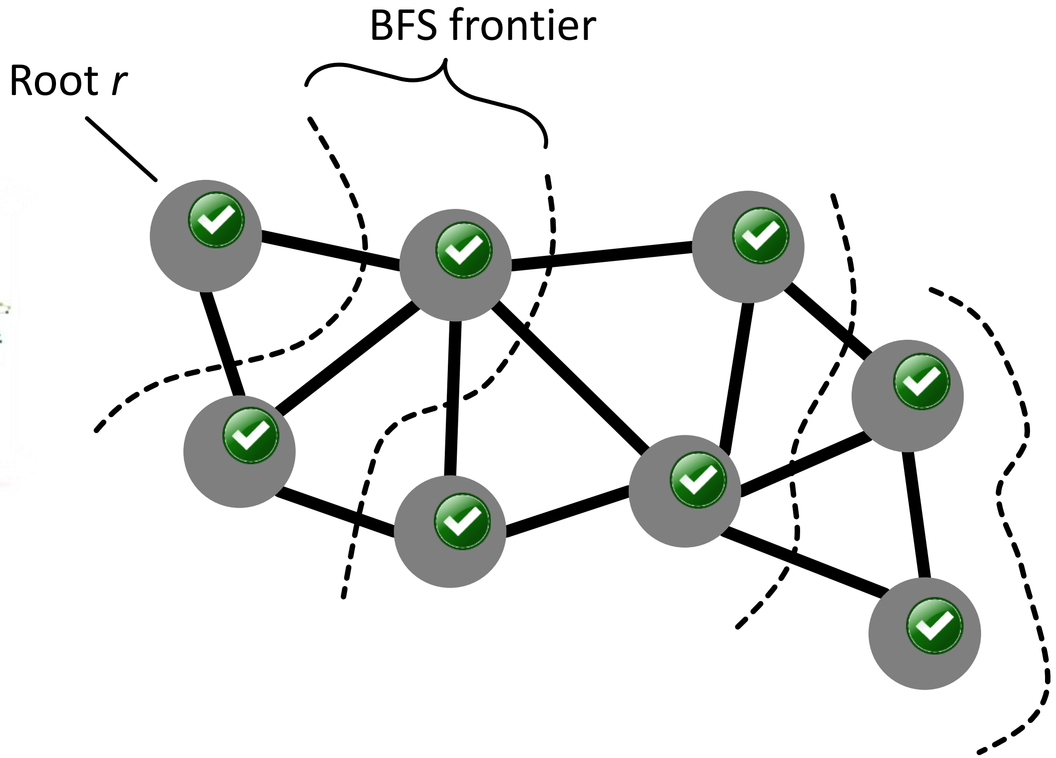


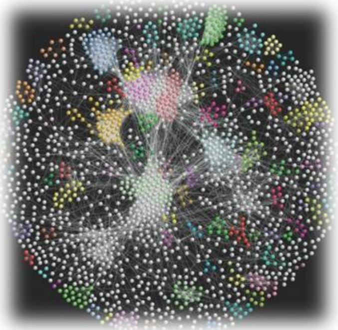
BFS
Top-Down vs. Bottom-Up [1]





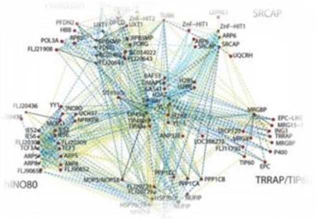


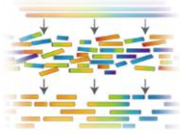


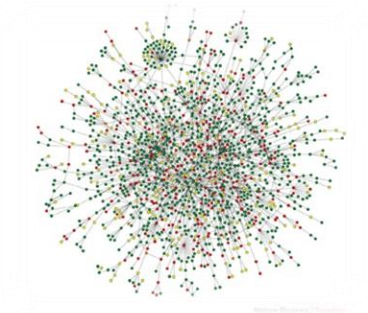


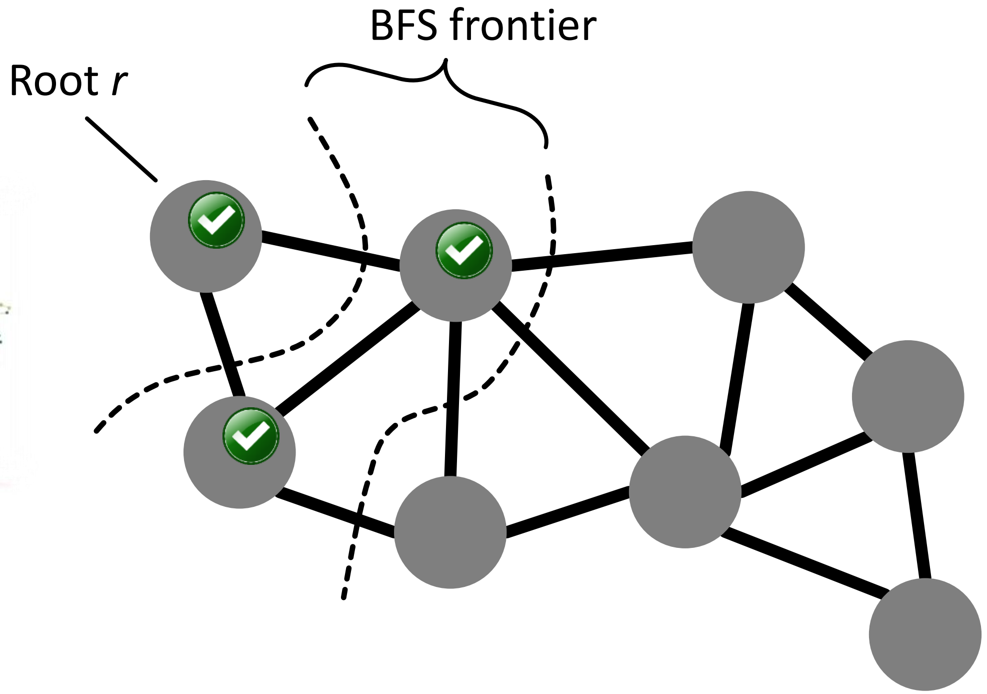
BFS

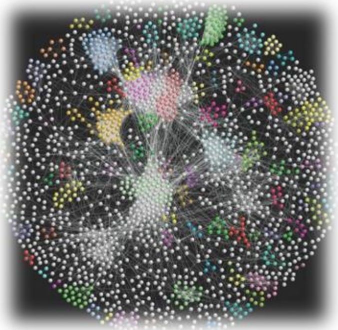
TOP-DOWN VS. BOTTOM-UP [1]



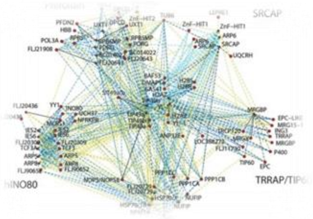


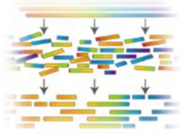




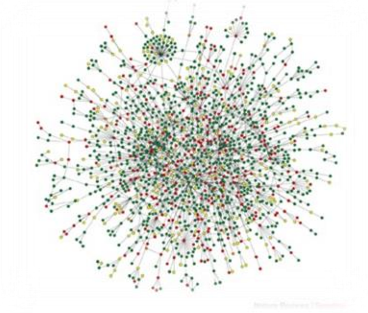


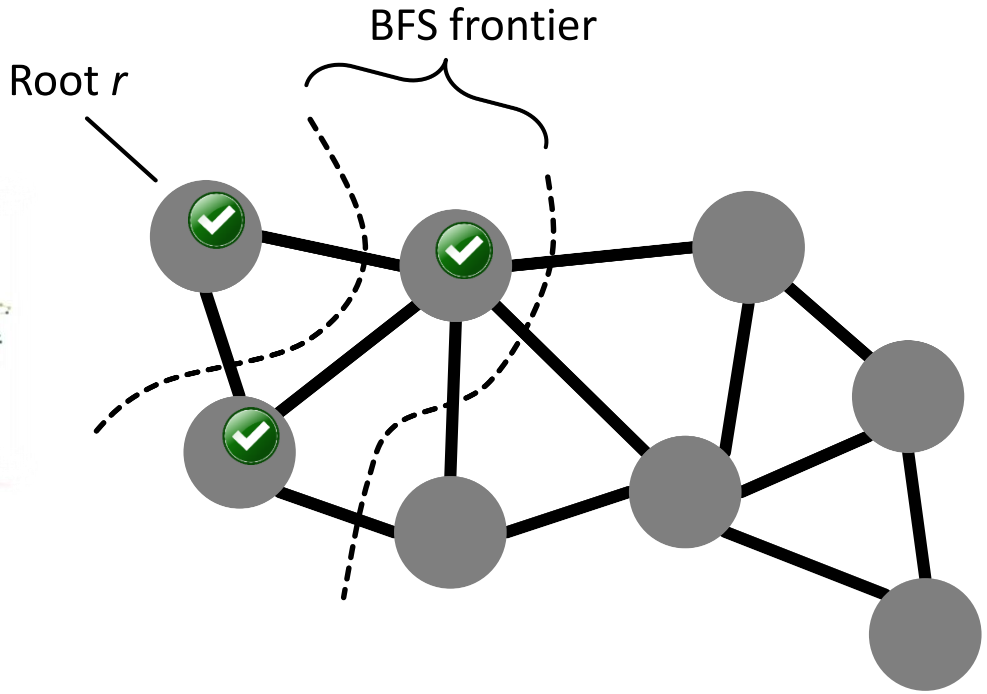
BFS
Top-Down vs. Bottom-Up [1]

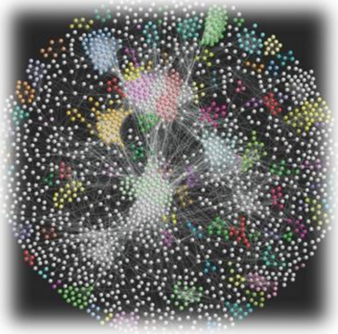




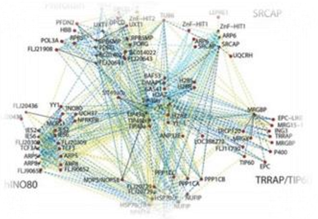
Pushing or pulling when expanding a frontier

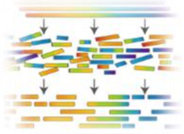


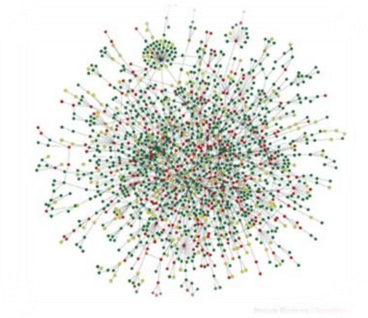




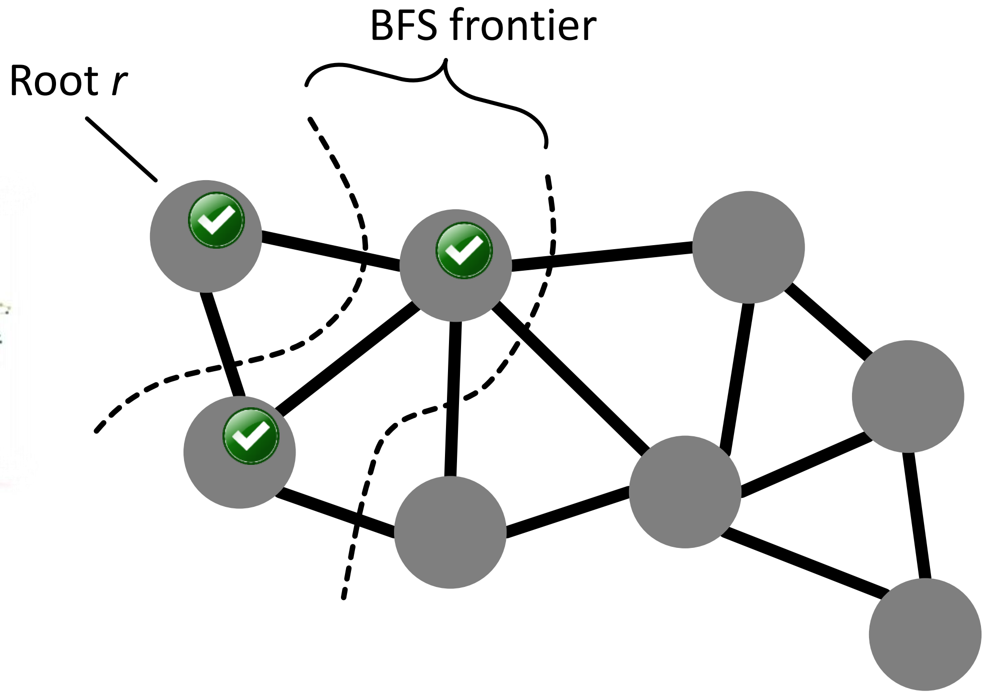
BFS
Top-Down vs. Bottom-Up [1]

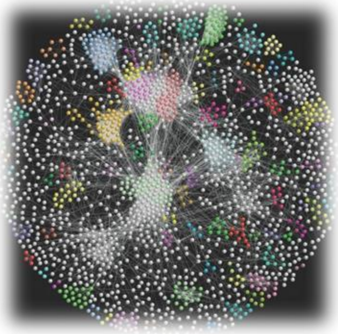






Pushing or pulling when expanding a frontier



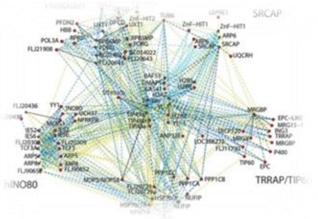


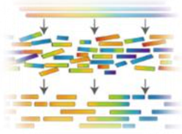
Pushing

[1] S. Beamer, K. Asanović, and D. Patterson. Direction-optimizing breadth-first search. SC12.

BFS

Top-Down vs. Bottom-Up [1]





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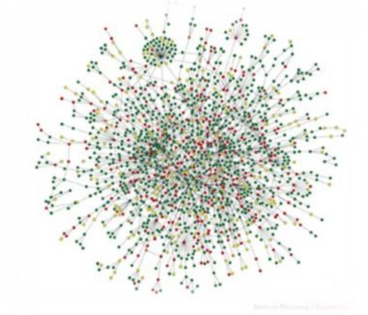

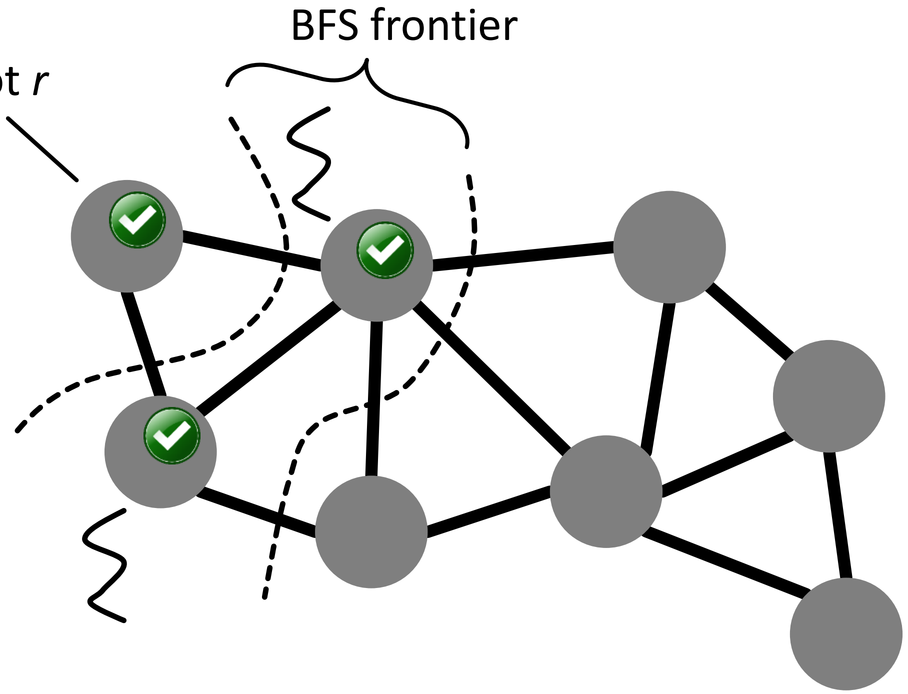

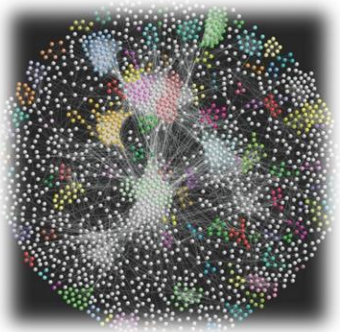

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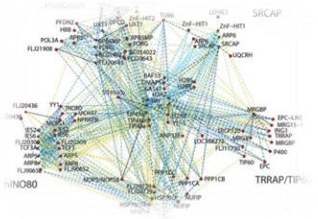
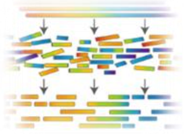
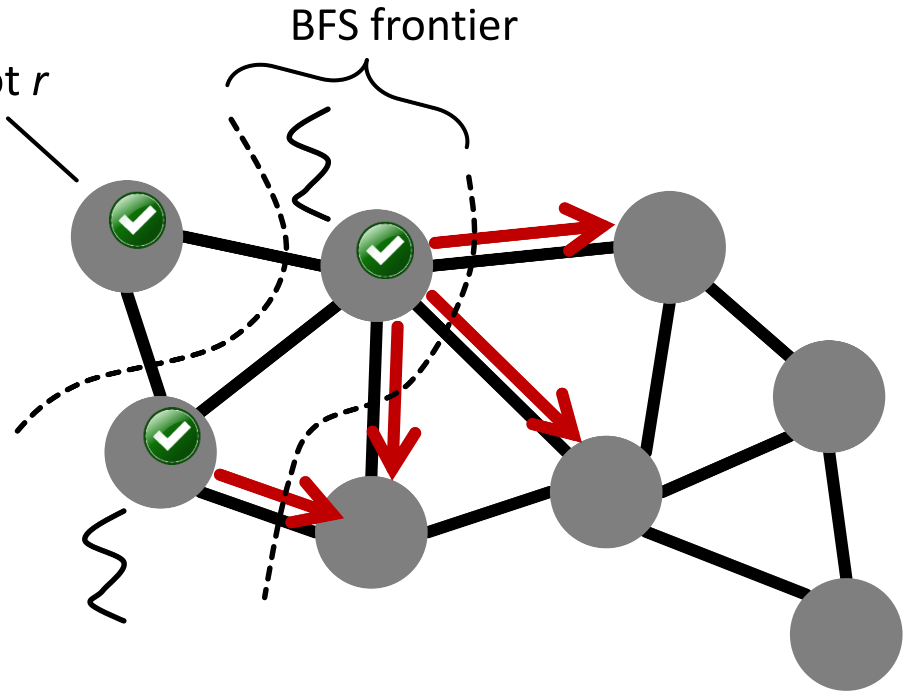
BFS

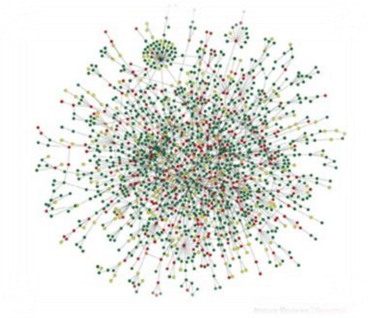
Top-Down vs. Bottom-Up [1]

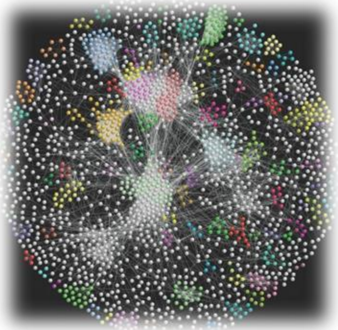


Pushing or pulling when expanding a frontier









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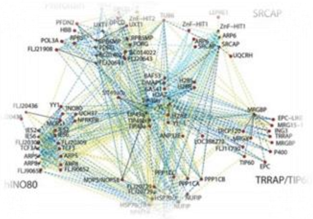
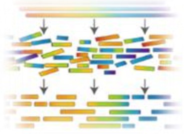
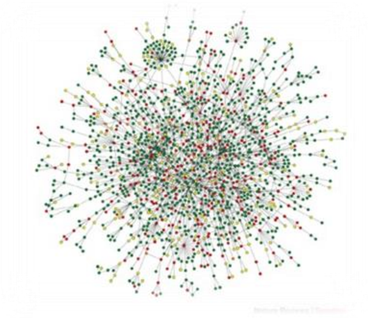
BFS

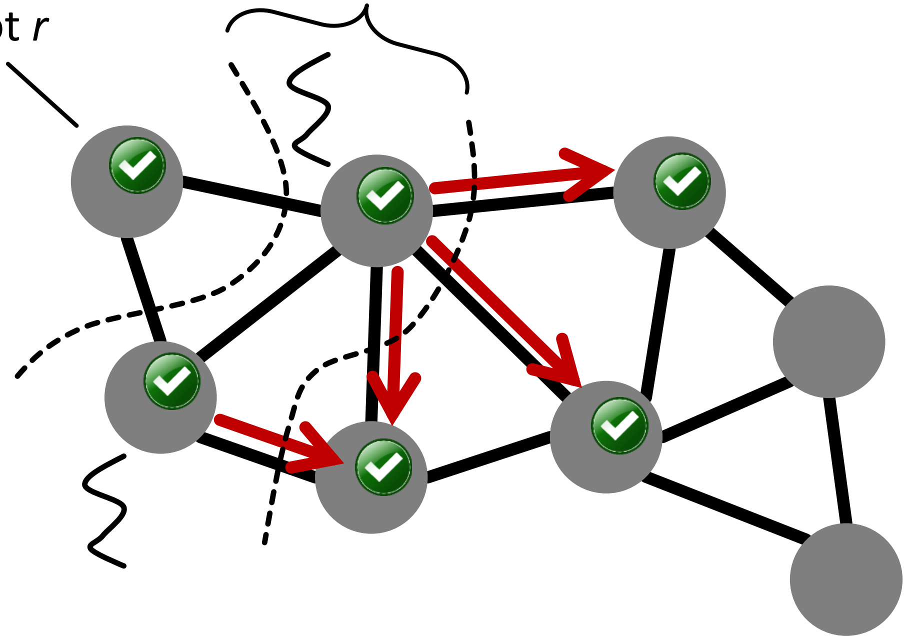
Top-Down vs. Bottom-Up [1]

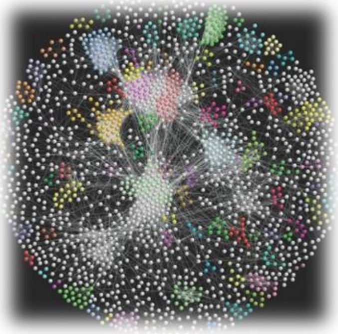




Pushing or pulling when expanding a frontier







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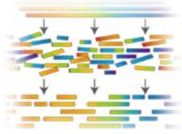
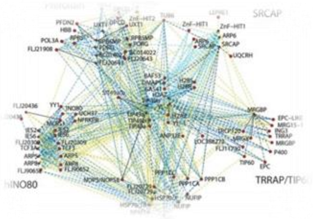
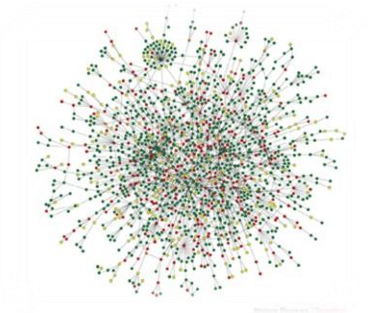
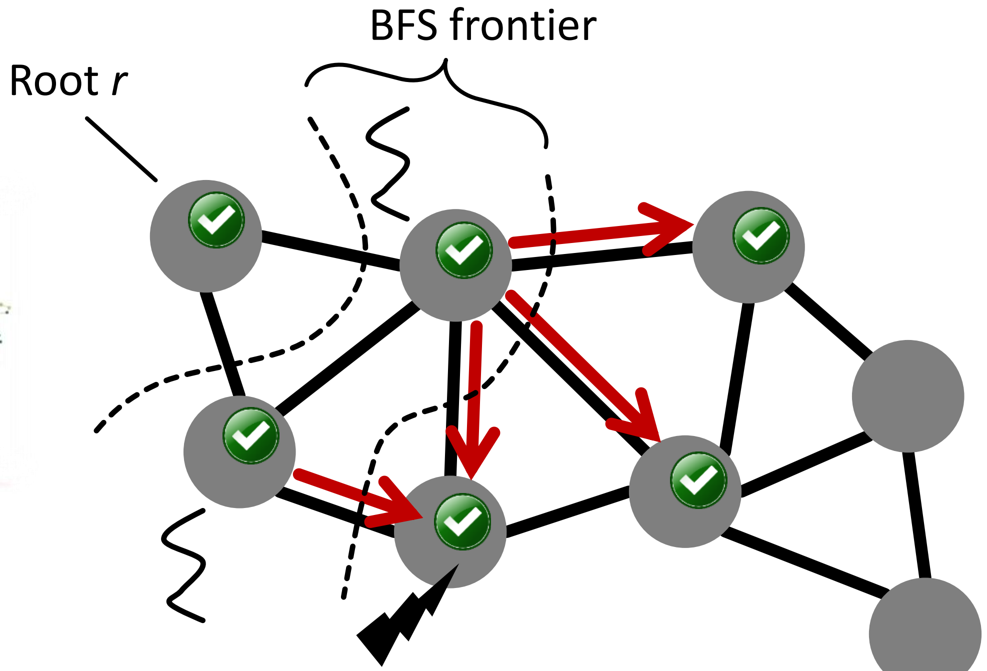
BFS
Top-Down vs. Bottom-Up [1]

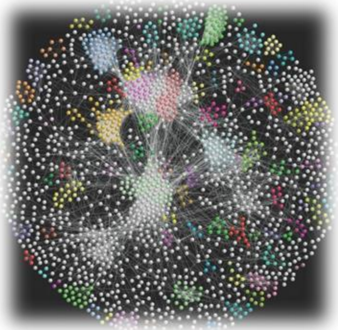




Pushing or pulling when expanding a frontier







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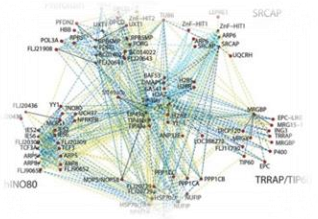
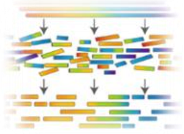
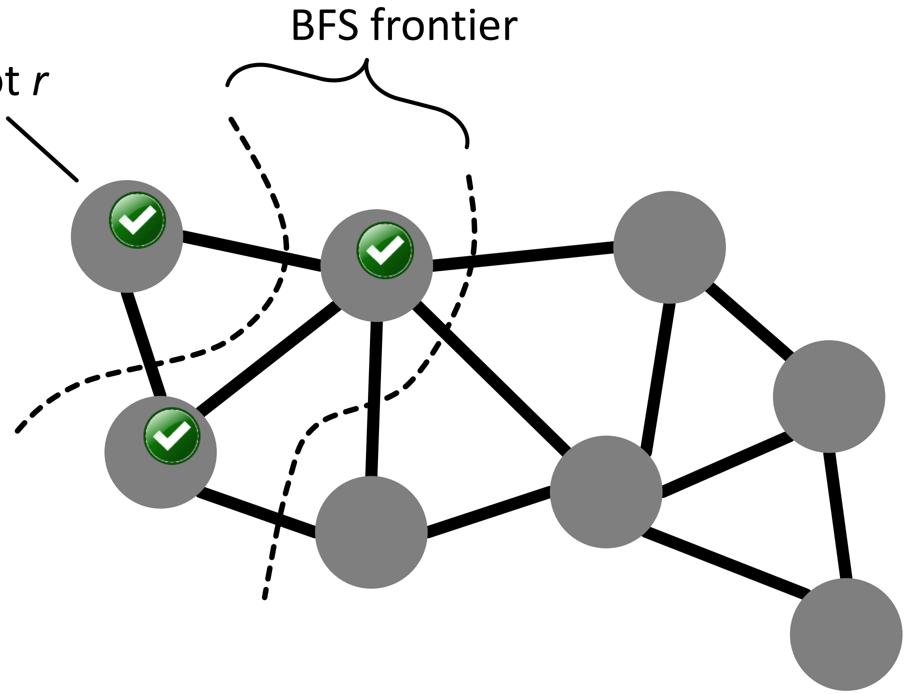
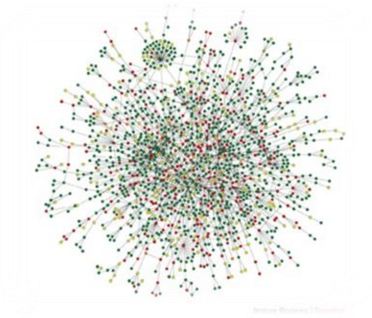
BFS
Top-Down vs. Bottom-Up [1]

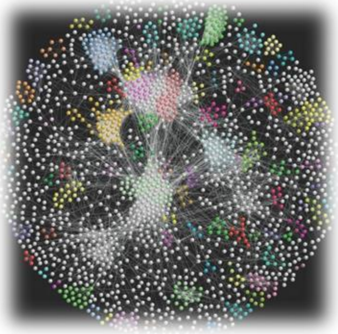




Pushing or pulling when expanding a frontier

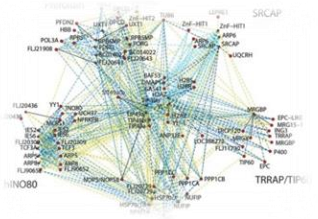


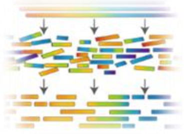




[1] S. Beamer, K. Asanović, and D. Patterson. Direction-optimizing breadth-first search. SC12.

BFS
Top-Down vs. Bottom-Up [1]





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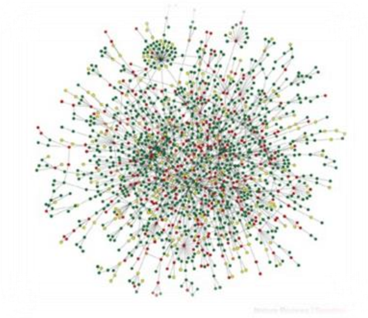

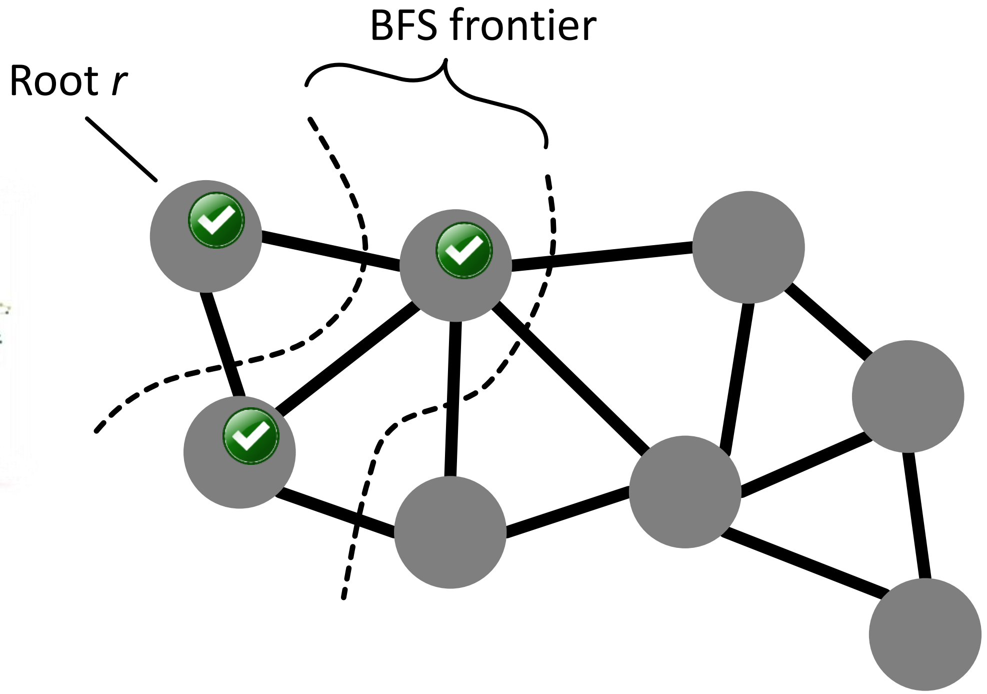

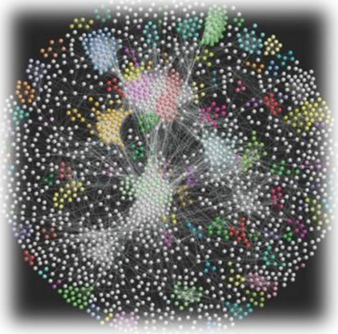

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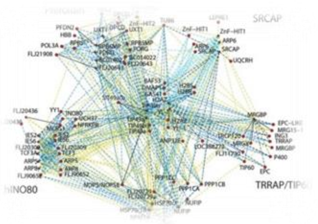
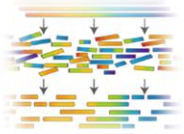
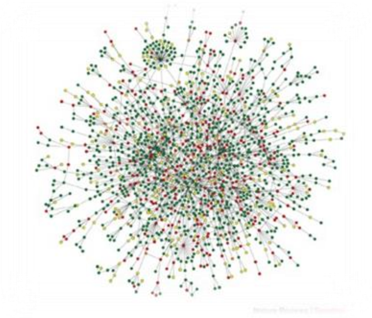
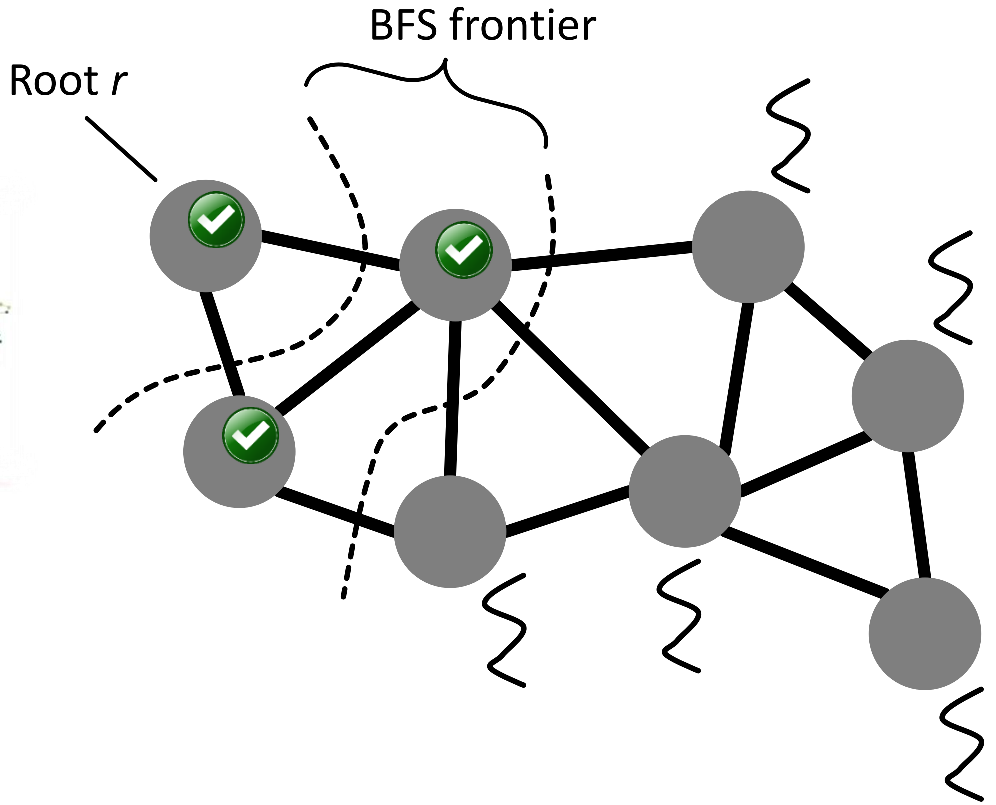
Top-Down vs. Bottom-Up [1]

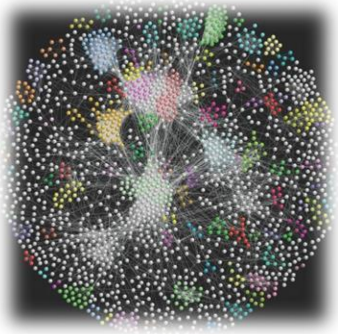






Pushing or pulling when expanding a frontier





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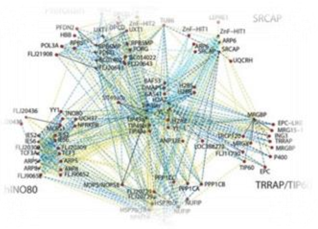
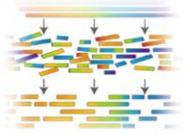
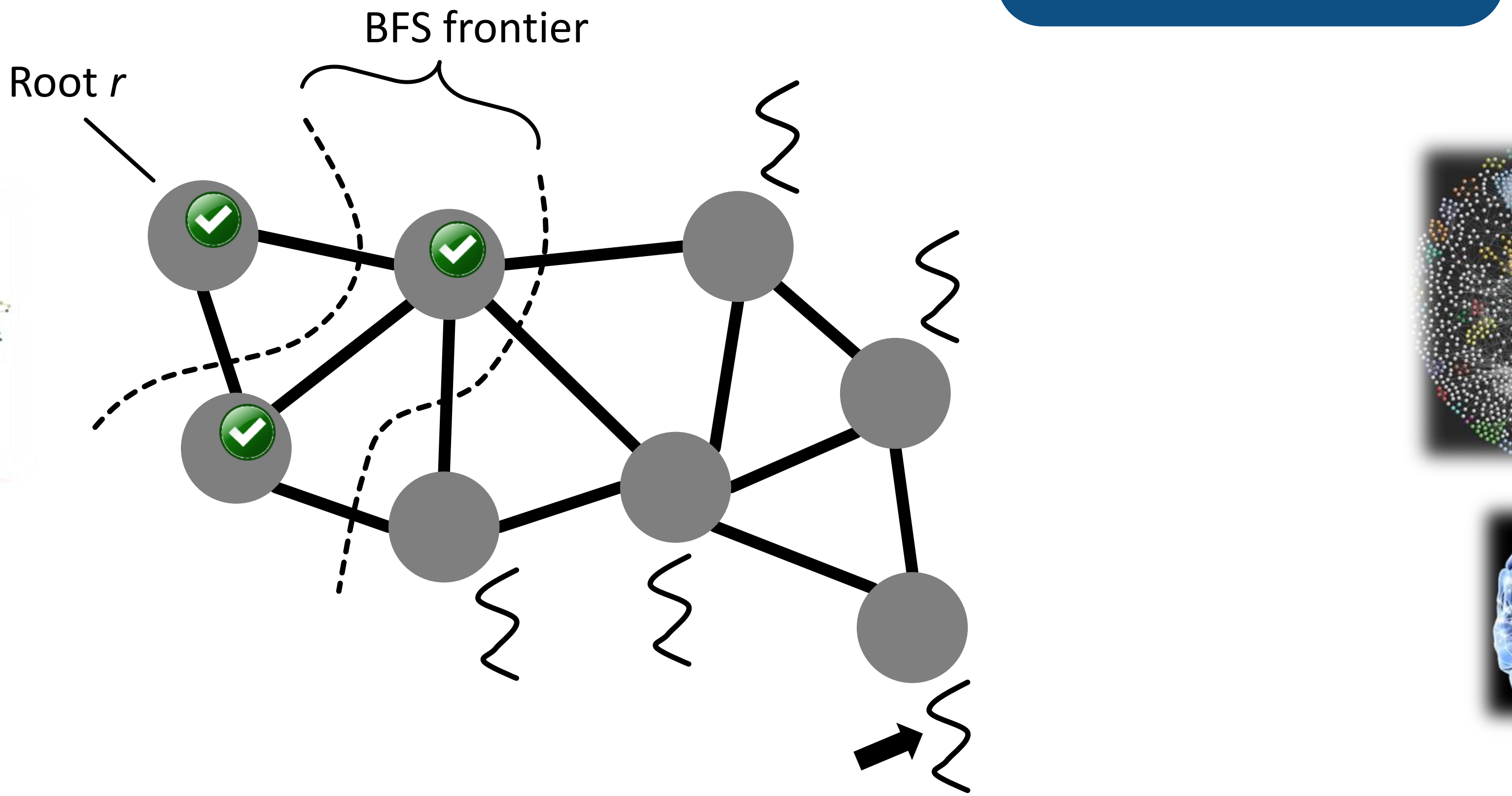
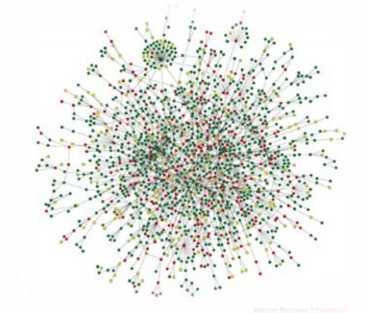
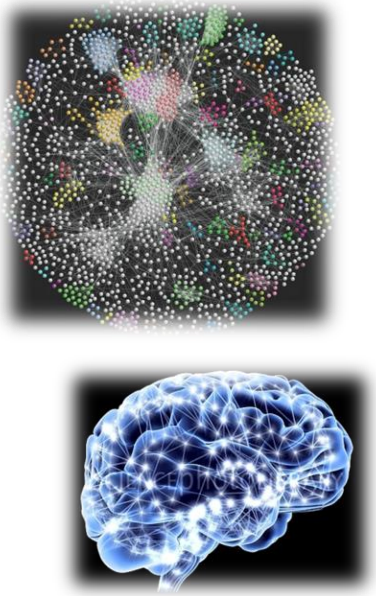
BFS
Top-Down vs. Bottom-Up [1]











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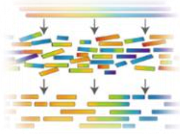
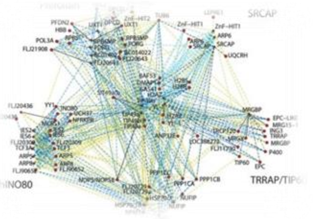
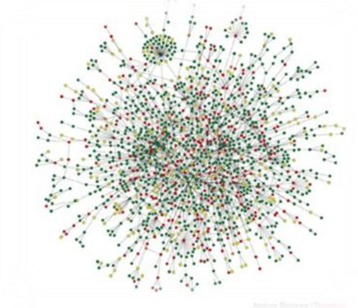
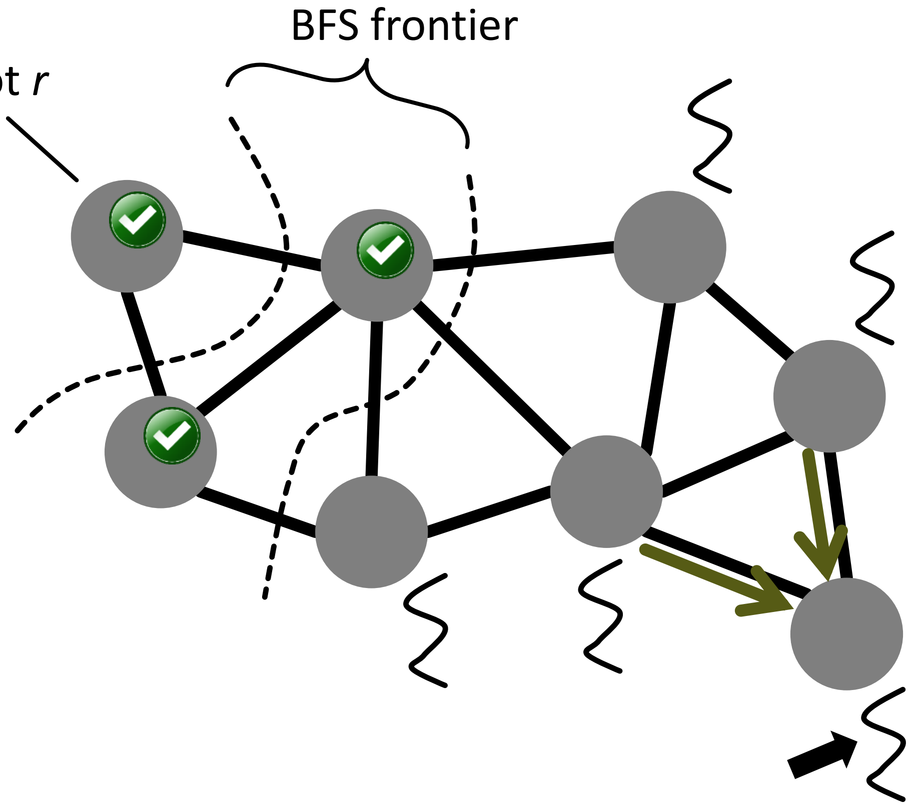
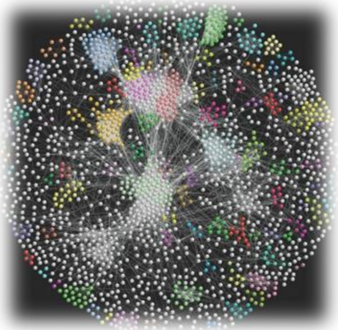




Pushing or pulling when expanding a frontier







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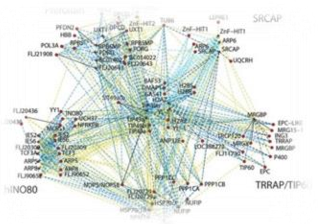

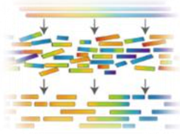

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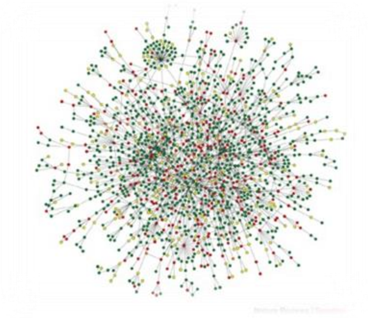

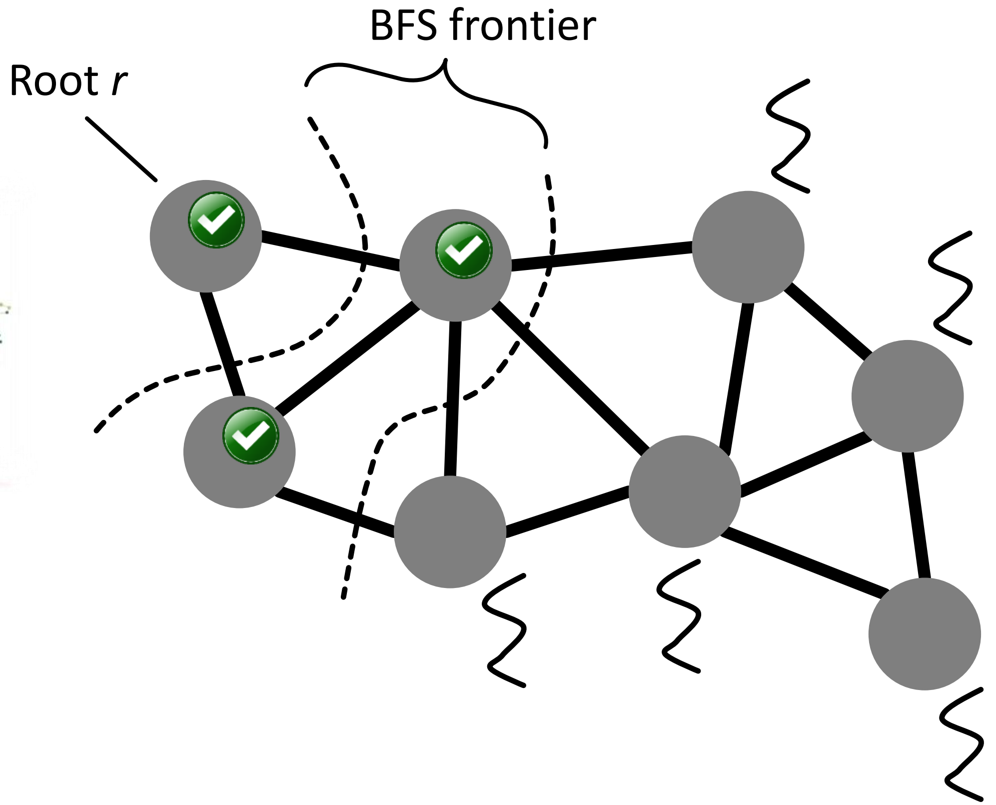

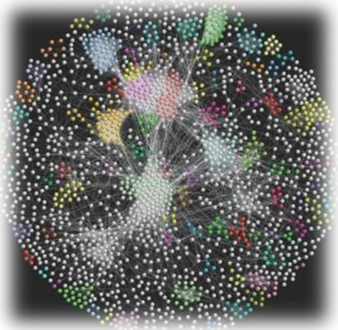

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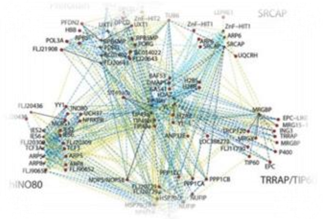

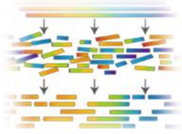

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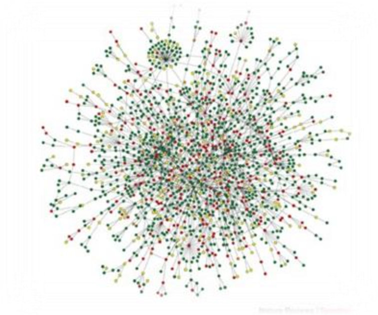

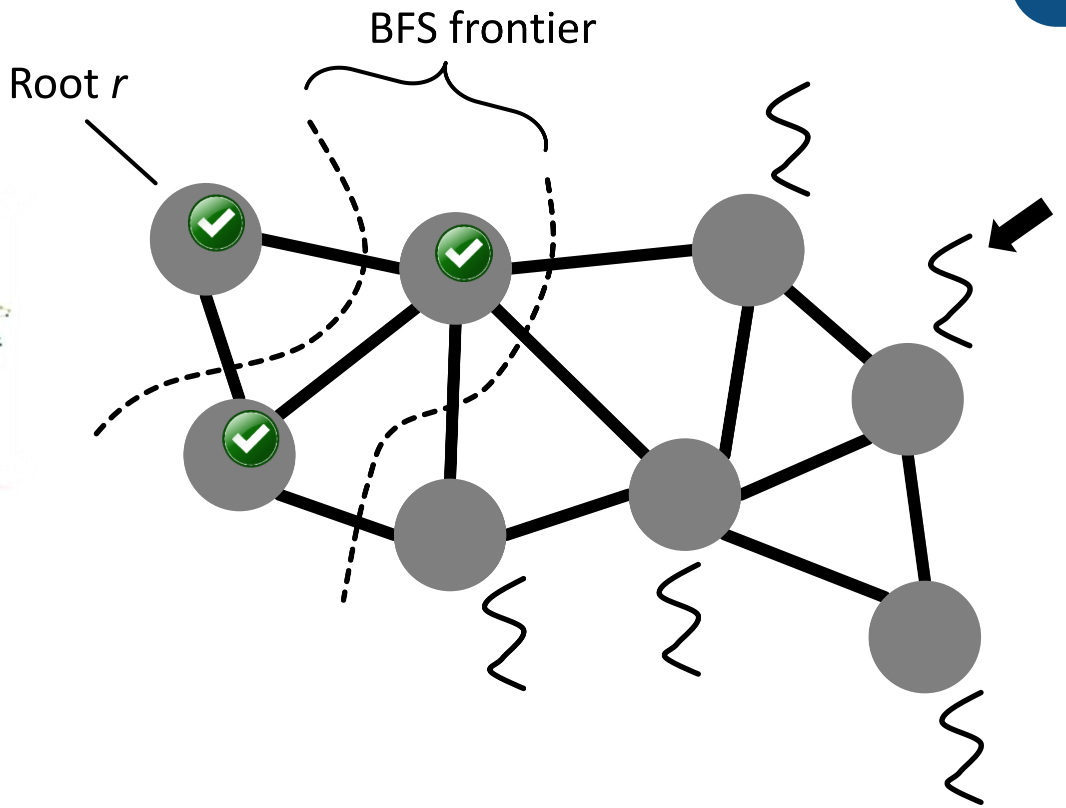

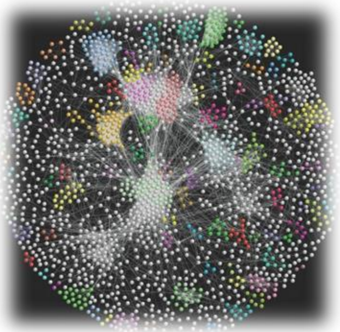

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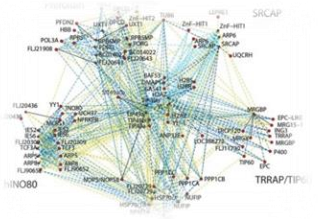

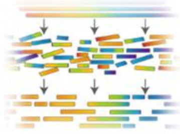

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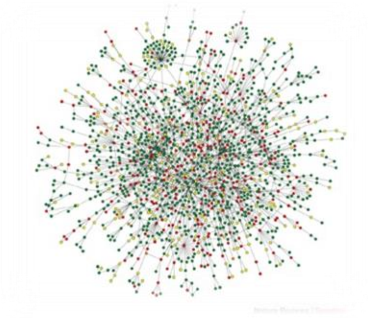

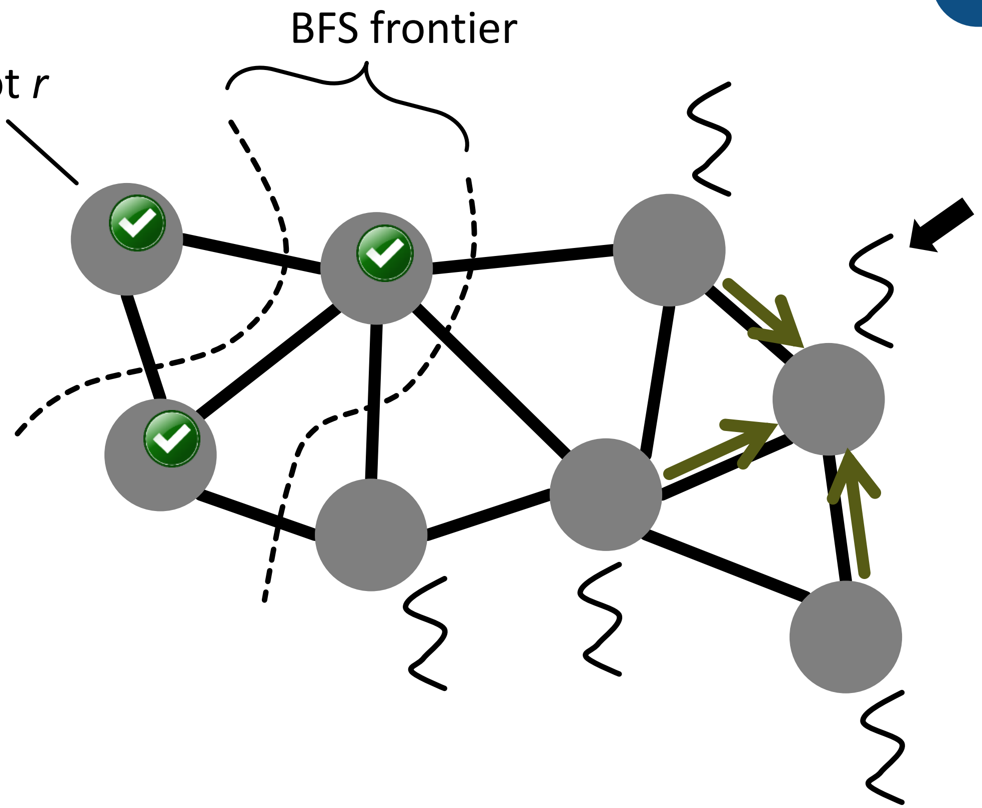

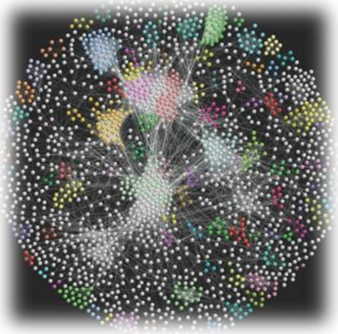

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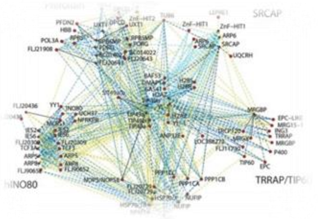
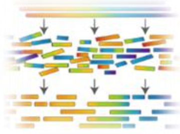
BFS
TOP-DOWN VS. BOTTOM-UP [1]





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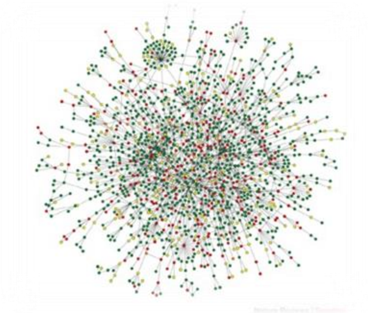

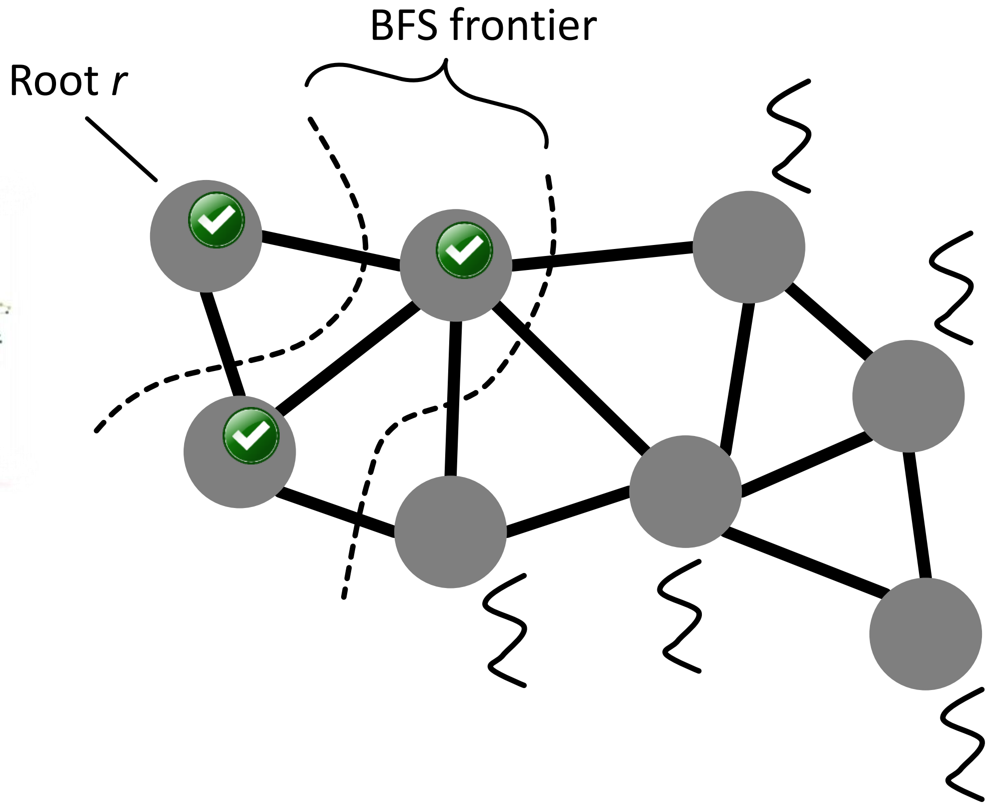

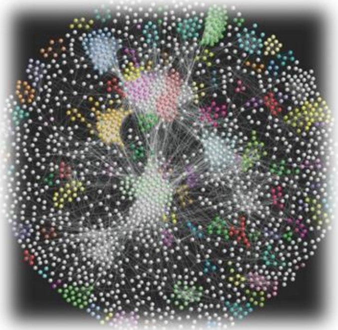

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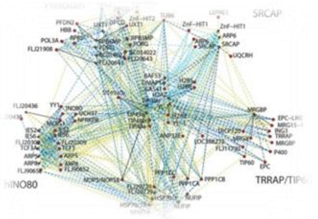

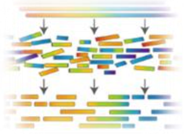

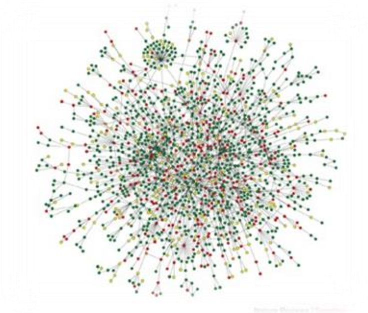

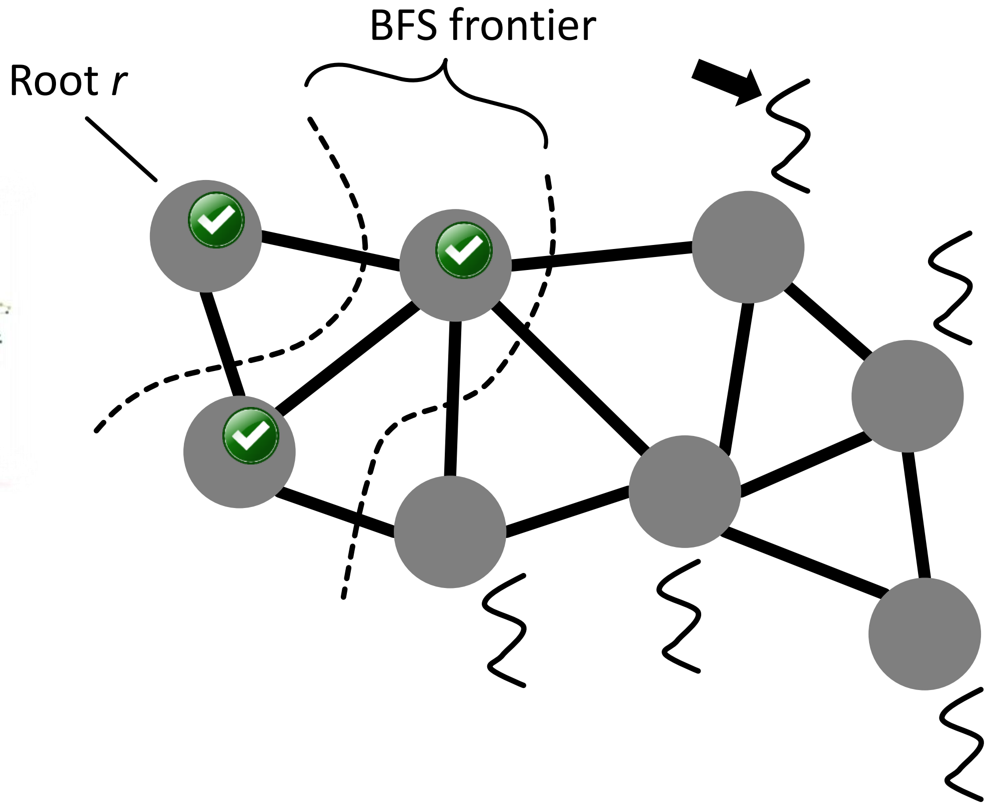

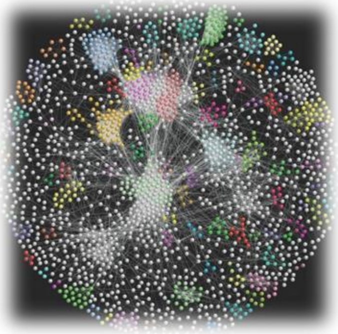

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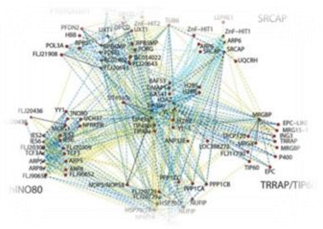

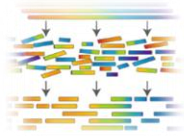

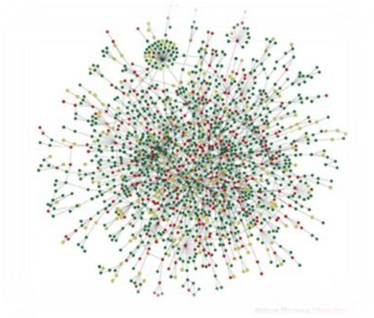

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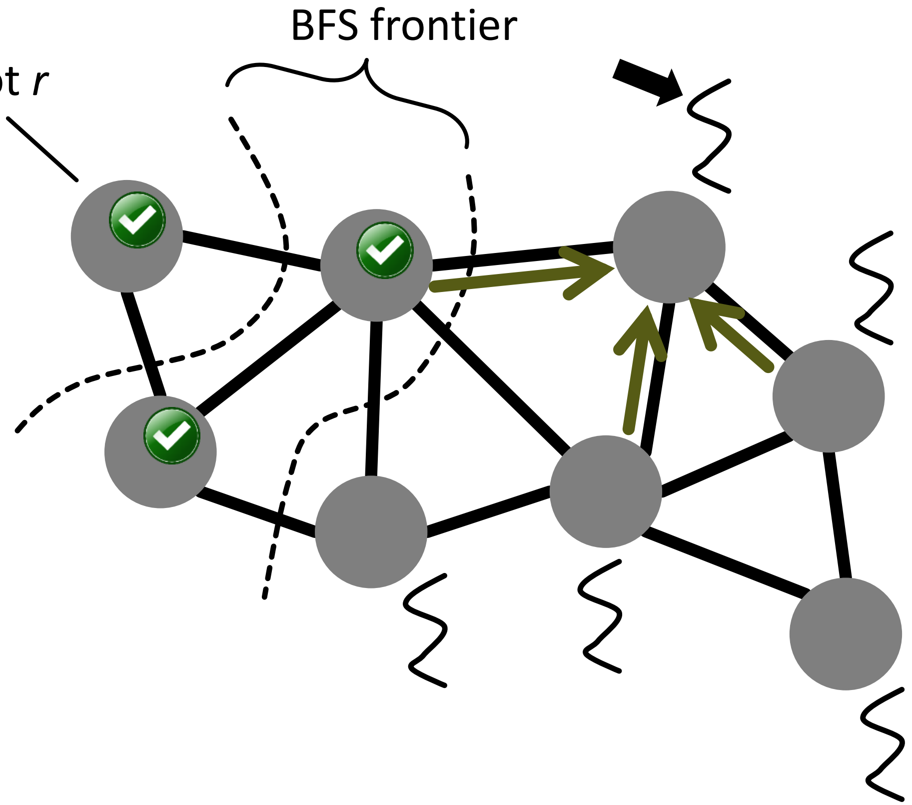

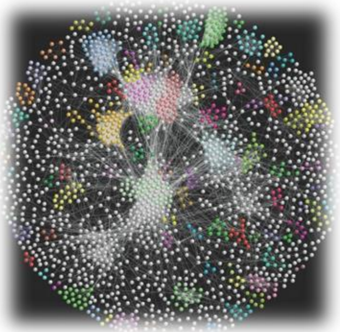

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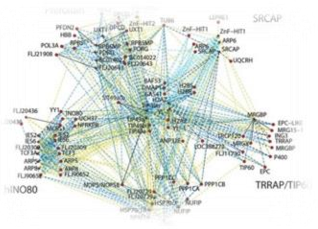

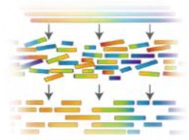

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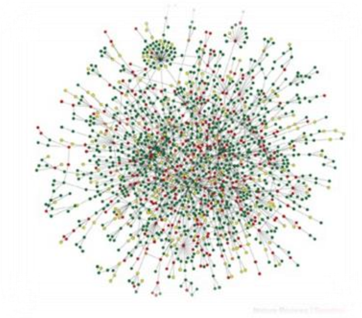

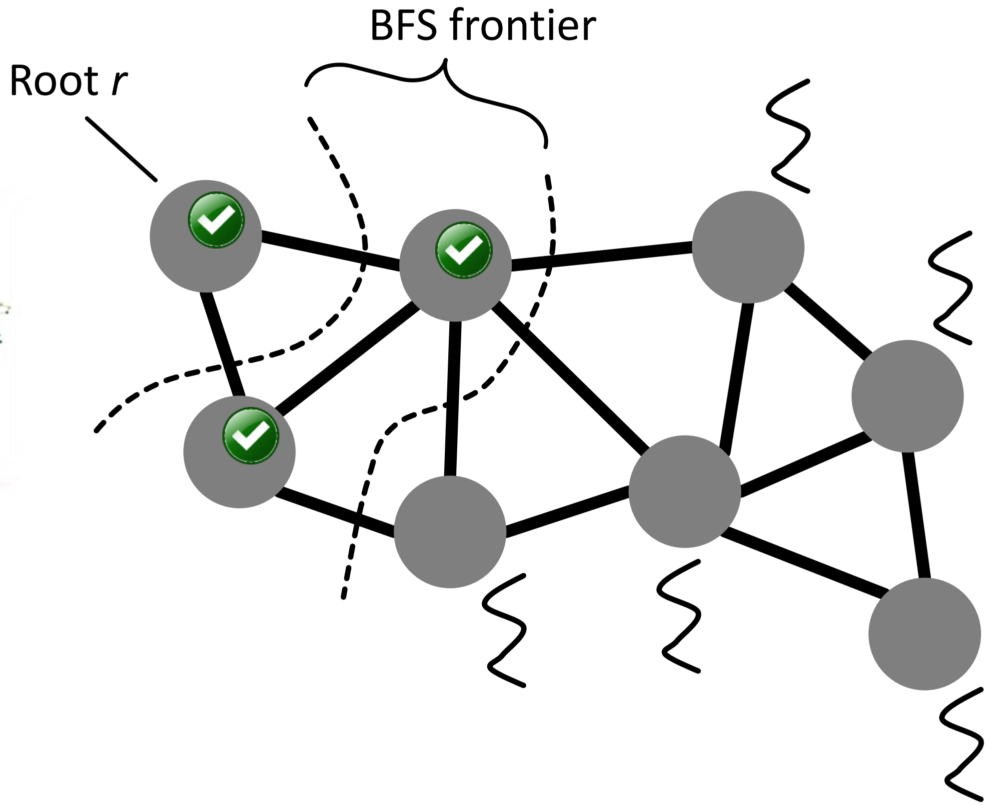

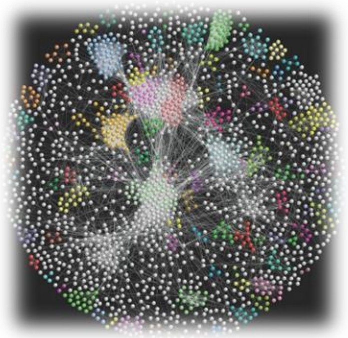

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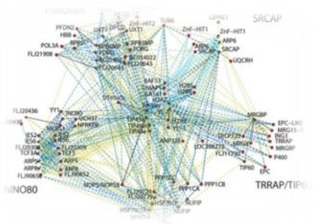

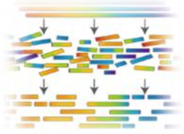

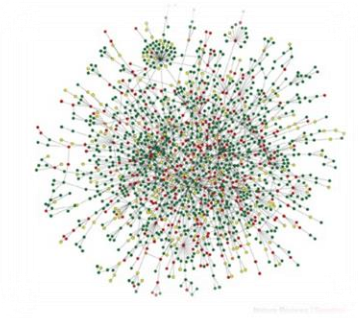

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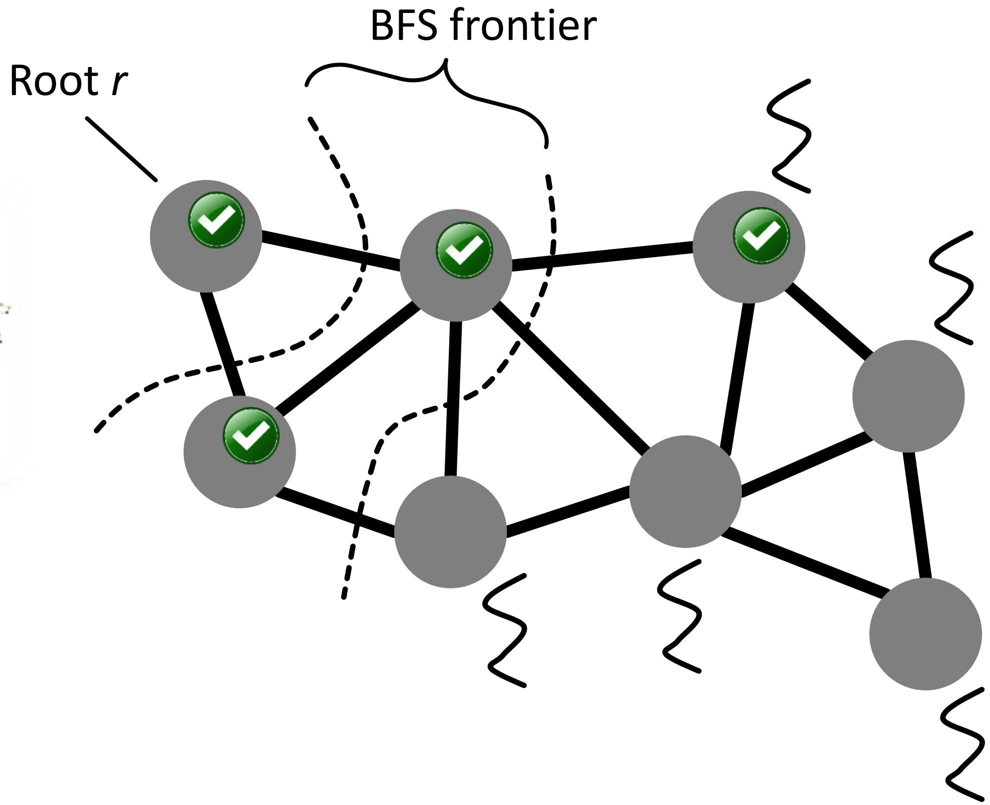

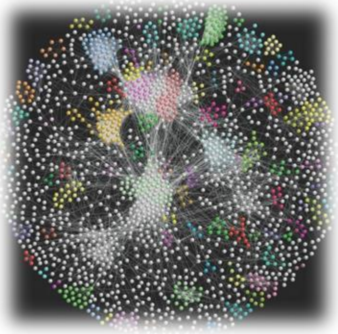

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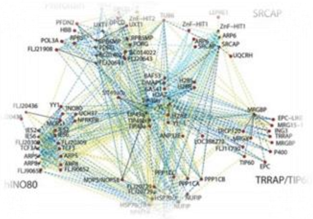

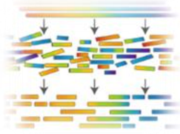

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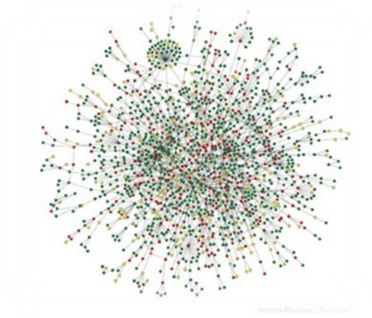

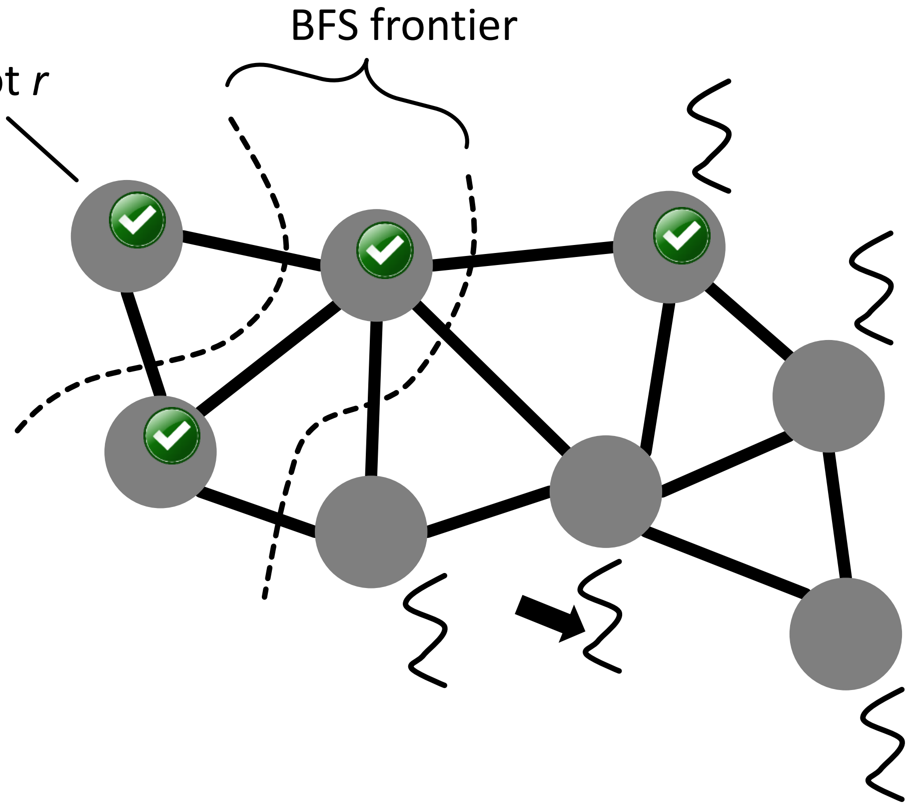

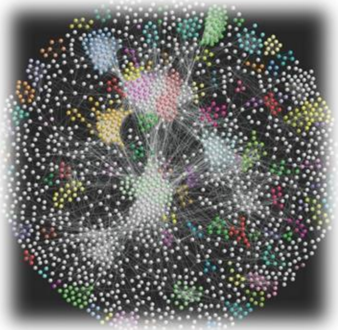

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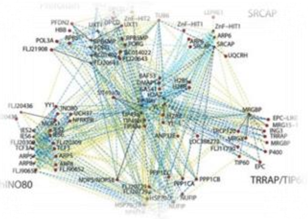
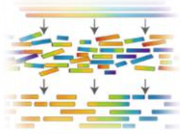
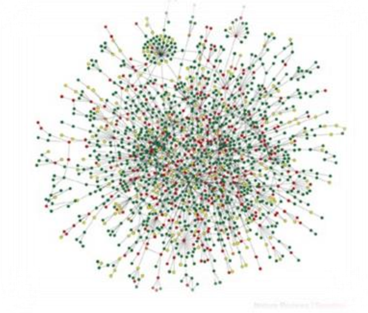
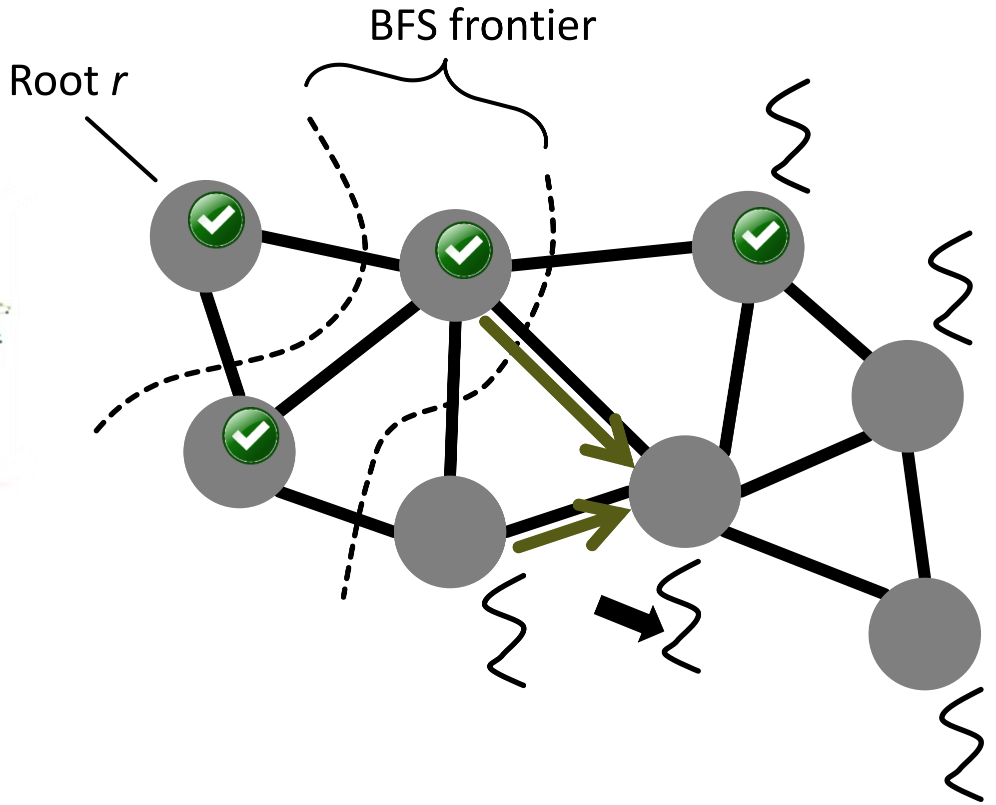
BFS
Top-Down vs. Bottom-Up [1]

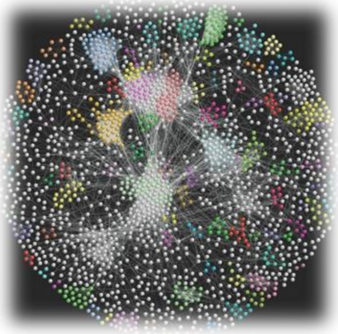




Pushing or pulling when expanding a frontier







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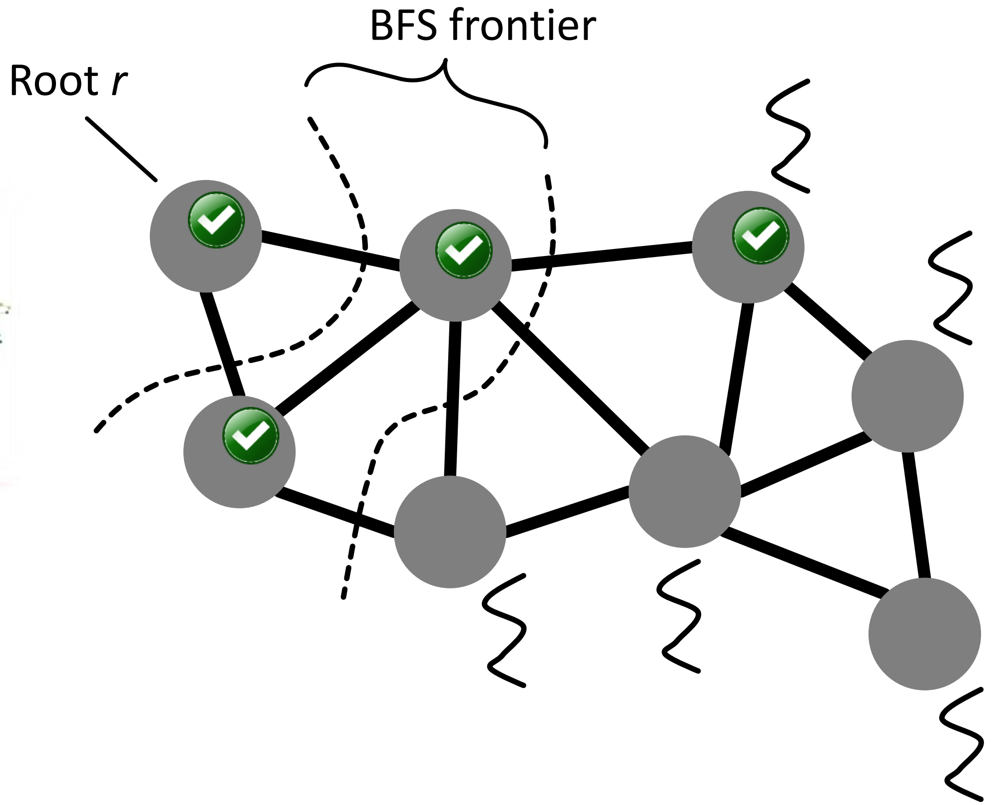

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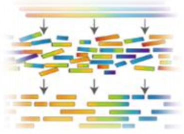

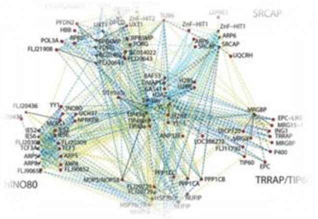

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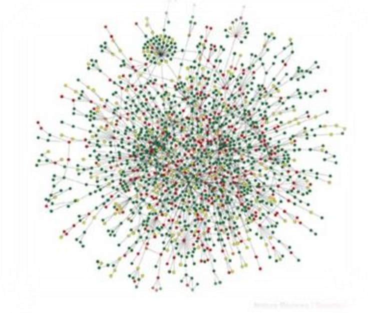

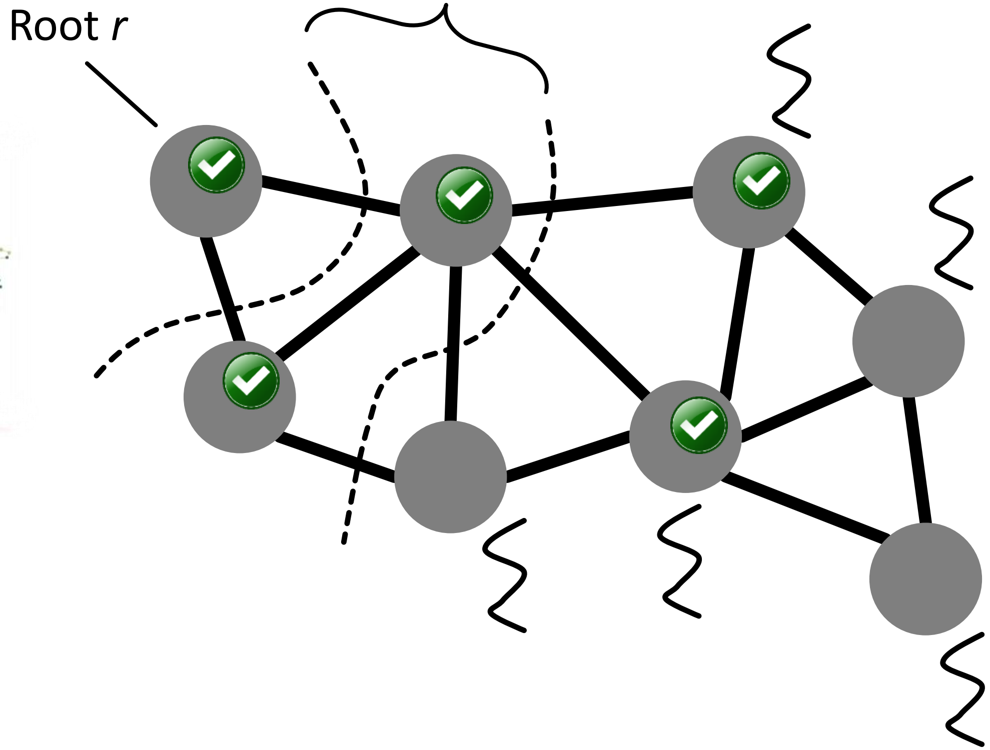

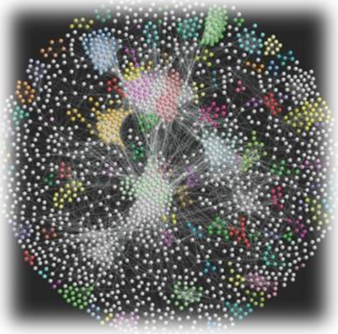

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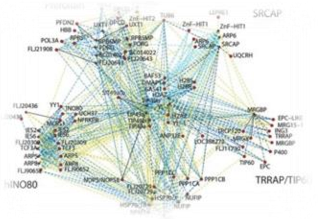

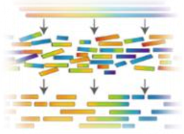

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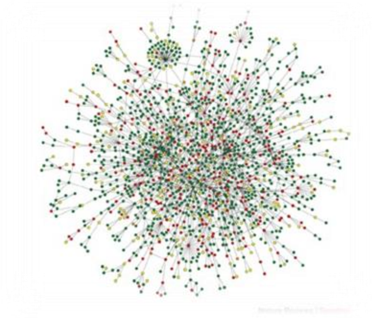

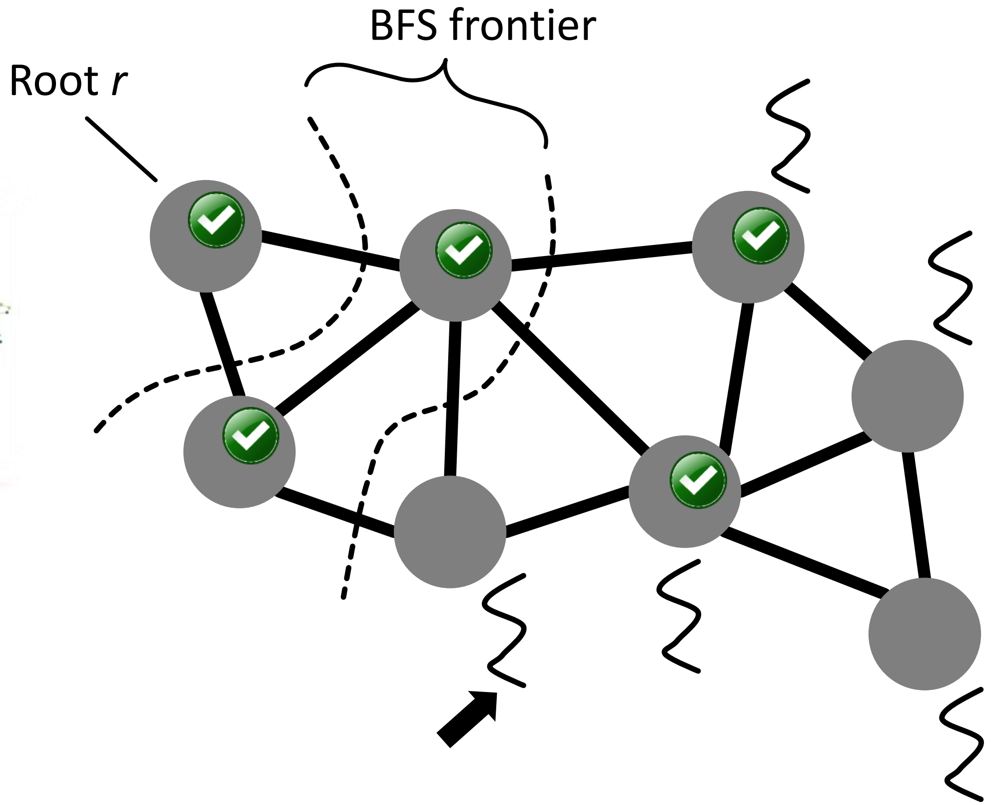

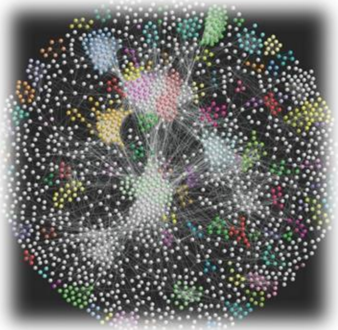

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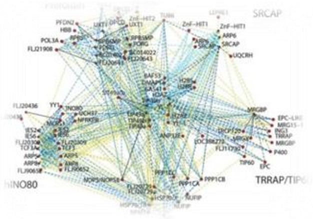

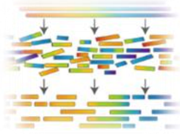

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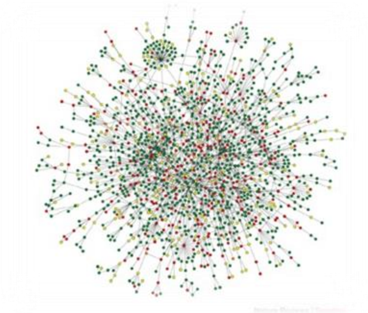

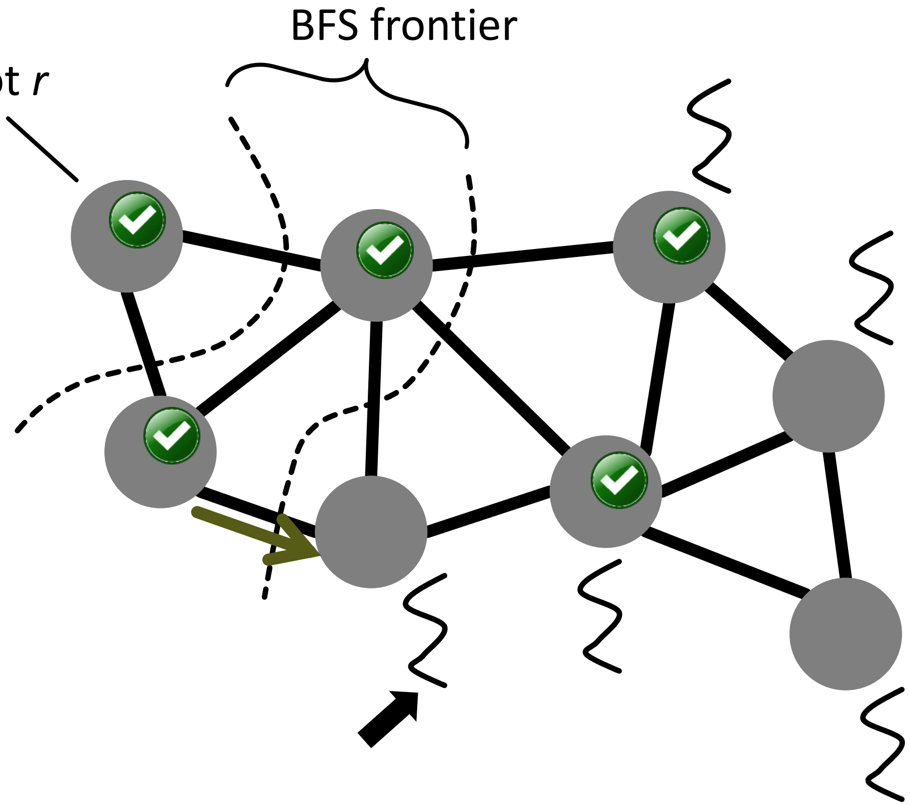

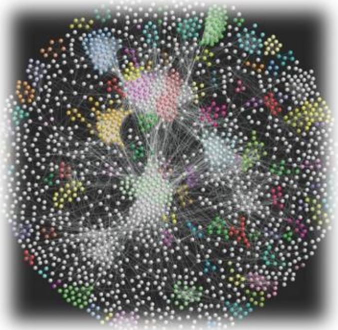

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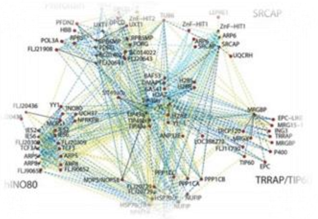

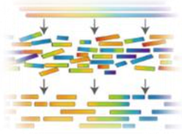

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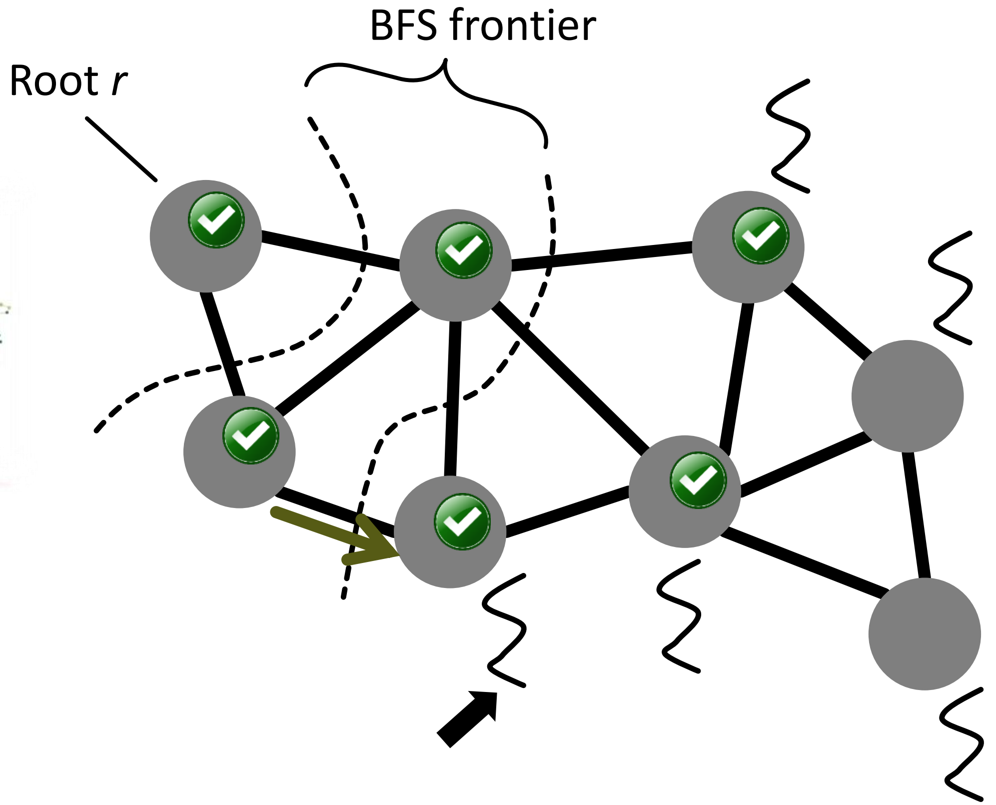

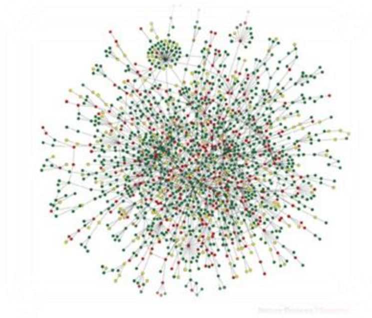

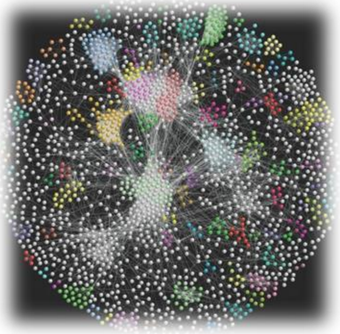

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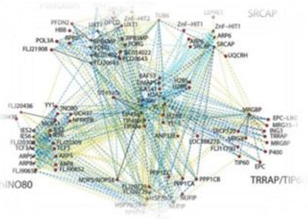

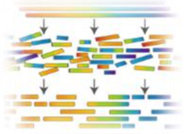

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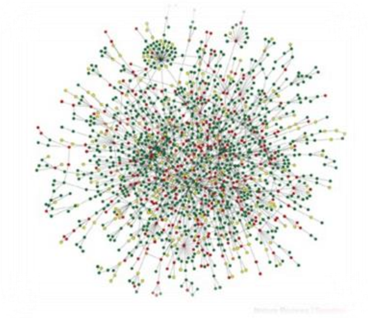

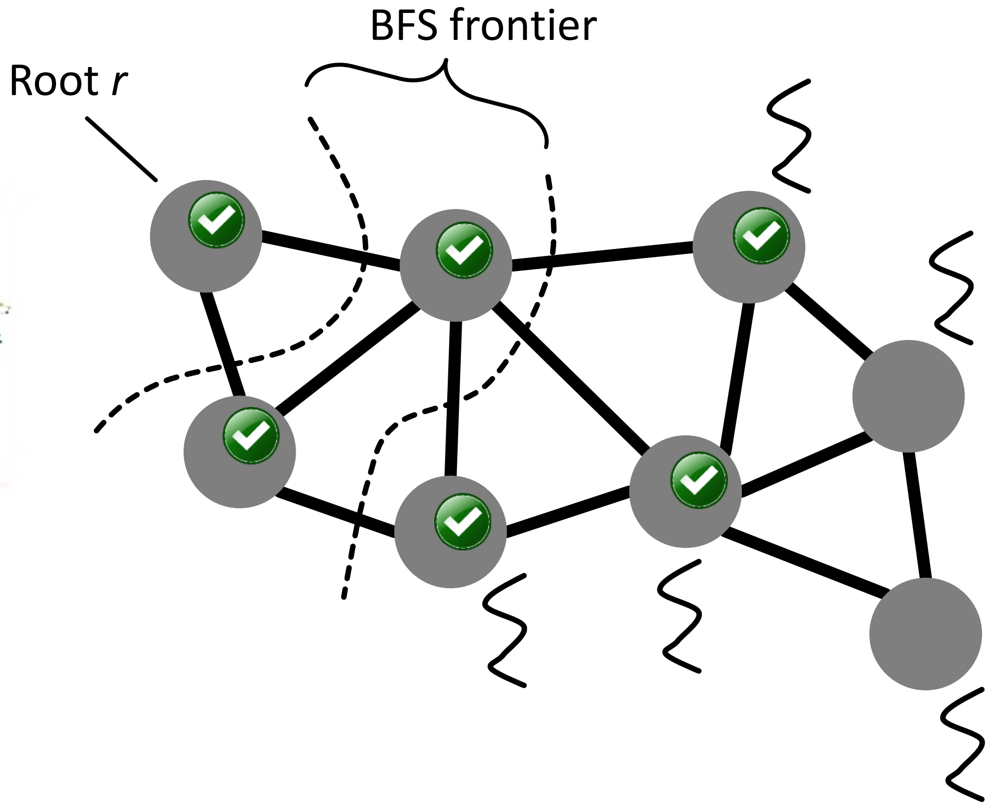

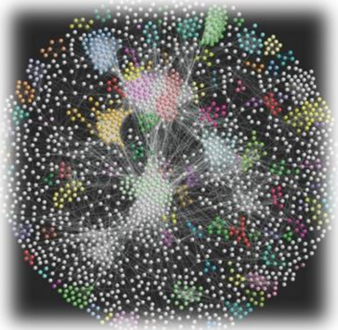

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OTHER ALGORITHMS & FORMULATIONS

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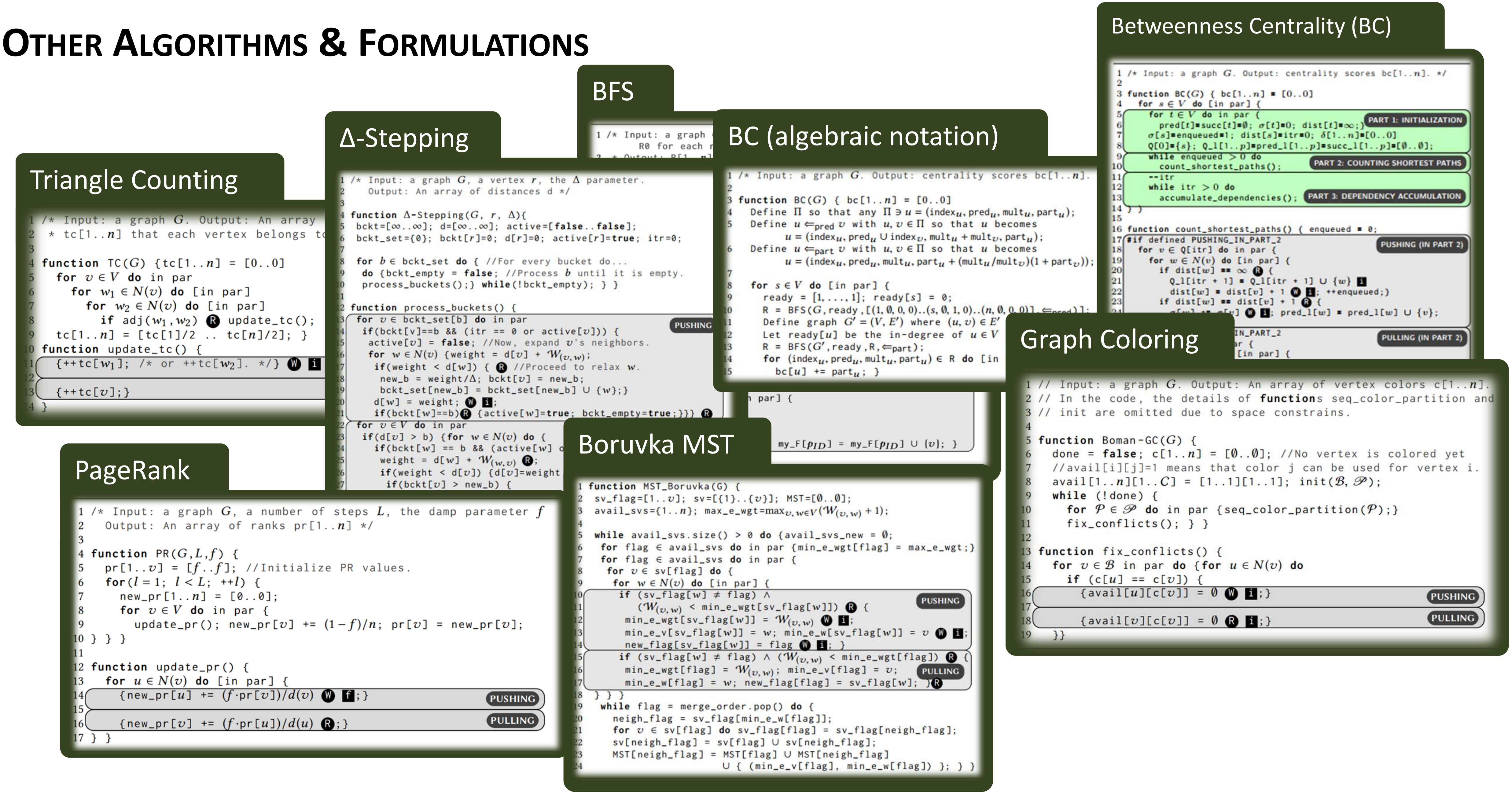
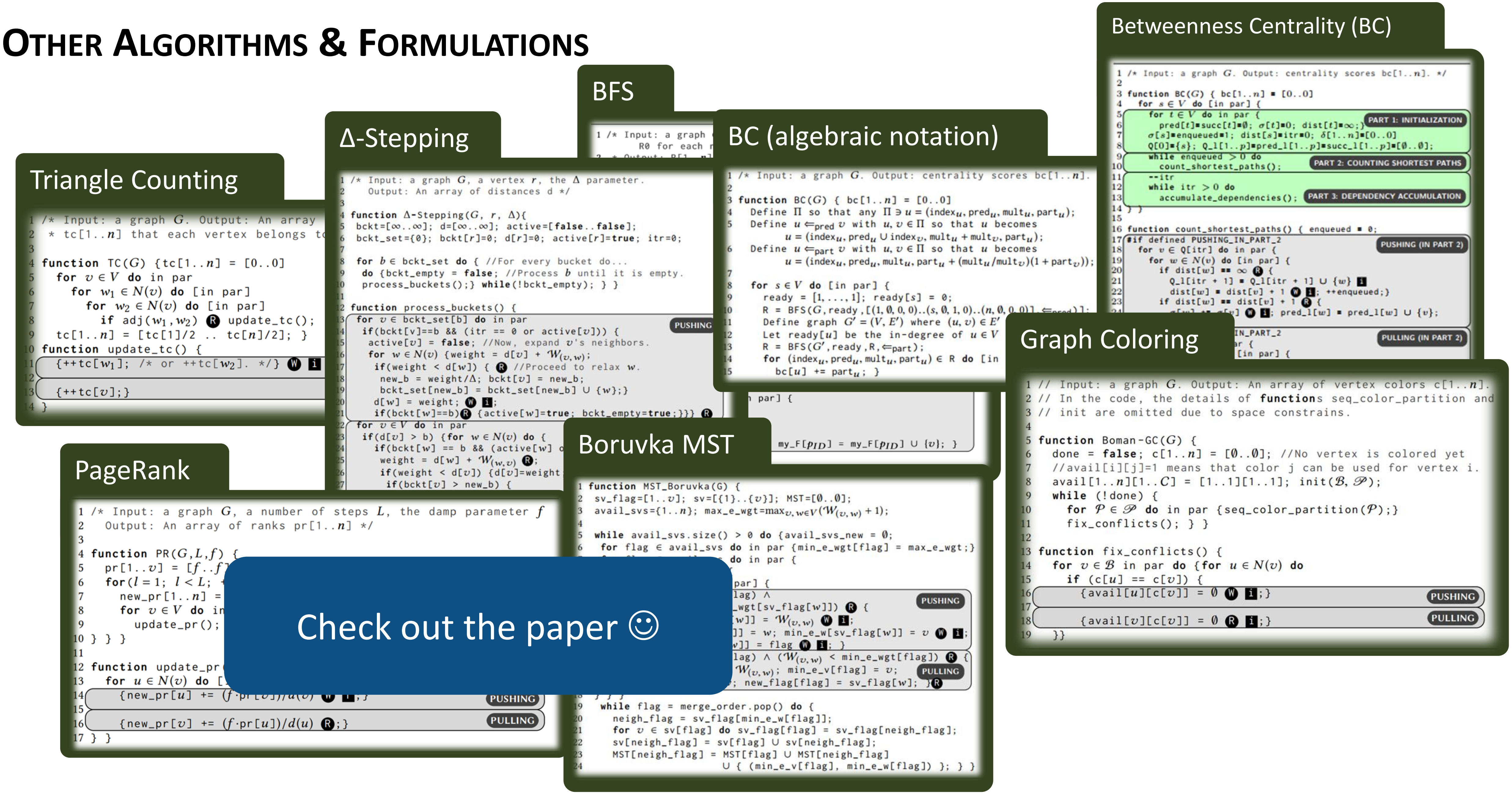


M. Besta et al.: “To Push or To Pull: On Reducing Communication and Synchronization in Graph Computations”, HPDC’17

Other Algorithms & Formulations



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PUSHING VS. PULLING

GENERIC DIFFERENCES

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PUSHING VS. PULLING

GENERIC DIFFERENCES

What pushing vs. pulling *really* is?

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Pushing vs. Pulling

Generic Differences

- Vertices: $v \in V$
- $t \leadsto v \Leftrightarrow t$ modifies v
- $t[v]$: a thread that owns v

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Algorithm uses pushing ⇔

$$(\exists t\ \exists v \in V:\ \ t \leadsto v \land t \neq t[v])$$

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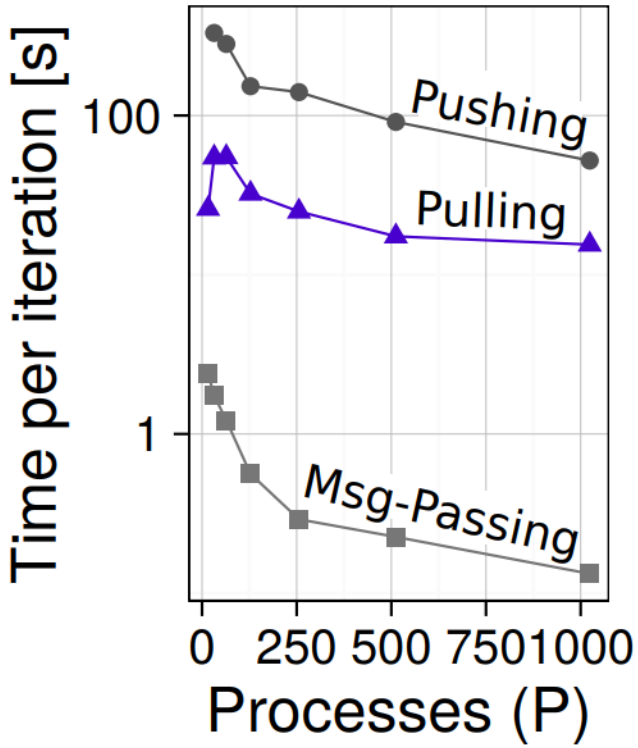
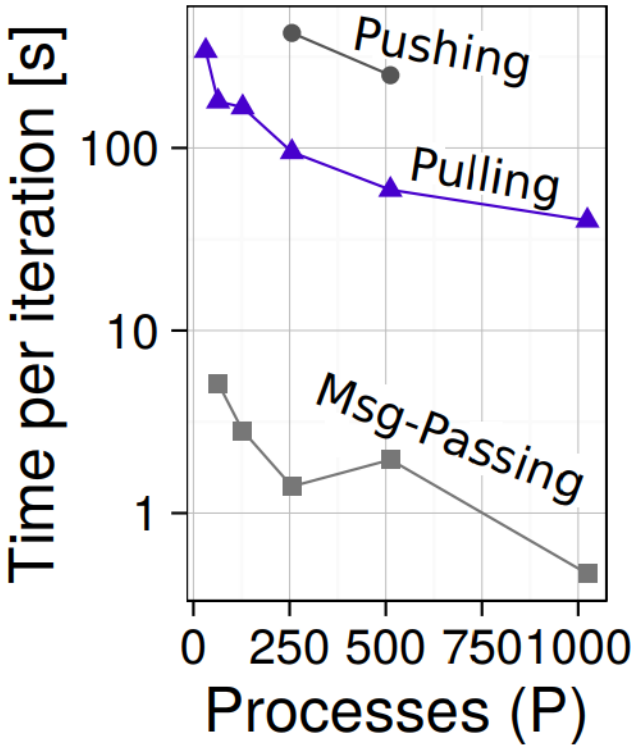
PERFORMANCE ANALYSIS
PAGERANK

Kronecker graphs

Distributed-Memory





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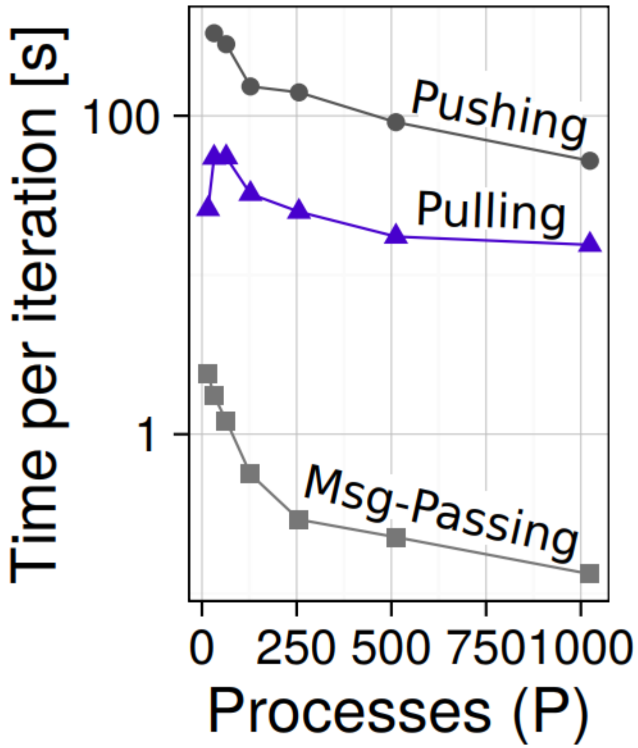
Performance Analysis

PageRank

Kronecker graphs

Distributed-Memory

$n = 2^{25}, m = 2^{27}$



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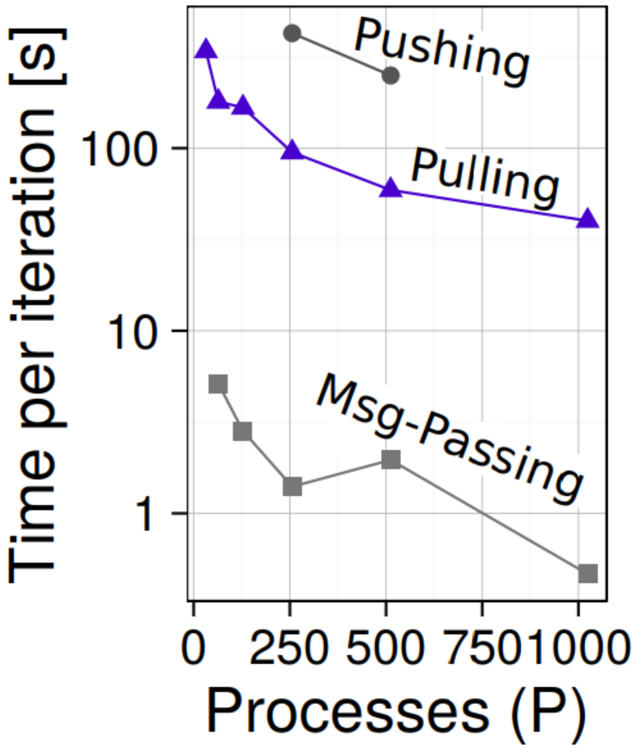

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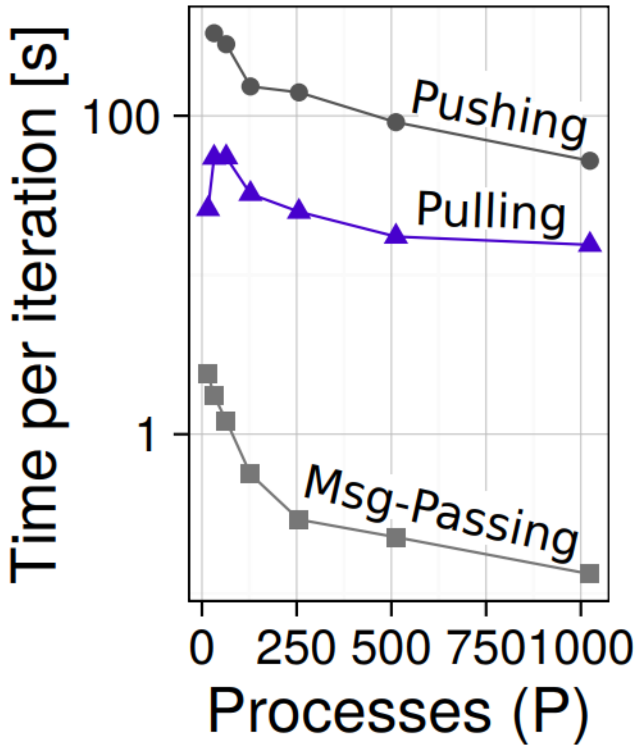

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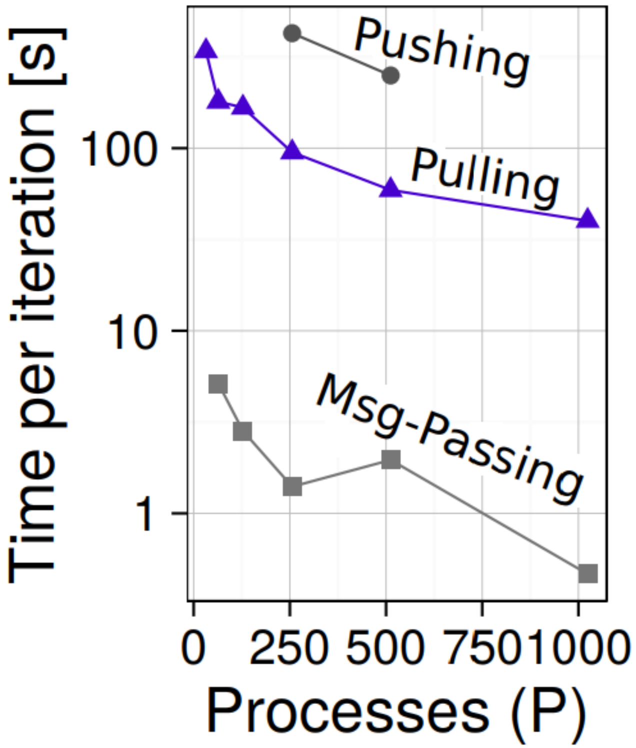

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Pulling incurs more communication while pushing expensive underlying locking

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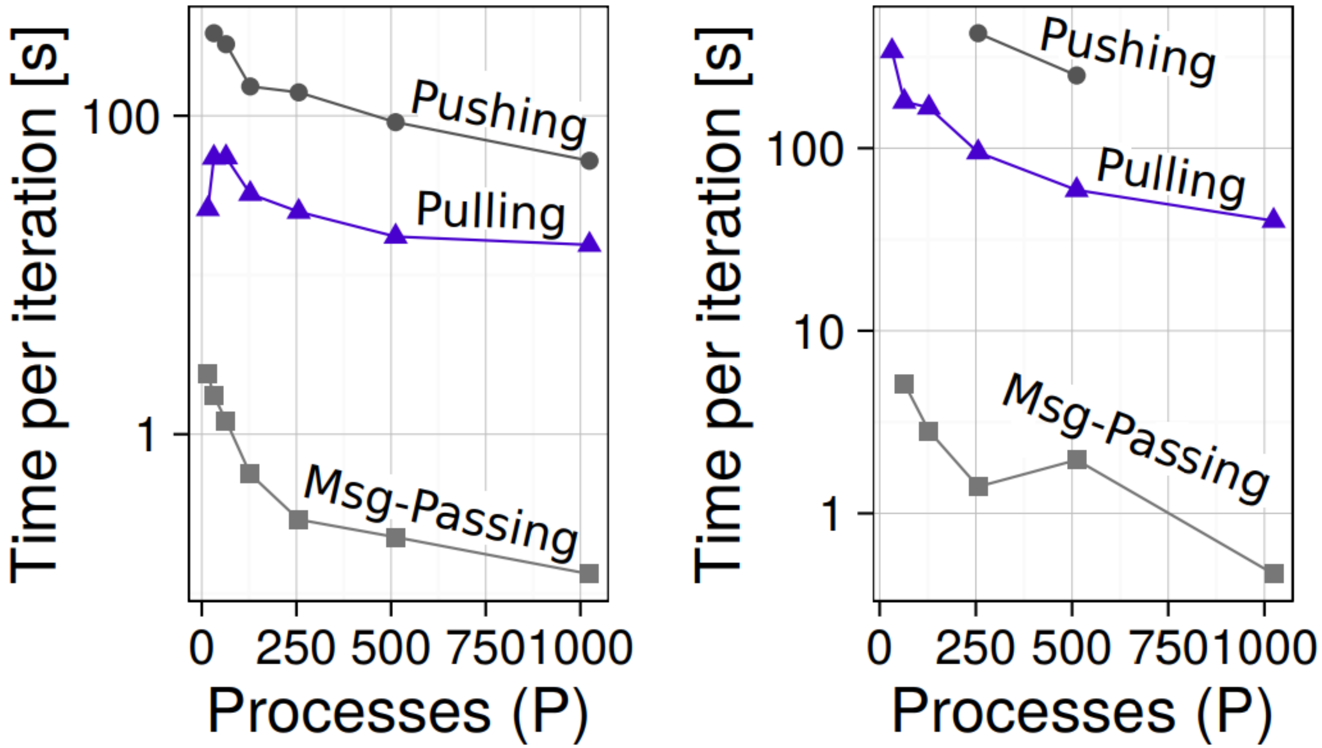

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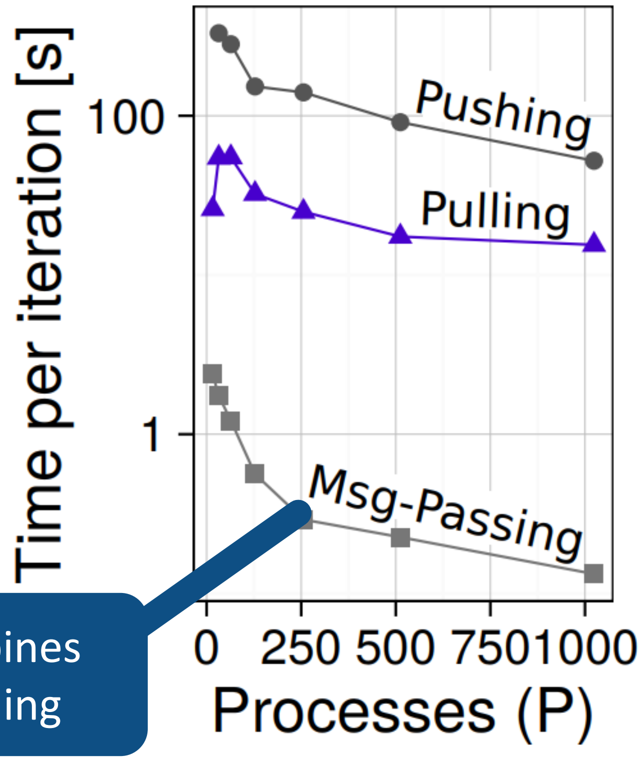

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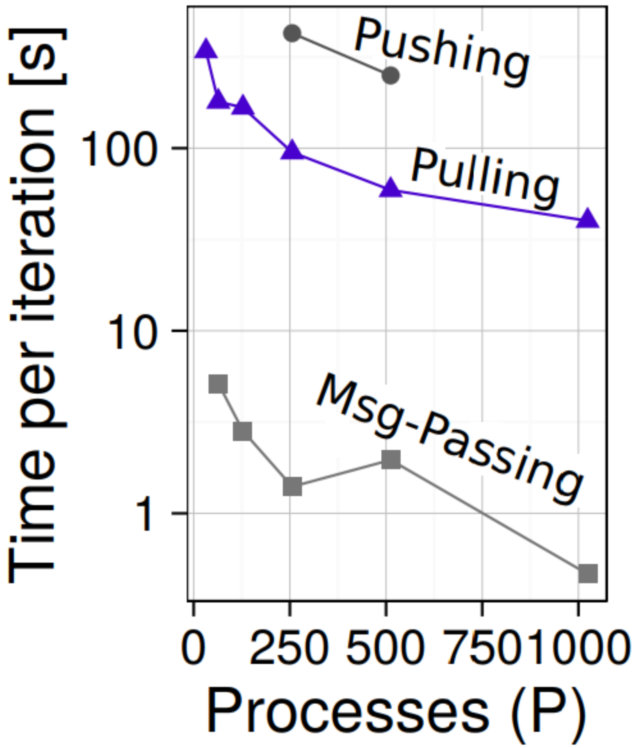

! Collectives: combines pushing and pulling

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To Push or To Pull?

M. Besta et al.: "To Push or To Pull: On Reducing Communication and Synchronization in Graph Computations", HPDC'17

To Push or To Pull?

If the complexities match: pull

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To Push or To Pull?

If the complexities match: pull

Otherwise: push

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To Push or To Pull?

If the complexities match: pull

Otherwise: push

+ check your hardware ☺

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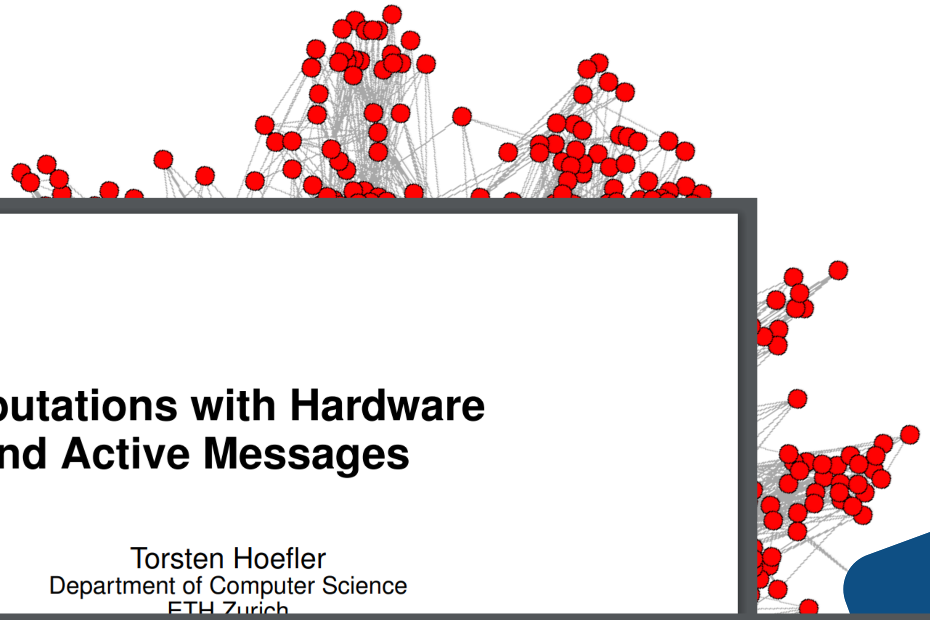
Moving on …



Accelerating Irregular Computations with Hardware Transactional Memory and Active Messages

Maciej Besta
Department of Computer Science
ETH Zurich

Torsten Hoefler
Department of Computer Science
ETH Zurich

…chronization-heavy

To Push or To…
Synchr…

Maciej Besta[1], Michał …
[1] Department of Computer Science…
[4] Departm…
maciej.besta@inf.ethz.ch, michal.p…

ABSTRACT

We reduce the cost of communication an…
processing by analyzing the fastest way t…
the updates to a shared state or pulling…

High-Performance Distributed RMA Locks

Patrick Schmid*
Department of Computer Science
ETH Zurich
patrick.schmid@ieffects.com

Maciej Besta*
Department of Computer Science
ETH Zurich
bestam@inf.ethz.ch

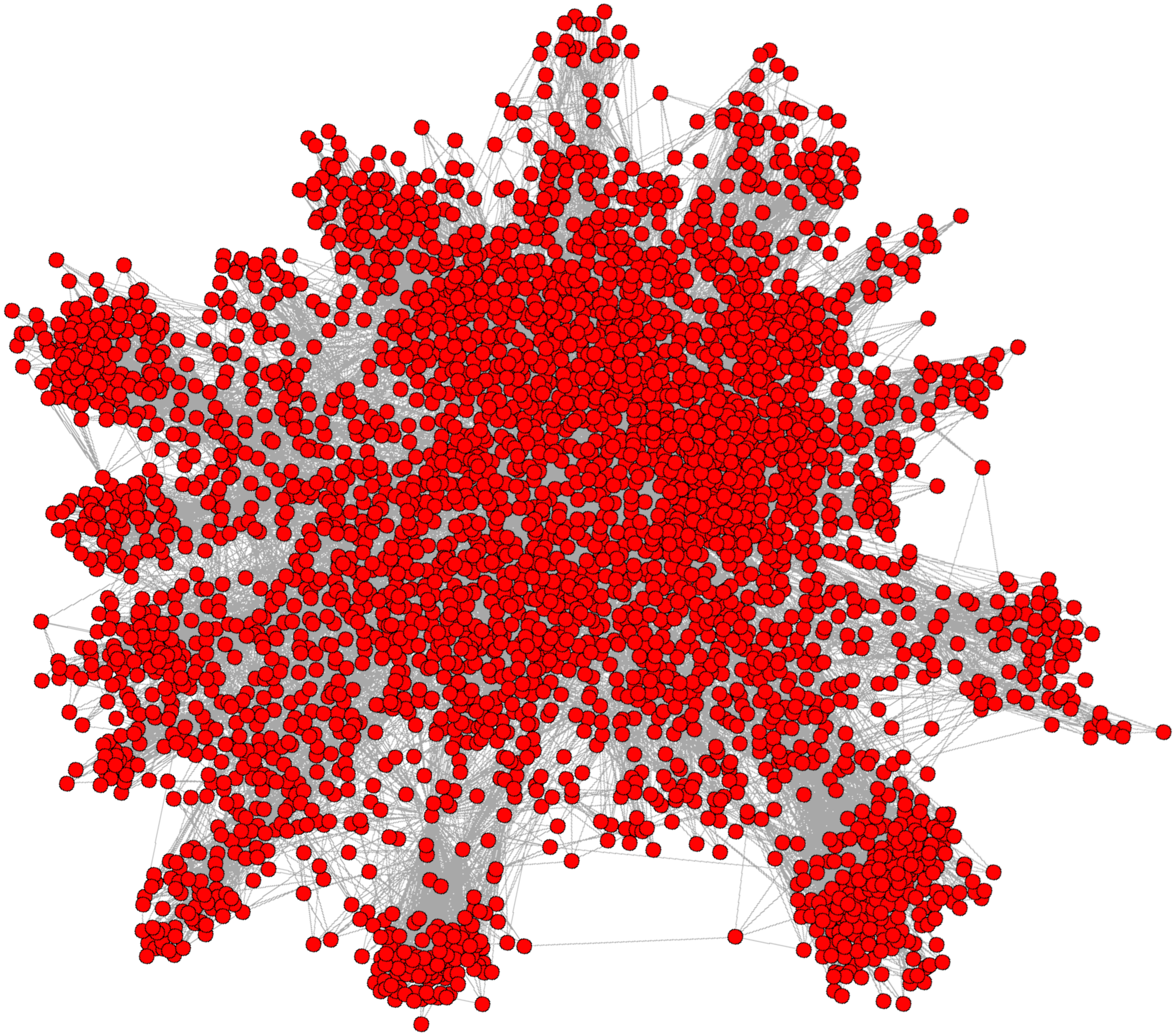
Torsten Hoefler
Department of Computer Science
ETH Zurich
htor@inf.ethz.ch

ABSTRACT

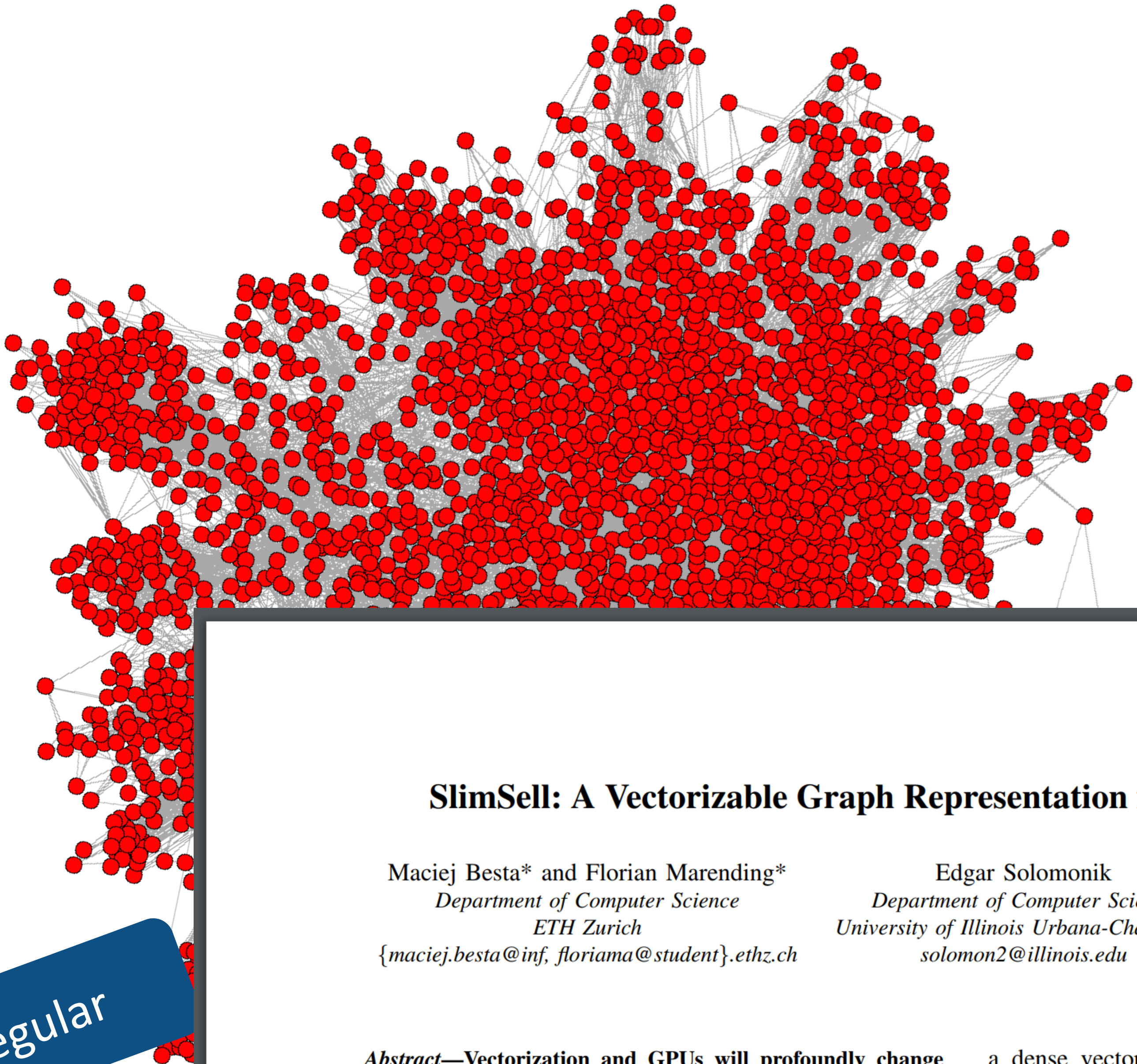
We propose a topology-aware distributed Reader-Writer lock

cesses competing for the same lock. Assume that two of them (A and B) run on one socket and the remaining two (C

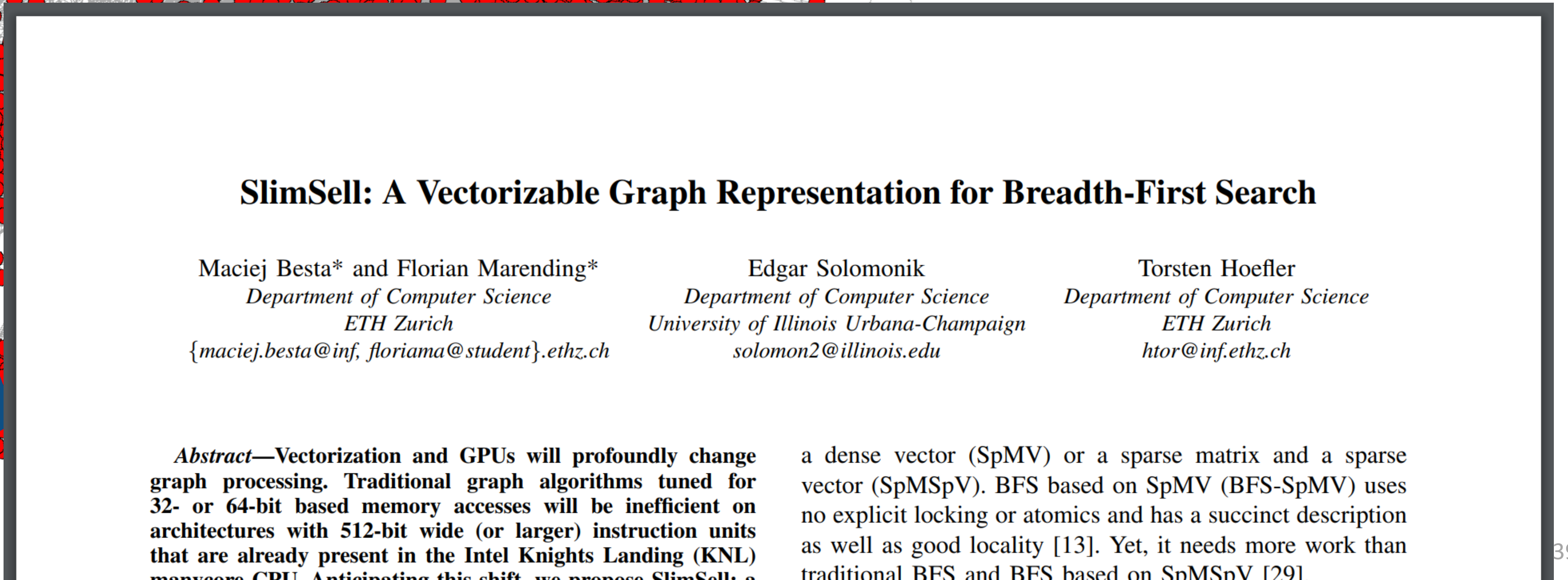
Moving on …



Moving on …



Irregular



SlimSell: A Vectorizable Graph Representation for Breadth-First Search

Maciej Besta* and Florian Marending*
Department of Computer Science
ETH Zurich
{maciej.besta@inf, floriama@student}.ethz.ch

Edgar Solomonik
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University of Illinois Urbana-Champaign
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Torsten Hoefler
Department of Computer Science
ETH Zurich
htor@inf.ethz.ch

***Abstract*—Vectorization and GPUs will profoundly change graph processing. Traditional graph algorithms tuned for 32- or 64-bit based memory accesses will be inefficient on architectures with 512-bit wide (or larger) instruction units that are already present in the Intel Knights Landing (KNL) manycore CPU. Anticipating this shift, we propose SlimSell: a**

a dense vector (SpMV) or a sparse matrix and a sparse vector (SpMSpV). BFS based on SpMV (BFS-SpMV) uses no explicit locking or atomics and has a succinct description as well as good locality [13]. Yet, it needs more work than traditional BFS and BFS based on SpMSpV [29].

VECTORIZATION

VECTORIZATION

- Deployed in various hardware

Vectorization

- Deployed in various hardware
- Becoming more popular

VECTORIZATION

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- Becoming more popular

AVX

$C = 8$ (SIMD width)

VECTORIZATION

- Deployed in various hardware
- Becoming more popular

AVX

$C\ = 16$ (SIMD width)

AVX

$C\ = 8$ (SIMD width)

Vectorization

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- Becoming more popular

AVX

$C = 16$ (SIMD width)

warps

$C = 32$ (warp size)

AVX

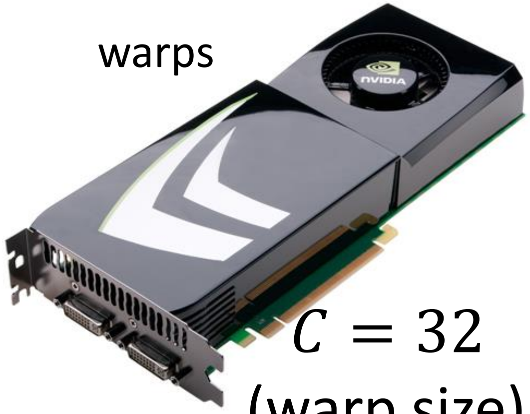
$C = 8$ (SIMD width)

C: „Chunk" size: SIMD width (CPUs, KNLs), warp size (GPUs)

VECTORIZATION

- Deployed in various hardware
- Becoming more popular
- Offers a lot of „regular" compute power

warps



$C\ = 32$
(warp size)

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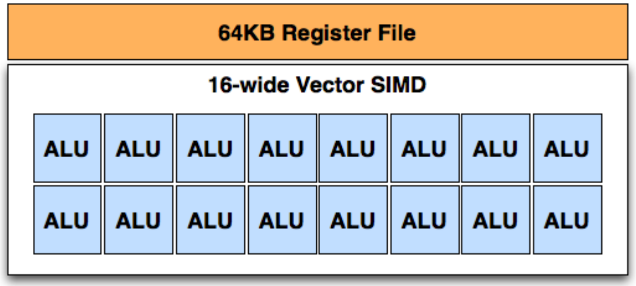
AVX

$C\ = 8$ (SIMD width)

C: **„Chunk" size: SIMD width (CPUs, KNLs), warp size (GPUs)**

VECTORIZATION

- Deployed in various hardware
- Becoming more popular
- Offers a lot of „regular” compute power



warps

$C = 32$ (warp size)

AVX

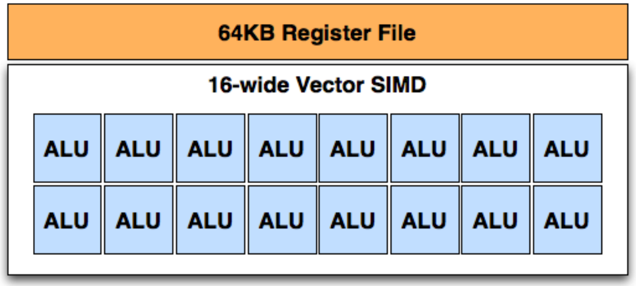
$C = 16$ (SIMD width)

AVX

$C = 8$ (SIMD width)

C: „Chunk” size: SIMD width (CPUs, KNLs), warp size (GPUs)

VECTORIZATION



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Regular

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AVX

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Vectorization

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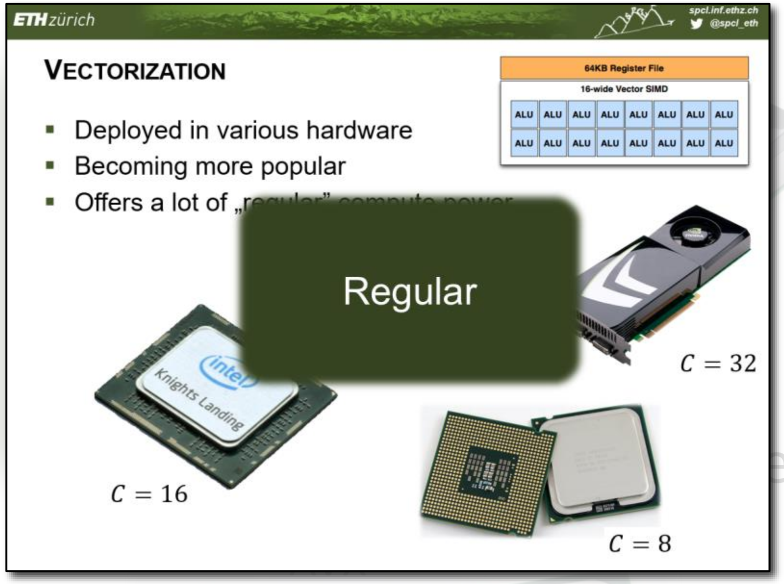
Regu

+



AVX

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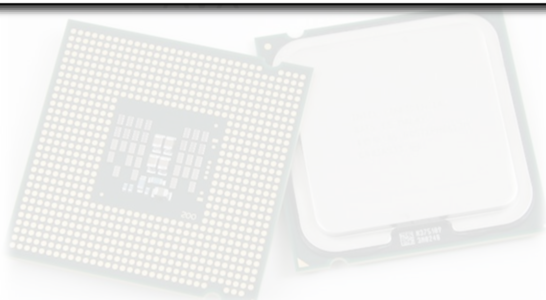

$C = 8$ (SIMD width)

C: „Chunk" size: SIMD width (CPUs, KNLs), warp size (GPUs)

Breadth-First Search

Traditional Formulation

Breadth-First Search

Traditional Formulation

- BFS is based on primitives such as queues

M. Besta et al.: “SlimSell: A Vectorized Graph Representation for Breadth-First Search”, IPDPS’17

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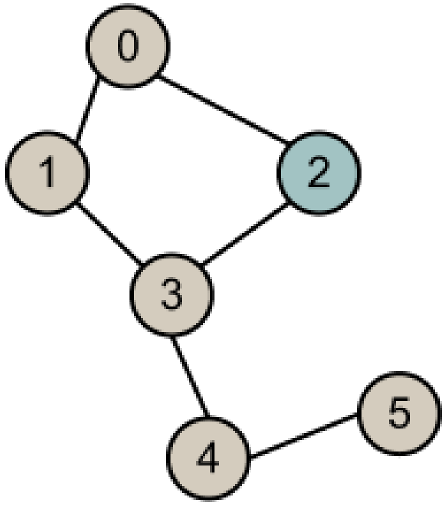

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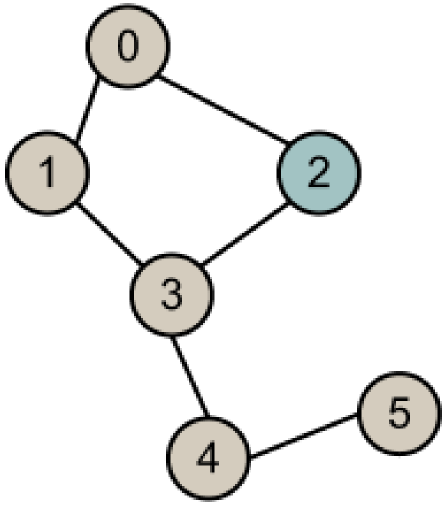

1) F = {}

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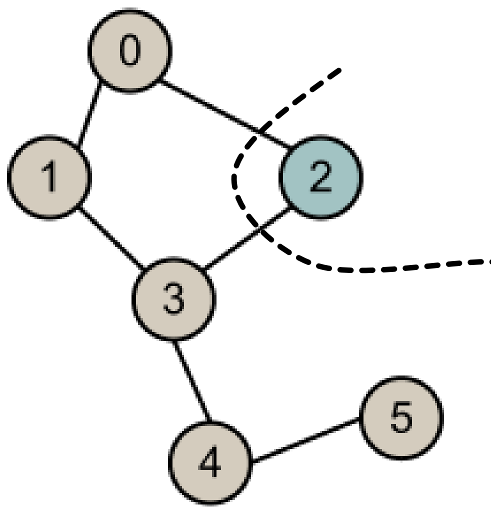

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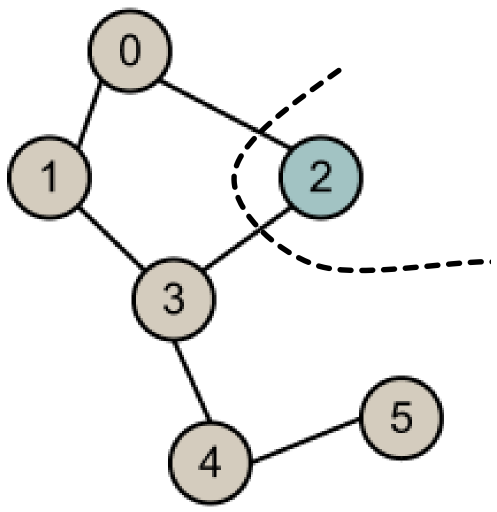

1) F = {}
2) F = {2}

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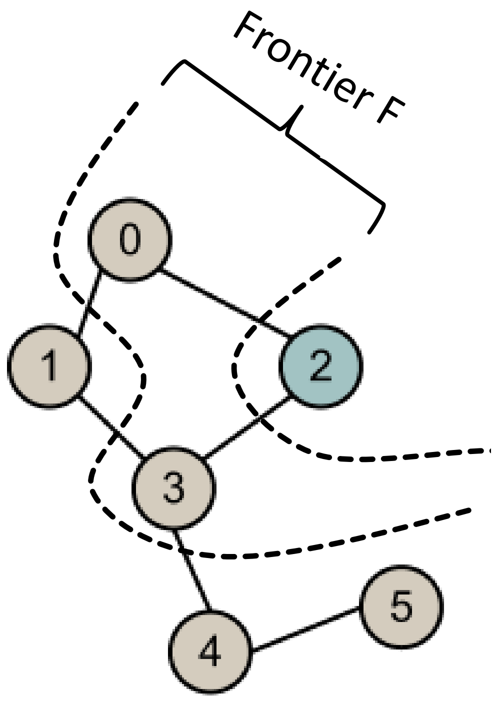

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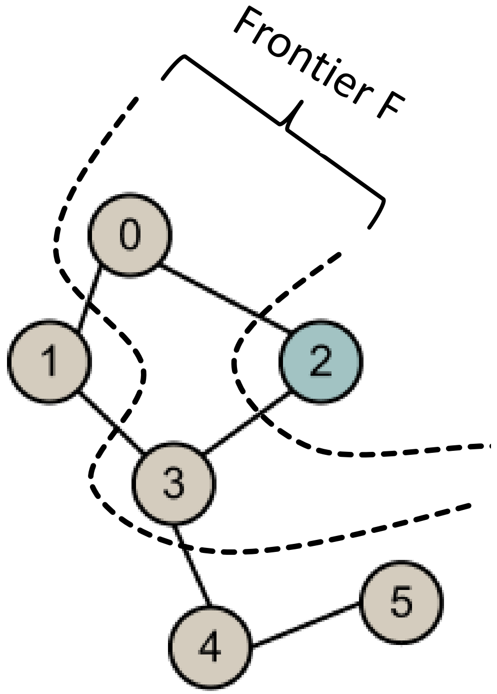

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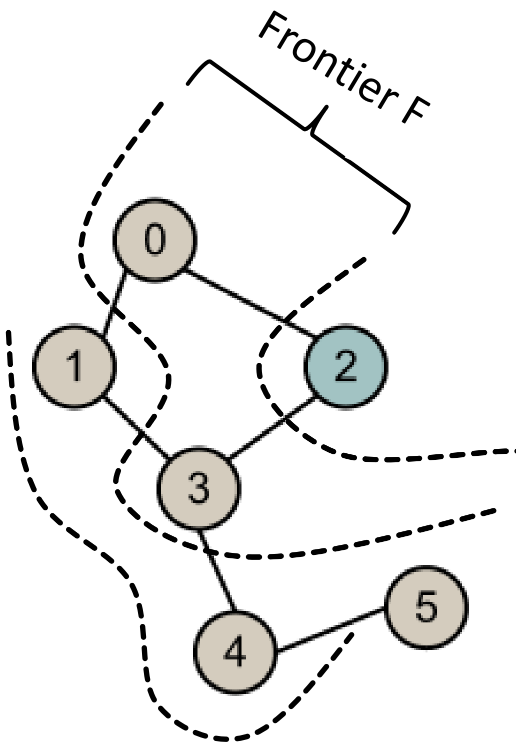

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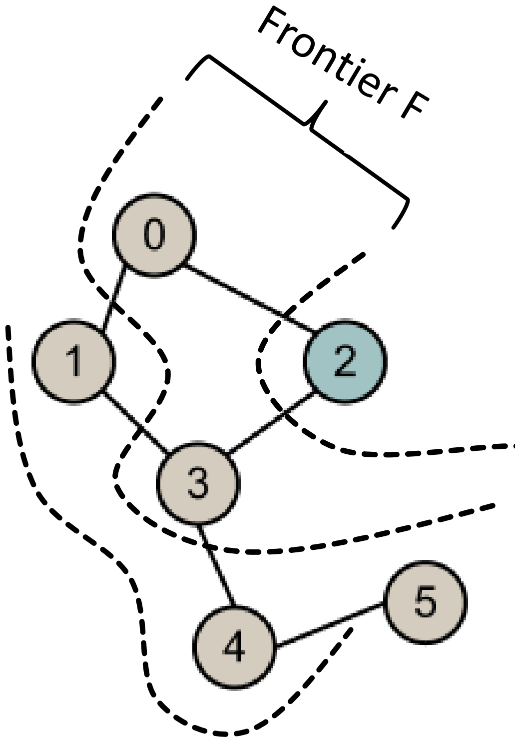

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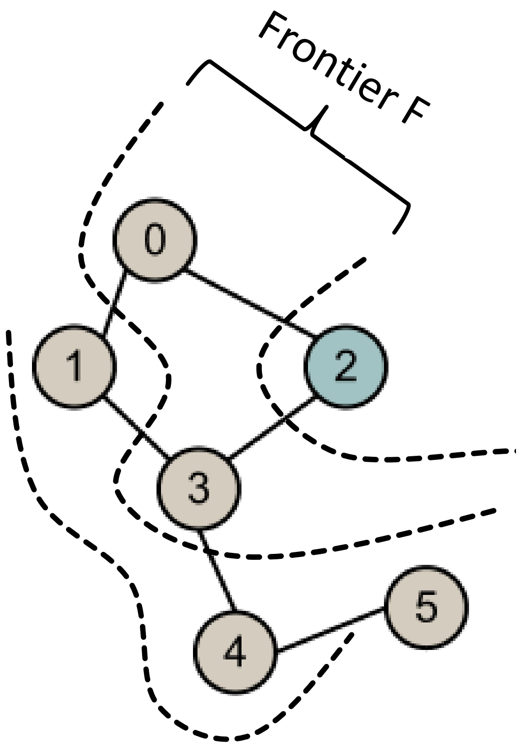

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Distances from the root

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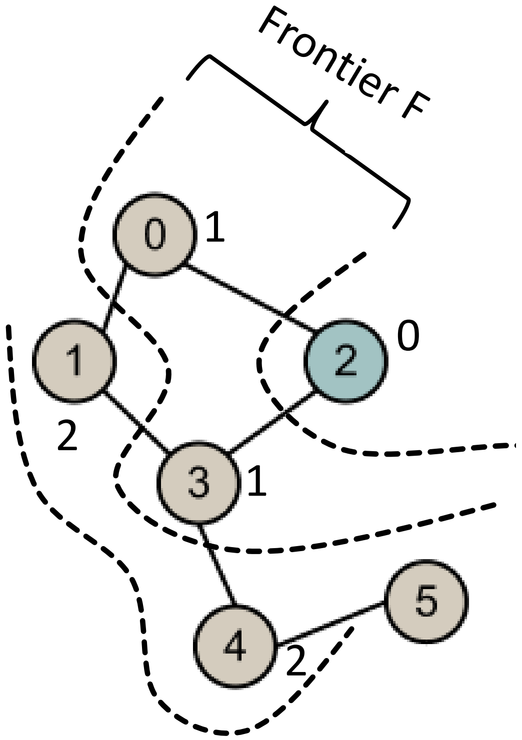

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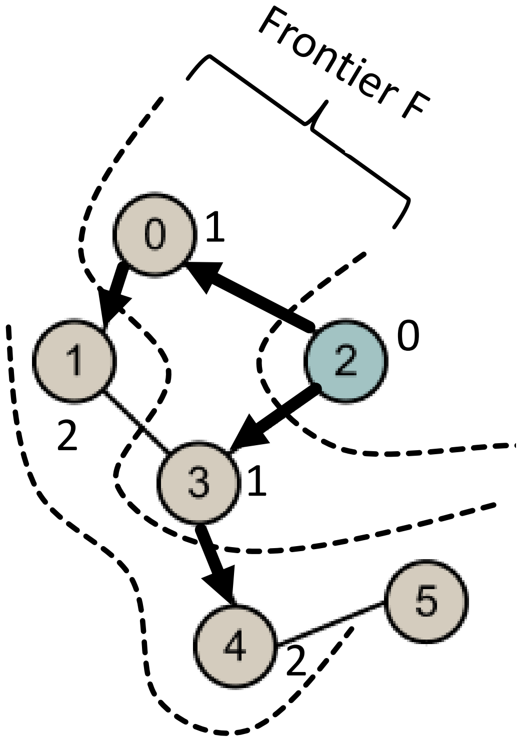

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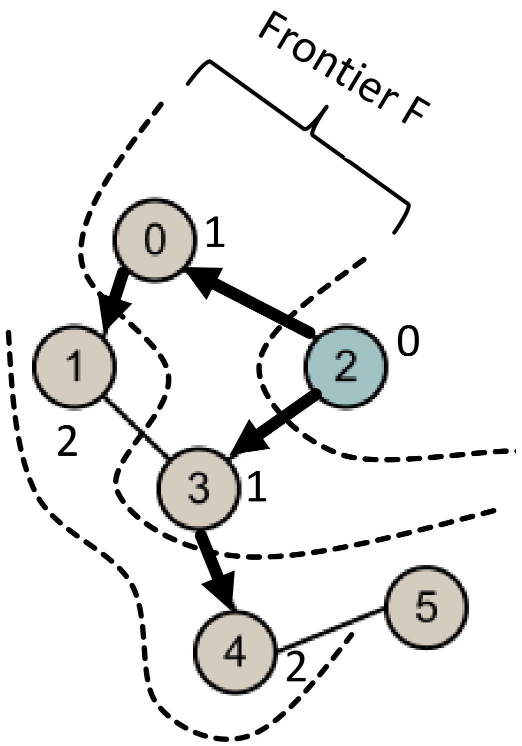

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Distances from the root

Parents (predecessors) in the traversal tree

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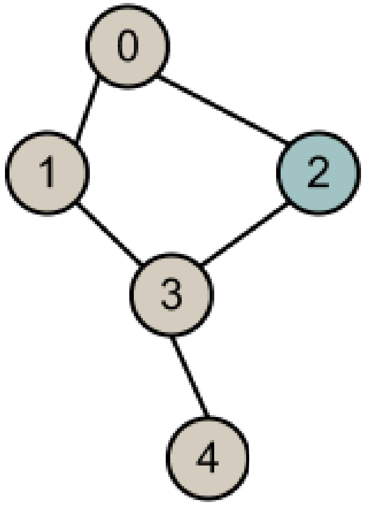
Breadth-First Search

Algebraic Formulation



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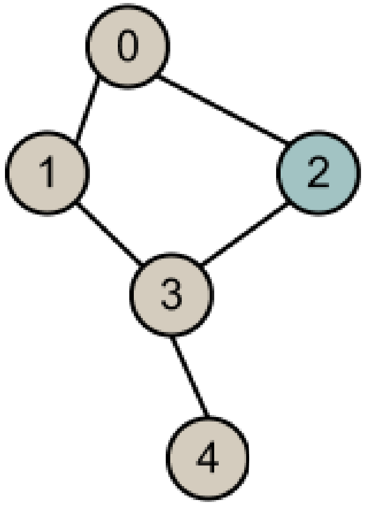
Breadth-First Search
Algebraic Formulation

- BFS is a series of matrix-vector products



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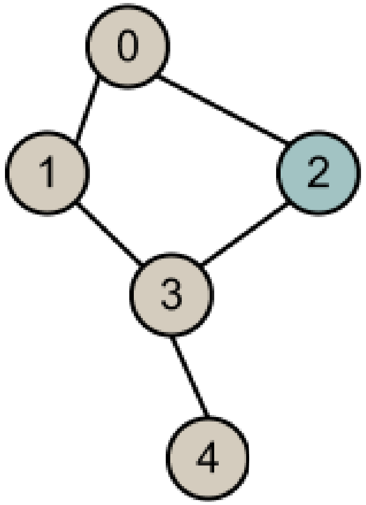
Breadth-First Search
Algebraic Formulation

- BFS is a series of matrix-vector products
- Graph is modeled by an adjacency matrix



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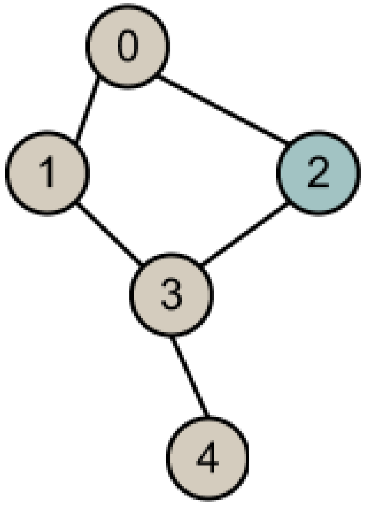

Adjacency Matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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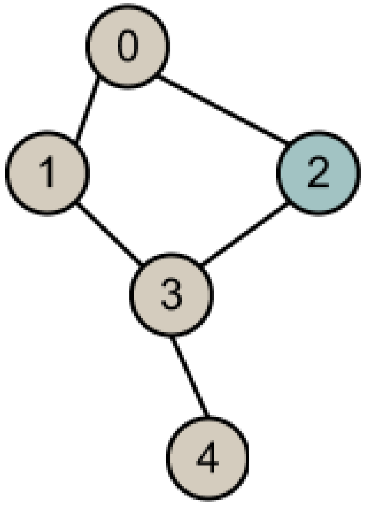
Breadth-First Search
Algebraic Formulation

- BFS is a series of matrix-vector products
- Graph is modeled by an adjacency matrix
- Multiplication is done over a semiring



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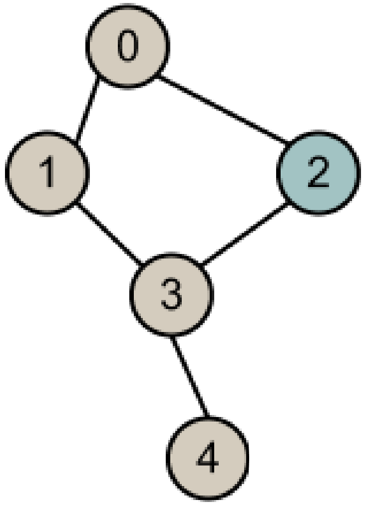

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Semiring:

$$(\mathbb{R}\,, op_1, op_2, el_1, el_2)$$

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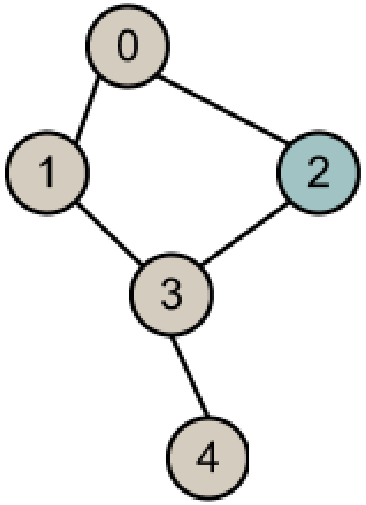

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Semiring:

$$(\mathbb{R}\,, op_1, op_2, el_1, el_2)$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

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Breadth-First Search

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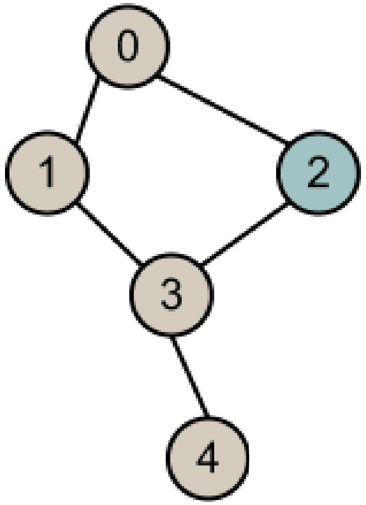

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$$(\mathbb{R}, +, \cdot, 0, 1)$$

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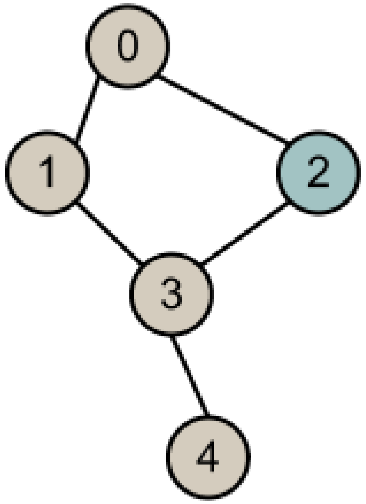
Breadth-First Search
Algebraic Formulation

Tropical Semiring

$(\mathbb{R} \cup \{\infty\}, min, +, \infty, 0)$



Breadth-First Search
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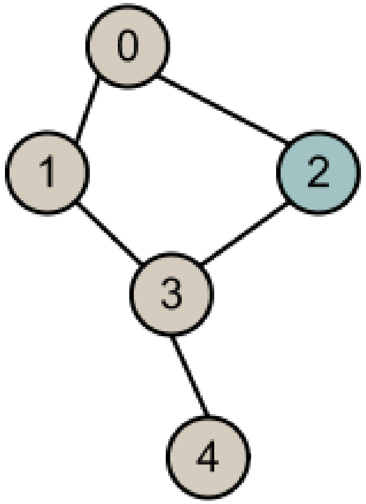

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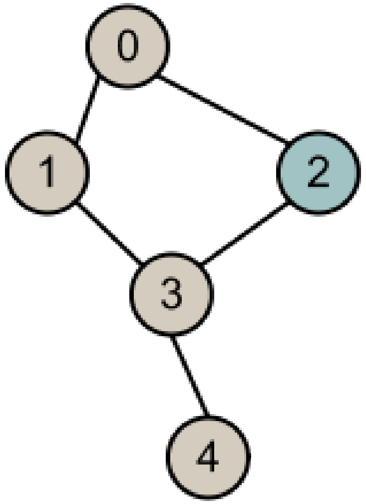

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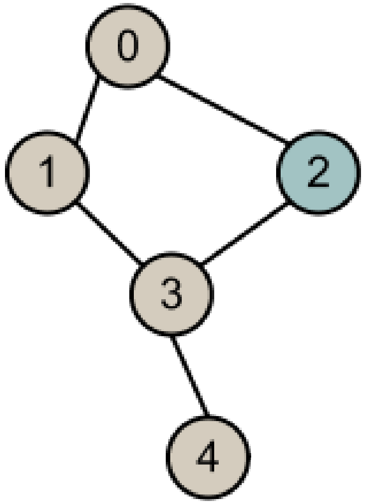
Usually stored using a sparse format

$$A' = \begin{bmatrix} 0 & 1 & 1 & \infty & \infty \\ 1 & 0 & \infty & 1 & \infty \\ 1 & \infty & 0 & 1 & \infty \\ \infty & 1 & 1 & 0 & 1 \\ \infty & \infty & \infty & 1 & 0 \end{bmatrix}$$



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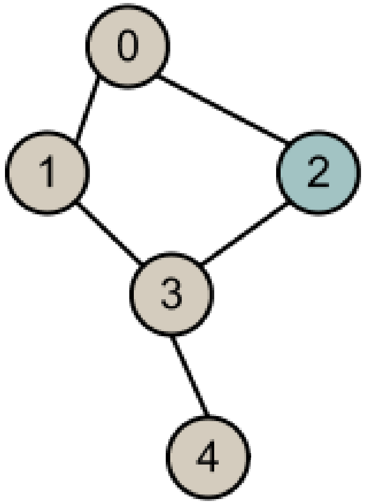

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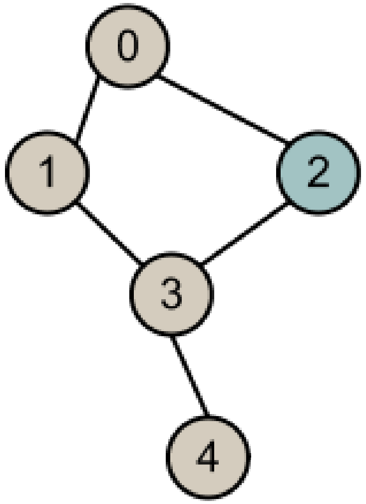

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Stored with a dense or a sparse format

$$f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

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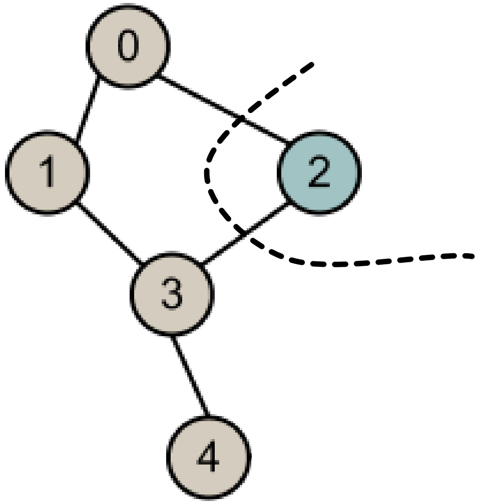

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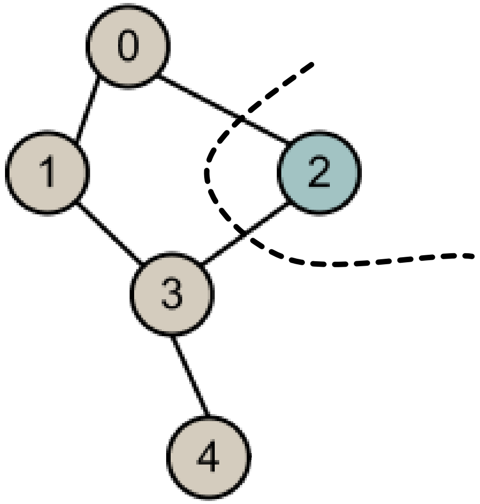

Usually stored using a sparse format

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$$f_1 = A'^T \otimes_T f_0 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

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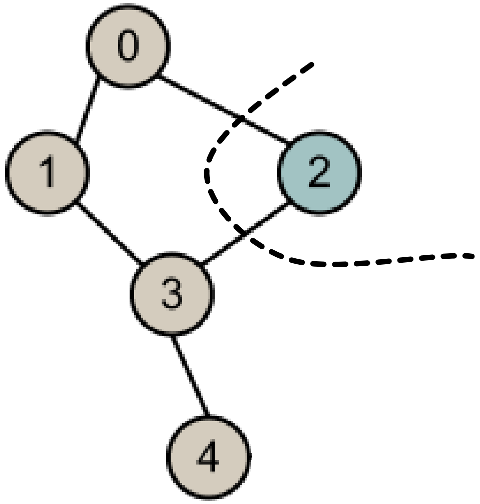

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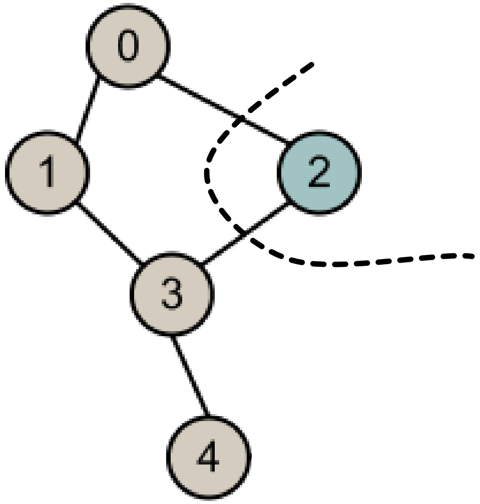
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Usually stored using a sparse format

Stored with a dense or a sparse format

$$A' = \begin{bmatrix} 0 & 1 & 1 & \infty & \infty \\ 1 & 0 & \infty & 1 & \infty \\ 1 & \infty & 0 & 1 & \infty \\ \infty & 1 & 1 & 0 & 1 \\ \infty & \infty & \infty & 1 & 0 \end{bmatrix} \qquad f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

$$f_1 = A'^T \otimes_T f_0 = \begin{pmatrix} 1 \\ \\ \\ \\ \end{pmatrix}$$

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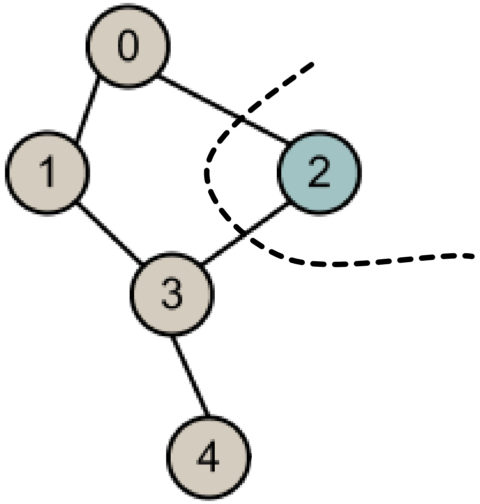
Breadth-First Search
Algebraic Formulation

Tropical Semiring

$(\mathbb{R} \cup \{\infty\}, min, +, \infty, 0)$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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$$A' = \begin{bmatrix} 0 & 1 & 1 & \infty & \infty \\ 1 & 0 & \infty & 1 & \infty \\ 1 & \infty & 0 & 1 & \infty \\ \infty & 1 & 1 & 0 & 1 \\ \infty & \infty & \infty & 1 & 0 \end{bmatrix} \quad f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

$$f_1 = A'^T \otimes_T f_0 = \begin{pmatrix} 1 \\ \infty \\ 0 \\ 1 \\ \infty \end{pmatrix}$$

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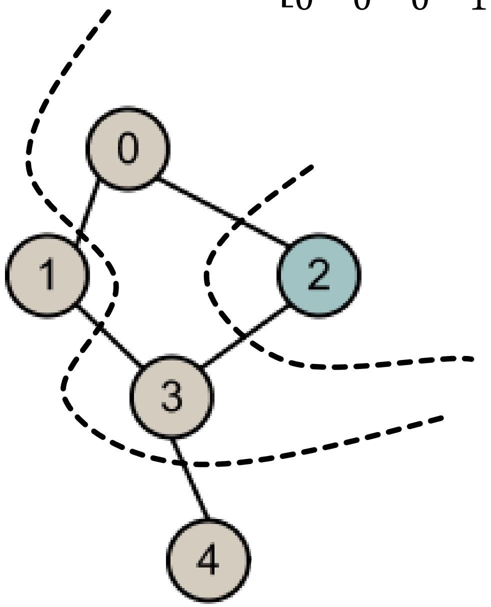
Breadth-First Search
Algebraic Formulation

Tropical Semiring

$(\mathbb{R} \cup \{\infty\}, min, +, \infty, 0)$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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$$A' = \begin{bmatrix} 0 & 1 & 1 & \infty & \infty \\ 1 & 0 & \infty & 1 & \infty \\ 1 & \infty & 0 & 1 & \infty \\ \infty & 1 & 1 & 0 & 1 \\ \infty & \infty & \infty & 1 & 0 \end{bmatrix} \qquad f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

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Breadth-First Search
Algebraic Formulation

Tropical Semiring

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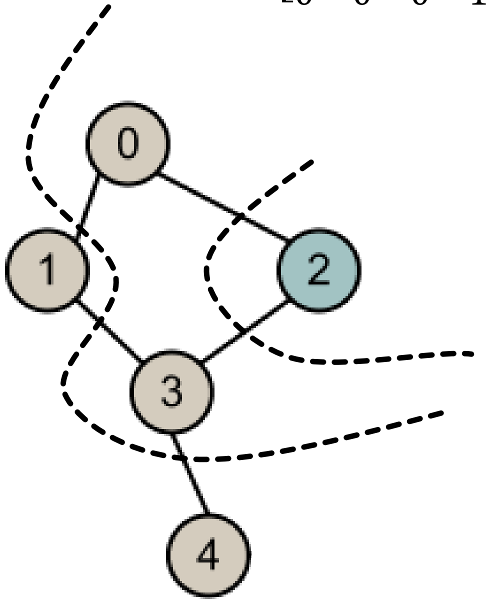
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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$$A' = \begin{bmatrix} 0 & 1 & 1 & \infty & \infty \\ 1 & 0 & \infty & 1 & \infty \\ 1 & \infty & 0 & 1 & \infty \\ \infty & 1 & 1 & 0 & 1 \\ \infty & \infty & \infty & 1 & 0 \end{bmatrix} \qquad f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

$$f_1 = A'^T \otimes_T f_0 = \begin{pmatrix} 1 \\ \infty \\ 0 \\ 1 \\ \infty \end{pmatrix} \qquad f_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

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Breadth-First Search

Algebraic Formulation

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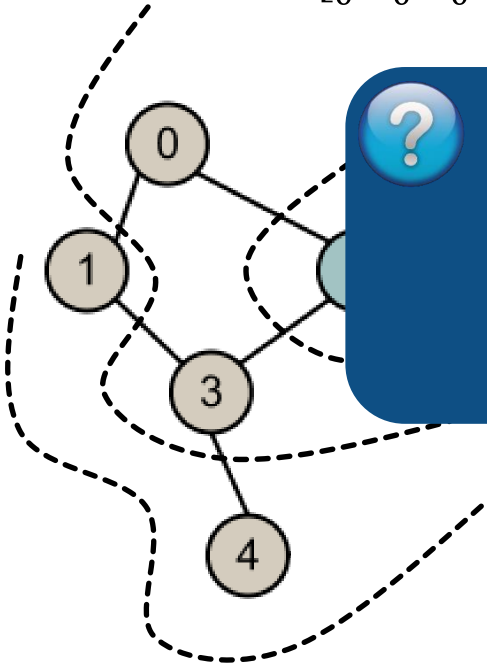
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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Stored with a dense or a sparse format

$$\begin{bmatrix} 0 & 1 & 1 & \infty & \infty \end{bmatrix}$$

$$f_0 = \begin{pmatrix} \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{pmatrix}$$

How to do this in practice?

$$f_1 = A'^T \otimes_T f_0 = \begin{pmatrix} \infty \\ 0 \\ 1 \\ \infty \end{pmatrix}$$

$$f_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

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GRAPH REPRESENTATIONS

COMPRESSED SPARSE ROW (CSR)

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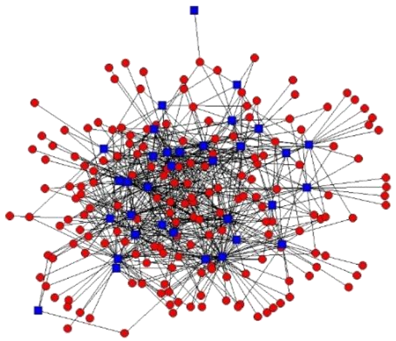
Graph Representations
Compressed Sparse Row (CSR)



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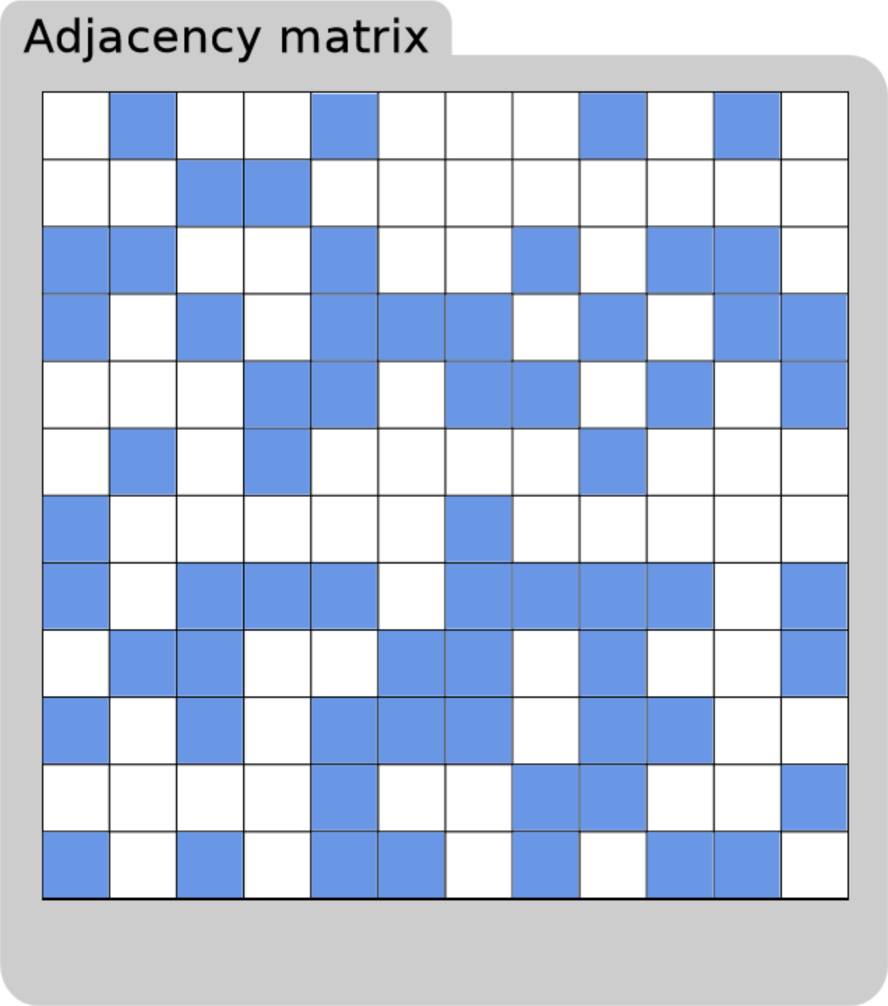

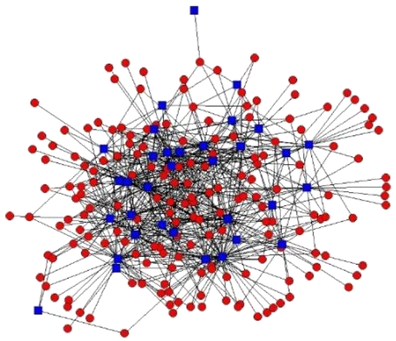

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Graph Representations

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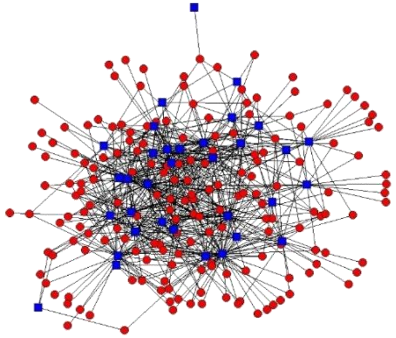

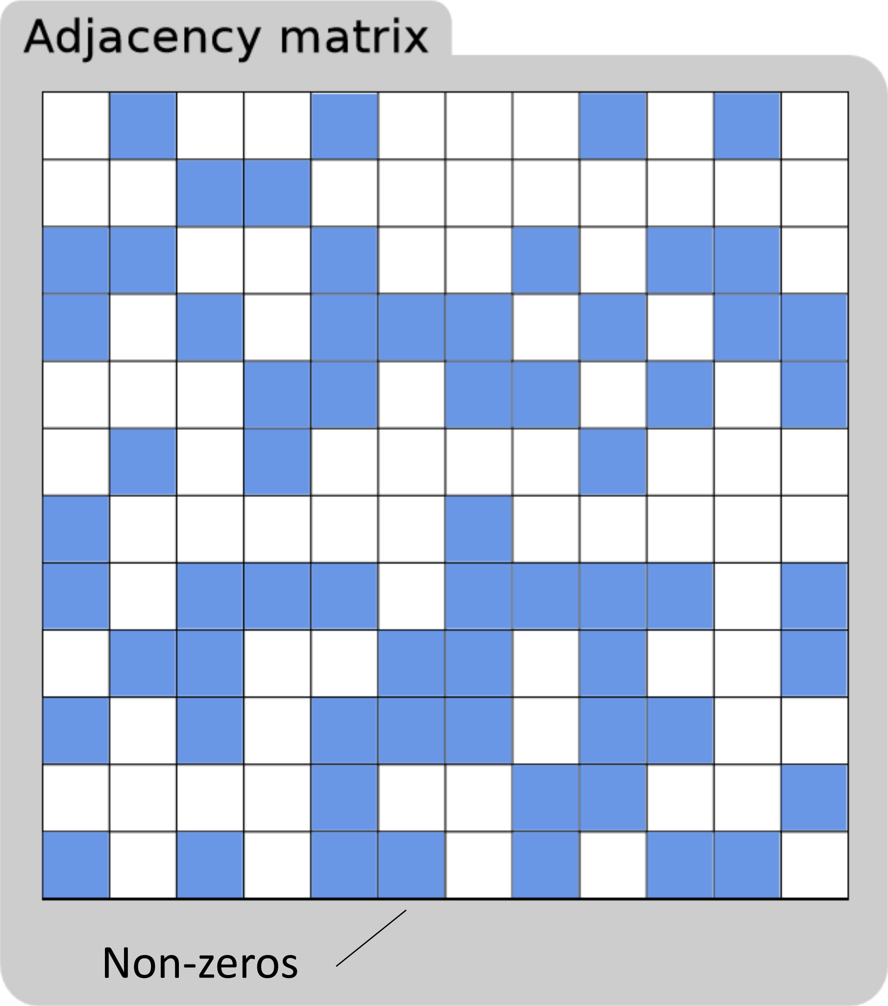

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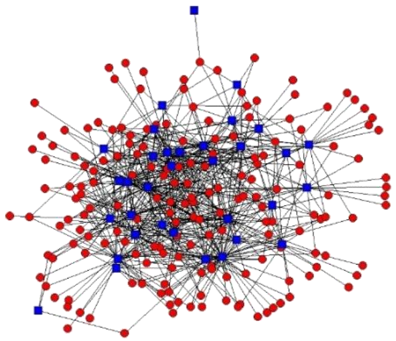
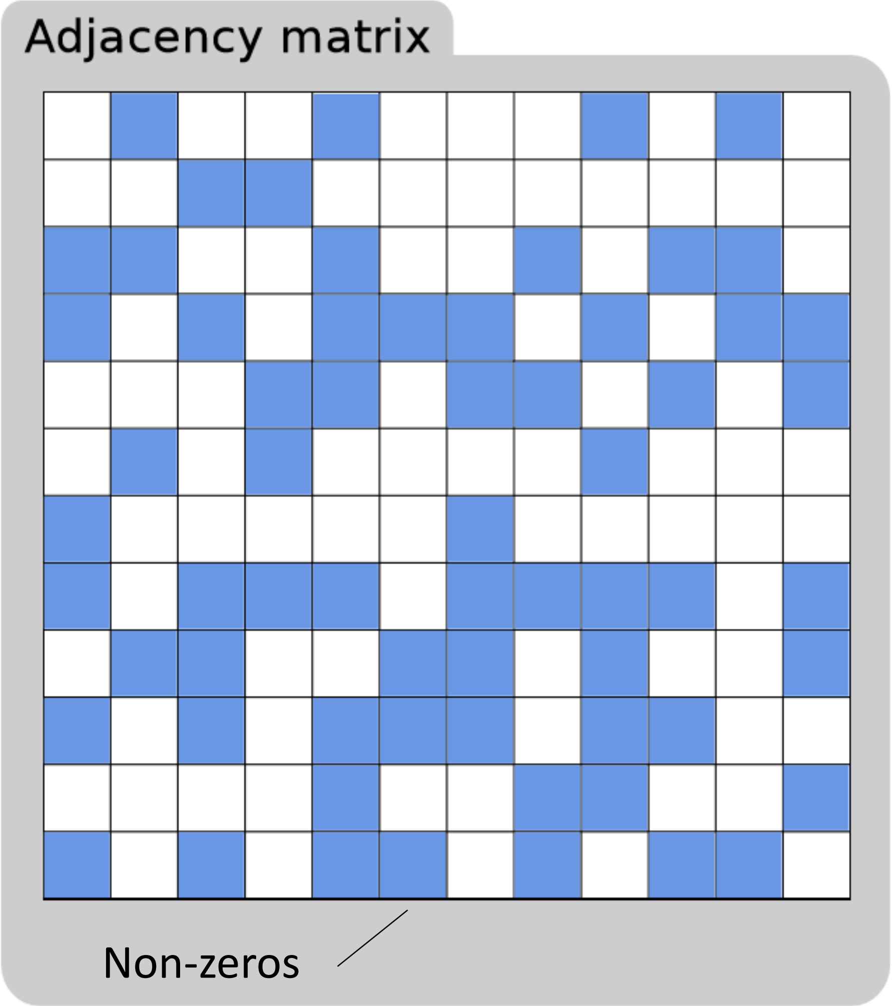
Graph Representations
Compressed Sparse Row (CSR)





Non-zeros are stored in the *val* array

size: *2m* cells

n: number of vertices
m: number of edges

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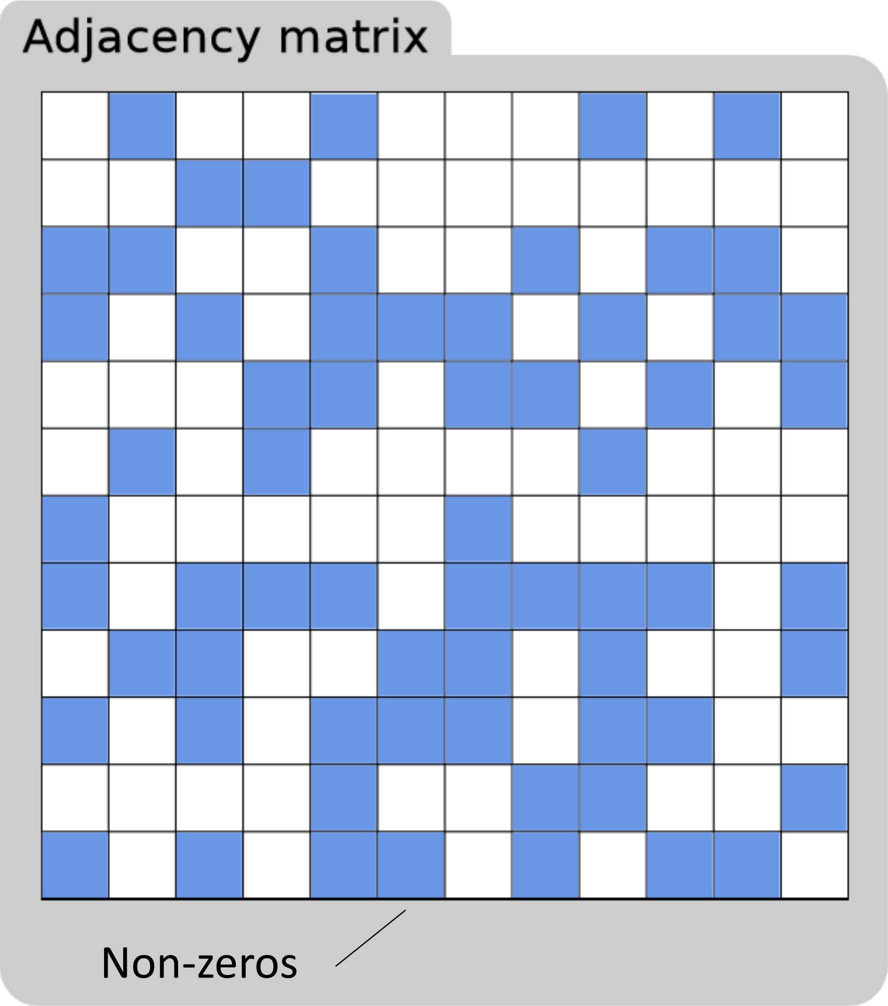
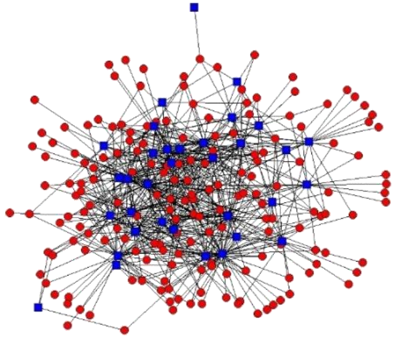
Graph Representations
Compressed Sparse Row (CSR)





Non-zeros are stored in the *val* array

size: *2m* cells

Column indices stored in the *col* array

size: *2m* cells

n: number of vertices
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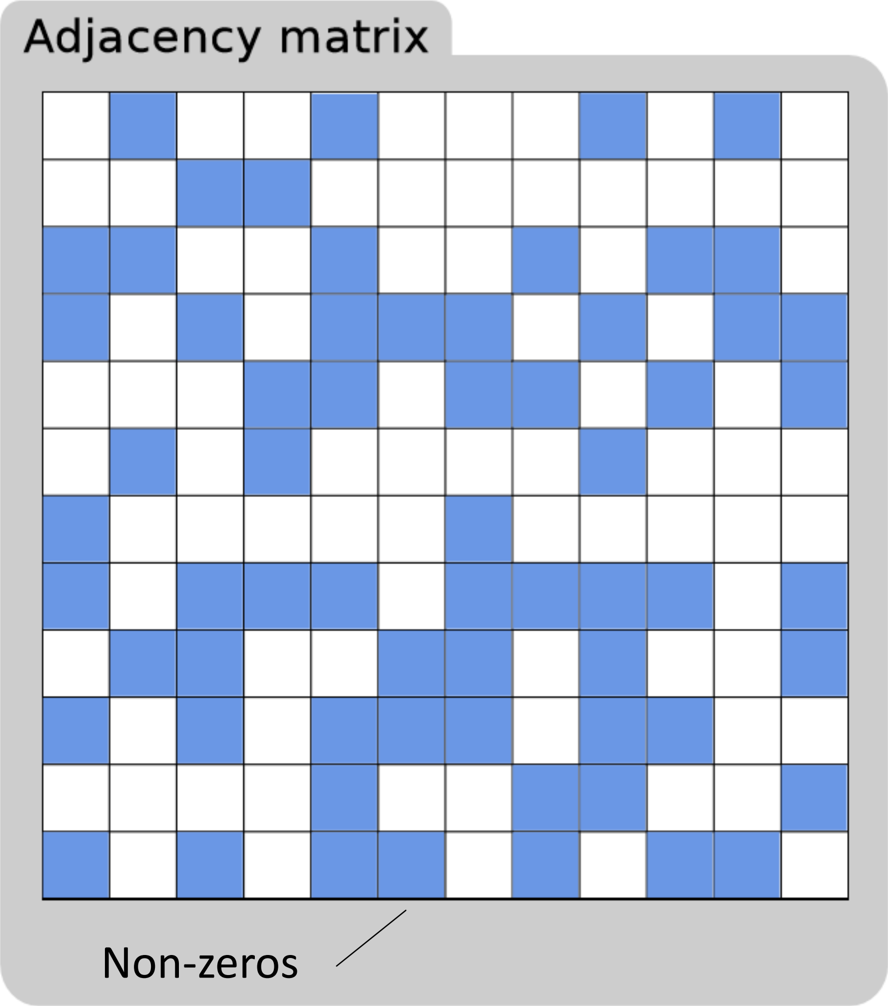
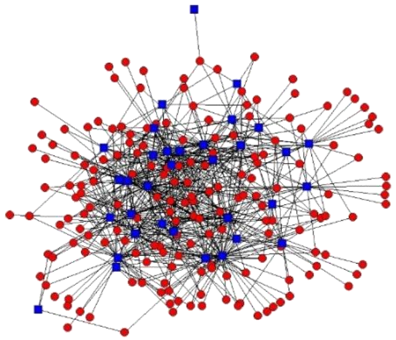
Graph Representations

Compressed Sparse Row (CSR)

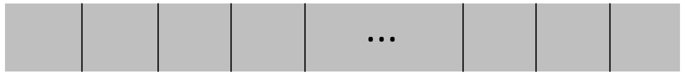




Non-zeros are stored in the *val* array — size: *2m* cells

Column indices stored in the *col* array — size: *2m* cells

Row indices are stored in the *row* array — size: *n* cells



n: number of vertices
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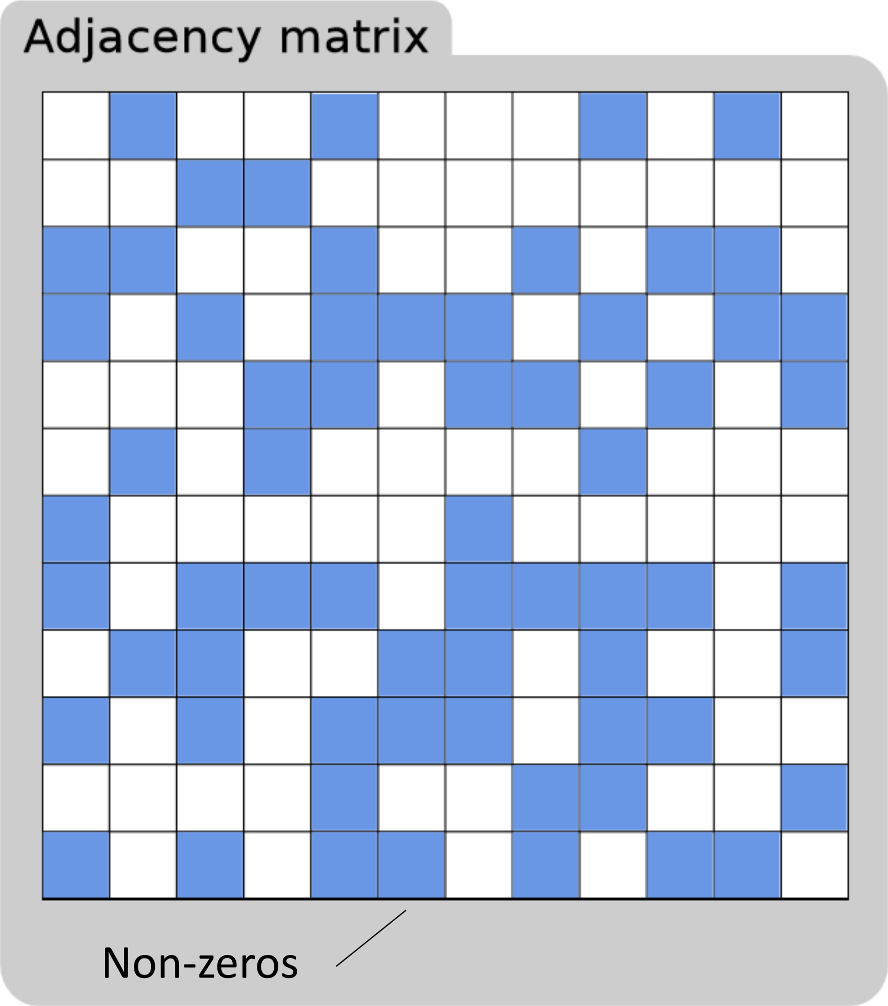
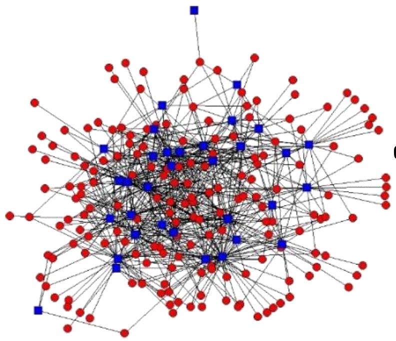
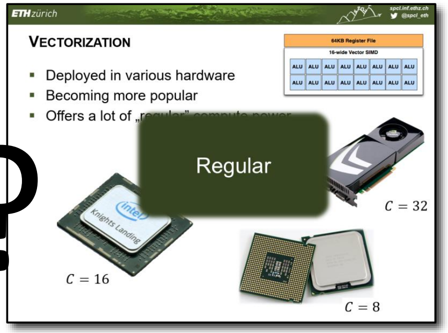
Graph Representations
Compressed Sparse Row (CSR)





?



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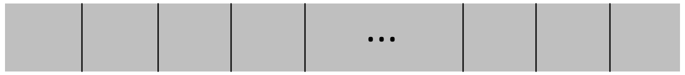
size: *n* cells



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Graph Representations

Sell-C-Sigma

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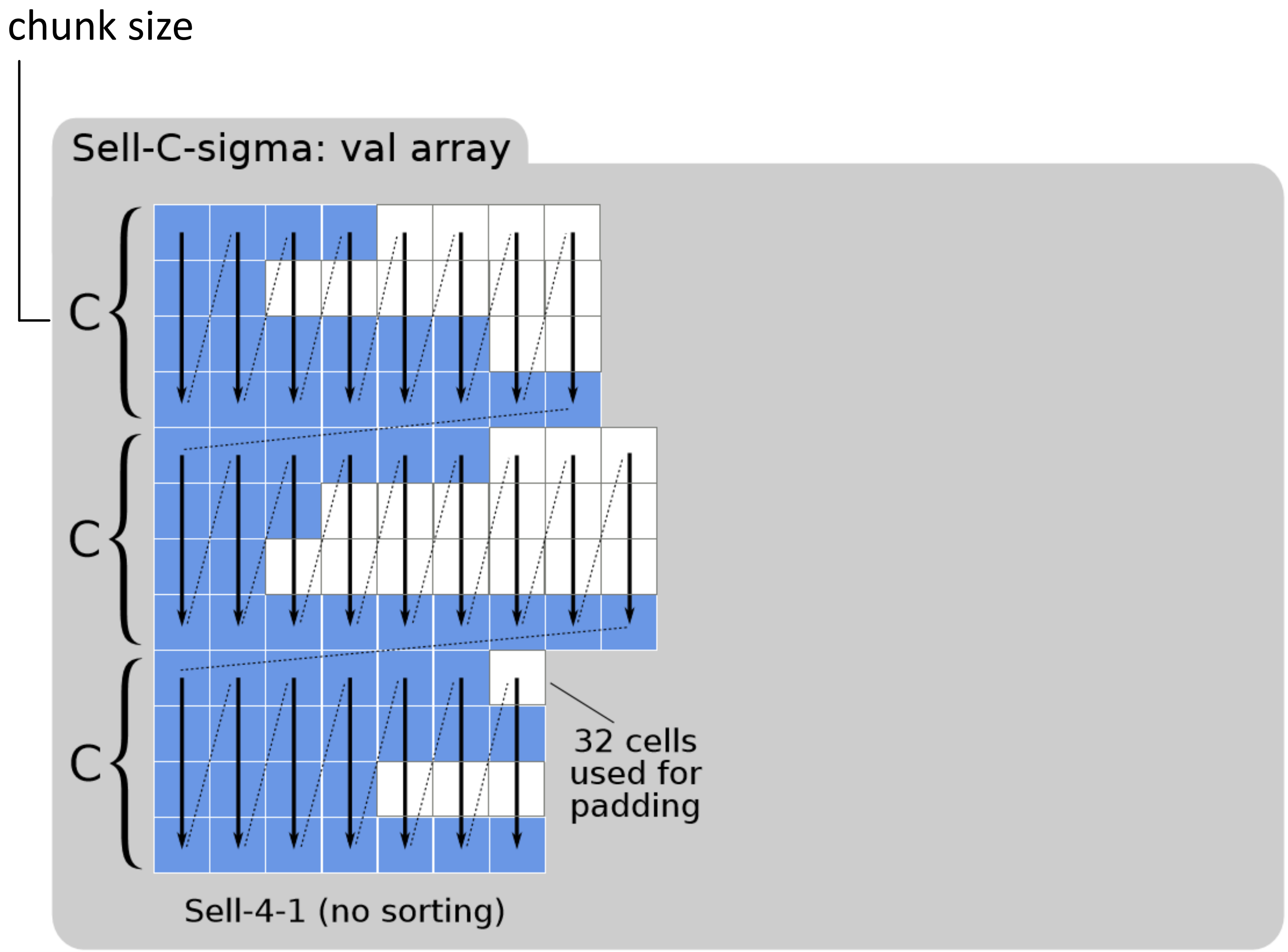
Graph Representations

Sell-C-Sigma



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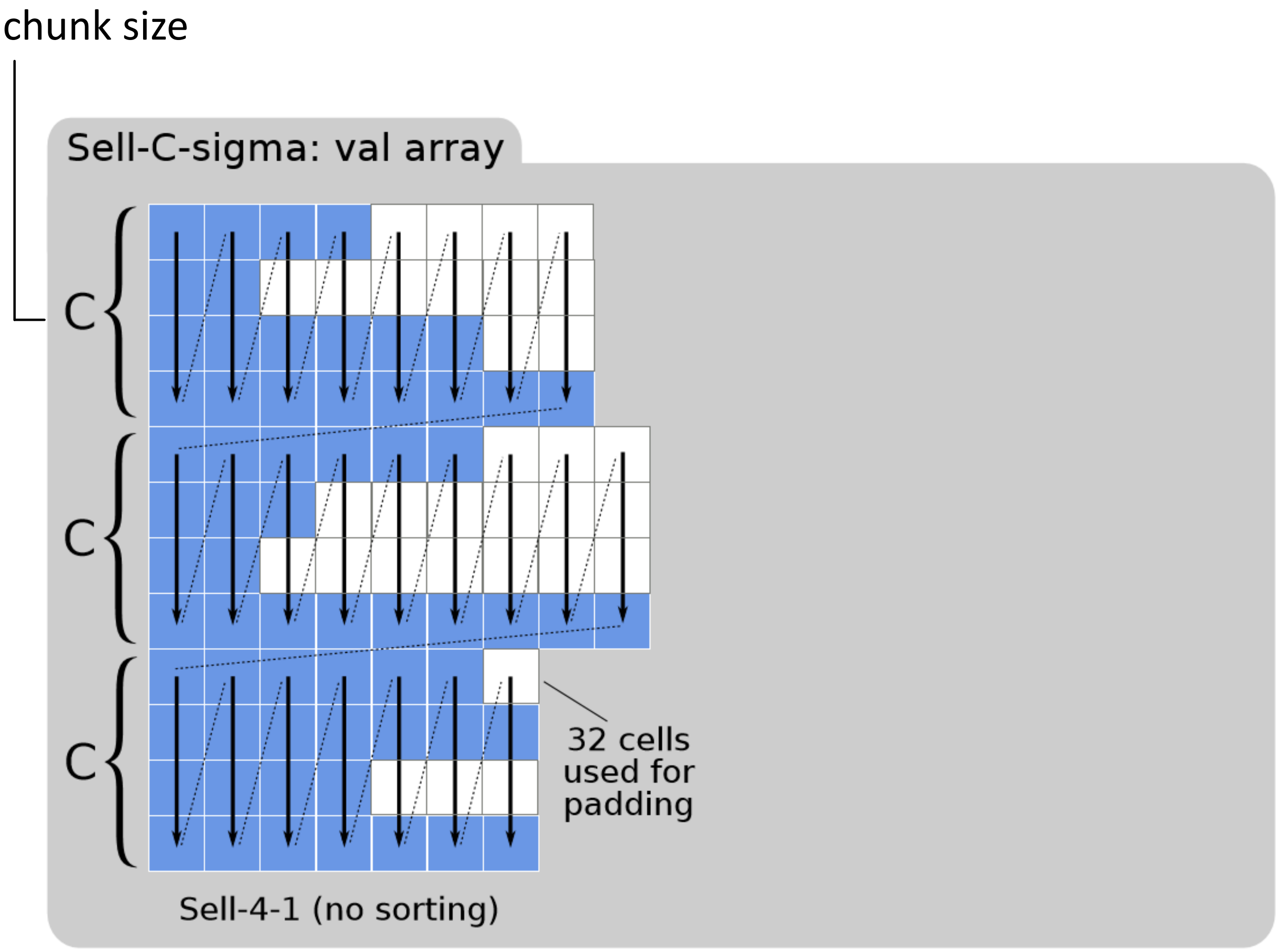
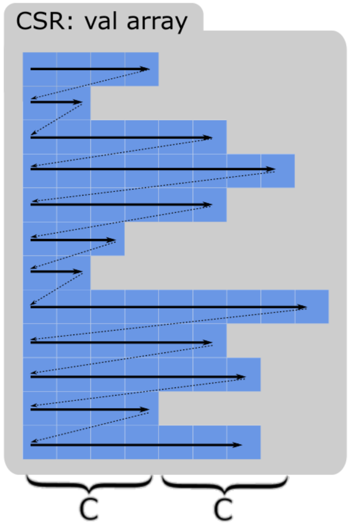
Graph Representations

Sell-C-Sigma





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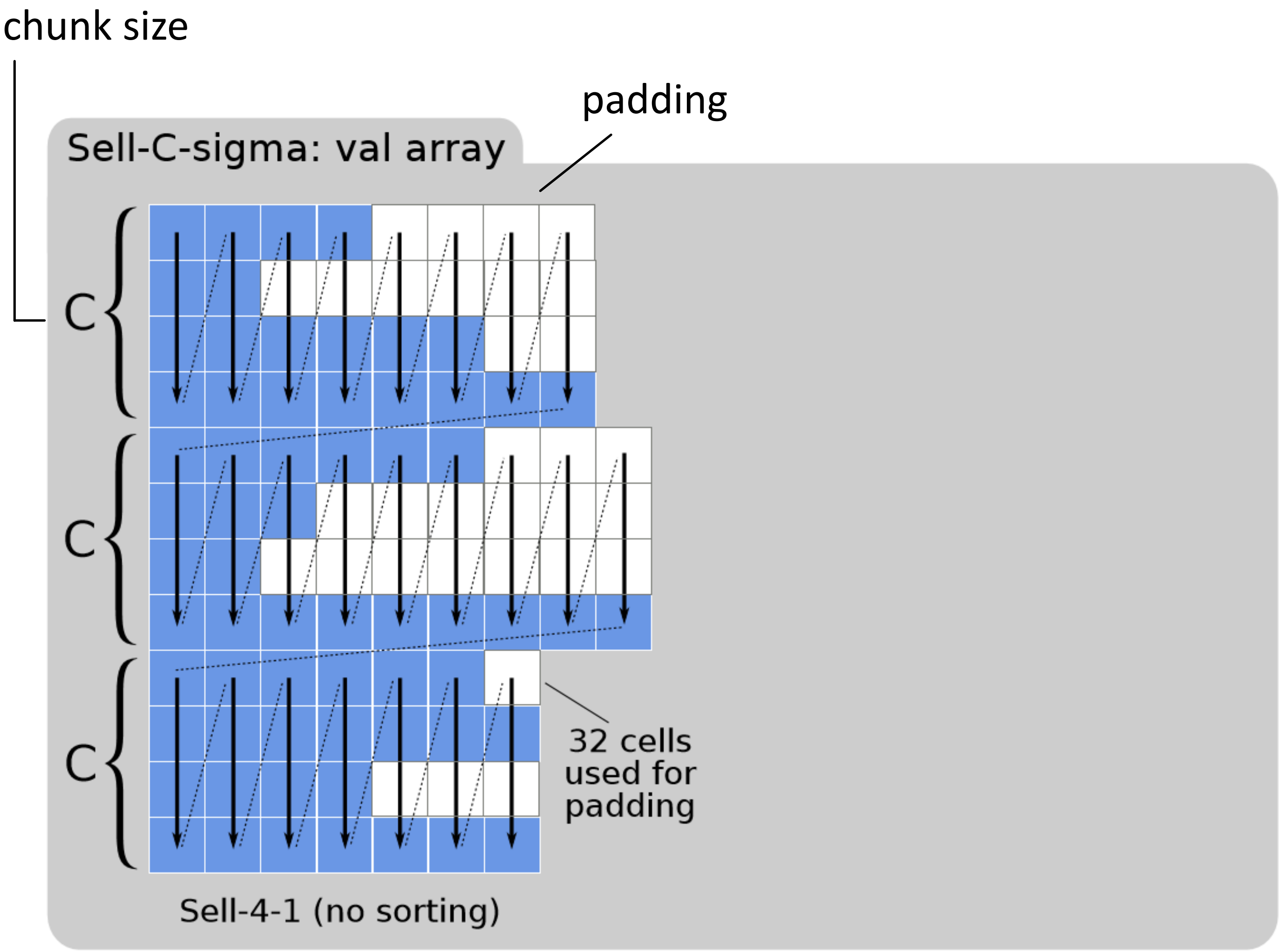
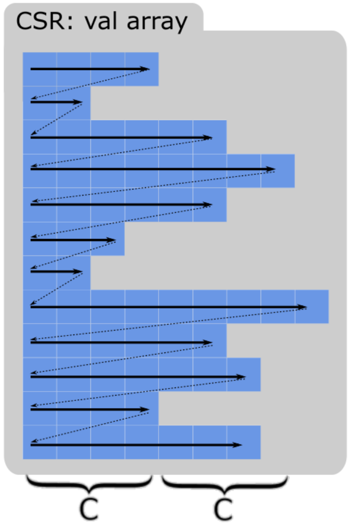
Graph Representations

Sell-C-Sigma





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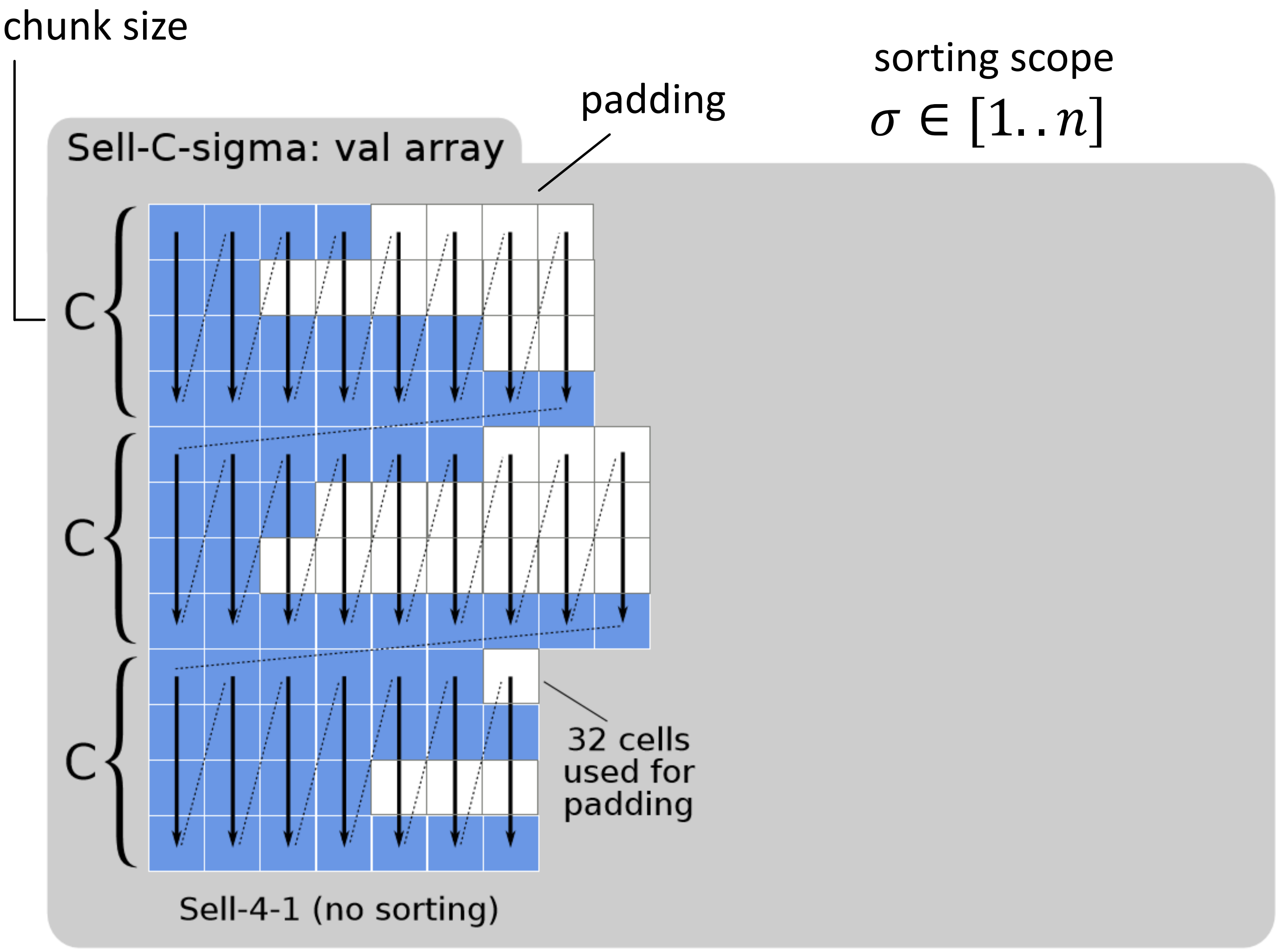
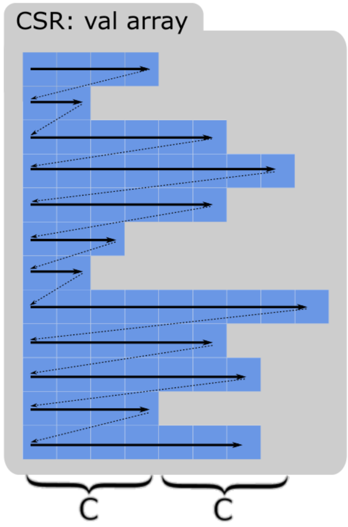
Graph Representations
Sell-C-Sigma





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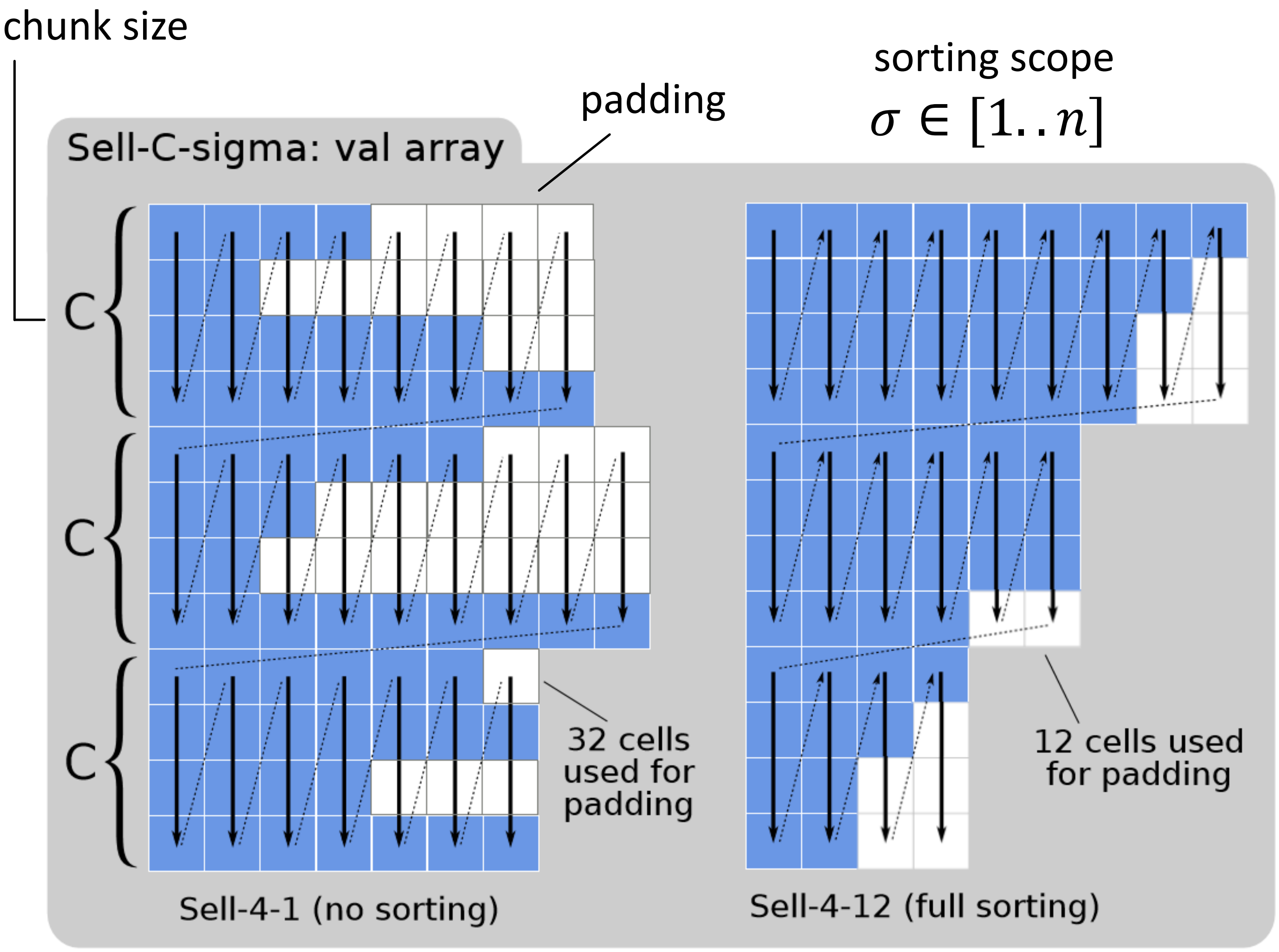
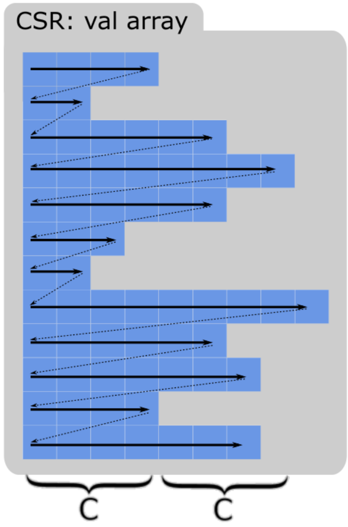
Graph Representations
Sell-C-Sigma





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Graph Representations

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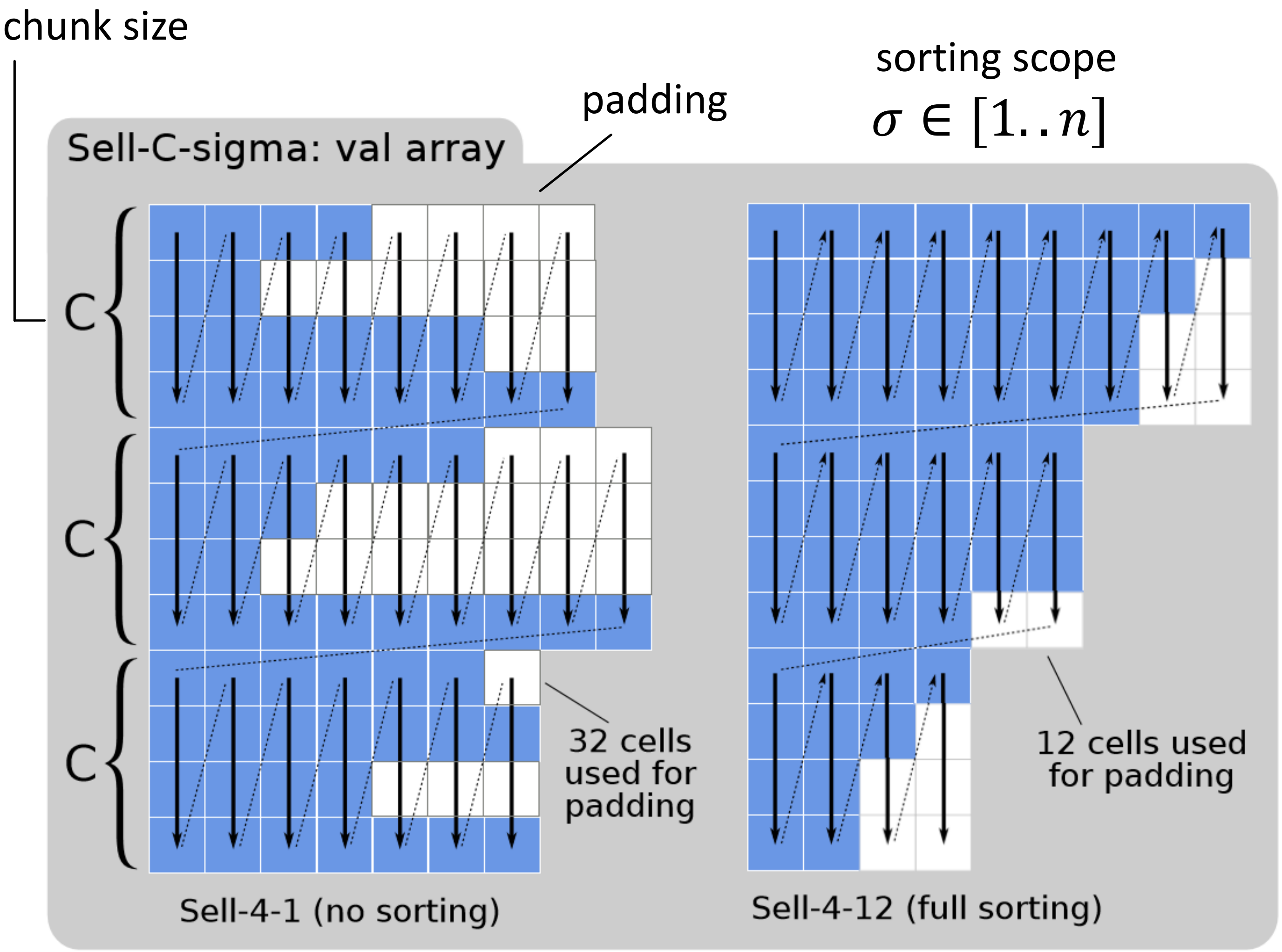

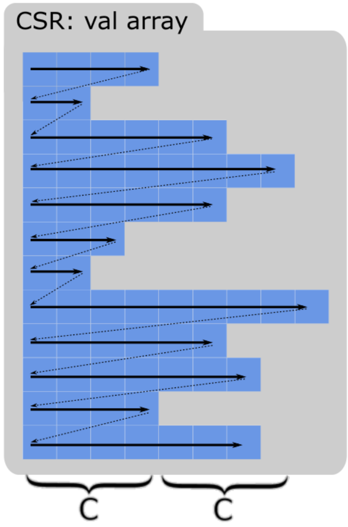

Reductions fast with SIMD operations

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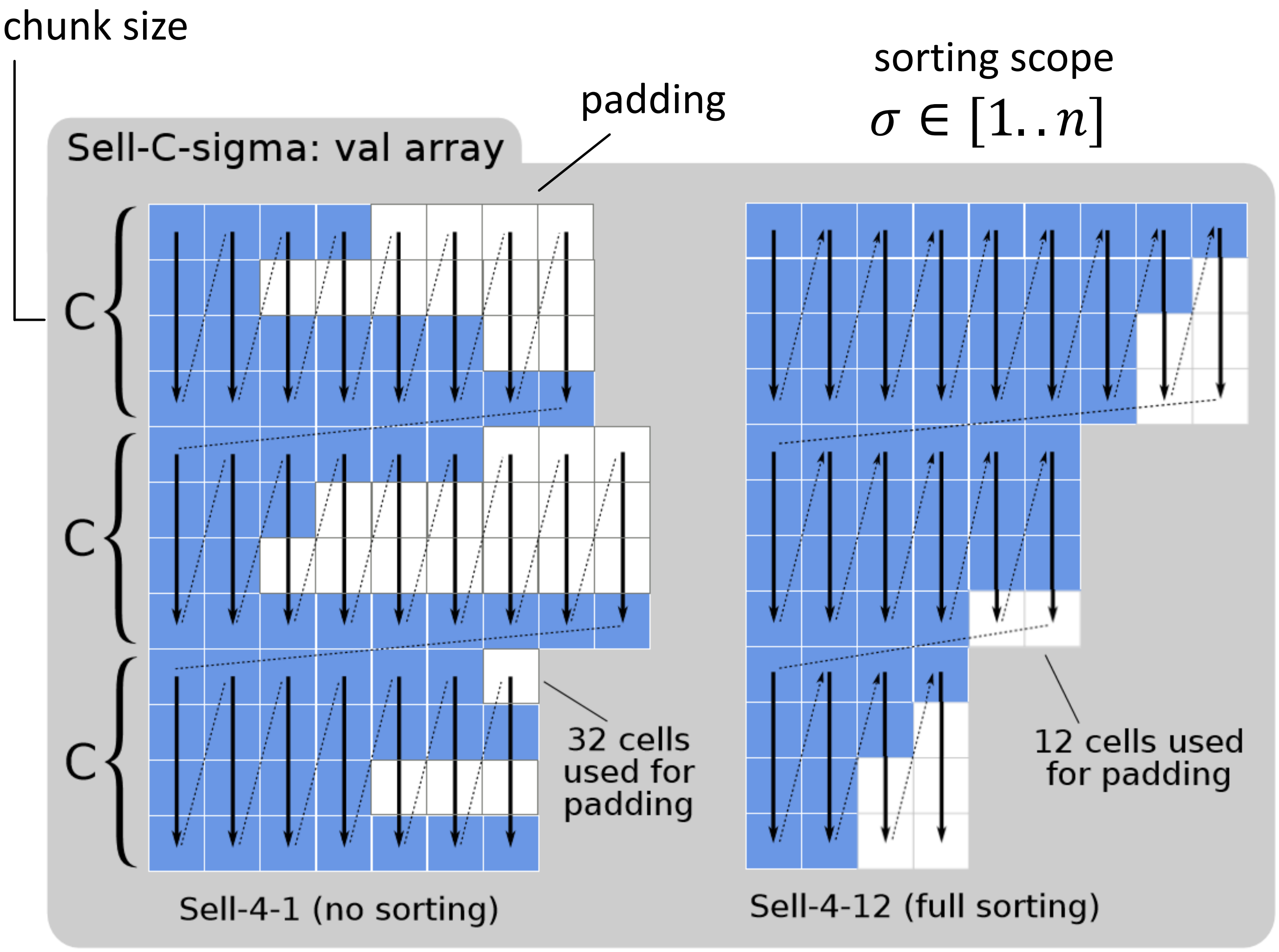
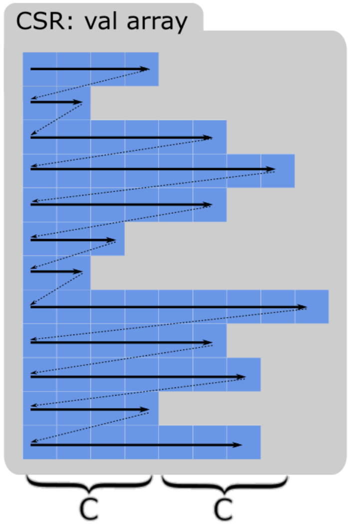
Graph Representations

Sell-C-Sigma





Reductions fast with SIMD operations

Portable

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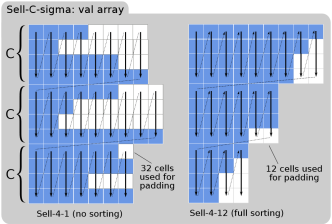
SELL-C-SIGMA + SEMIRINGS + (…) = SLIMSELL

FORMULATIONS

SELL-C-SIGMA + SEMIRINGS + (…) = SLIMSELL

FORMULATIONS



+

$(X, op_1, op_2, el_1, el_2)$

$(\mathbb{R} \cup \{\infty\}, min, +, \infty, 0)$

$(\mathbb{R}, +, \cdot, 0, 1)$

$(\{0,1\}, |, \&, 0, 1)$

$(\mathbb{R}, max, \cdot, -\infty, 1)$

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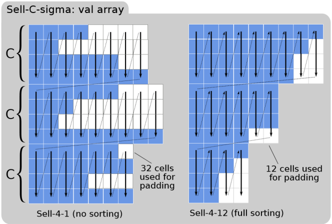
SELL-C-SIGMA + SEMIRINGS + (…) = SLIMSELL

FORMULATIONS



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$(\mathbb{R}, +, \cdot, 0, 1)$

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$(\mathbb{R}, max, \cdot, -\infty, 1)$

```
    // Compute x_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING
    x = MIN(ADD(rhs, vals), x);
```
TROPICAL SEMIRING

```
#elif defined USE_BOOLEAN_SEMIRING
    x = OR(AND(rhs, vals), x);
```
BOOLEAN SEMIRING

```
#elif defined USE_SELMAX_SEMIRING
    x = MAX(MUL(rhs, vals), x);
```
SEL-MAX SEMIRING

```
#endif
    index += C;
  }
  // Now, derive f_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING
  STORE(&f_k[i*C], x); // Just a store.
```
TROPICAL SEMIRING

```
#elif defined USE_BOOLEAN_SEMIRING
  // First, derive f_k using filtering.
  V g = LOAD(&g_{k-1}[i*C]); // Load the filter g_{k-1}.
  x = CMP(AND(x, g), [0,0,...0], NEQ); STORE(&x_k[i*C], x);

  // Second, update distances d; depth is the iteration number.
  V x_mask = x; x = MUL(x, [depth,...,depth]);
  x = BLEND(LOAD(&d[i*C]), x, x_mask); STORE(&d[i*C], x);

  // Third, update the filtering term.
  g = AND(NOT(x_mask), g); STORE(&g_k[i*C], g);
```
BOOLEAN SEMIRING

```
#elif defined USE_SELMAX_SEMIRING:
  // Update parents.
  V pars = LOAD(&p_{k-1}[i*C]); // Load the required part of p_{k-1}
  V pnz = CMP(pars, [0,0,...,0], NEQ);
  pars = BLEND([0,0,...,0], pars, pnz); STORE(&p_k[i*C], pars);

  // Set new x_k vector.
  V tmpnz = CMP(x, [0,0,...,0], NEQ);
  x = BLEND(x, &v[i*C], tmpnz); STORE(&x_k[i*C], x);
#endif
```
SEL-MAX SEMIRING

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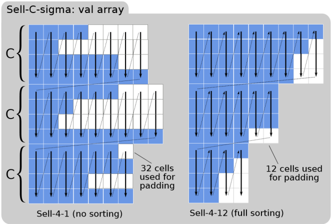
SELL-C-SIGMA + SEMIRINGS + (…) = SLIMSELL

FORMULATIONS



+

$(X, op_1, op_2, el_1, el_2)$

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#elif defined USE_BOOLEAN_SEMIRING                          BOOLEAN SEMIRING
    x = OR(AND(rhs, vals), x);
#elif defined USE_SELMAX_SEMIRING                           SEL-MAX SEMIRING
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  // First, derive f_k using filtering.
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  x = BLEND(LOAD(&d[i*C]), x, x_mask); STORE(&d[i*C], x);

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#elif defined USE_SELMAX_SEMIRING:                          SEL-MAX SEMIRING
  // Update parents.
  V pars = LOAD(&p_{k-1}[i*C]); // Load the required part of p_{k-1}
  V pnz = CMP(pars, [0,0,...,0], NEQ);
  pars = BLEND([0,0,...,0], pars, pnz); STORE(&p_k[i*C], pars);

  // Set new x_k vector.
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  x = BLEND(x, &v[i*C], tmpnz); STORE(&x_k[i*C], x);
#endif
```

! Detailed formulations are in the paper ☺

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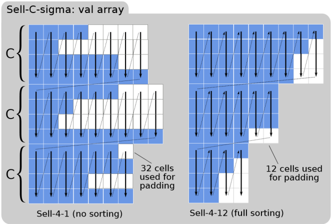
Sell-C-Sigma + Semirings + (…) = SlimSell

Formulations



+

$(X, op_1, op_2, el_1, el_2)$

$(\mathbb{R} \cup \{\infty\}, min, +, \infty, 0)$

$(\mathbb{R}, +, \cdot, 0, 1)$

$(\{0,1\}, |, \&, 0, 1)$

$(\mathbb{R}, max, \cdot, -\infty, 1)$

```
    // Compute x_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING
    x = MIN(ADD(rhs, vals), x);
```

TROPICAL SEMIRING

```
#elif defined USE_BOOLEAN_SEMIRING
    x = OR(AND(rhs, vals), x);
```

BOOLEAN SEMIRING

```
#elif defined USE_SELMAX_SEMIRING
    x = MAX(MUL(rhs, vals), x);
```

SEL-MAX SEMIRING

```
#endif
    index += C;
  }
  // Now, derive f_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING
  STORE(&f_k[i*C], x); // Just a store.
```

TROPICAL SEMIRING

```
#elif defined USE_BOOLEAN_SEMIRING
  // First, derive f_k using filtering.
  V g = LOAD(&g_{k-1}[i*C]); // Load the filter g_{k-1}.
  x = CMP(AND(x, g), [0,0,...0], NEQ); STORE(&x_k[i*C], x);

  // Second, update distances d; depth is the iteration number.
  V x_mask = x; x = MUL(x, [depth,...,depth]);
  x = BLEND(LOAD(&d[i*C]), x, x_mask); STORE(&d[i*C], x);

  // Third, update the filtering term.
  g = AND(NOT(x_mask), g); STORE(&g_k[i*C], g);
```

BOOLEAN SEMIRING

```
#elif defined USE_SELMAX_SEMIRING:
  // Update parents.
  V pars = LOAD(&p_{k-1}[i*C]); // Load the required part of p_{k-1}
  V pnz = CMP(pars, [0,0,...,0], NEQ);
  pars = BLEND([0,0,...,0], pars, pnz); STORE(&p_k[i*C], pars);

  // Set new x_k vector.
  V tmpnz = CMP(x, [0,0,...,0], NEQ);
  x = BLEND(x, &v[i*C], tmpnz); STORE(&x_k[i*C], x);
#endif
```

SEL-MAX SEMIRING

What vector operations are required for each semiring when using Sell-C-sigma

Detailed formulations are in the paper ☺

M. Besta et al.: “SlimSell: A Vectorized Graph Representation for Breadth-First Search”, IPDPS’17

SELL-C-SIGMA + SEMIRINGS + (…) = SLIMSELL

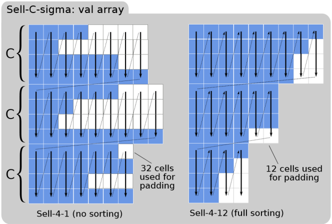
FORMULATIONS

```
   // Compute x_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING                                   TROPICAL SEMIRING
     x = MIN(ADD(rhs, vals), x);
#elif defined USE_BOOLEAN_SEMIRING                             BOOLEAN SEMIRING
     x = OR(AND(rhs, vals), x);
#elif defined USE_SELMAX_SEMIRING                              SEL-MAX SEMIRING
     x = MAX(MUL(rhs, vals), x);
#endif
     index += C;
  }
  // Now, derive f_k (versions differ based on the used semiring):
#ifdef USE_TROPICAL_SEMIRING                                   TROPICAL SEMIRING
  STORE(&f_k[i*C], x); // Just a store.
#elif defined USE_BOOLEAN_SEMIRING                             BOOLEAN SEMIRING
  // First, derive f_k using filtering.
  V g = LOAD(&g_{k-1}[i*C]); // Load the filter g_{k-1}.
  x = CMP(AND(x, g), [0,0,...0], NEQ); STORE(&x_k[i*C], x);

  // Second, update distances d; depth is the iteration number.
  V x_mask = x; x = MUL(x, [depth,...,depth]);
  x = BLEND(LOAD(&d[i*C]), x, x_mask); STORE(&d[i*C], x);

  // Third, update the filtering term.
  g = AND(NOT(x_mask), g); STORE(&g_k[i*C], g);
#elif defined USE_SELMAX_SEMIRING:                             SEL-MAX SEMIRING
  // Update parents.
  V pars = LOAD(&p_{k-1}[i*C]); // Load the required part of p_{k-1}
  V pnz = CMP(pars, [0,0,...,0], NEQ);
  pars = BLEND([0,0,...,0], pars, pnz); STORE(&p_k[i*C], pars);

  // Set new x_k vector.
  V tmpnz = CMP(x, [0,0,...,0], NEQ);
  x = BLEND(x, &v[i*C], tmpnz); STORE(&x_k[i*C], x);
#endif
```



+

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! What vector operations are required for each semiring when using Sell-C-sigma

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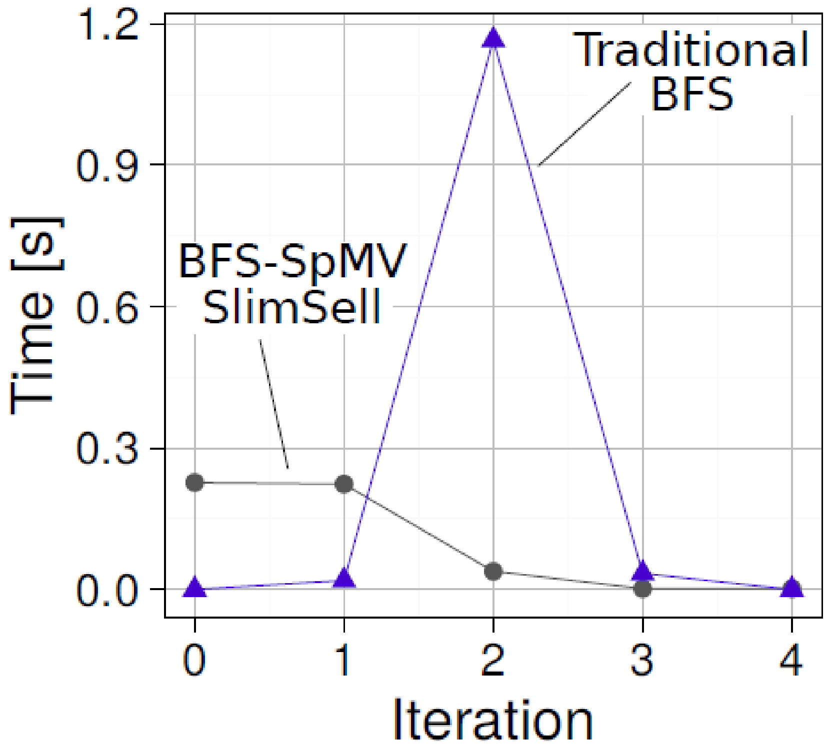
PERFORMANCE ANALYSIS
COMPARISON TO GRAPH500

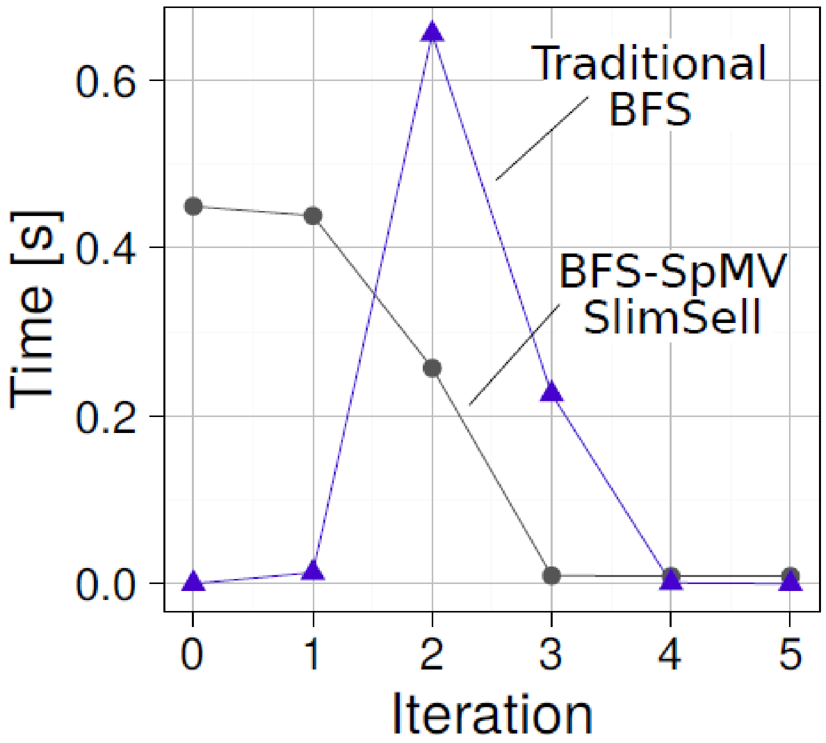
Kronecker power-law graphs

Intel KNL, $C = 16$

$\log \sigma \in \{20,21,22\}$

Dynamic scheduling





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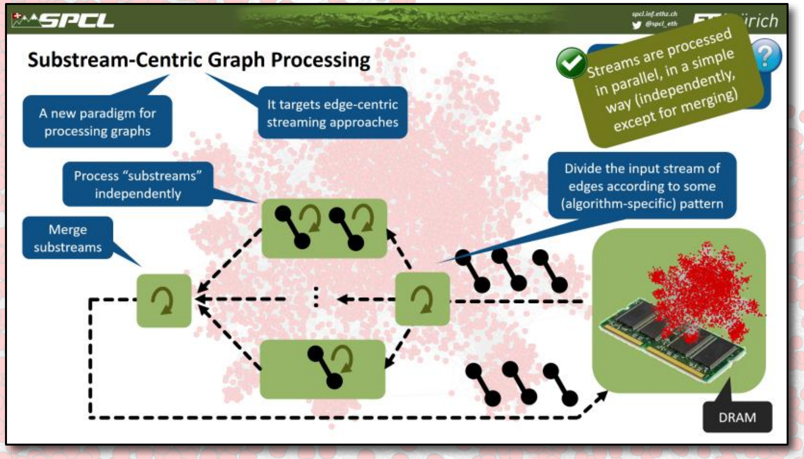
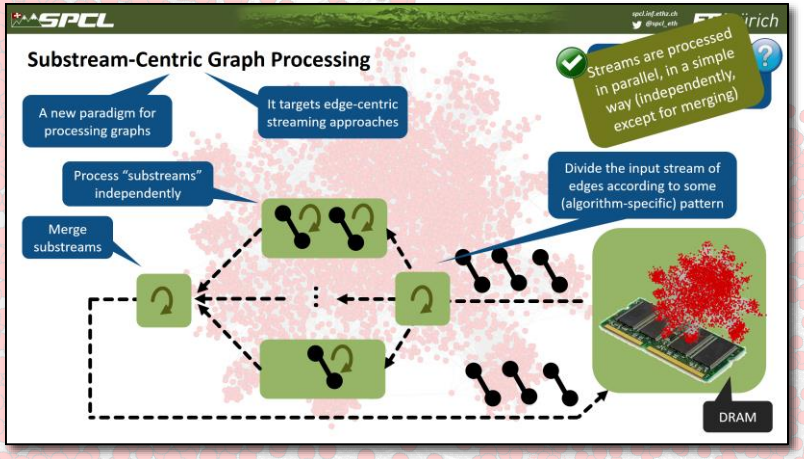


SUBSTREAM-CENTRIC GRAPH PROCESSING PARADIGM:

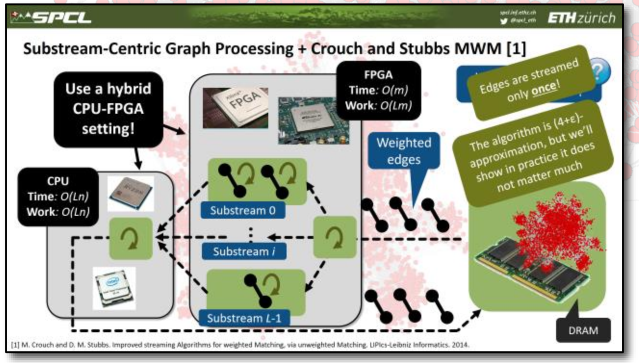
EXPOSES PARALLELISM, ENABLES EASY PIPELINING, SUPPORTS APPROXIMATION

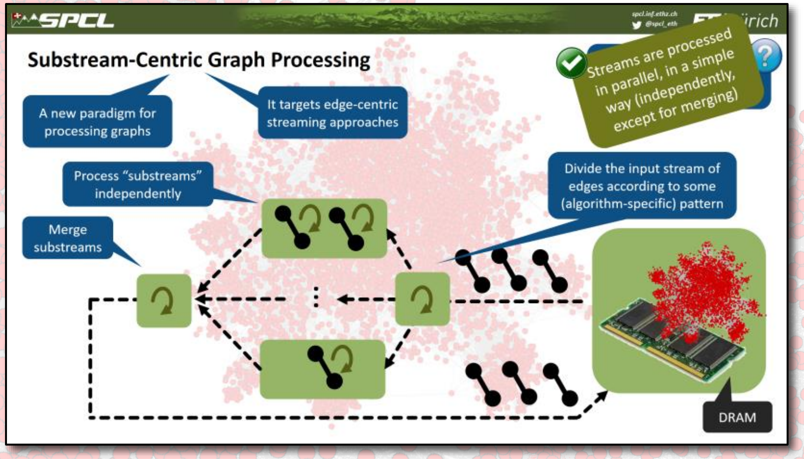


THEORY-INSPIRED MWM APPROXIMATE ALGORITHM ON A HYBRID CPU-FPGA SETTING

SUBSTREAM-CENTRIC GRAPH PROCESSING PARADIGM:

EXPOSES PARALLELISM, ENABLES EASY PIPELINING, SUPPORTS APPROXIMATION





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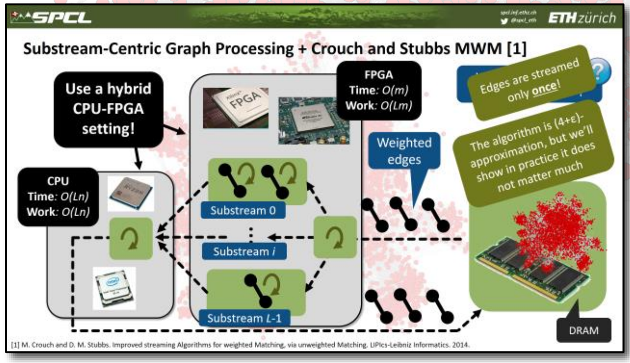
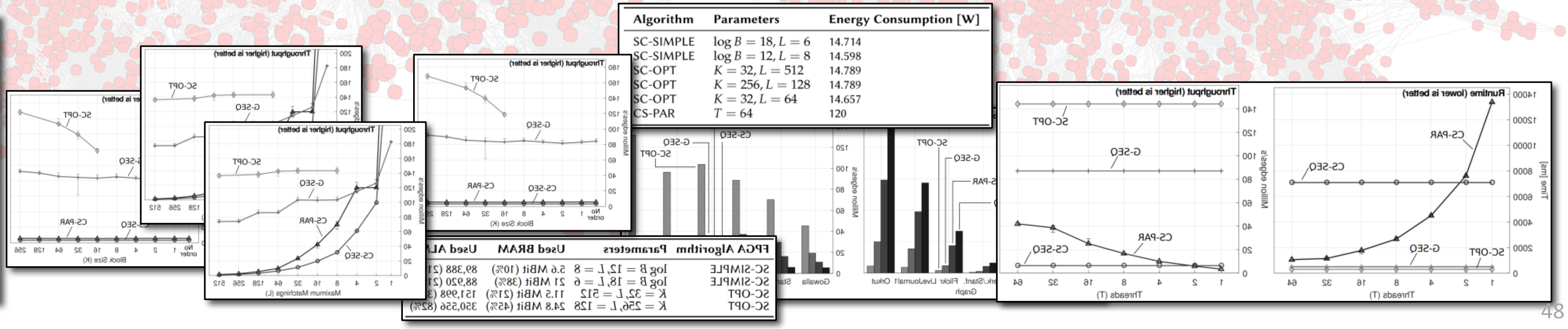
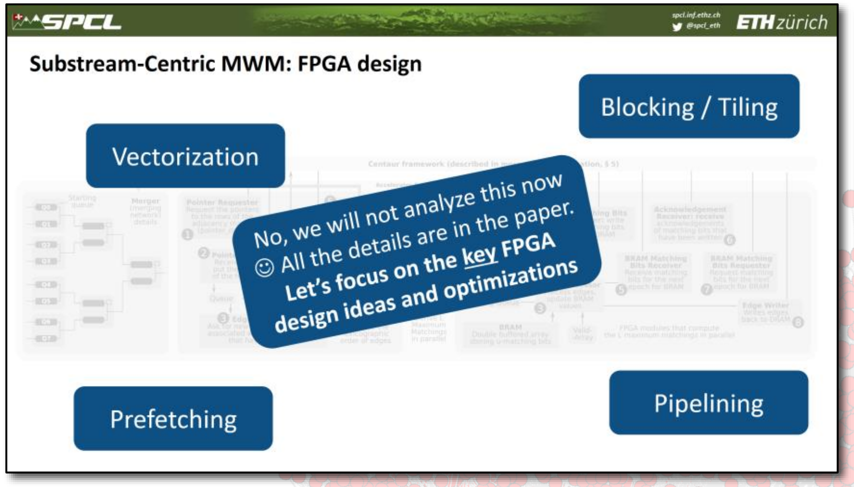
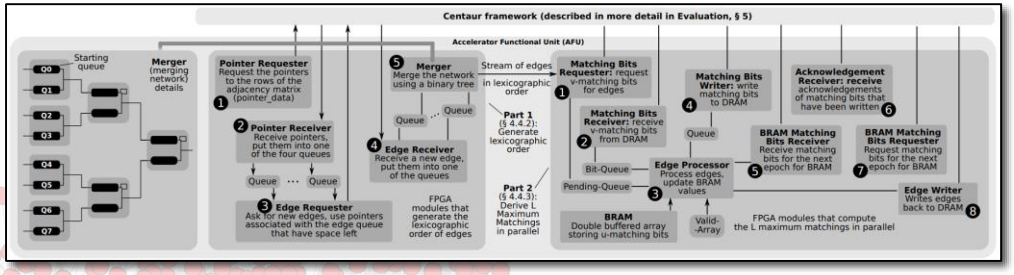


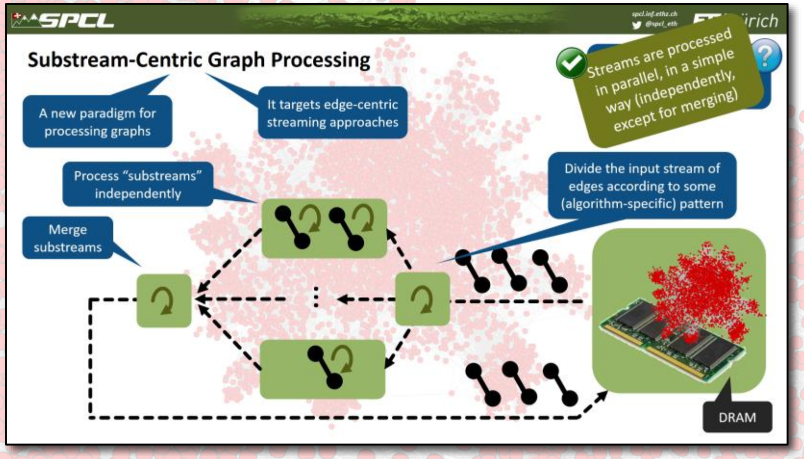


Algorithm	Parameters	Energy Consumption [W]
SC-SIMPLE	$\log B = 18, L = 6$	14.714
SC-SIMPLE	$\log B = 12, L = 8$	14.598
SC-OPT	$K = 32, L = 512$	14.789
SC-OPT	$K = 256, L = 128$	14.789
SC-OPT	$K = 32, L = 64$	14.657
CS-PAR	$T = 64$	120





GENERIC FPGA DESIGN, CODE AVAILABLE



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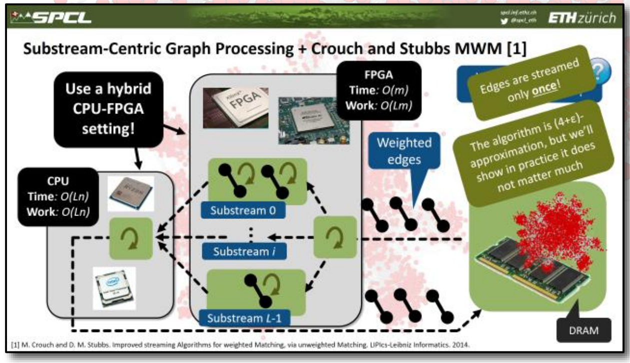

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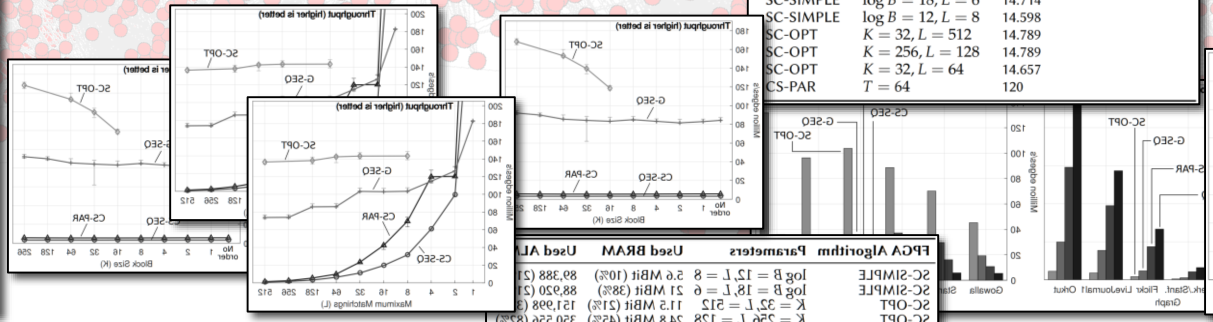

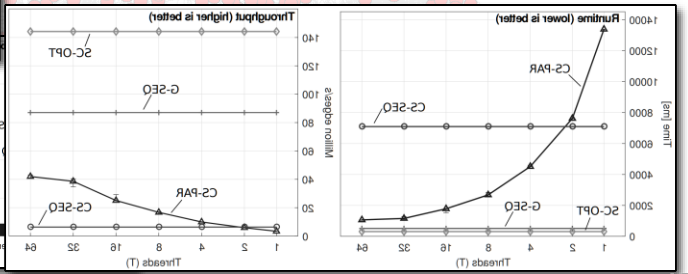

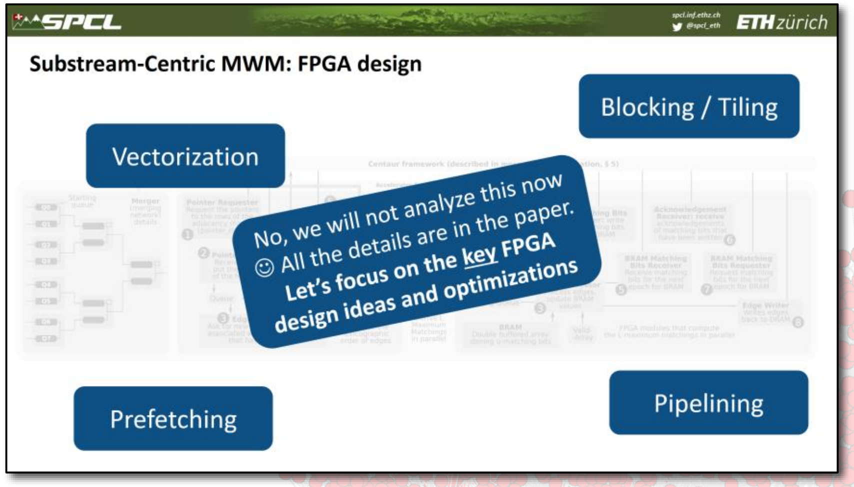

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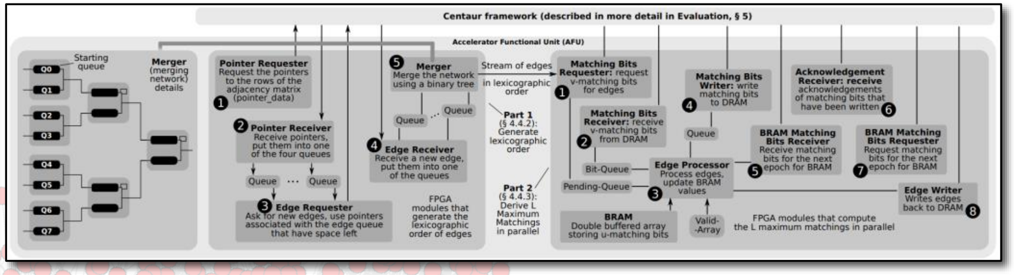
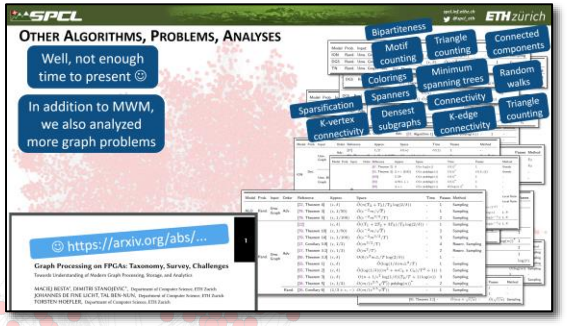
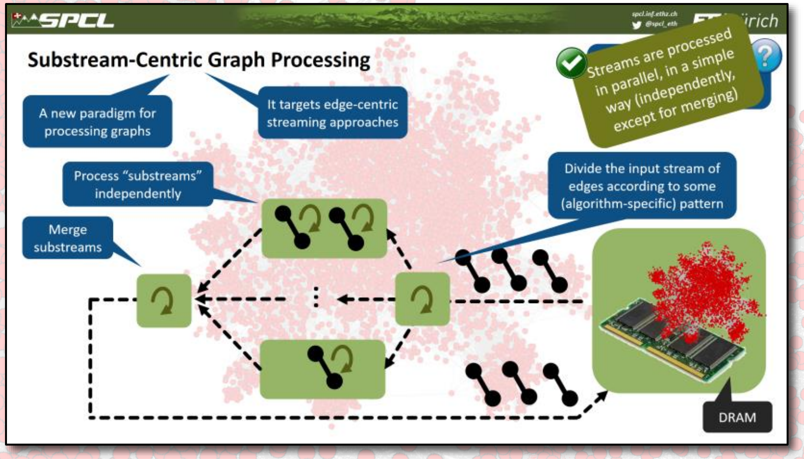




GENERALIZABILITY TO OTHER GRAPH PROBLEMS AND SETTINGS



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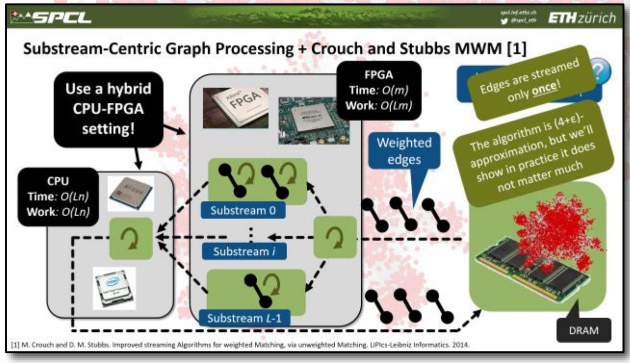

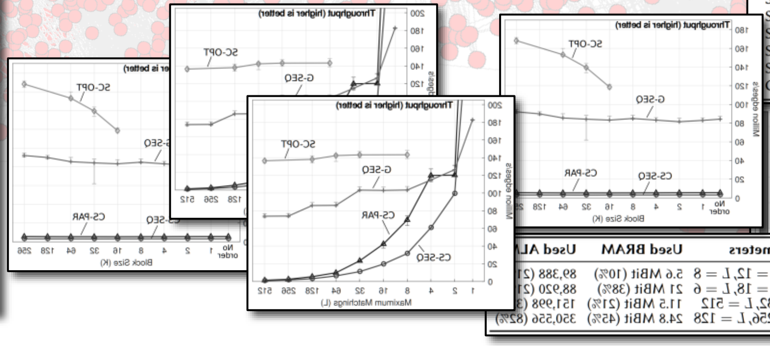

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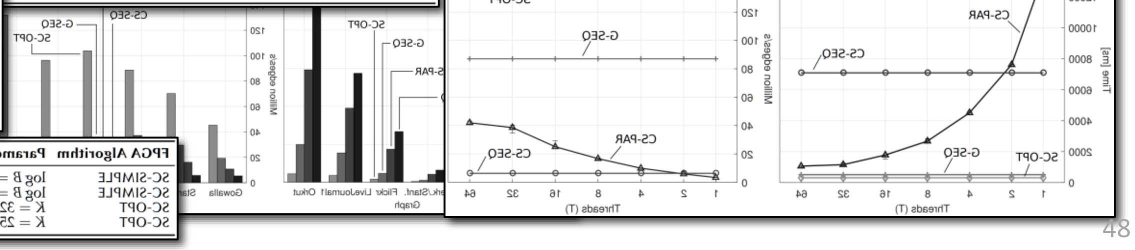
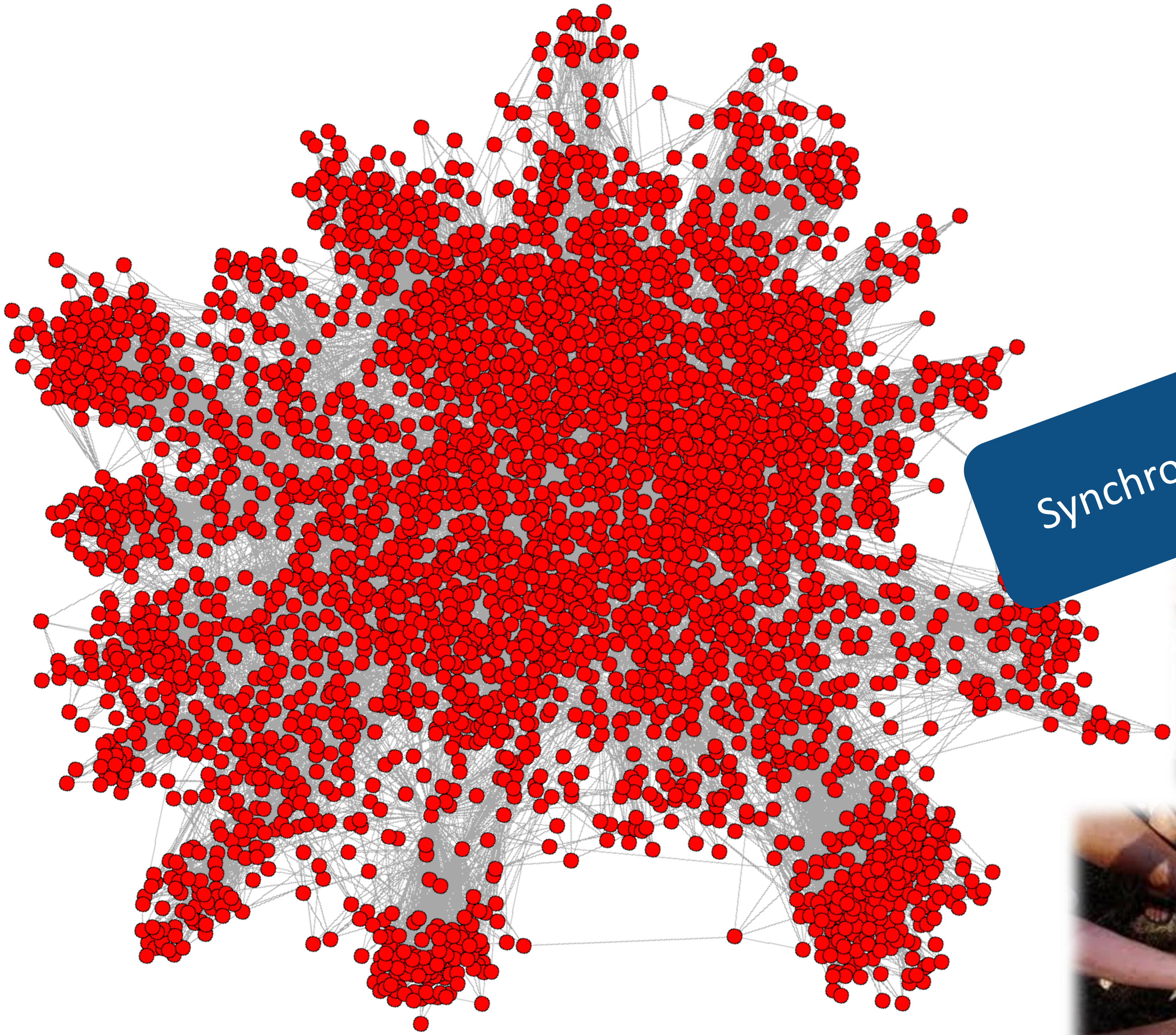
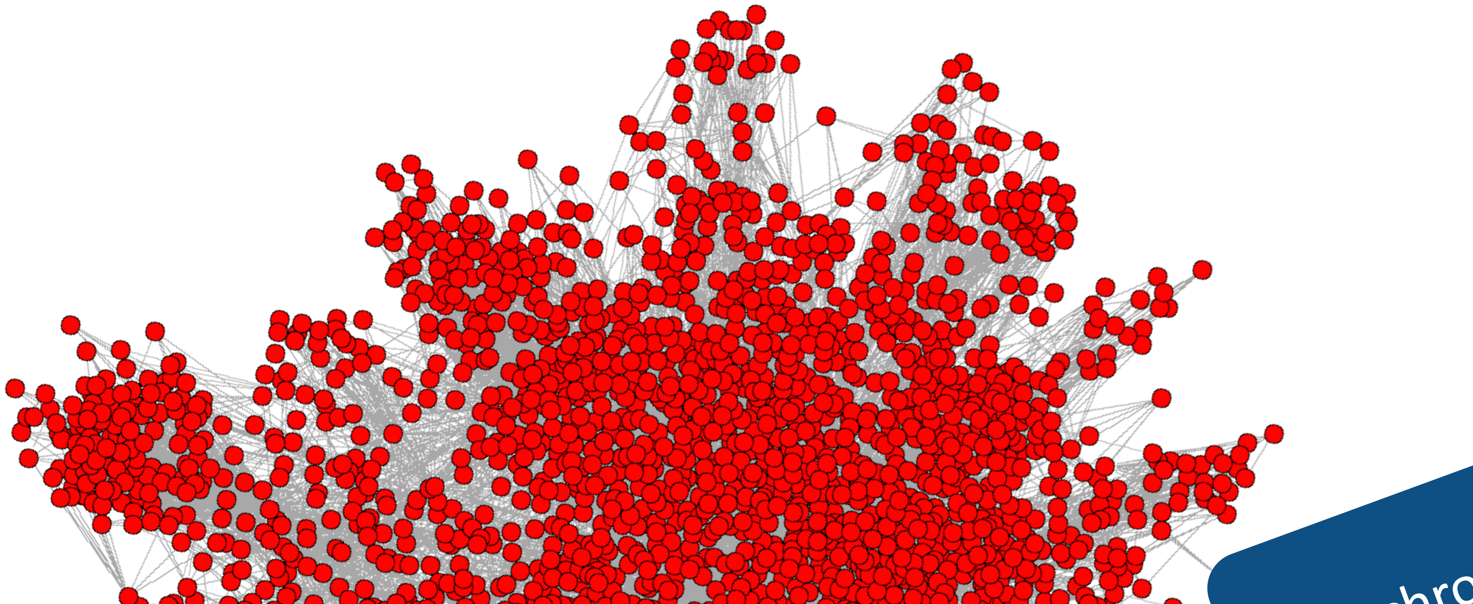


Problems!

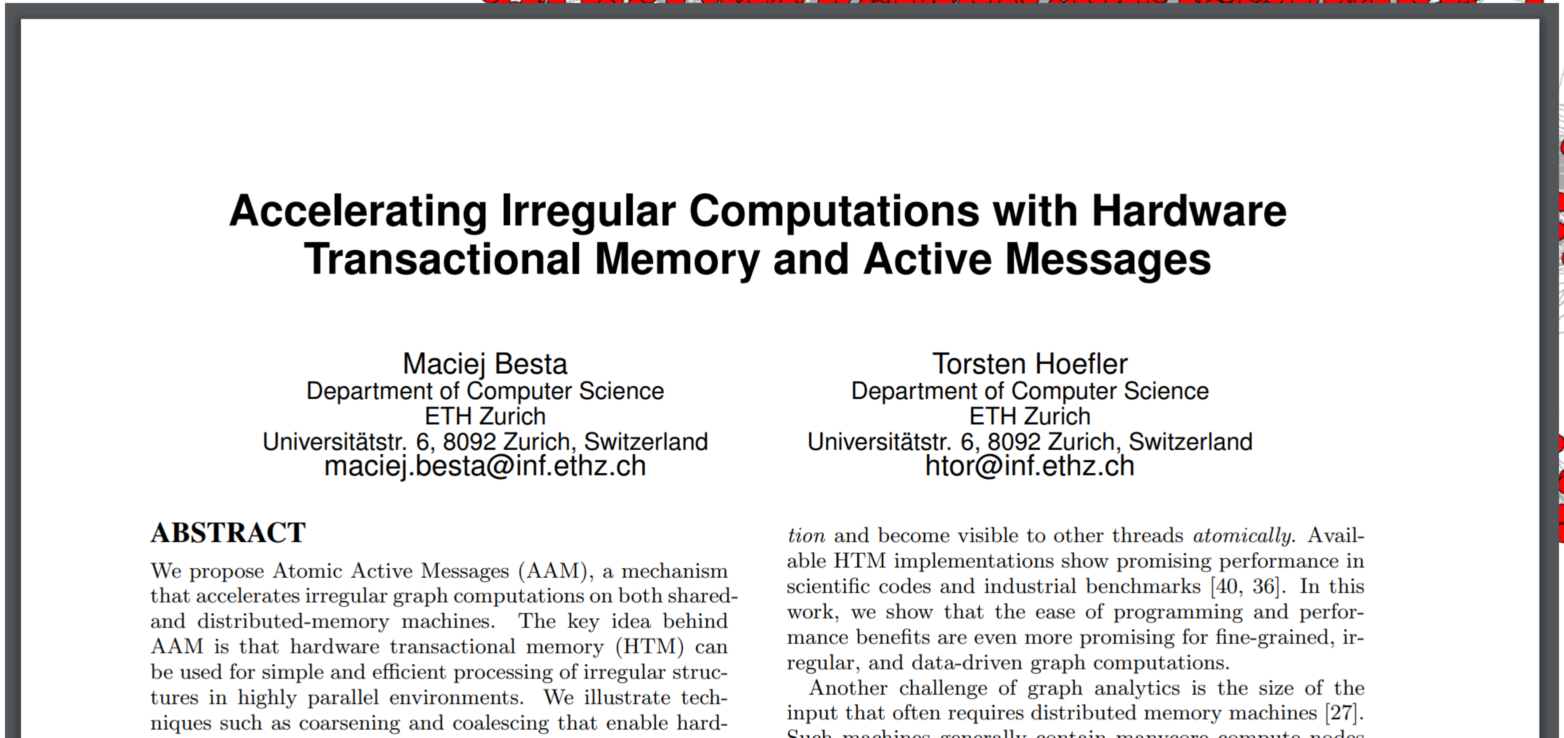


Synchronization-heavy

Problems!



Synchronization-heavy



Accelerating Irregular Computations with Hardware Transactional Memory and Active Messages

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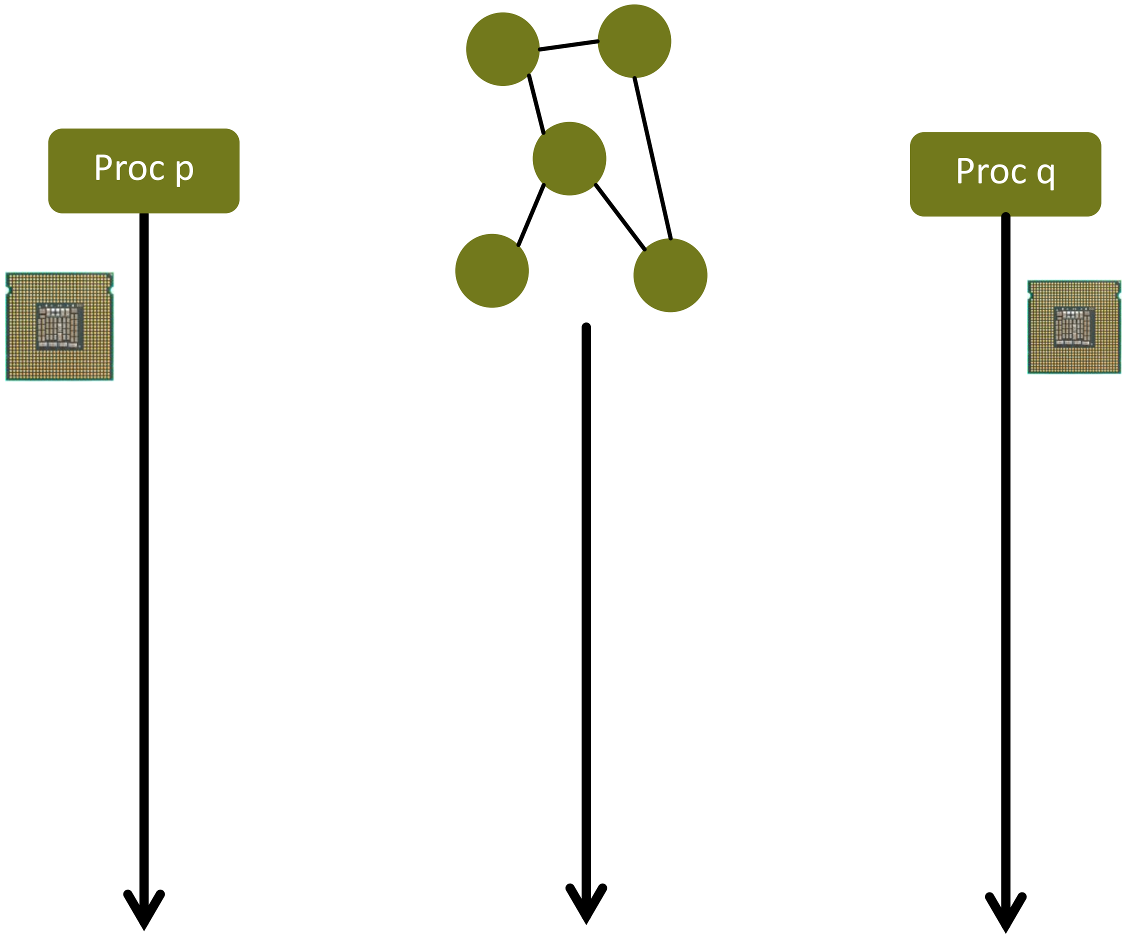
ABSTRACT

We propose Atomic Active Messages (AAM), a mechanism that accelerates irregular graph computations on both shared- and distributed-memory machines. The key idea behind AAM is that hardware transactional memory (HTM) can be used for simple and efficient processing of irregular structures in highly parallel environments. We illustrate techniques such as coarsening and coalescing that enable hard-

tion and become visible to other threads *atomically*. Available HTM implementations show promising performance in scientific codes and industrial benchmarks [40, 36]. In this work, we show that the ease of programming and performance benefits are even more promising for fine-grained, irregular, and data-driven graph computations.

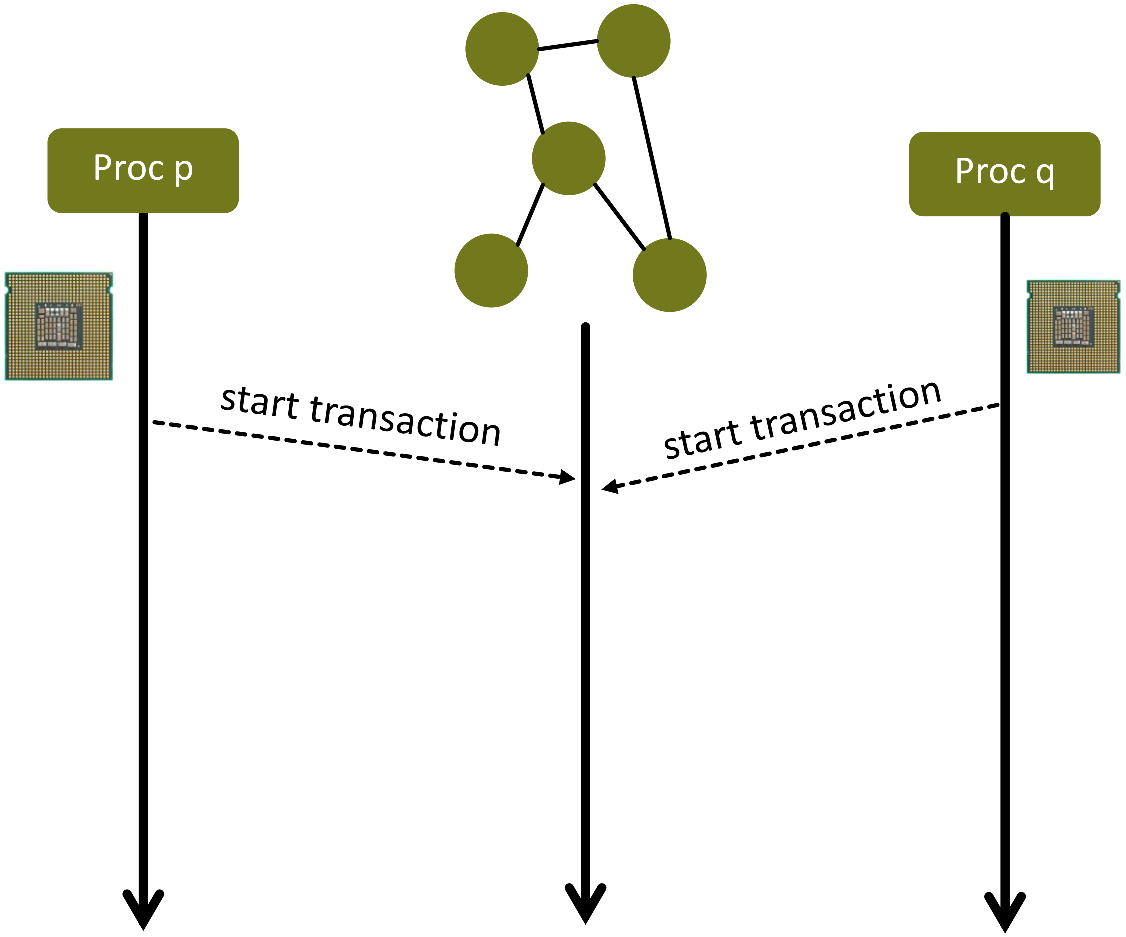
Another challenge of graph analytics is the size of the input that often requires distributed memory machines [27].

TRANSACTIONAL MEMORY [1]



[1] N. Shavit and D. Touitou. Software transactional memory. PODC'95.

Transactional Memory [1]



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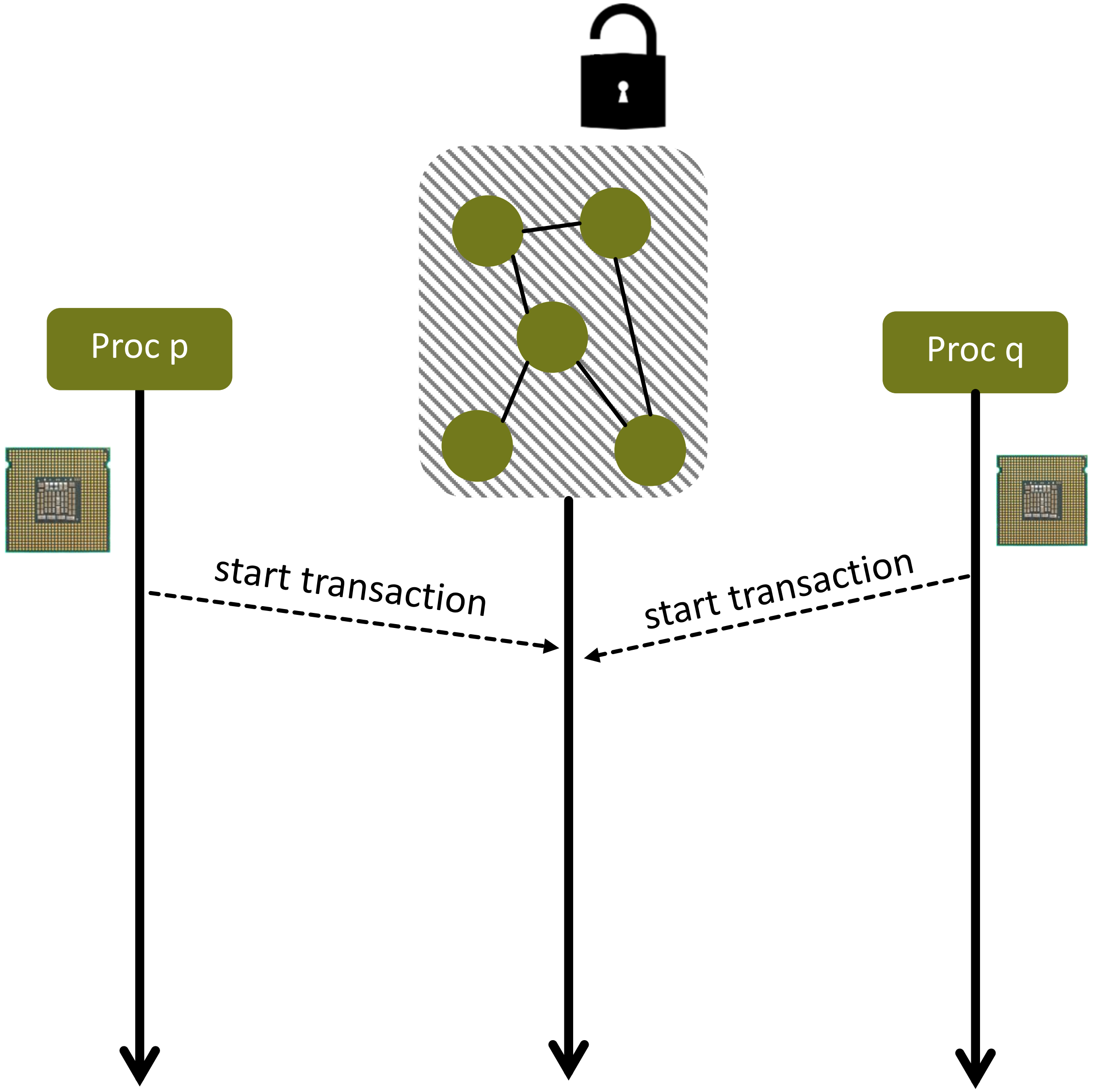

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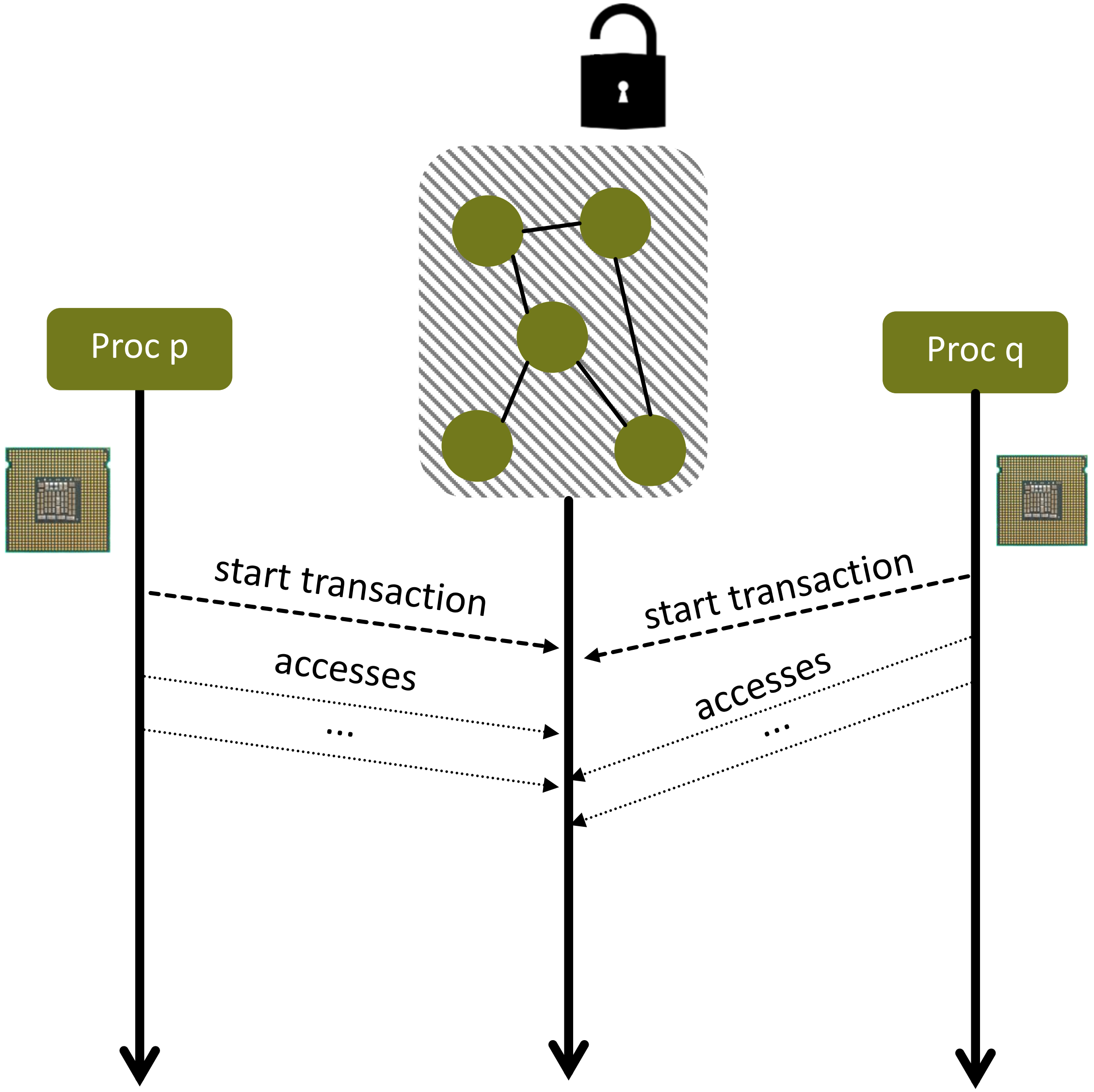

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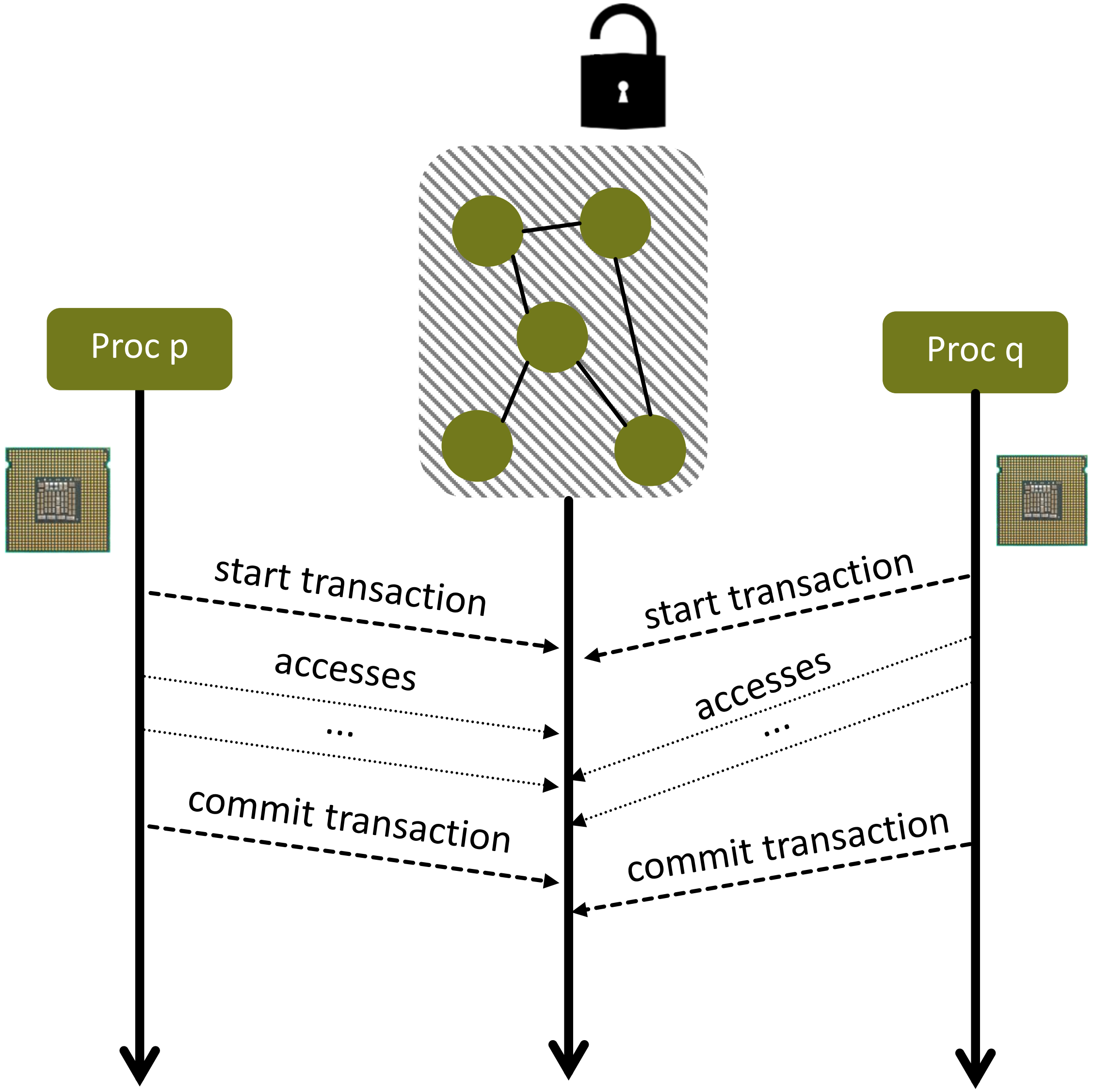

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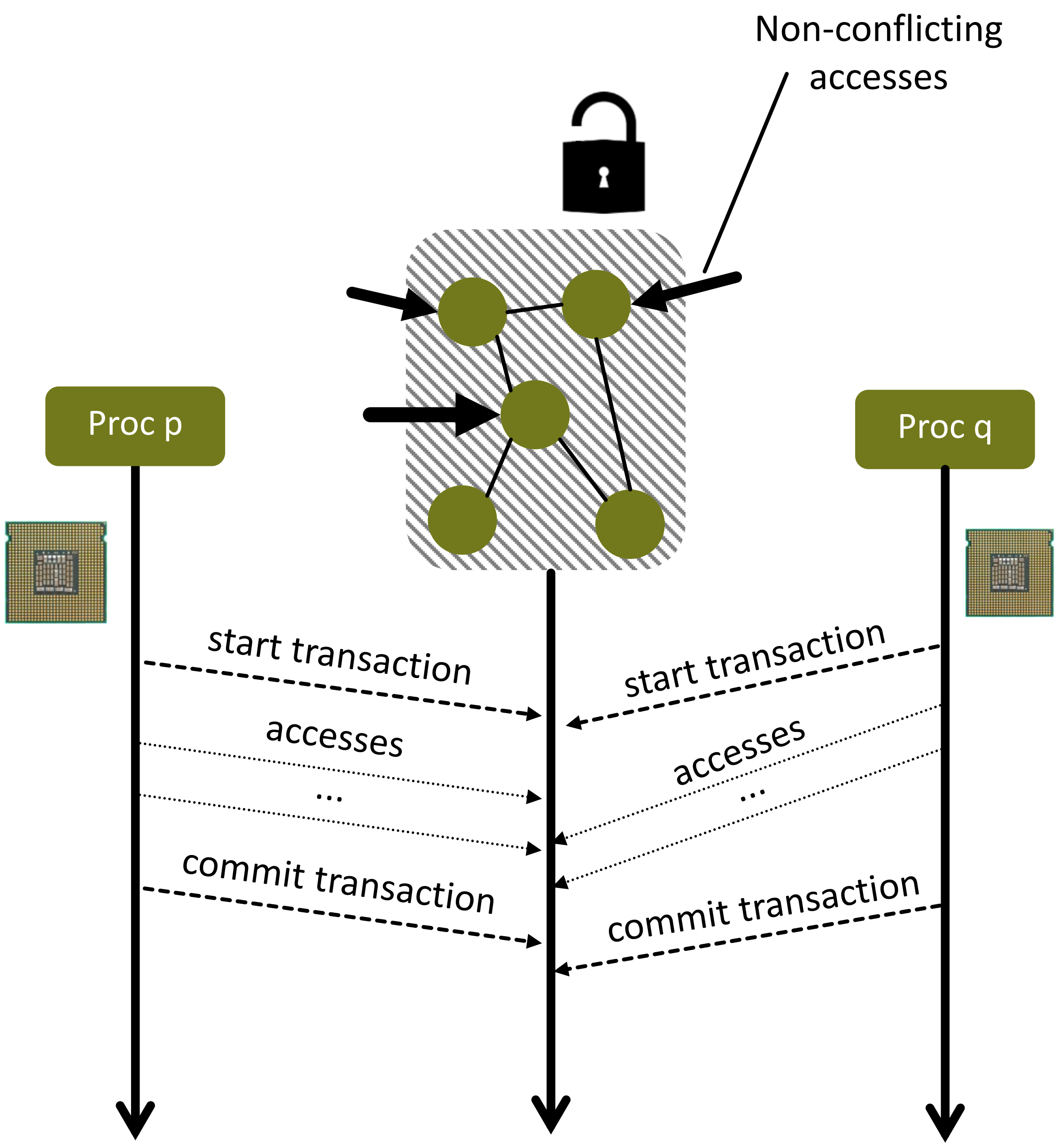

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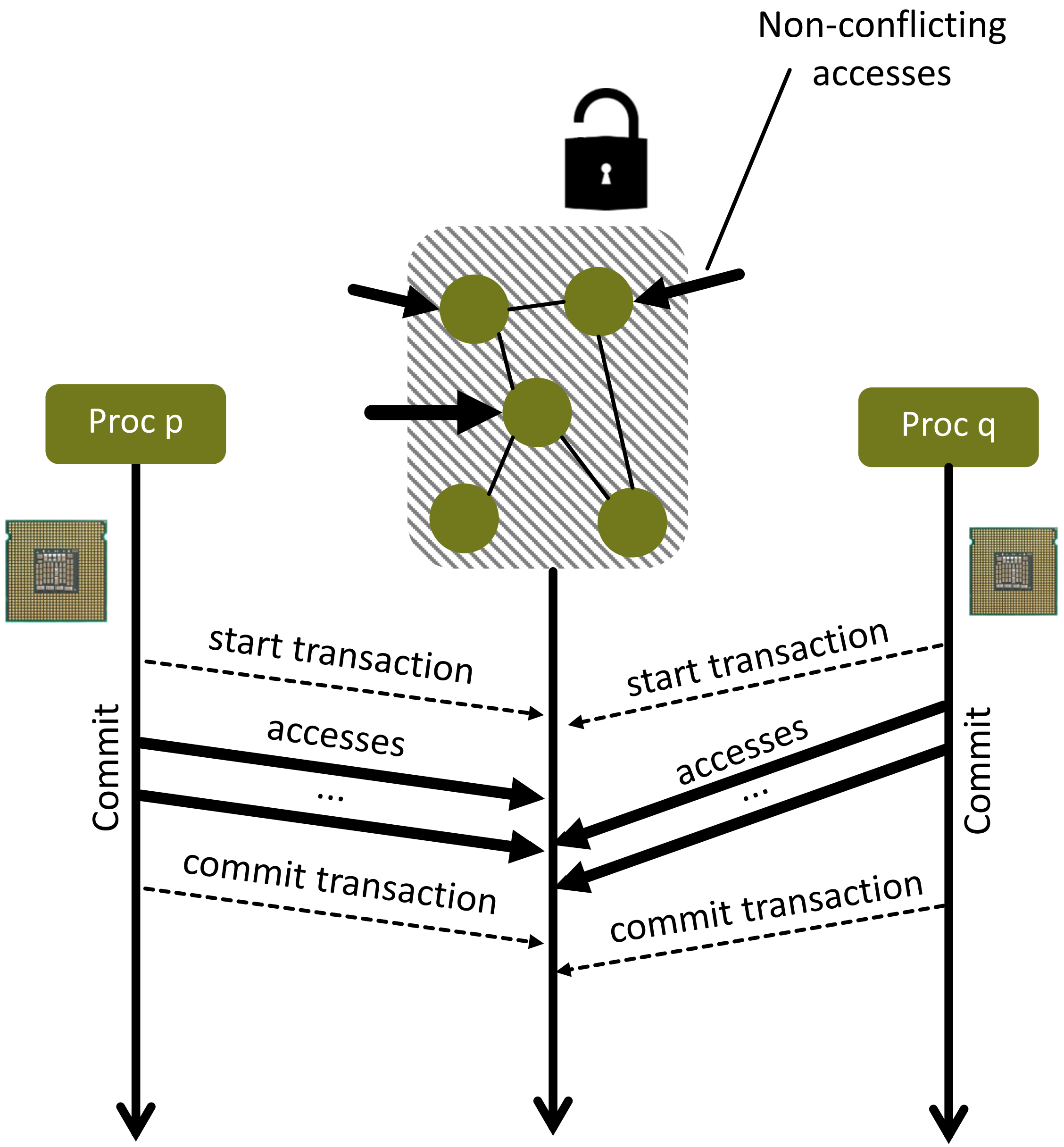
TRANSACTIONAL MEMORY [1]



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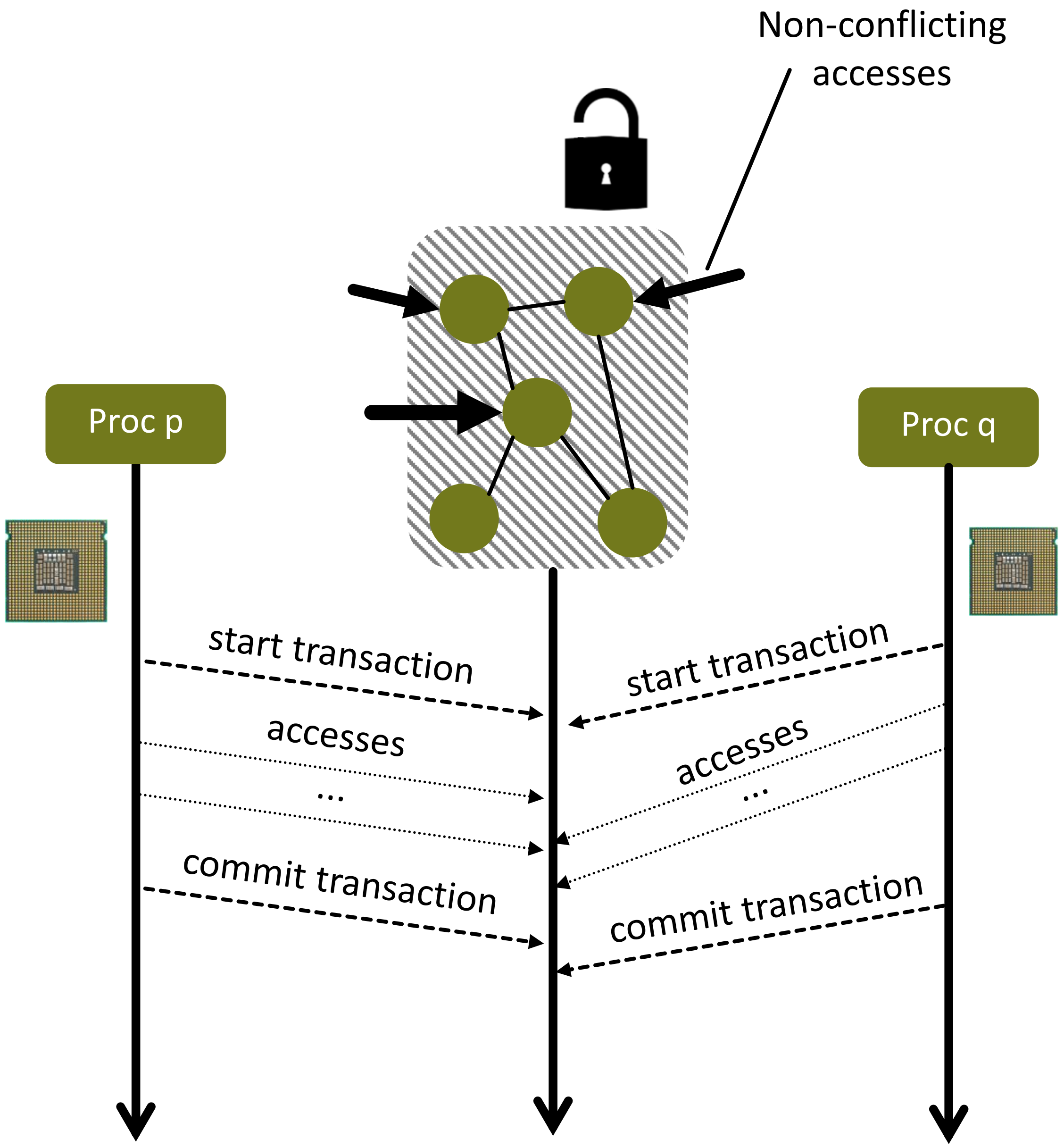

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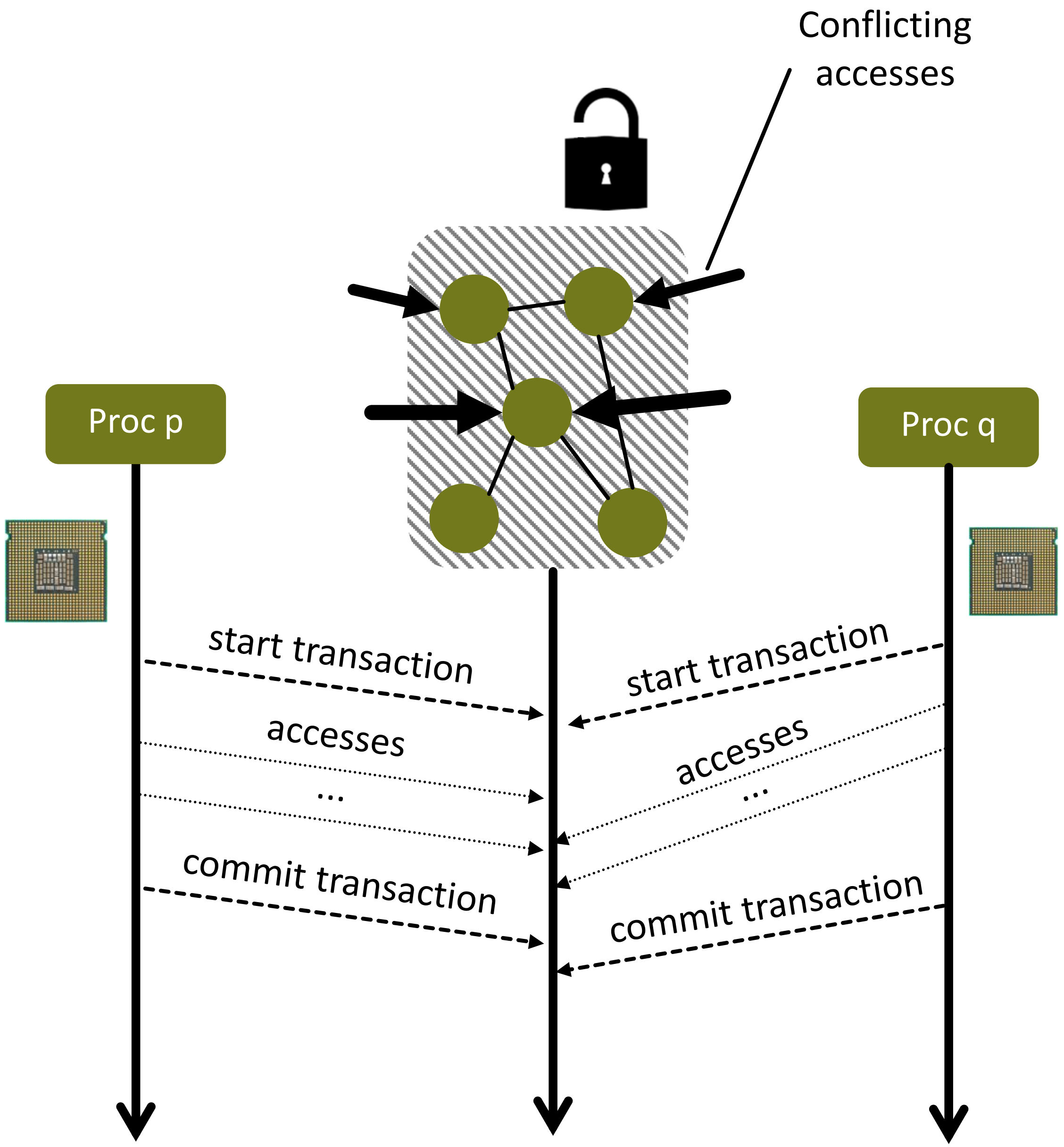

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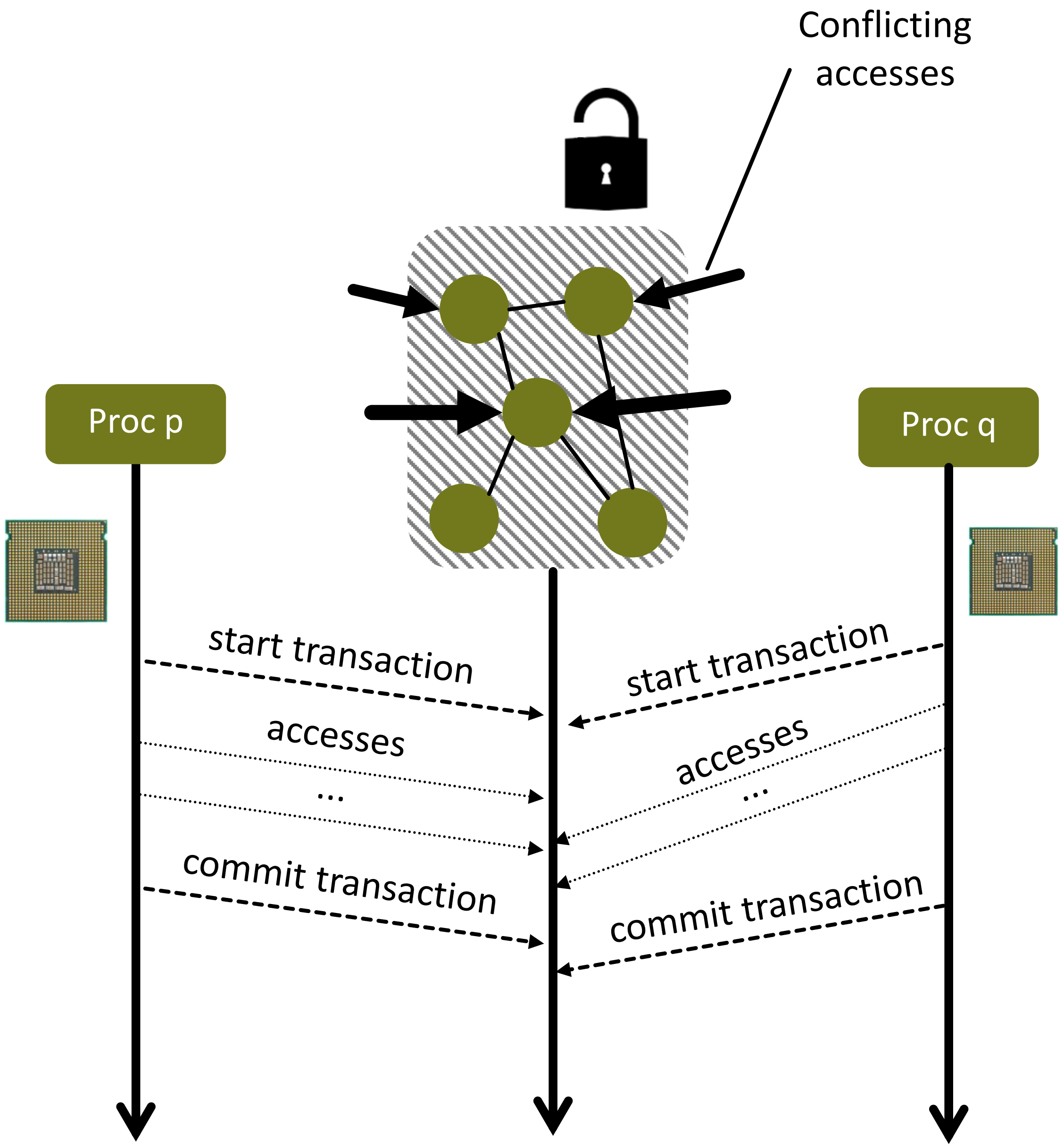
Transactional Memory [1]

Conflicts solved with rollbacks and/or serialization.



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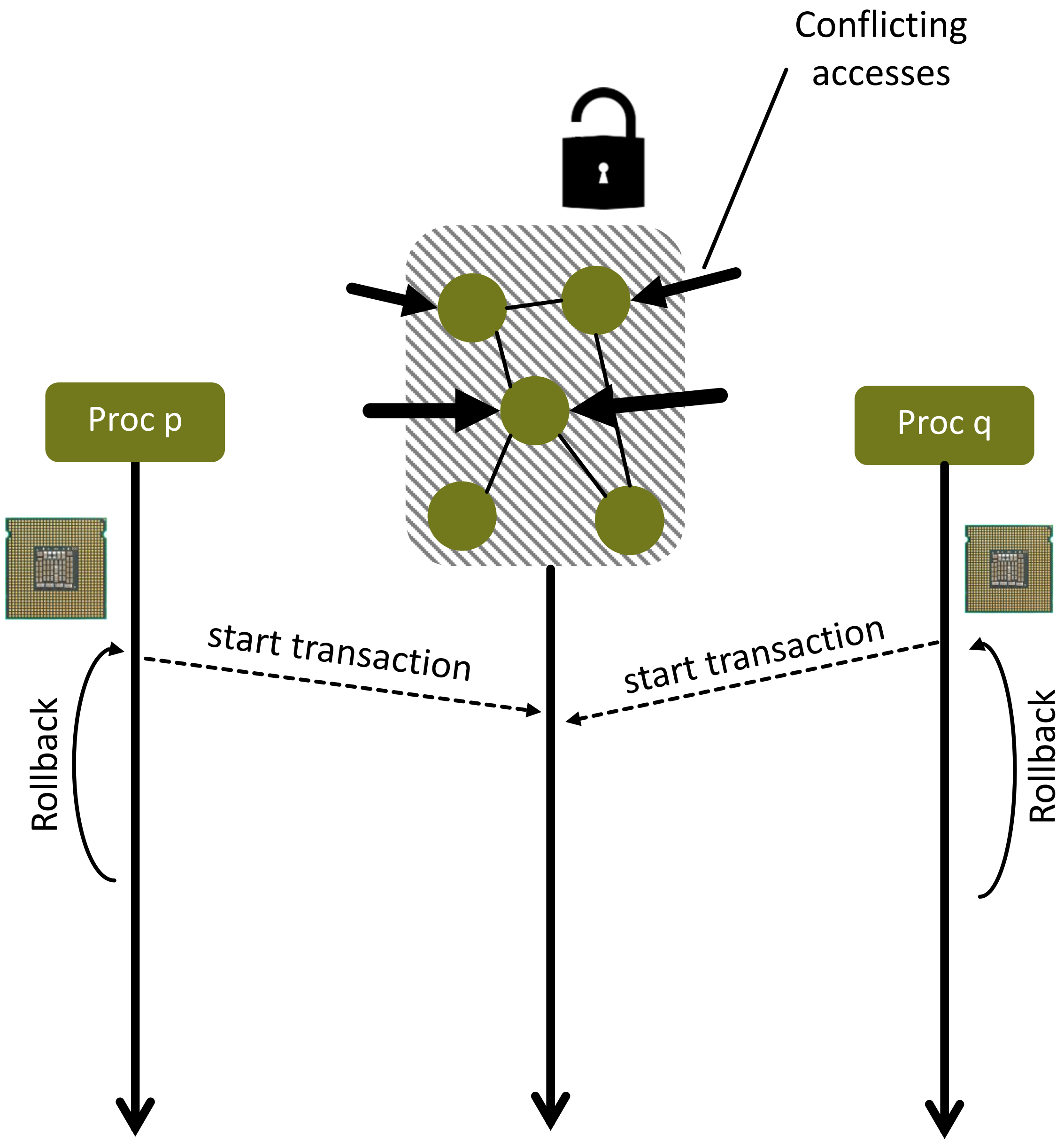

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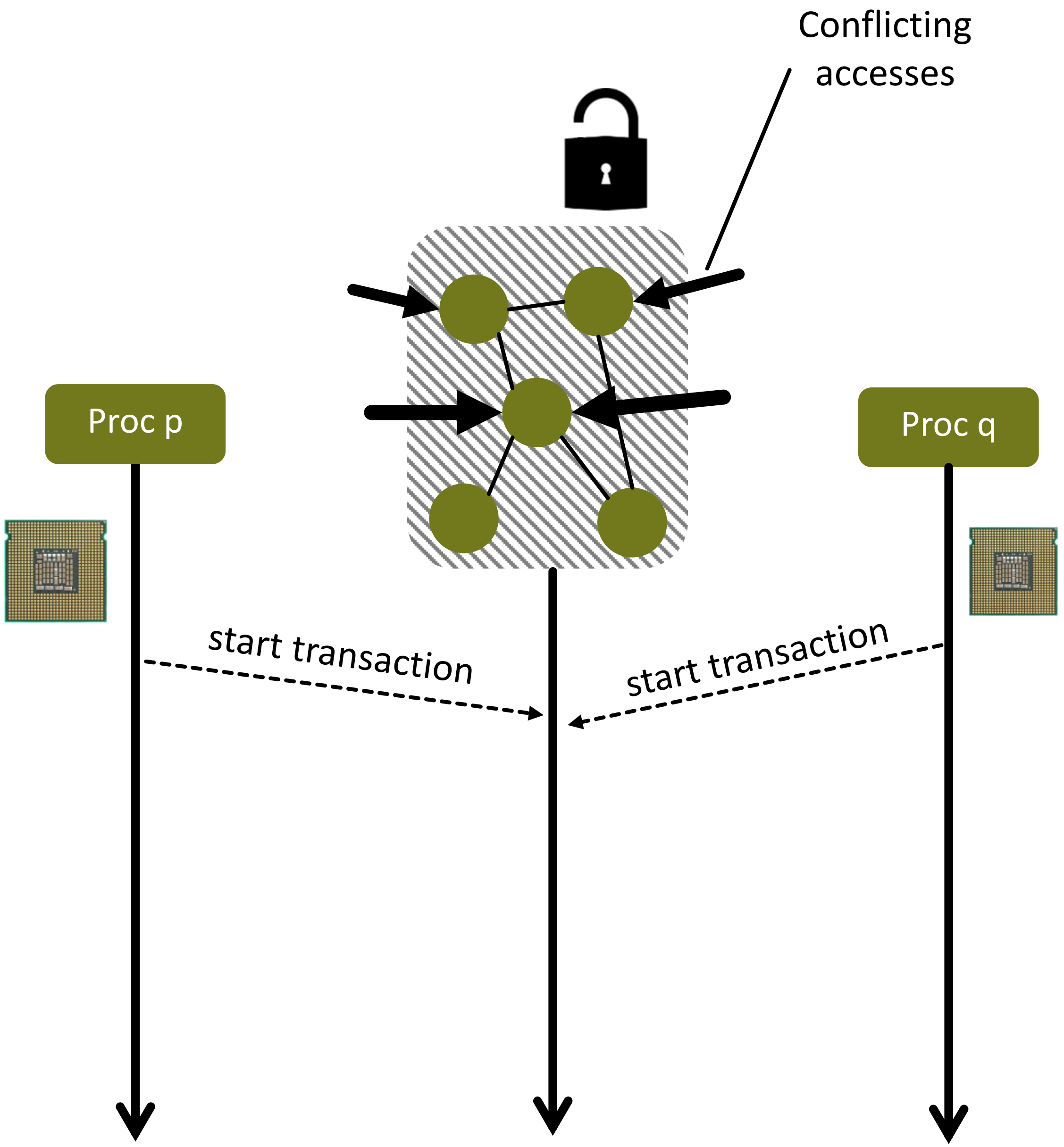

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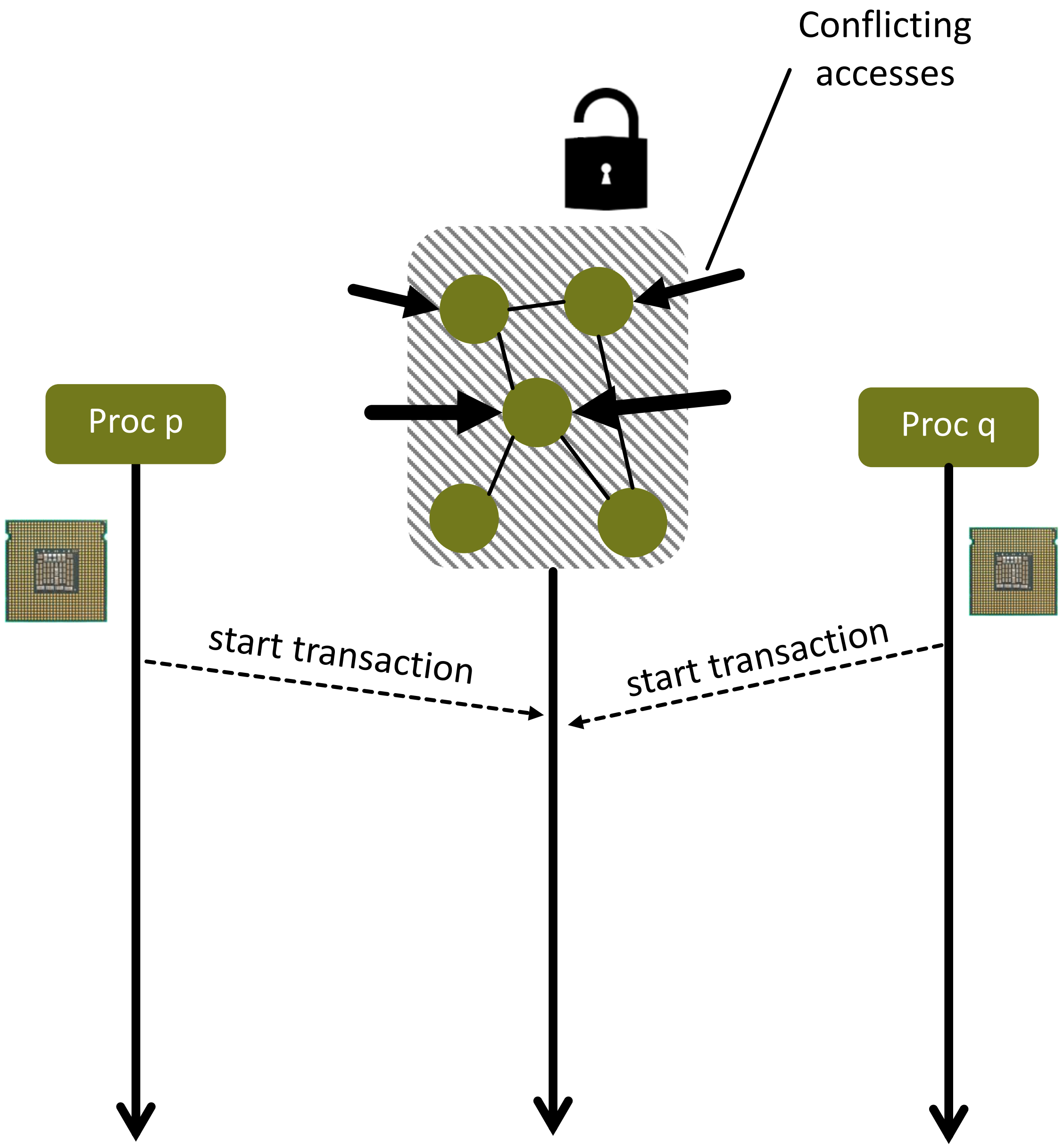


Simple protocols

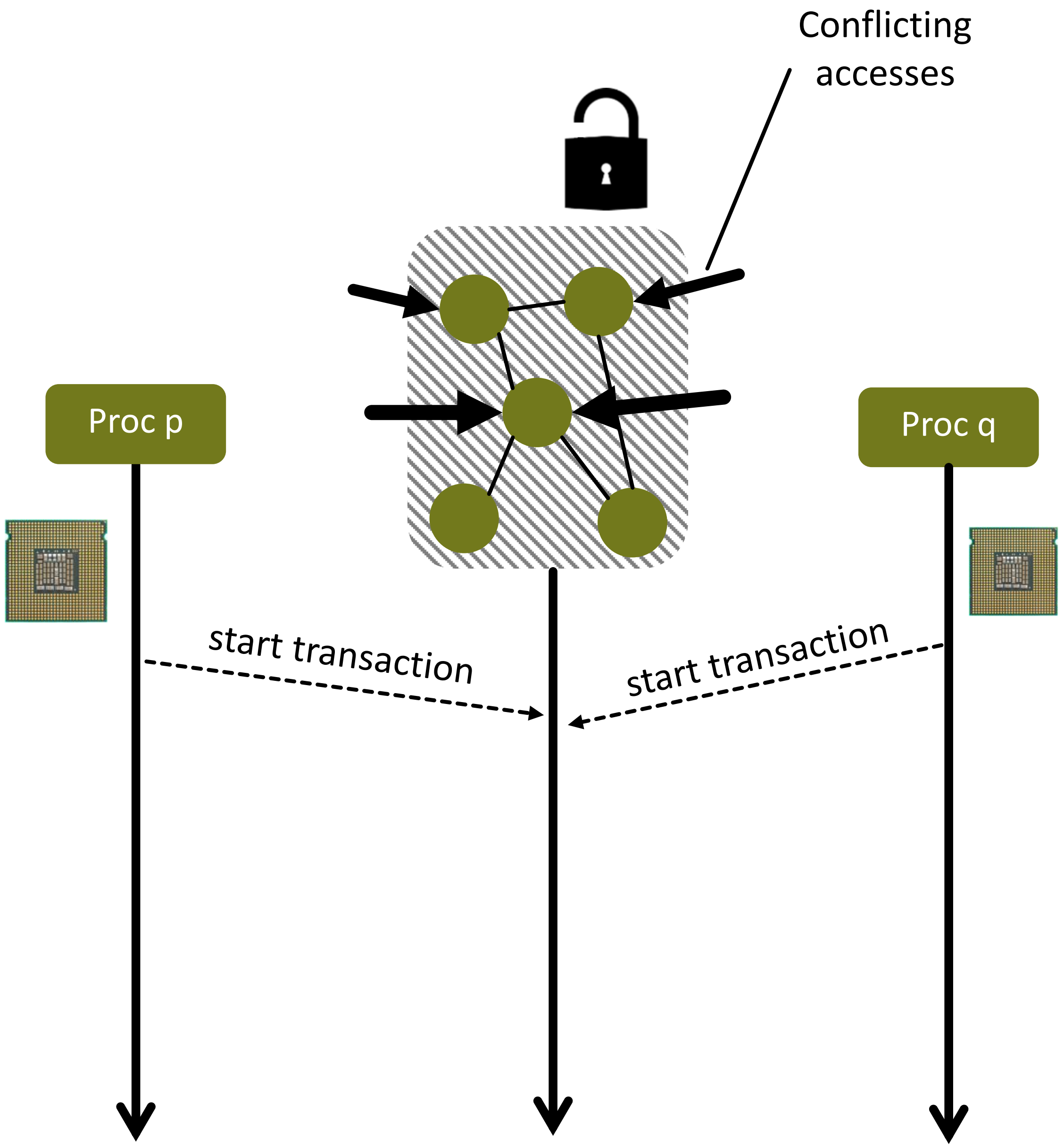
[1] N. Shavit and D. Touitou. Software transactional memory. PODC’95.

TRANSACTIONAL MEMORY [1]

Conflicts solved with rollbacks and/or serialization.

Software overheads

Simple protocols



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TRANSACTIONAL MEMORY [1]

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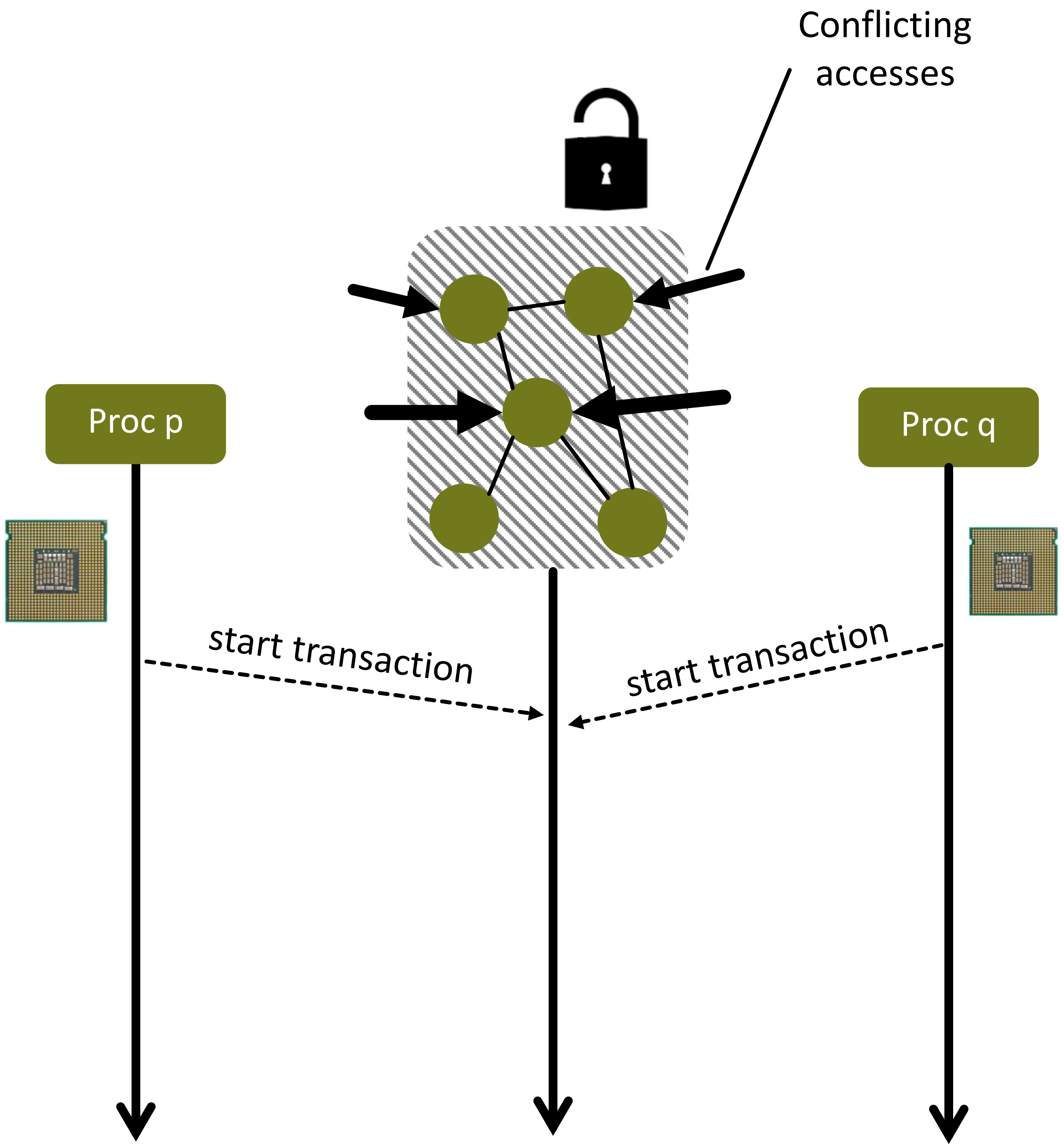
High performance? For graphs?

Software overheads

Simple protocols



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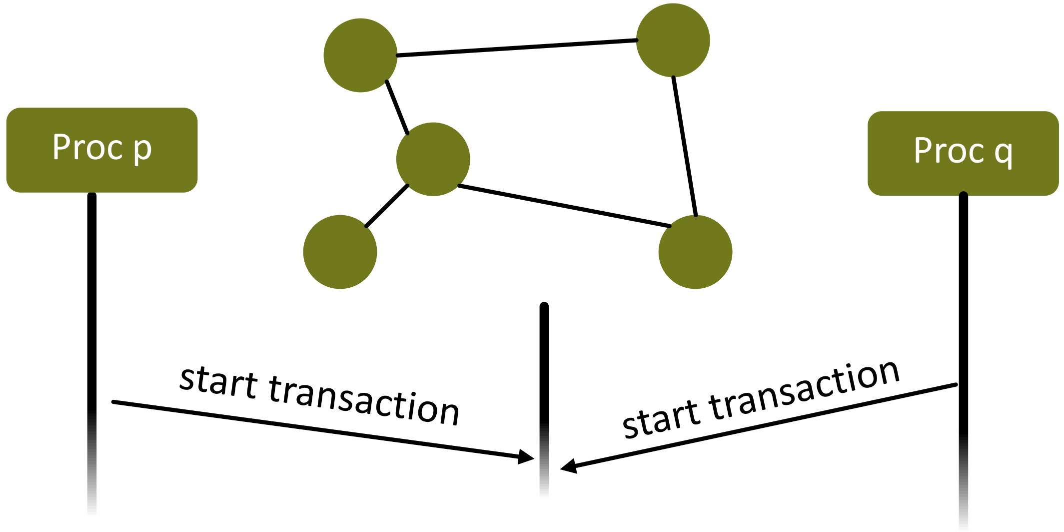
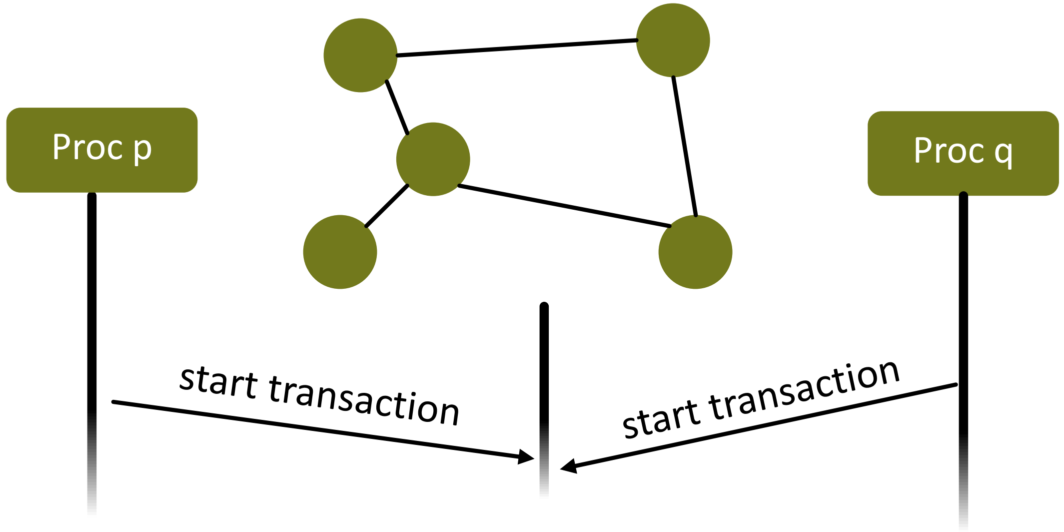
Shared- & Distributed-memory Machines



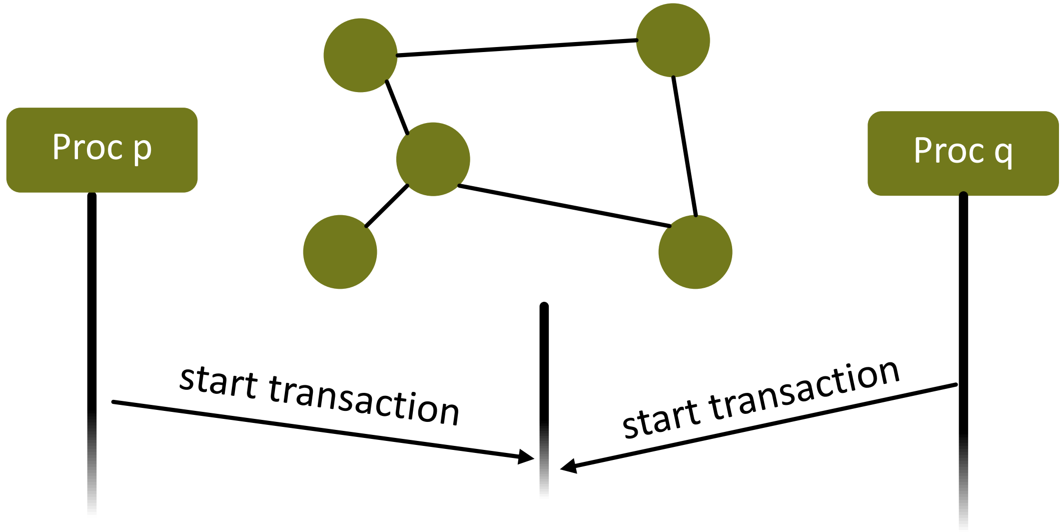
Shared- & Distributed-memory Machines

- HTM works fine for single shared-memory domains



SHARED- & DISTRIBUTED-MEMORY MACHINES

- HTM works fine for single shared-memory domains
  - Most graphs fit in such machines [1]



[1] Y. Perez et al. Ringo: Interactive Graph Analytics on Big-Memory Machines. SIGCOMM'14.

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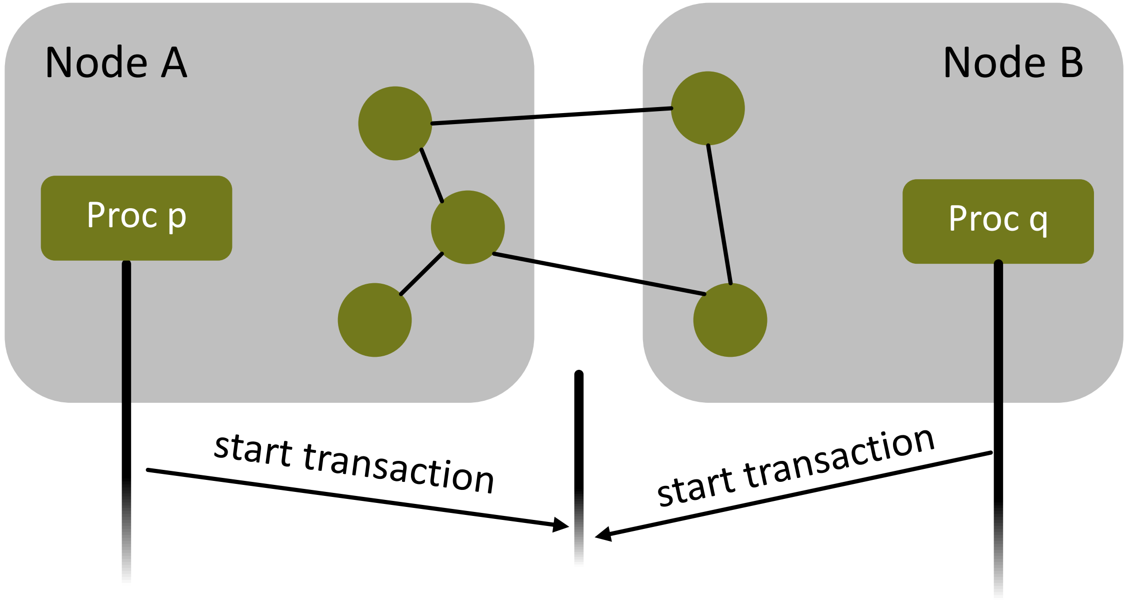

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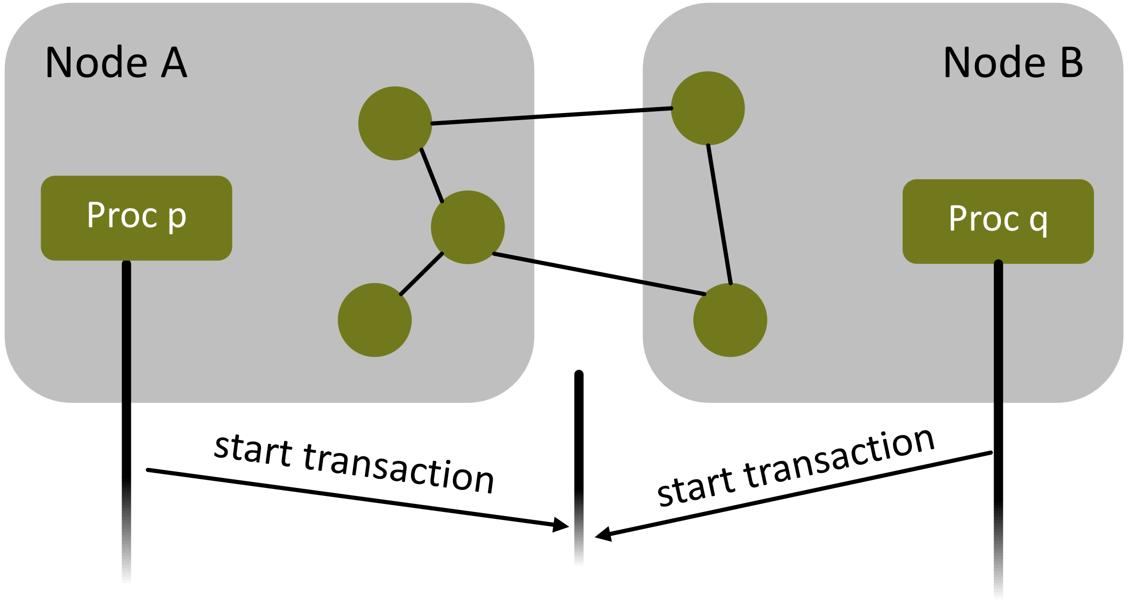

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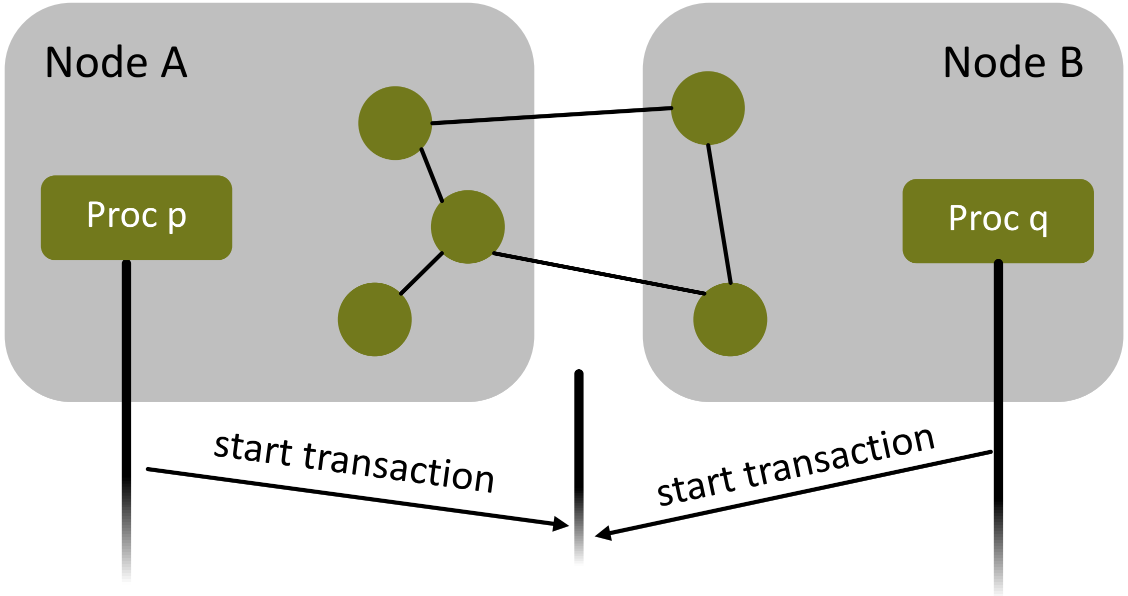

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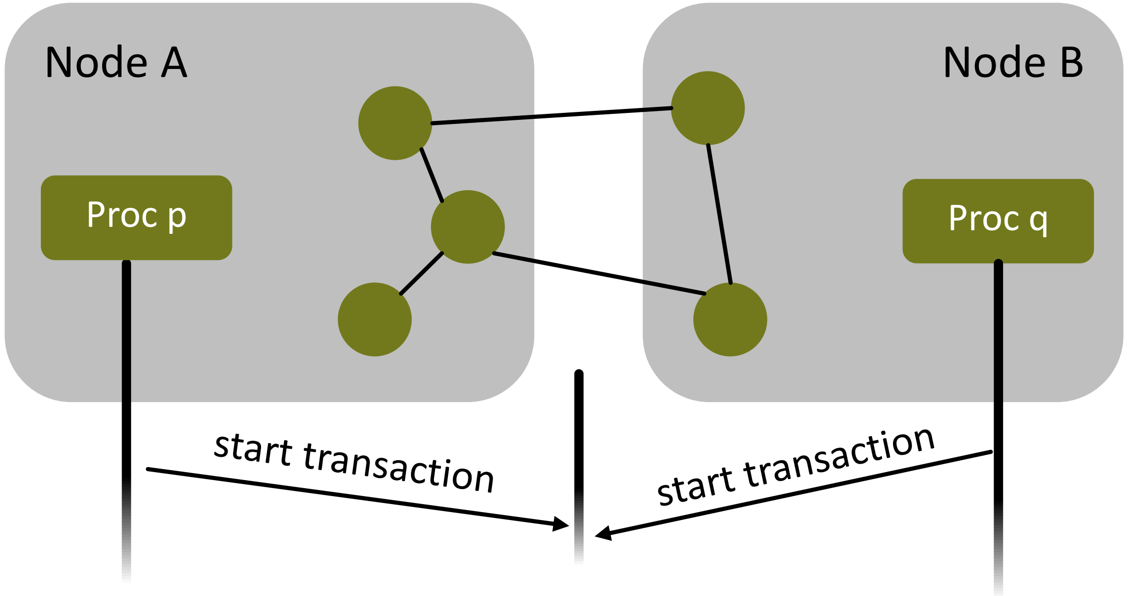

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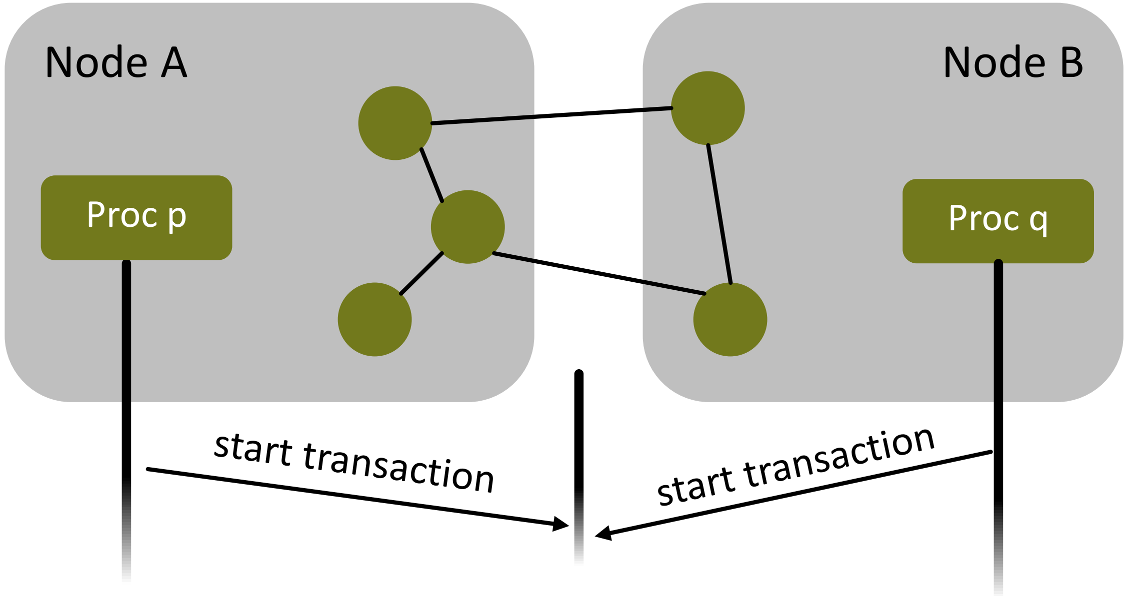

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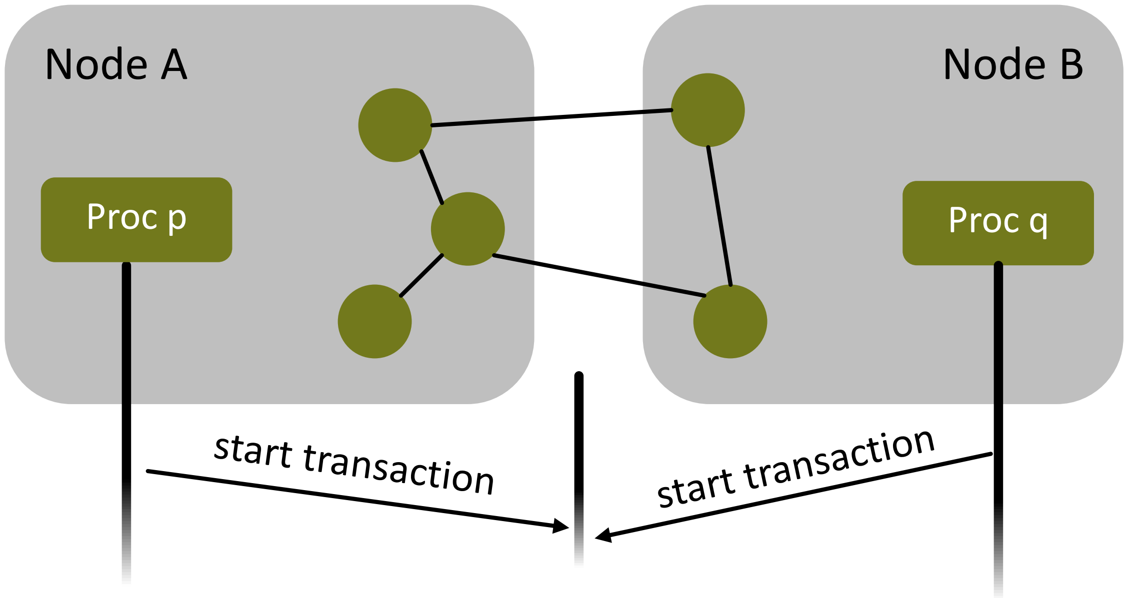
How to apply HTM in such a setting?



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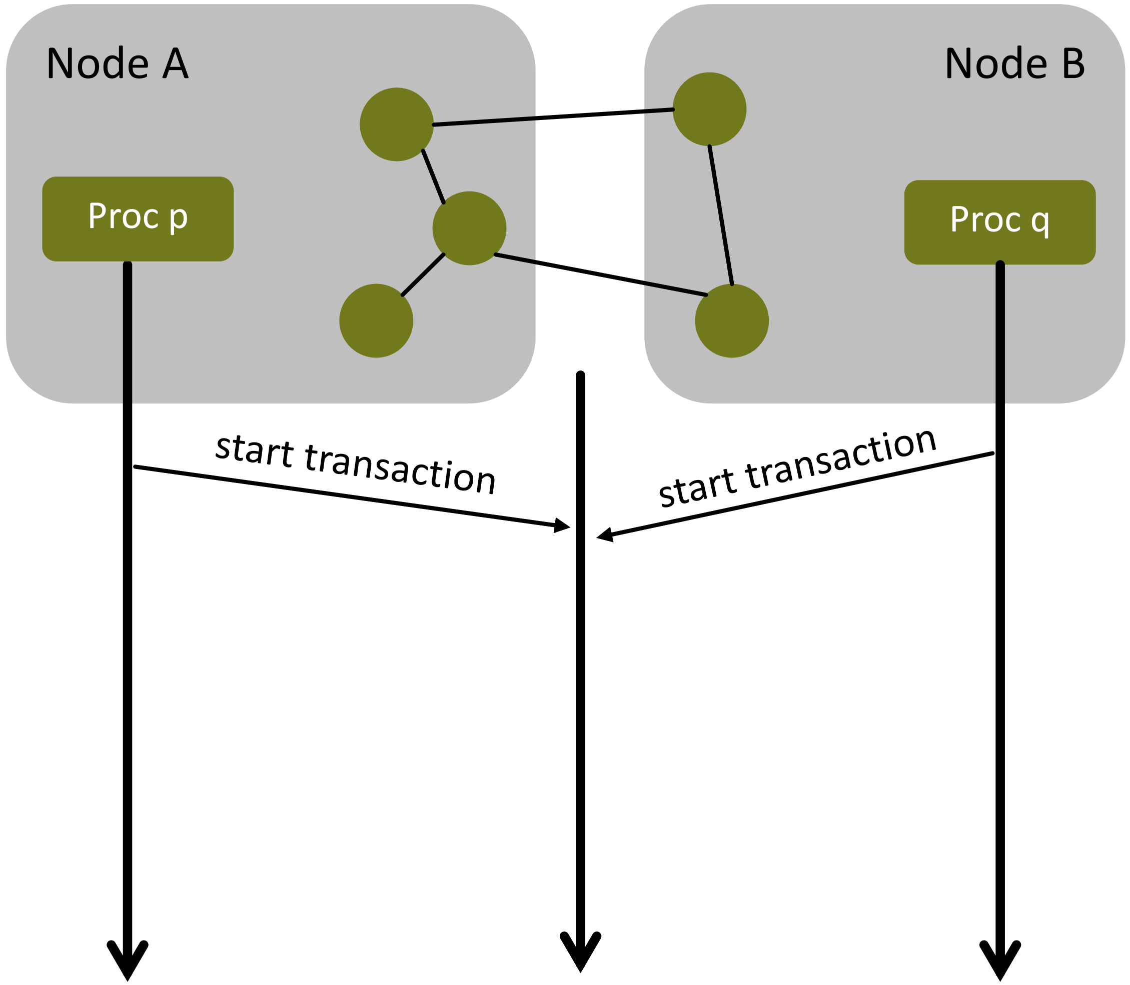
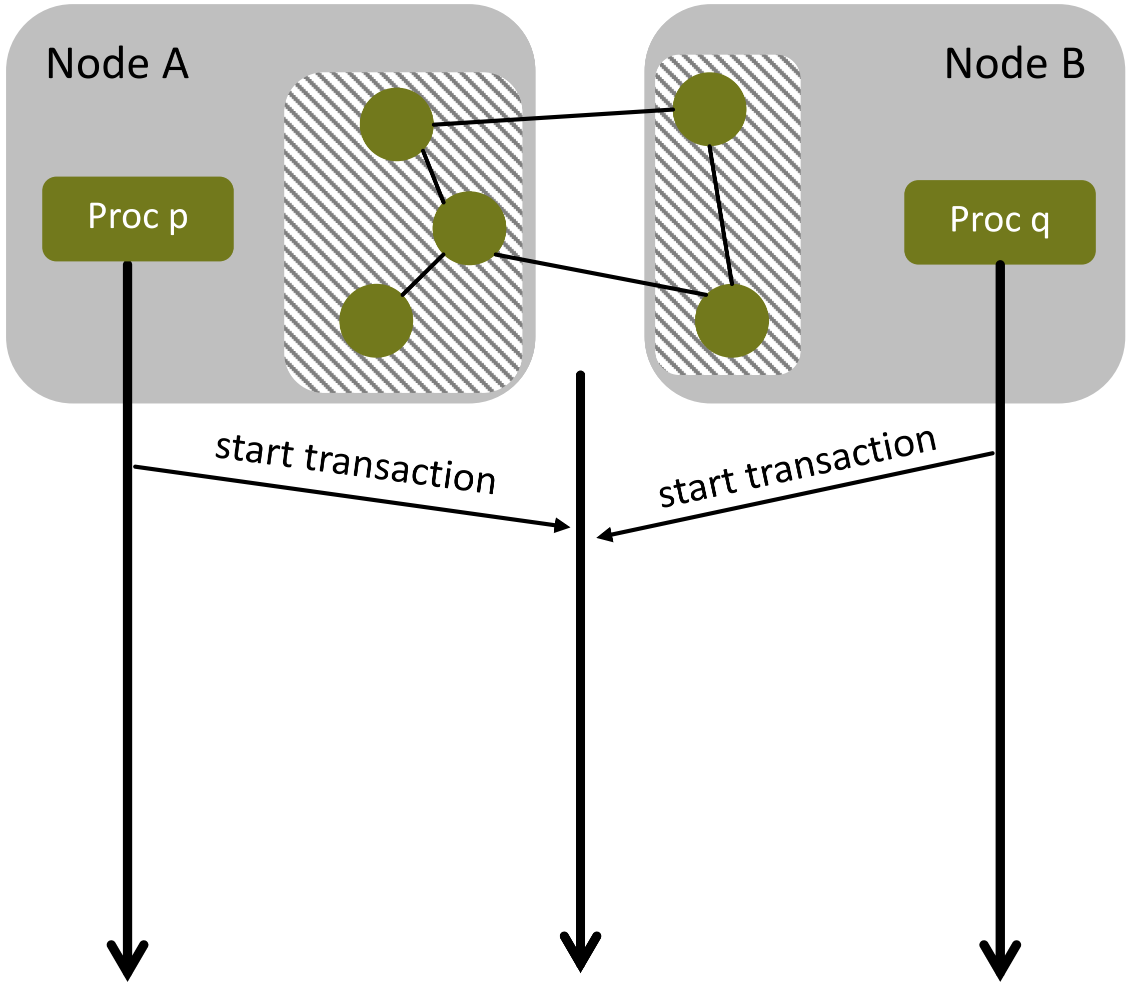
AM + HTM = ...


Node A
Node B
Proc p
Proc q
start transaction
start transaction

AM + HTM = ...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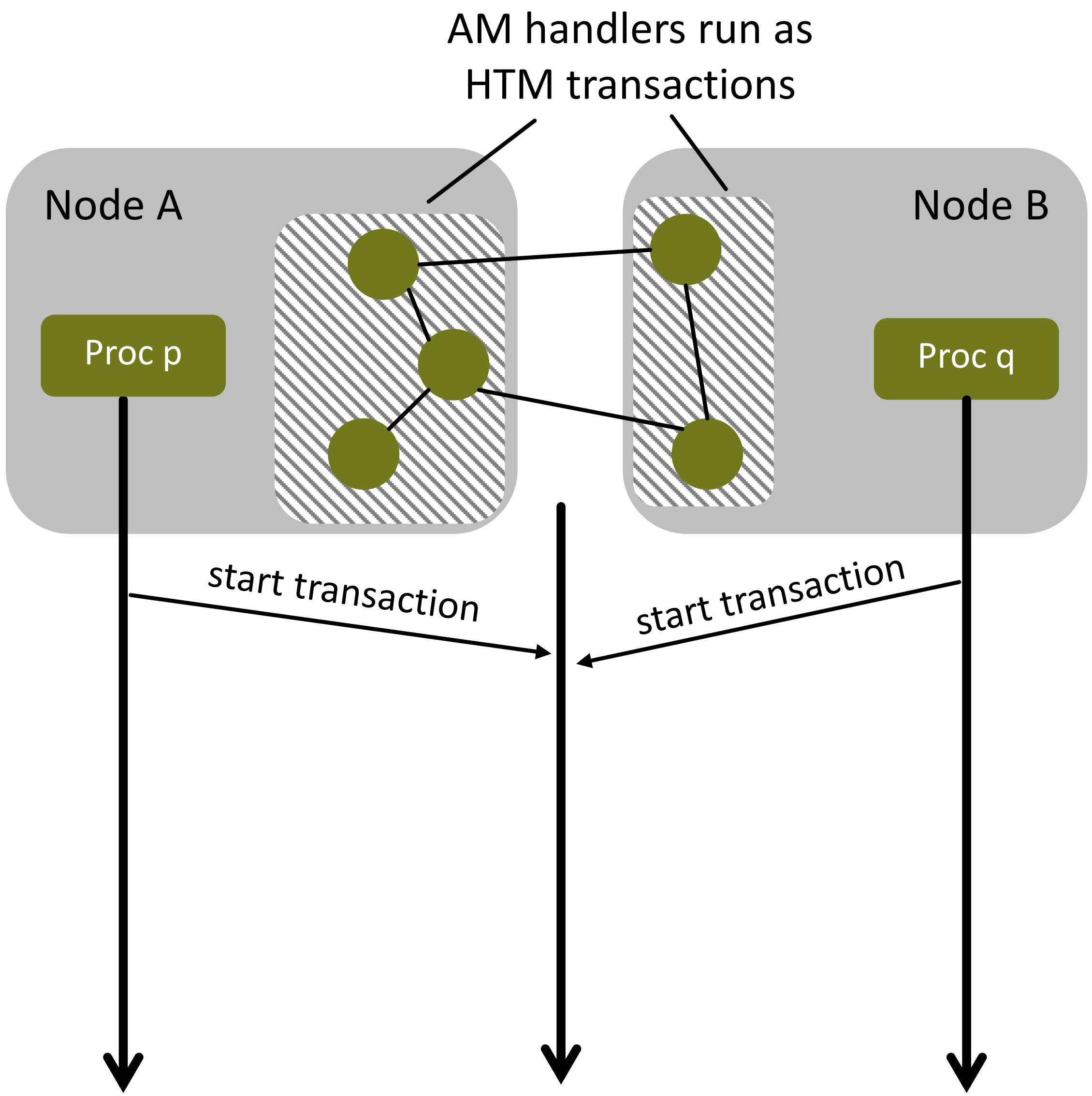
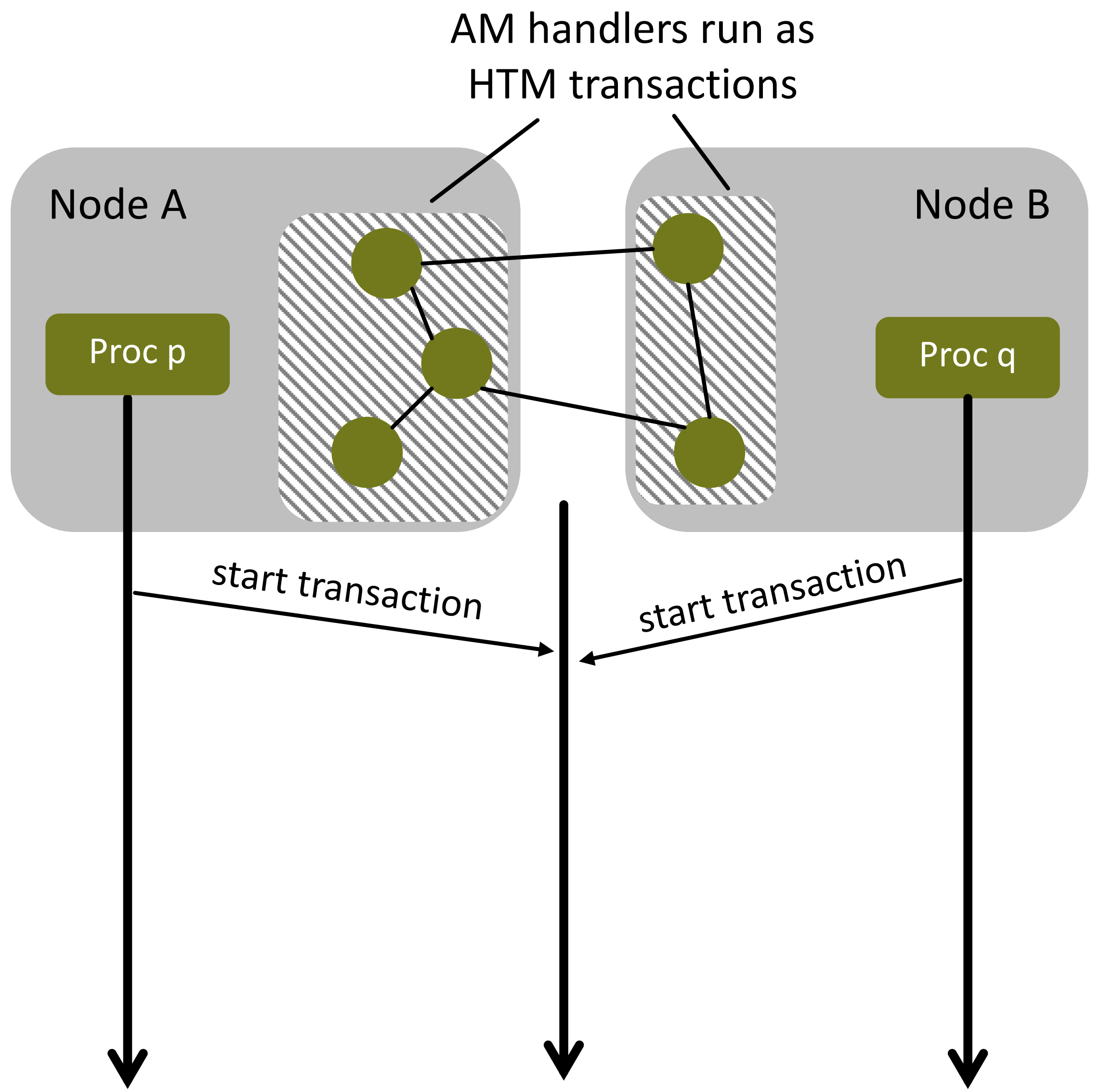
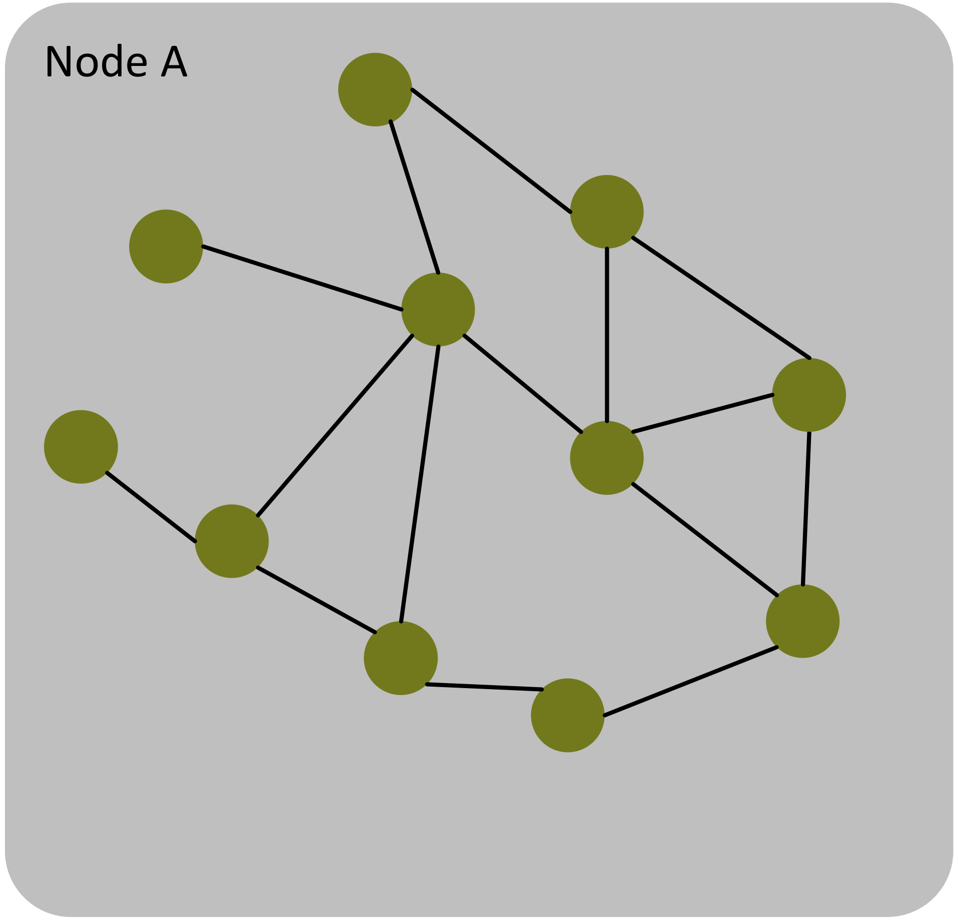

AM handlers run as
HTM transactions
Node A
Node B
Proc p
Proc q
start transaction
start transaction

AM + HTM = Atomic Active Messages



Accessing Multiple Vertices Atomically

Example: BFS



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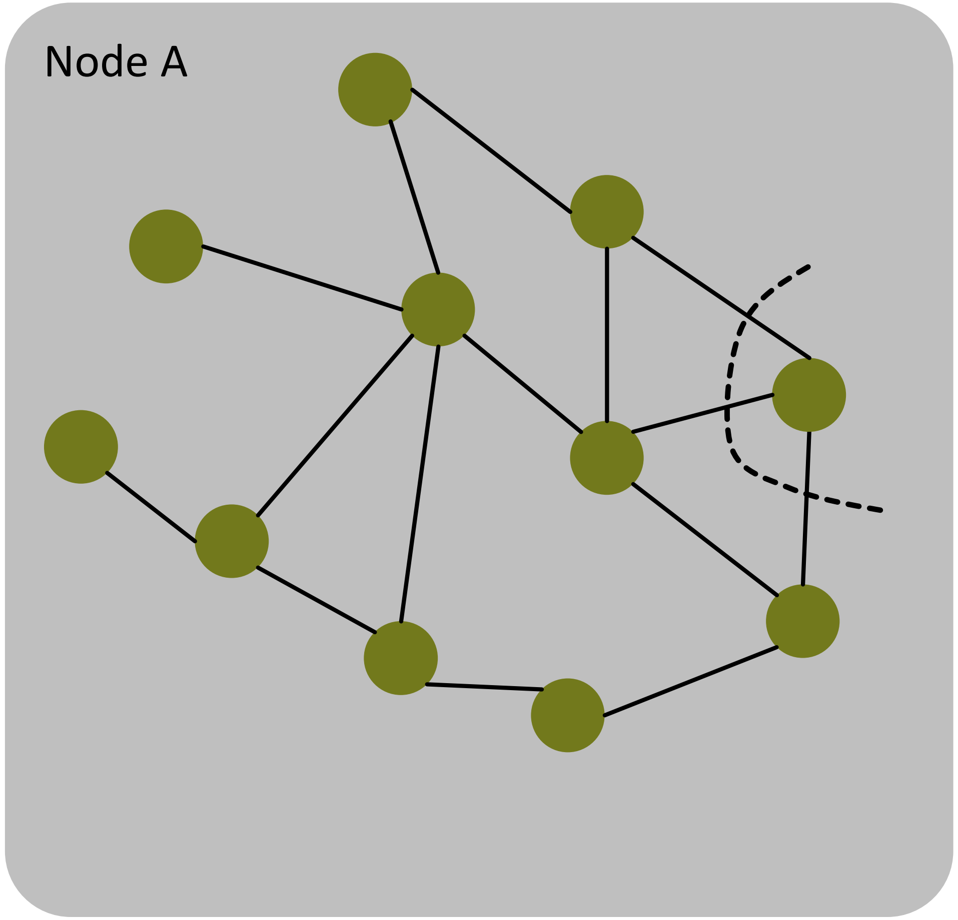

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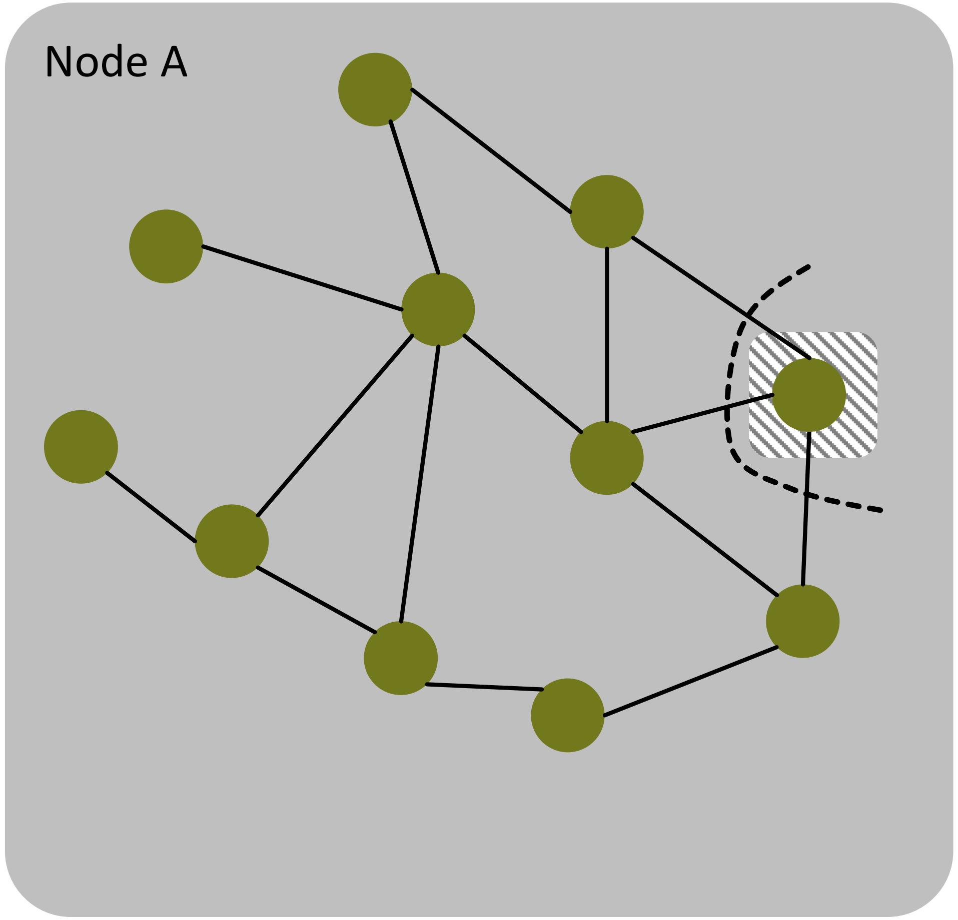

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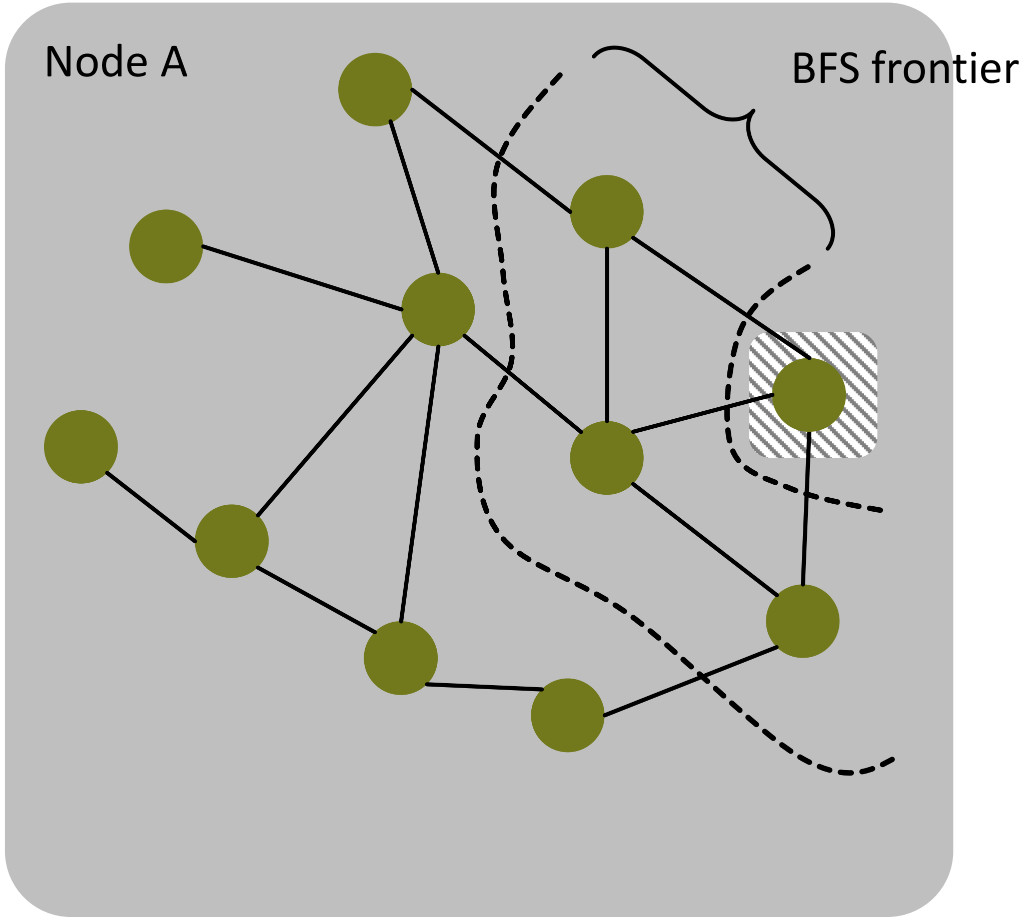

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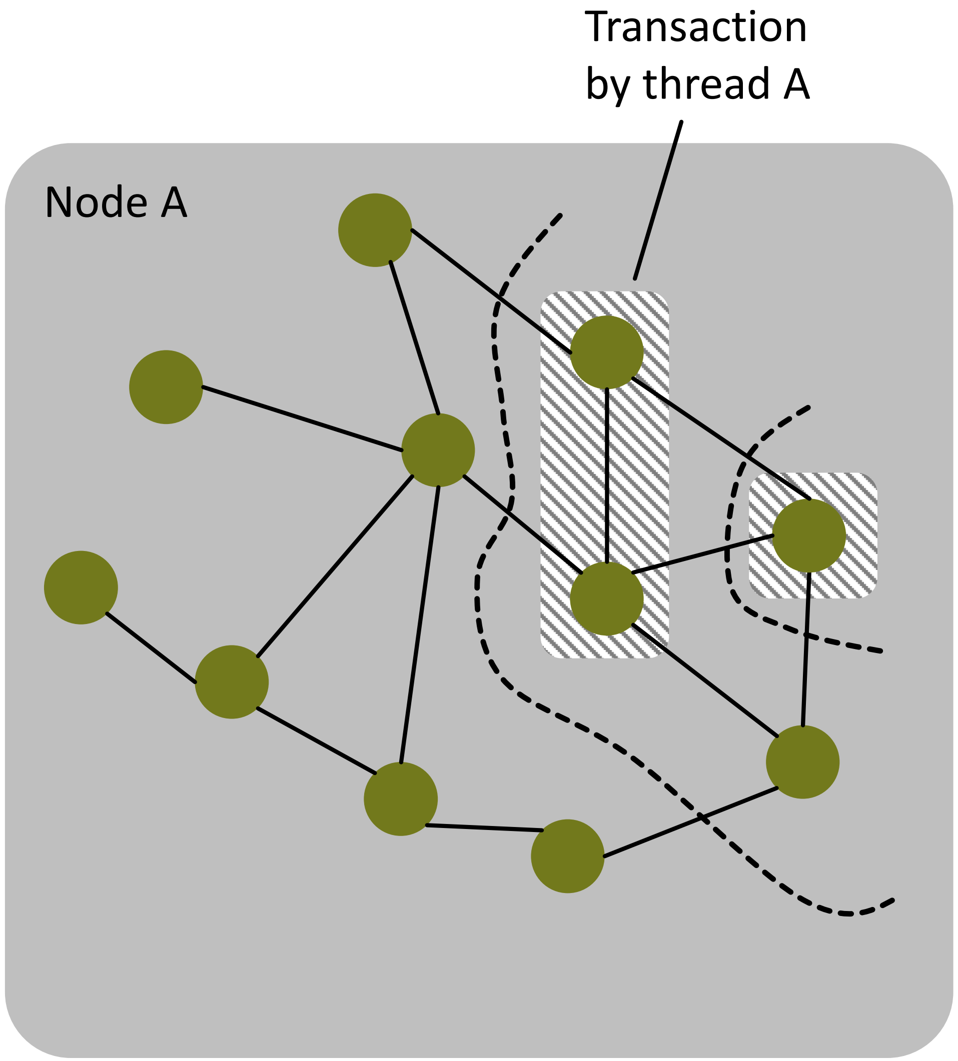

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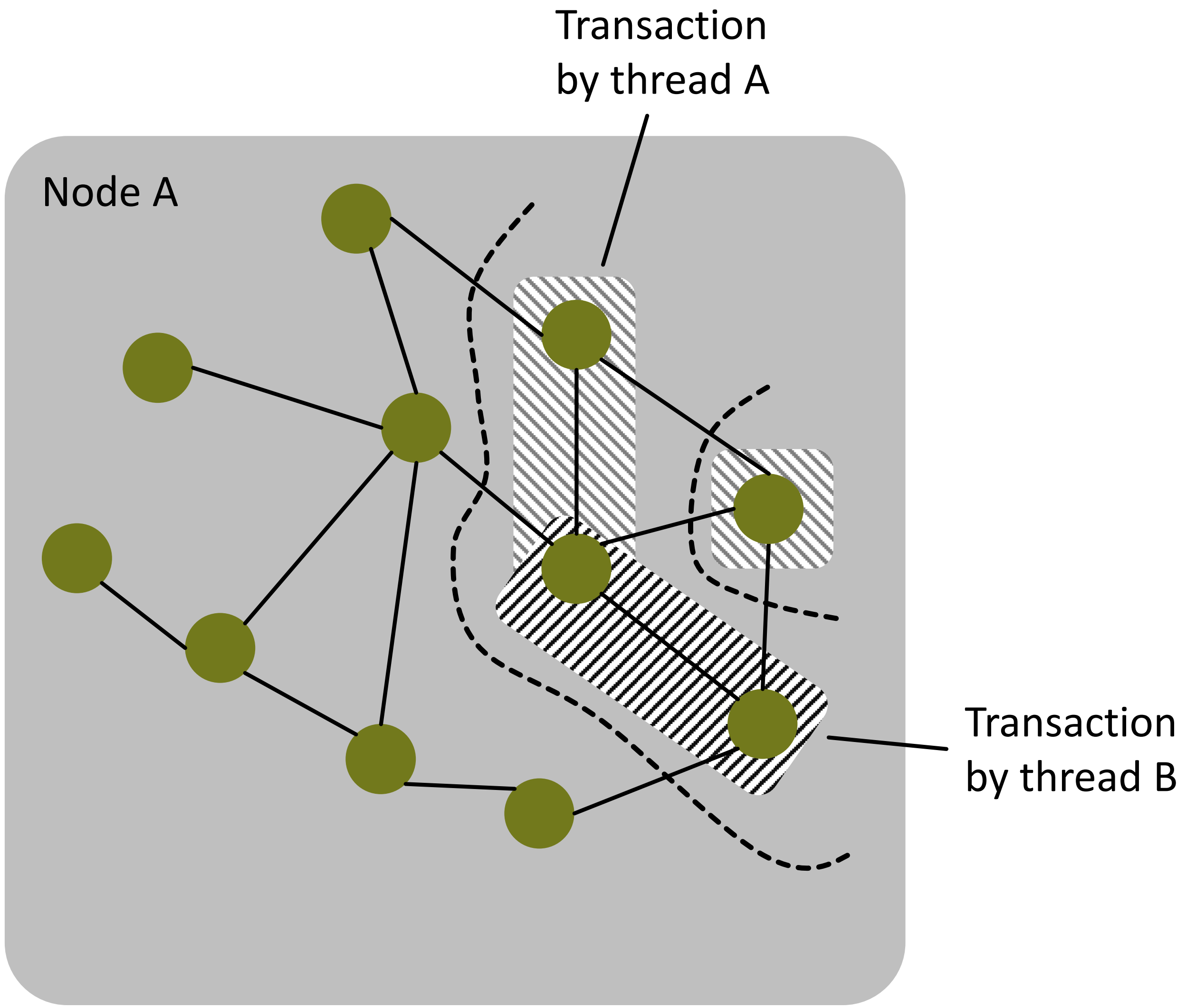

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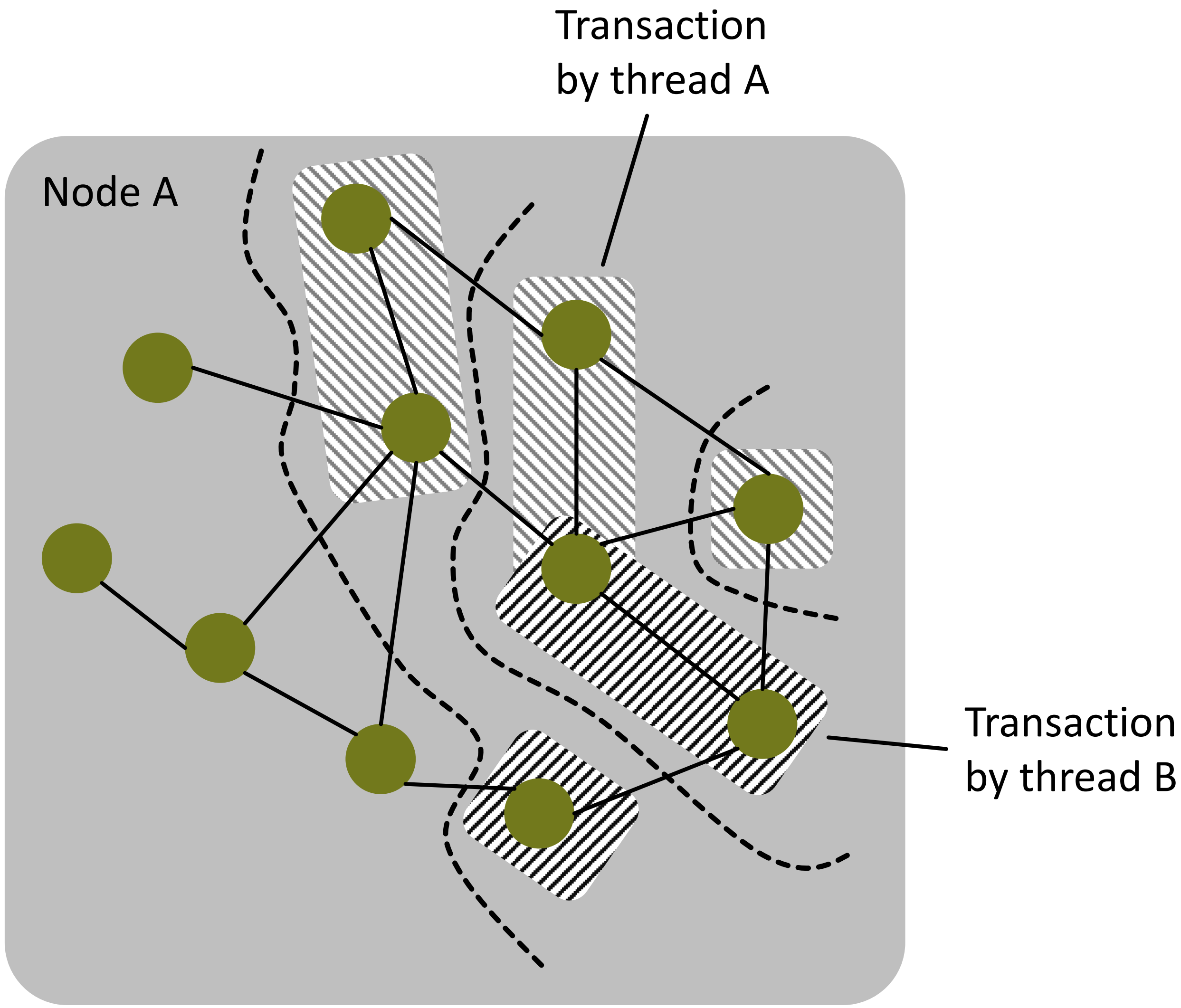

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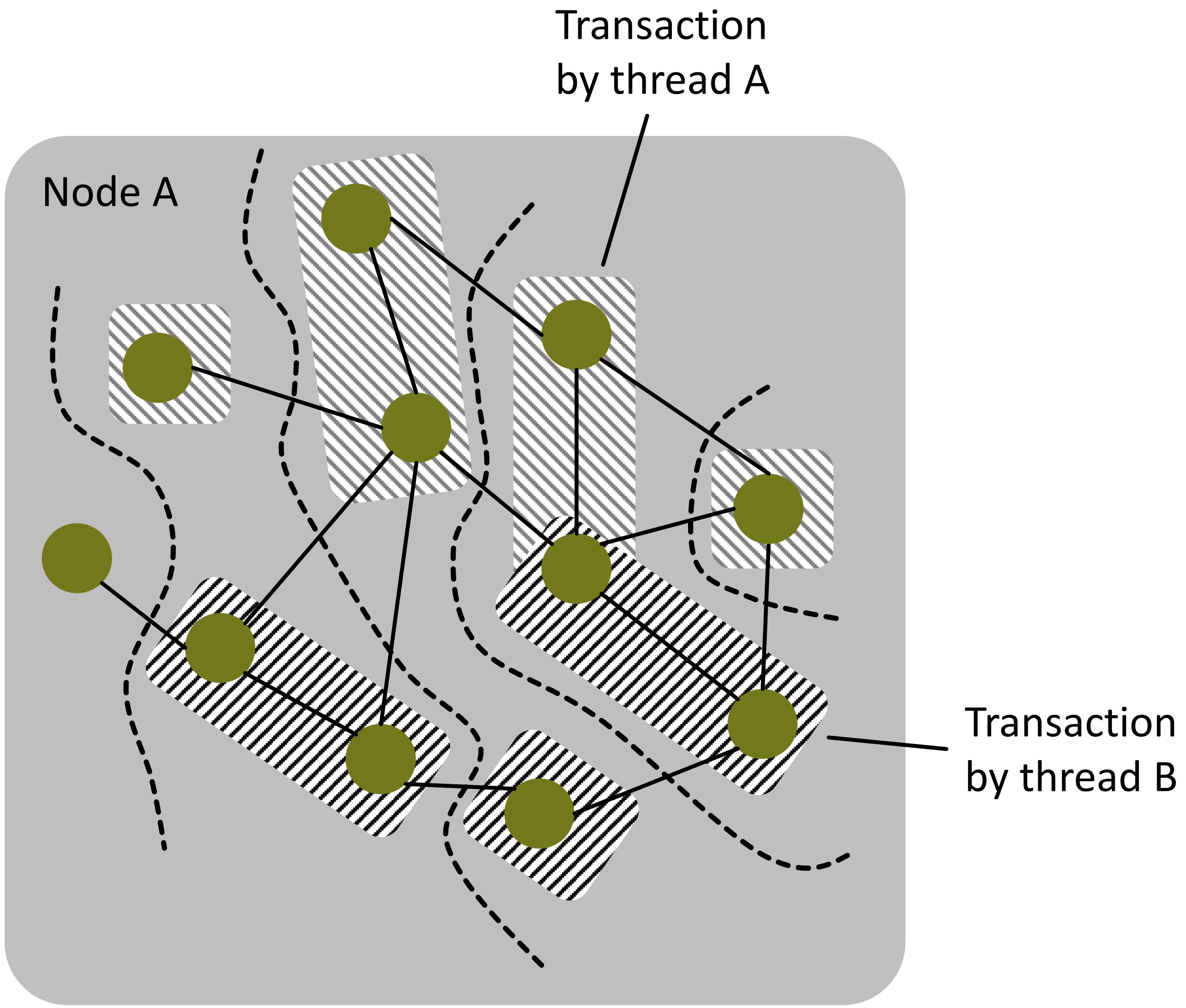

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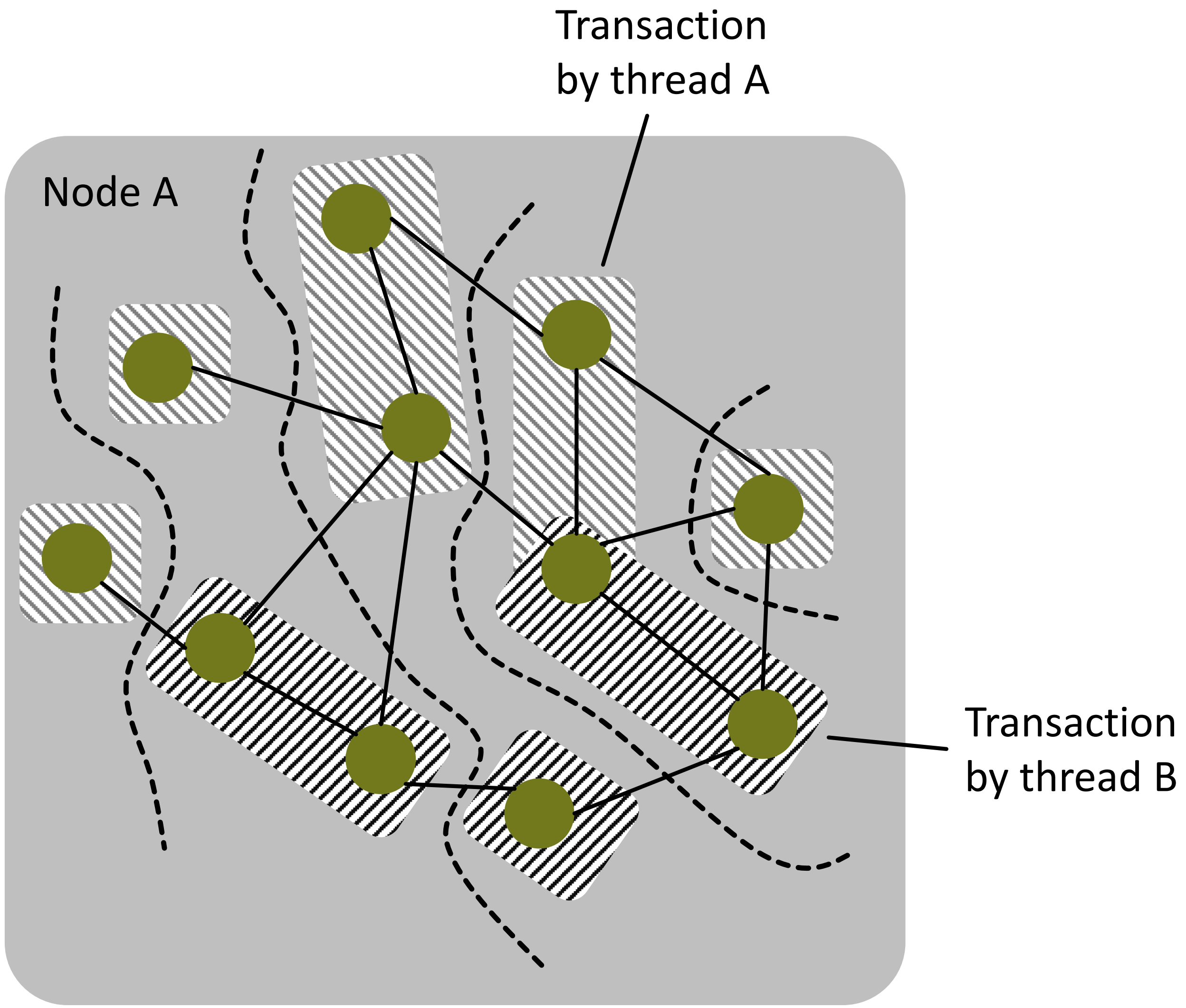

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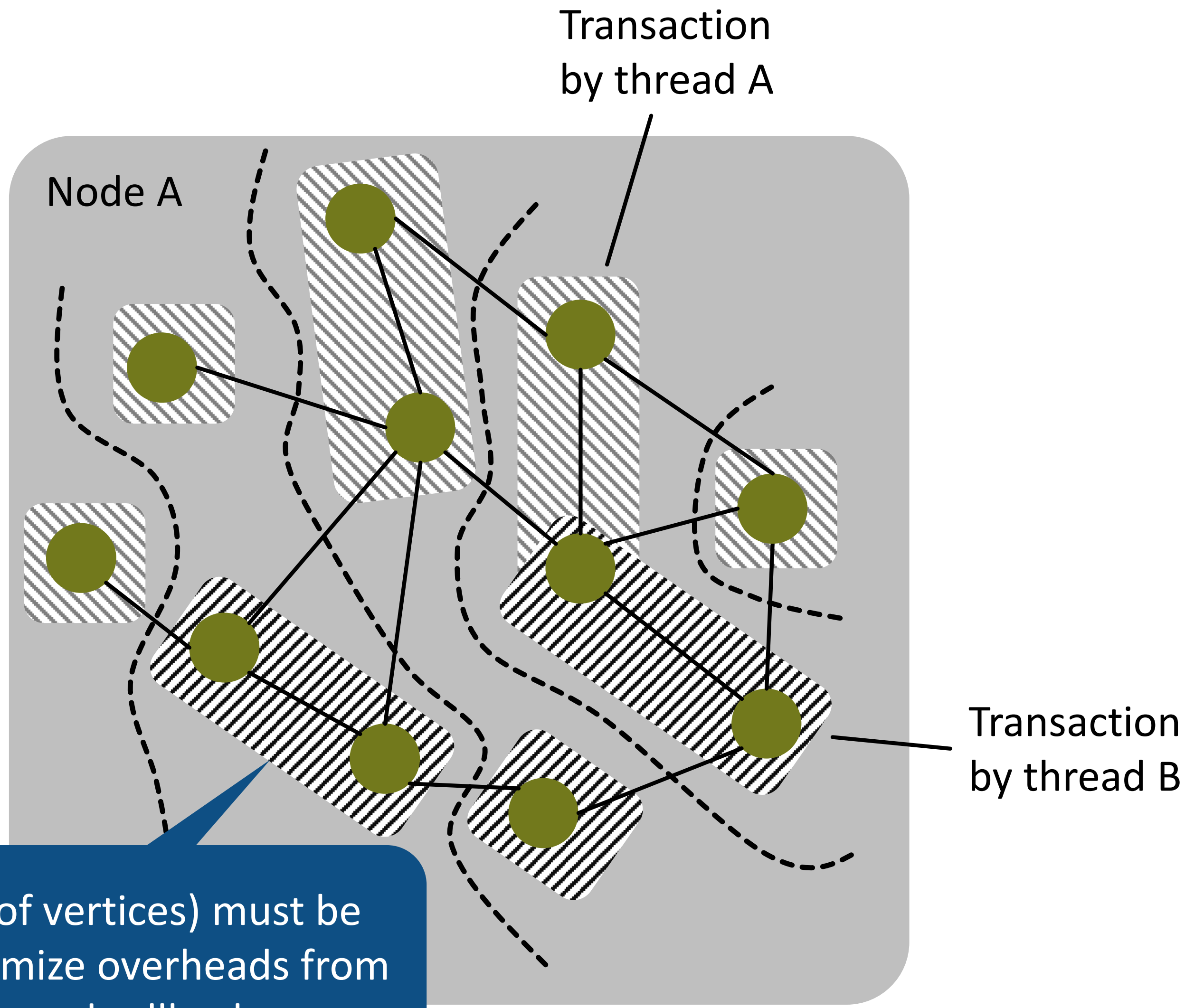
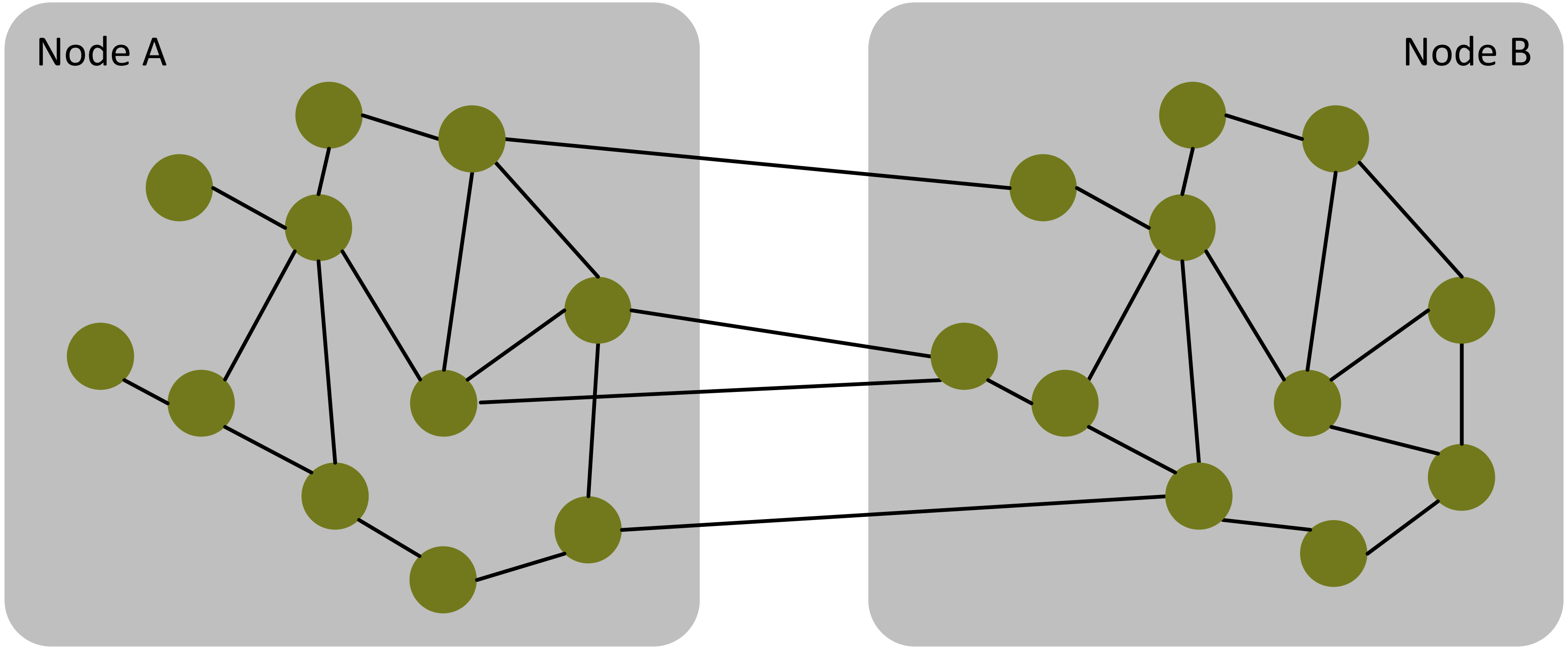


Size (the number of vertices) must be appropriate to minimize overheads from both commits and rollbacks

Executing Transactions on Multiple Nodes



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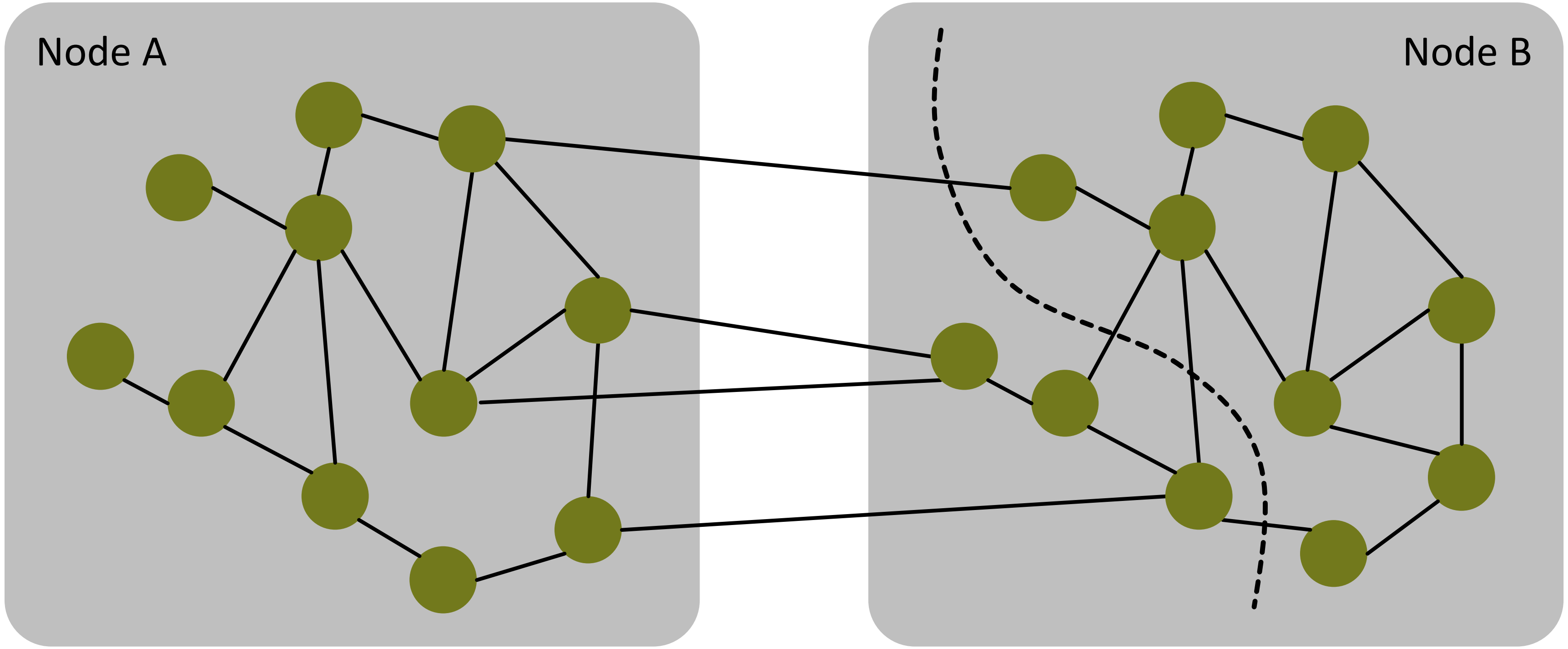

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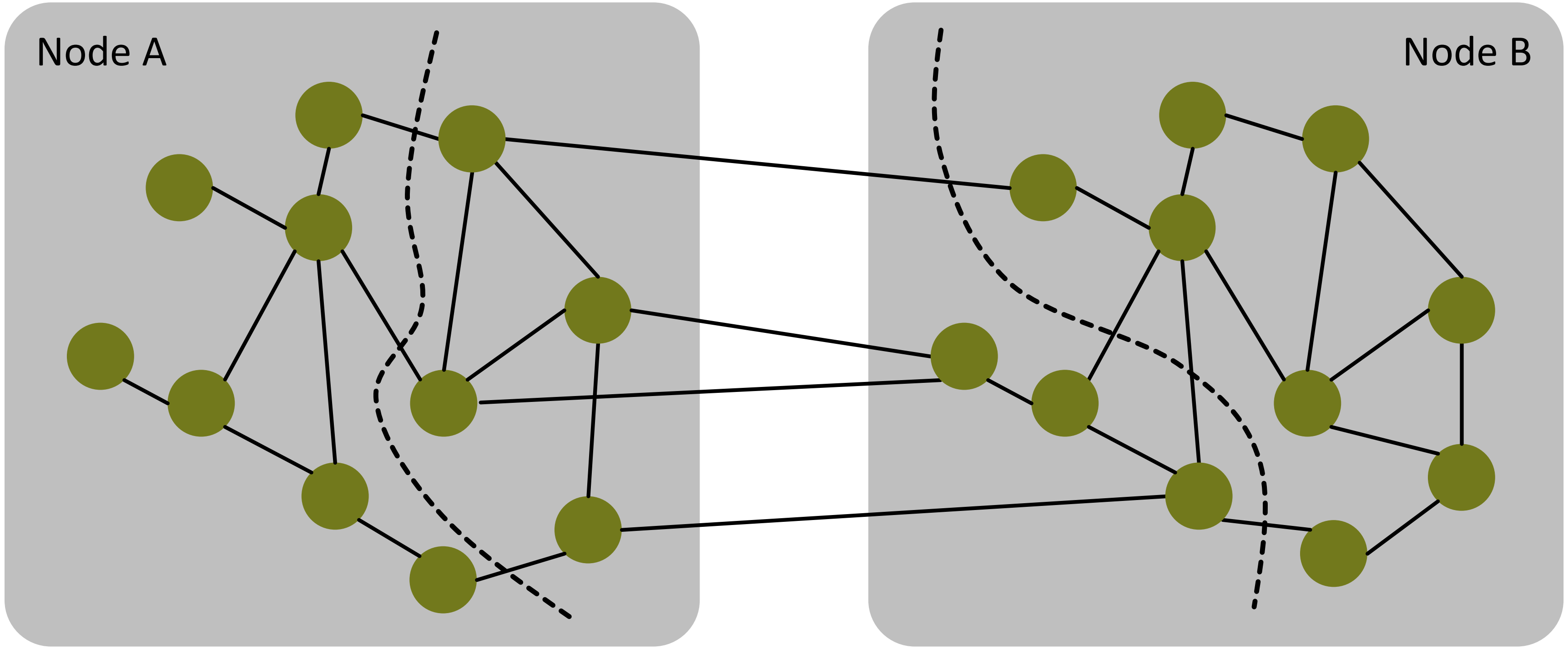

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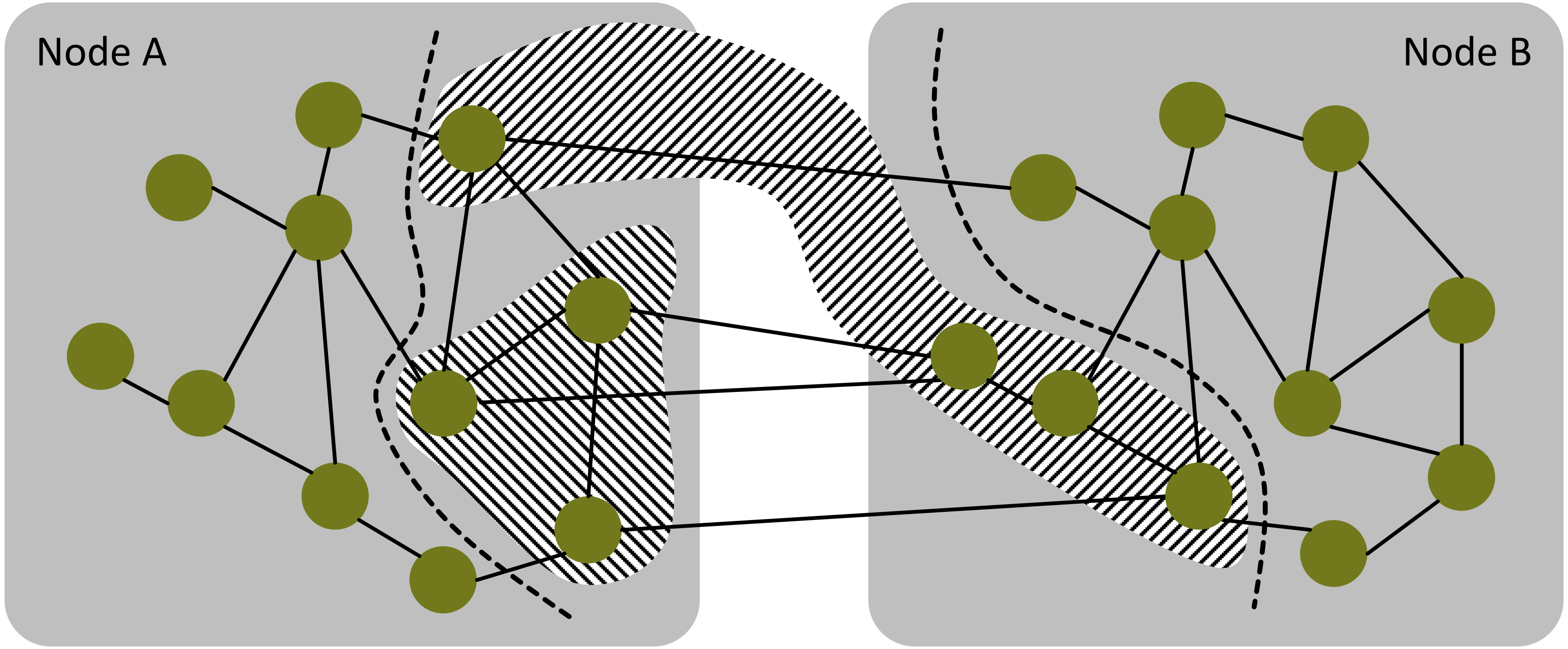

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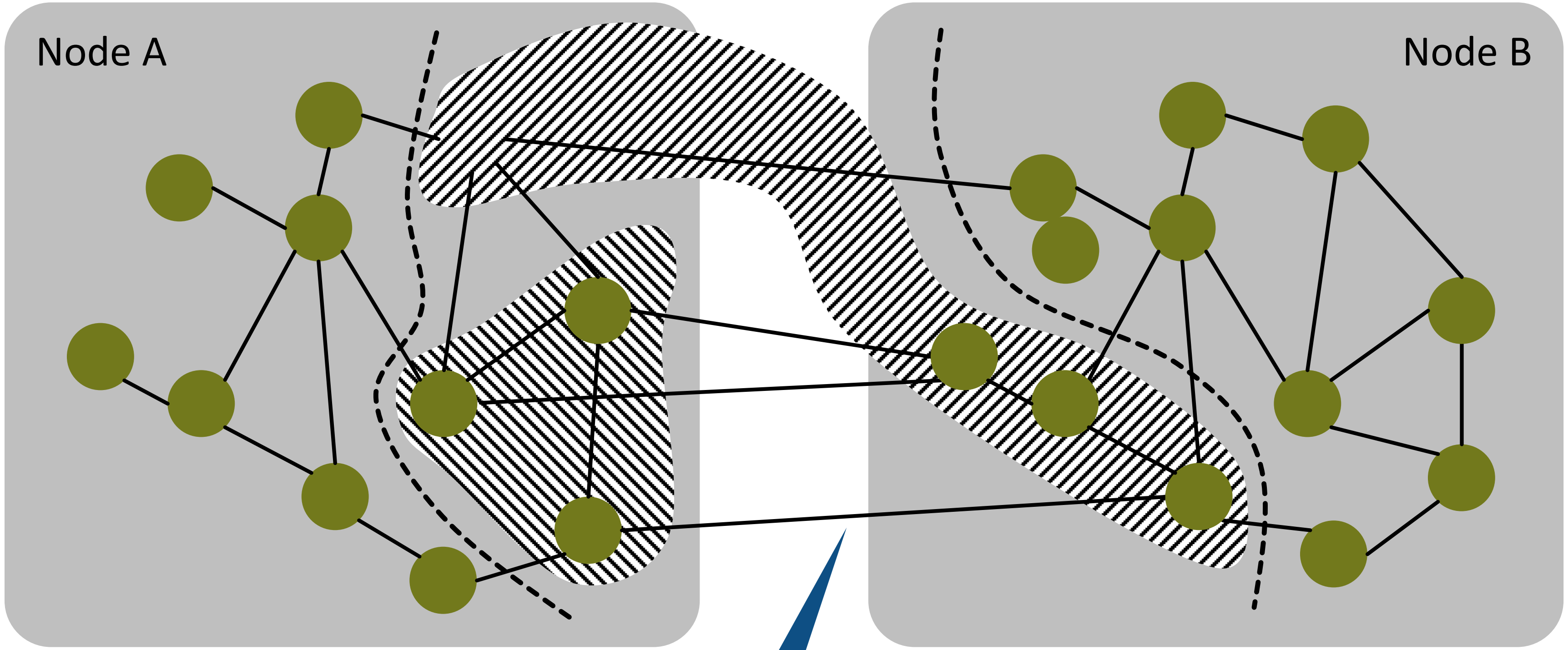

Vertices must be appropriately relocated to enable execution of a hardware transaction

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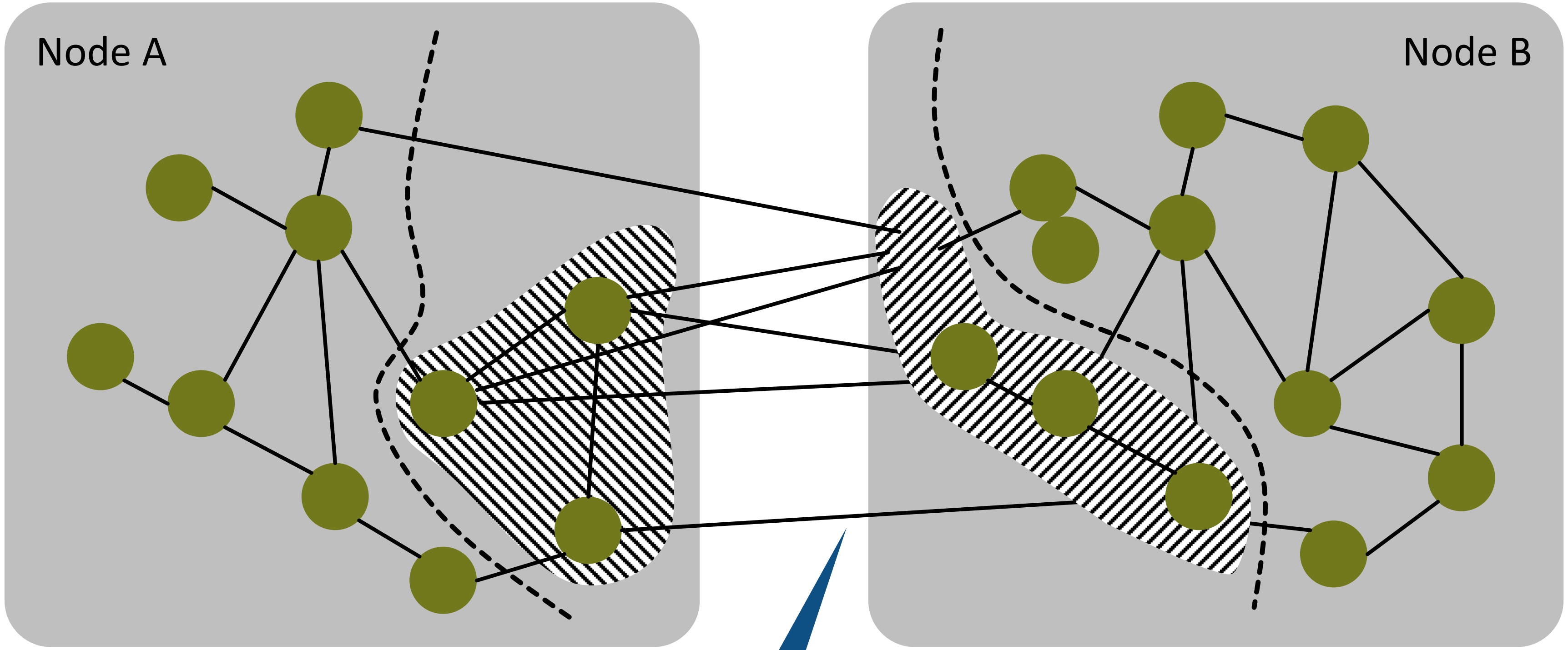

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Performance Model

Atomics vs transactions

Performance Model

Atomics vs transactions

Time to modify *N* vertices with atomics:

$$T_{AT}(N) = A_{AT}N + B_{AT}$$

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ATOMICS VS TRANSACTIONS

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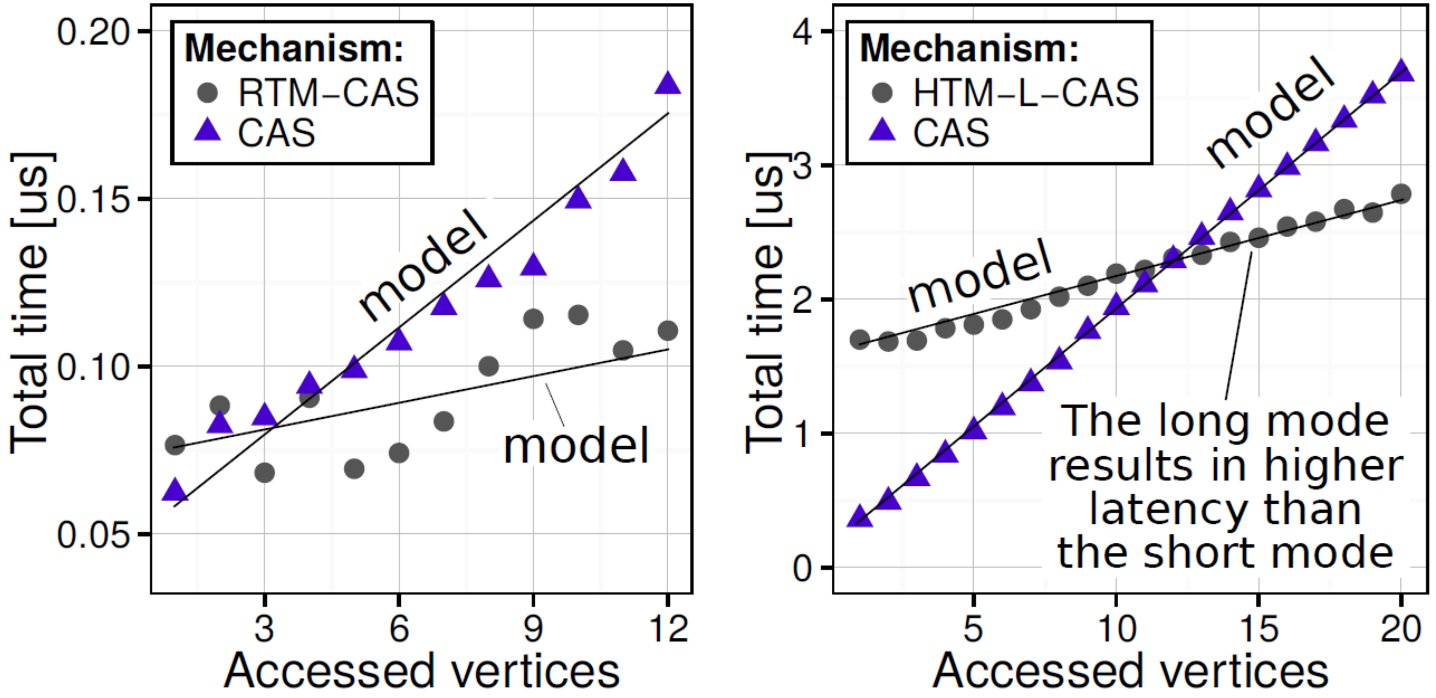

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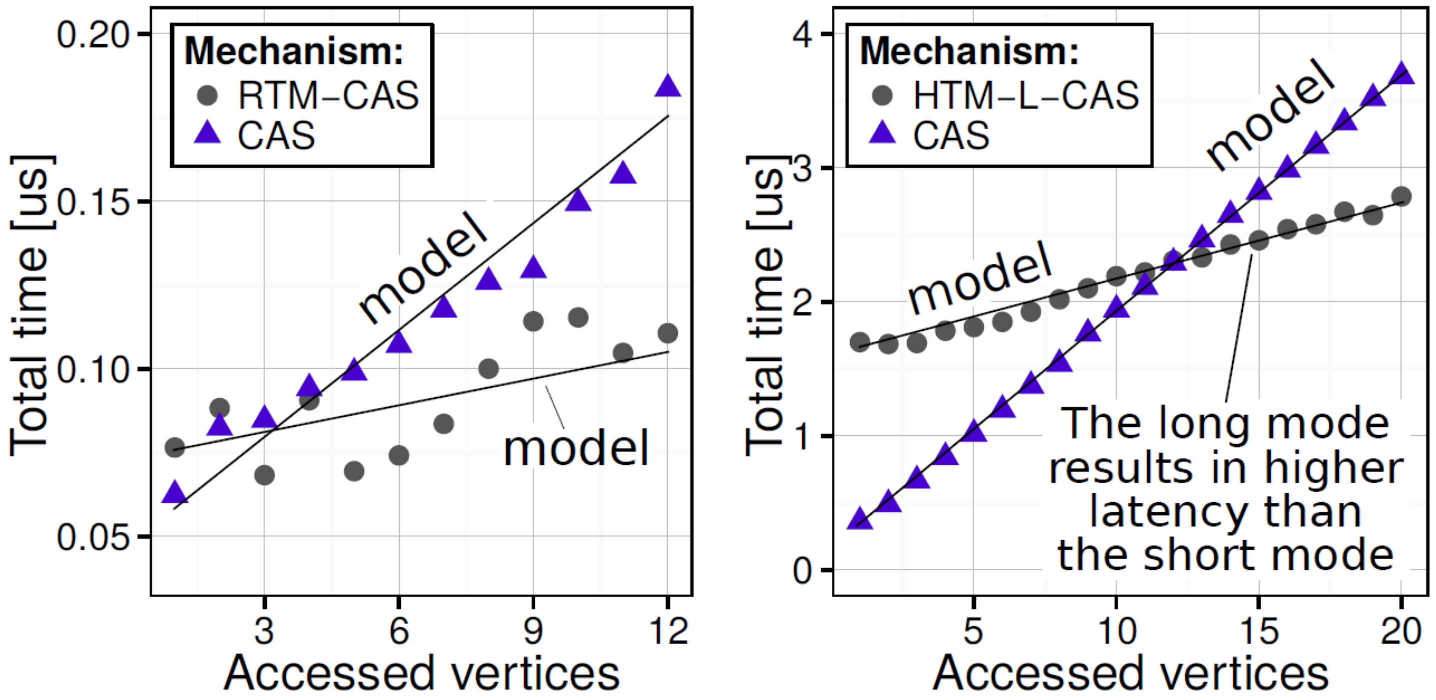

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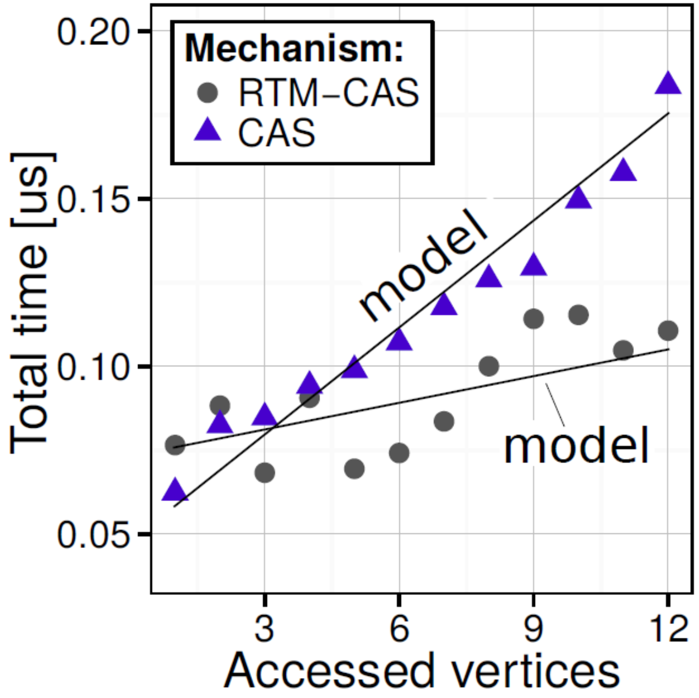

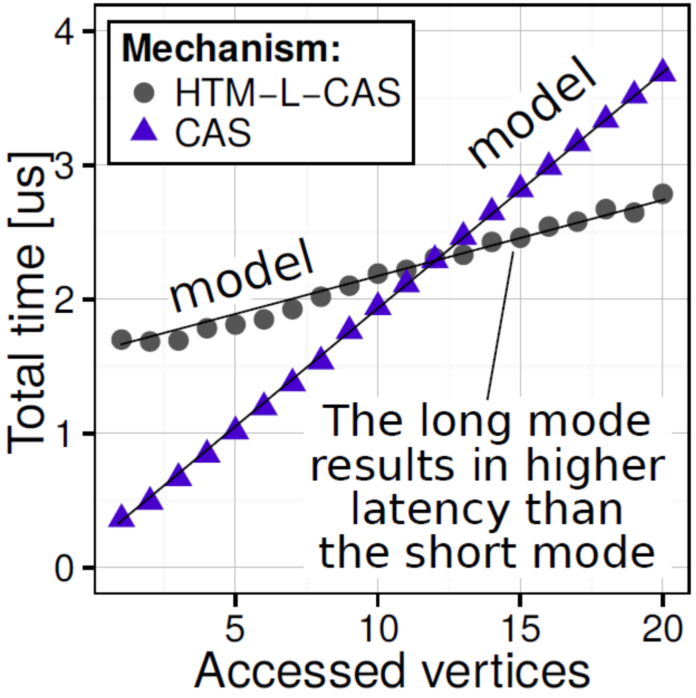

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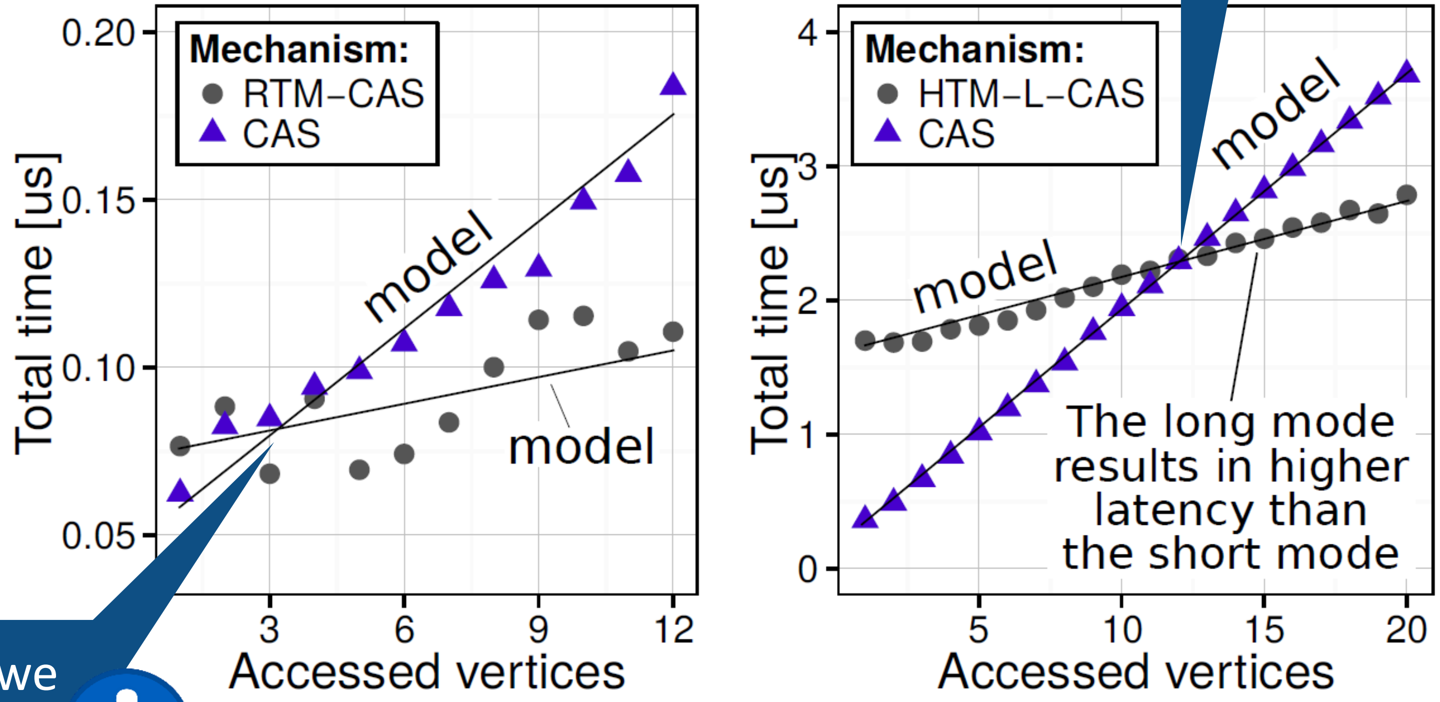

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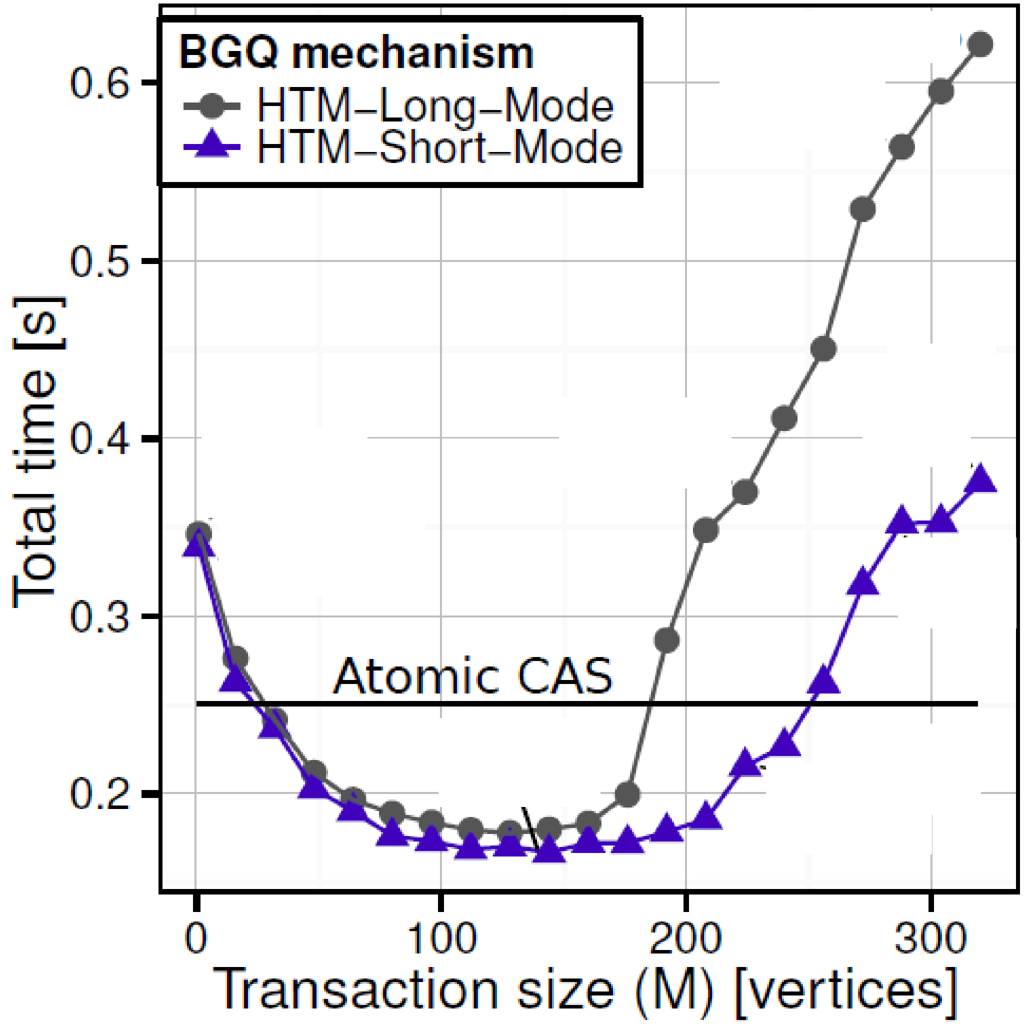
MULTI-VERTEX TRANSACTIONS

MARKING VERTICES AS VISITED

Multi-Vertex Transactions

Marking vertices as visited



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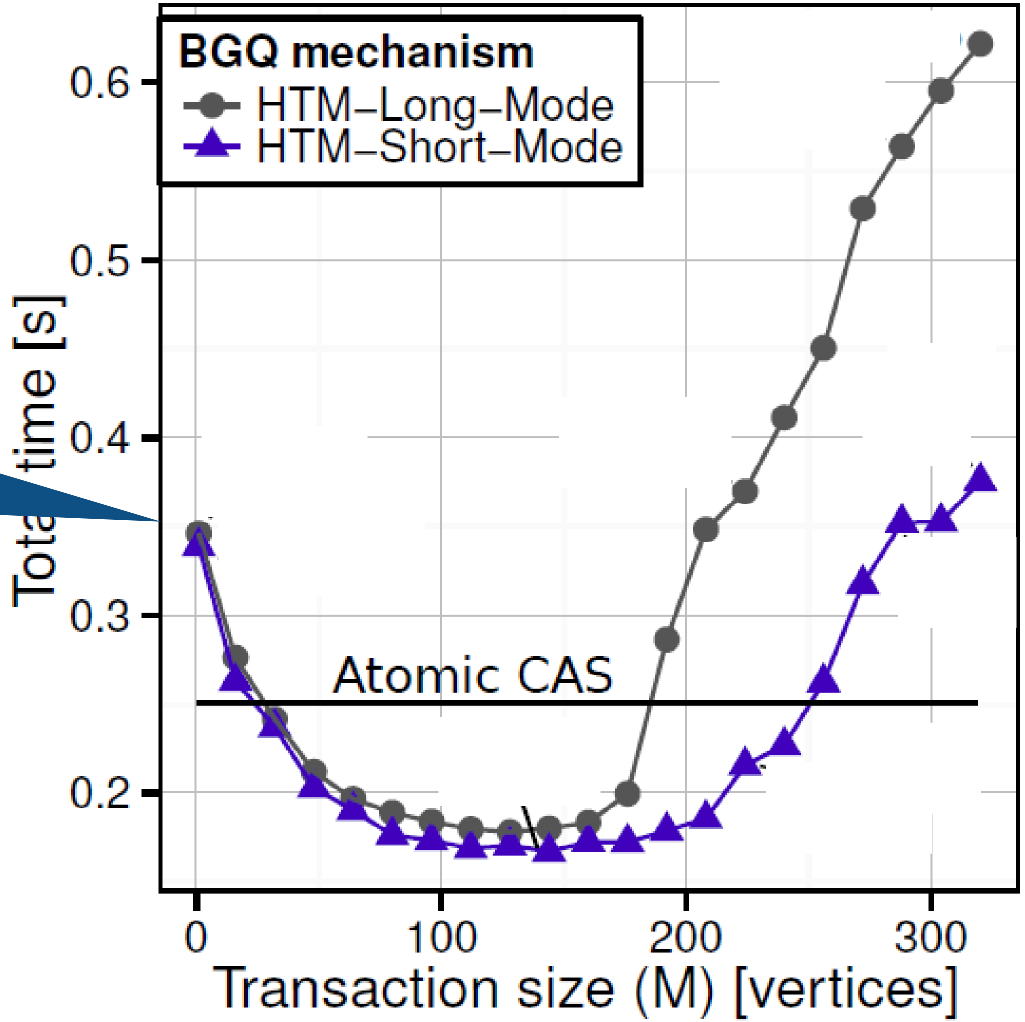
MARKING VERTICES AS VISITED

Startup and commit overheads



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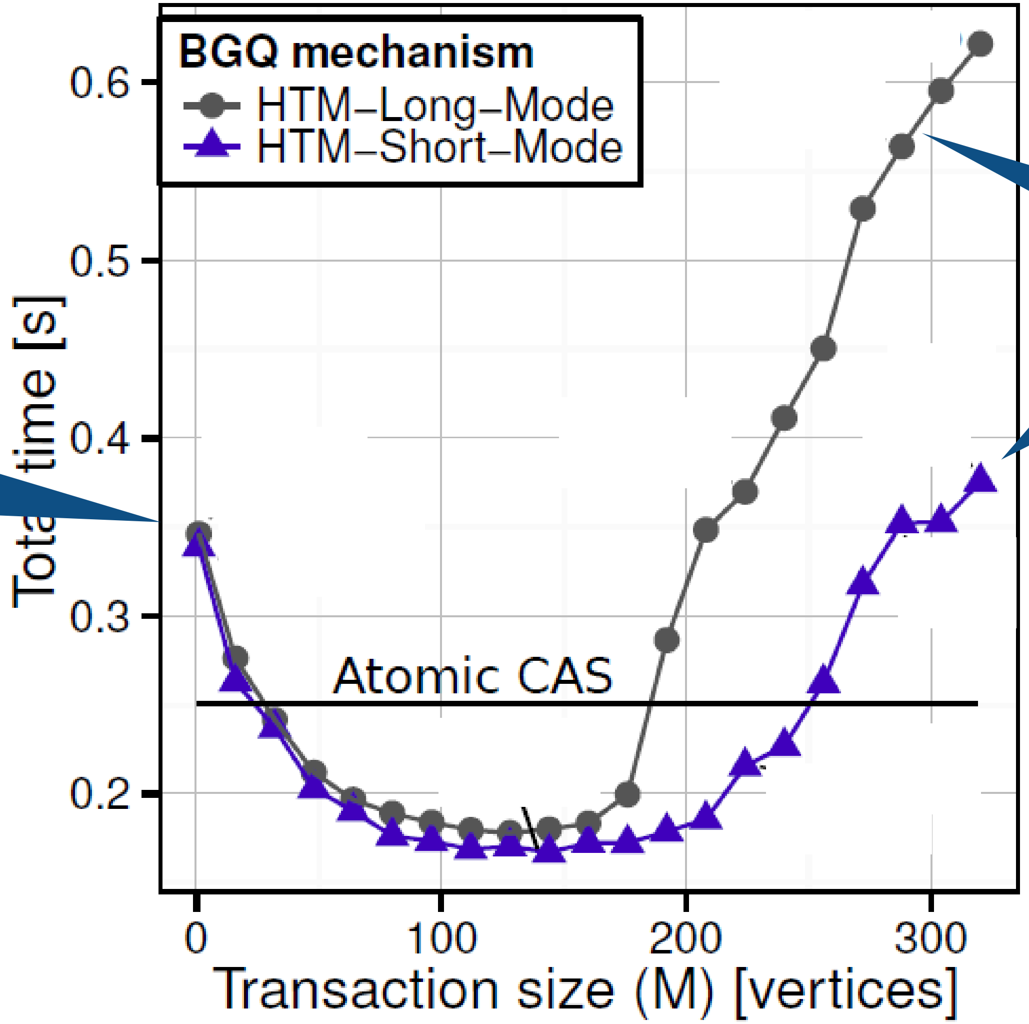

Startup and commit overheads

Abort and rollback overheads

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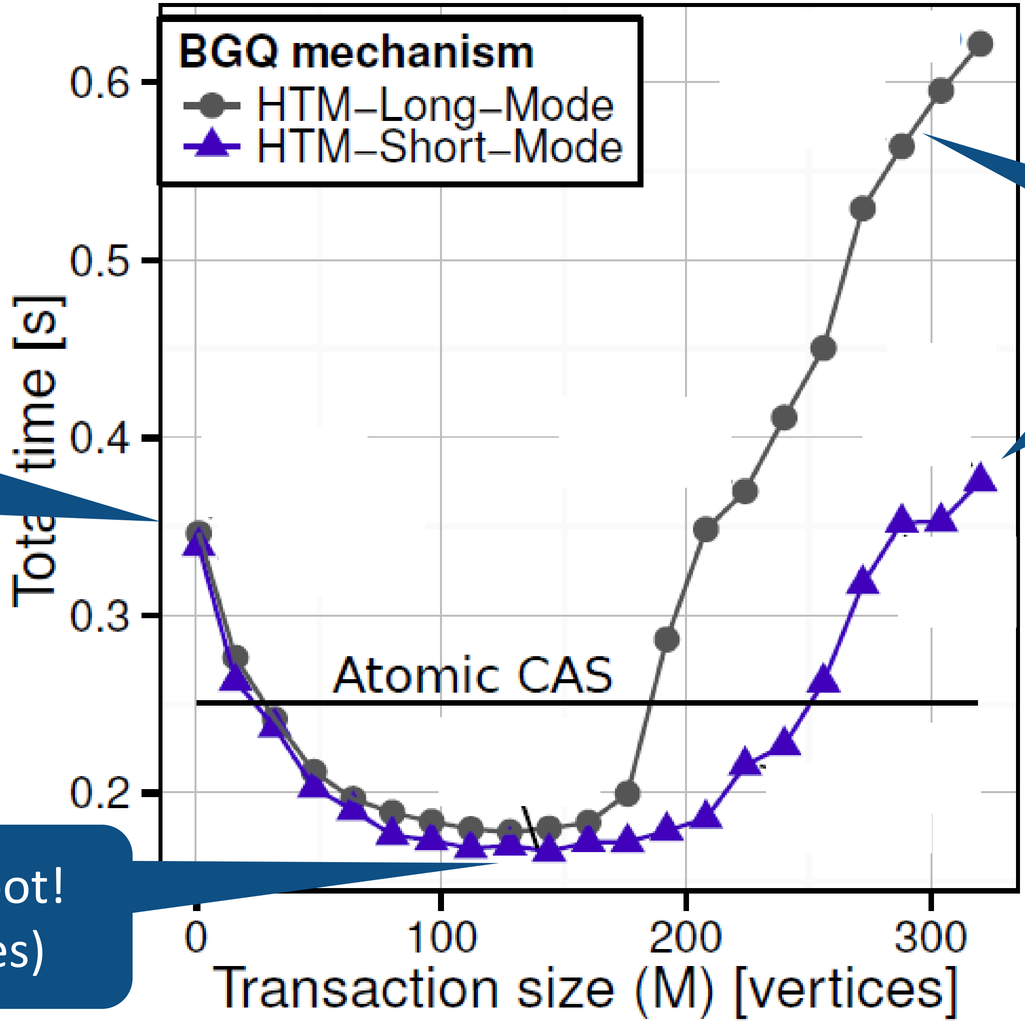

Startup and commit overheads

Abort and rollback overheads

The sweetspot! (144 vertices)